

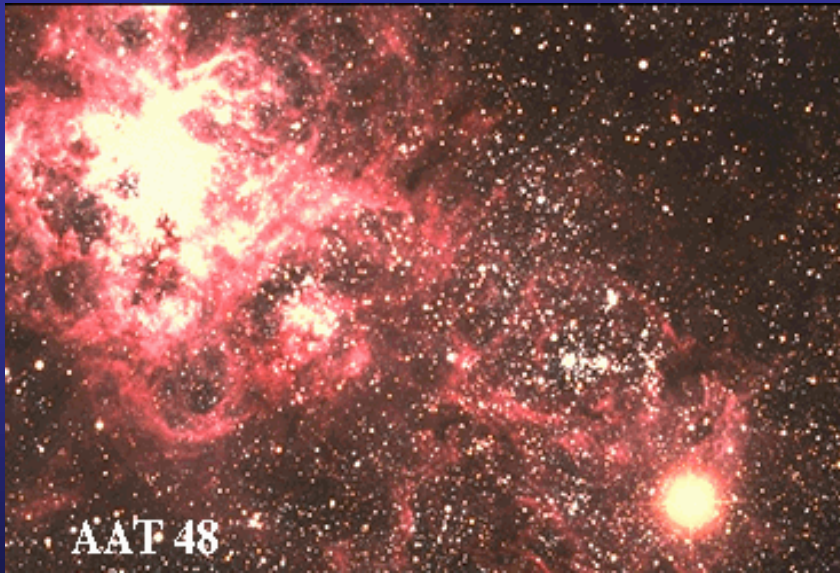
# Appearance of a quark matter phase in hybrid stars

*Strangeness in Quark Matter  
SQM2013, Birmingham*

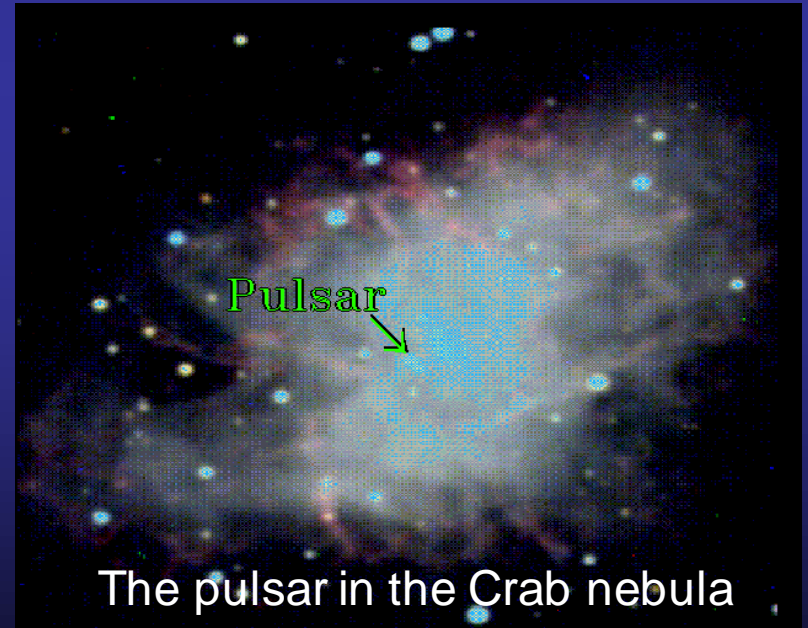
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Kagawa National College of Technology, Japan

Neutron star : supernova remnant



SN 1987A in the Large Magellanic Cloud  
But we cannot observe any pulsar



The pulsar in the Crab nebula  
This explosion was observed more than  
900 years ago...

客星古現例  
 皇極天皇九年秋七月甲寅客星入月  
 陽氏院自觀十九年正月廿五日丙戌時客星  
 在御見西方  
 字多天皇寬平三年三月九日巳申時  
 客星互東向星東方相三十二  
 醍醐天皇延長八年正月一後七月一初客星  
 入御林中  
 一惟院寬弘三年六月二日庚寅夜一降駱駝  
 中有大客星如紫或死明動雅建於上且  
 南方數日動河將軍是夜在櫻陰交  
 後冷泉院承平二年八月廿四日庚申時客星和  
 此方直至萬守備金守



“Meigetsu-ki(明月記)”

Teika Fujiwara (藤原定家)

“May 1054, a star appeared in the east sky.  
 The size was as same as Jupiter”.

The historical record in China “宋史”  
 Chinese people also saw this explosion

Quark?  
Hyperon?

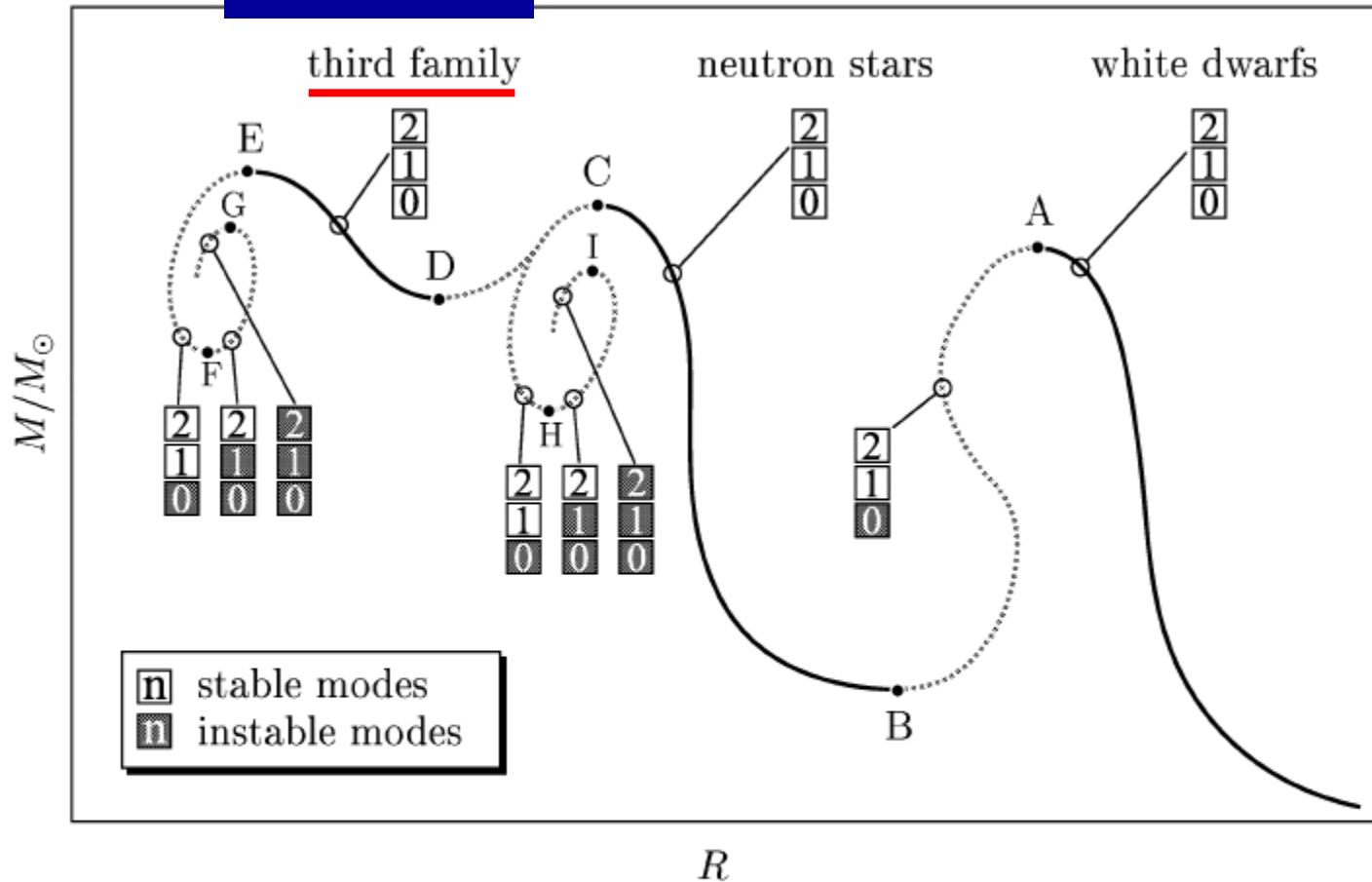
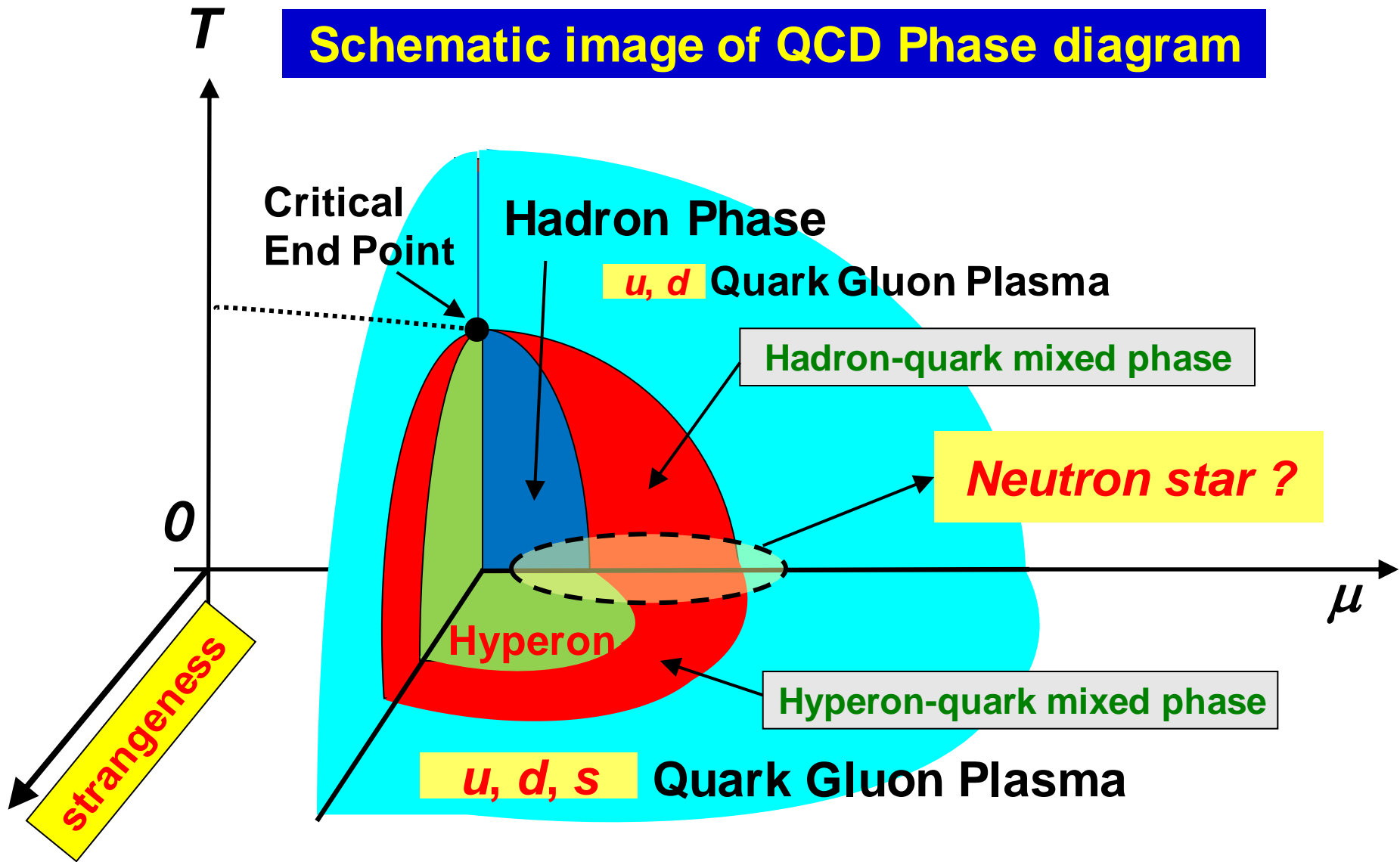
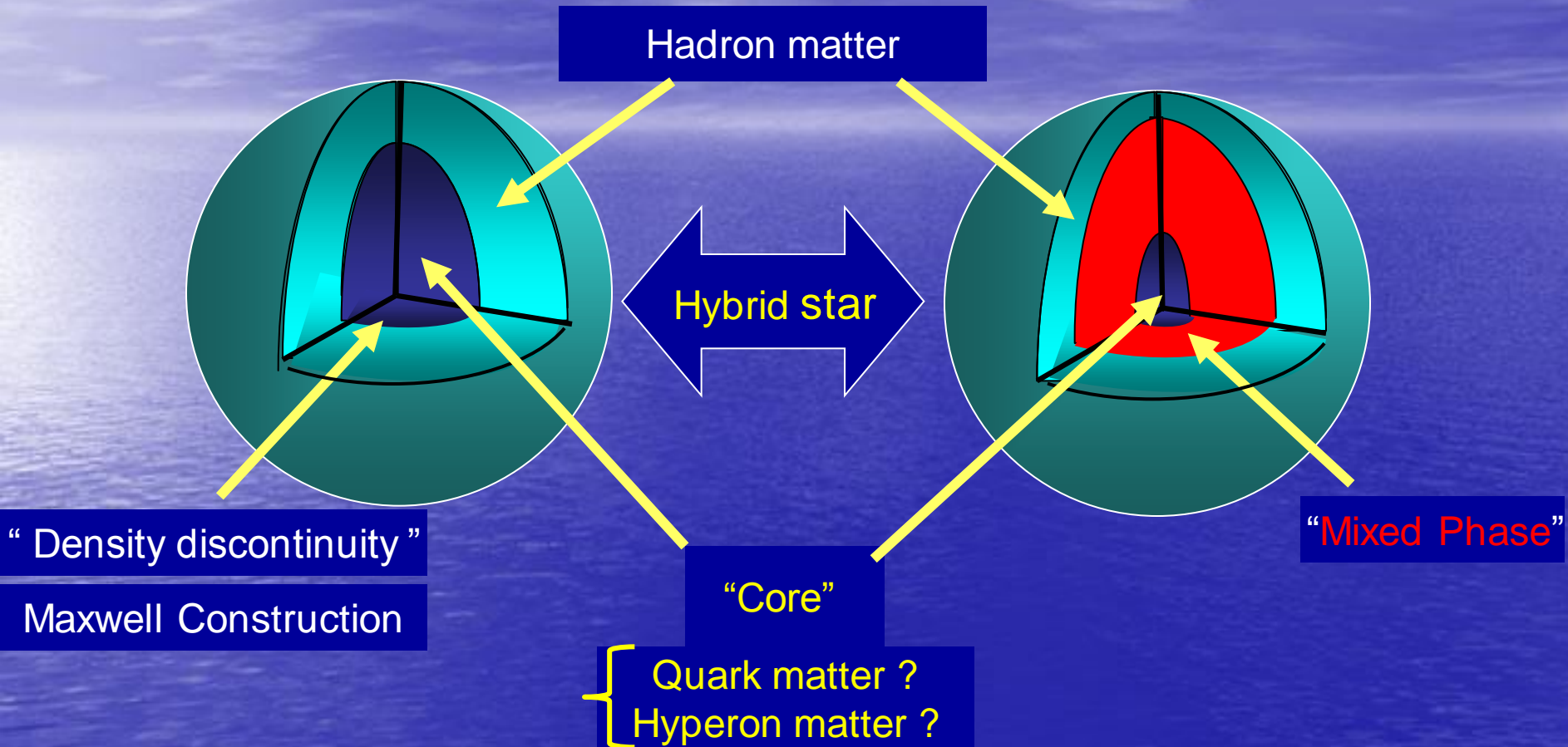


Fig. 14. Schematic mass–radius relation showing three stable families of compact stars. The letters A, B, . . . , I refer to critical points (turning points) where a vibrational mode changes stability [45,69,70]. The stability (solid lines) or instability (dotted lines) of the three lowest-lying modes ( $n = 0, 1, 2$ ) is depicted by the numbers. Higher modes are stable. See text for more details.

# Schematic image of QCD Phase diagram



# Inner structures of the neutron star...



Inner structures strongly depend on EOS of the matter

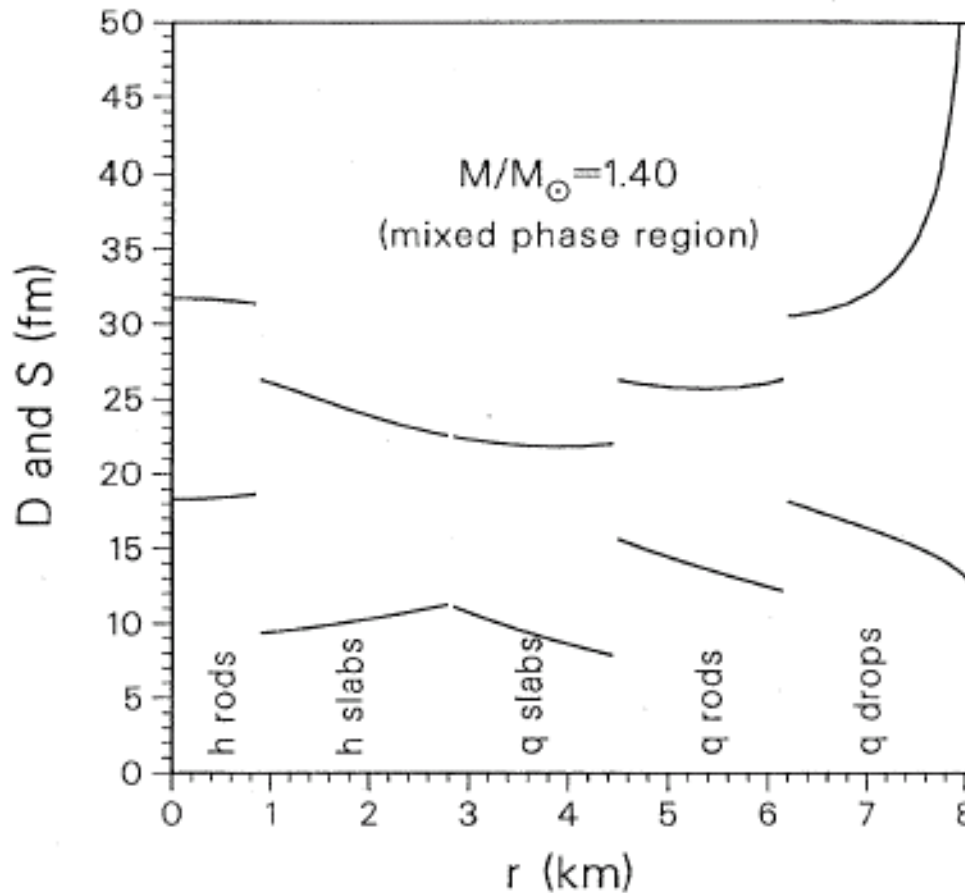


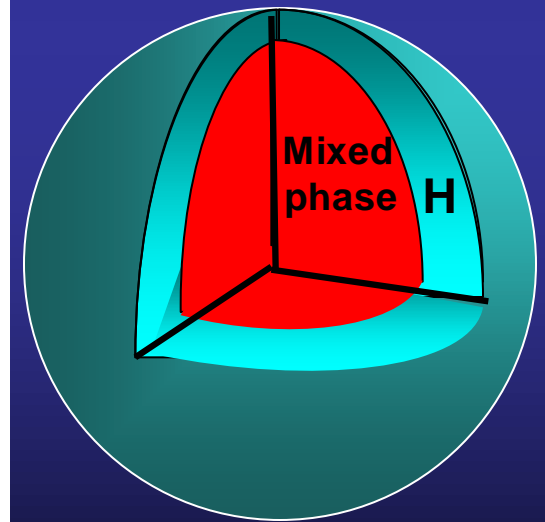
FIG. 2. Similar to Fig. 1 but for slightly less massive star. Mixed crystalline phase now extends to star's center. Radius is 12.3 km.

$$P_{\text{quark}} = P_{\text{Hadron}}$$

$$T_{\text{quark}} = T_{\text{Hadron}}$$

$$\mu_{\text{quark}}^B = \mu_{\text{Hadron}}^B$$

$$\mu_{\text{Hadron}}^e = \mu_{\text{quark}}^e$$





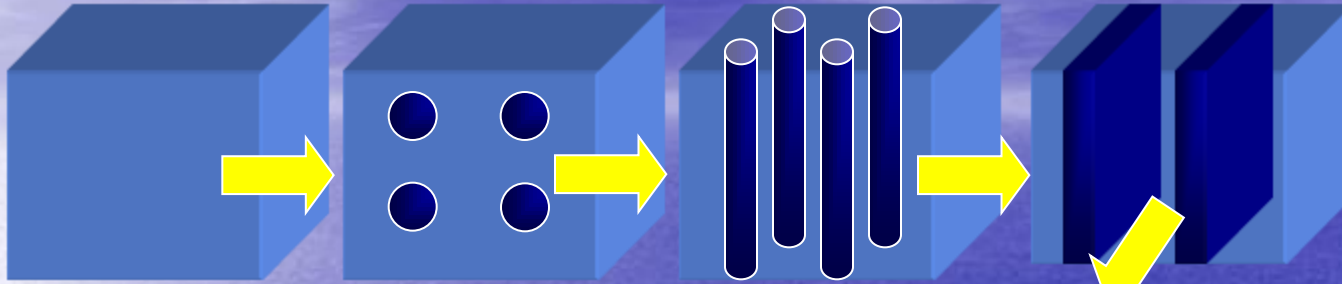
“Crystalline structure appear”

Uniform (nucleon)

drop

rod

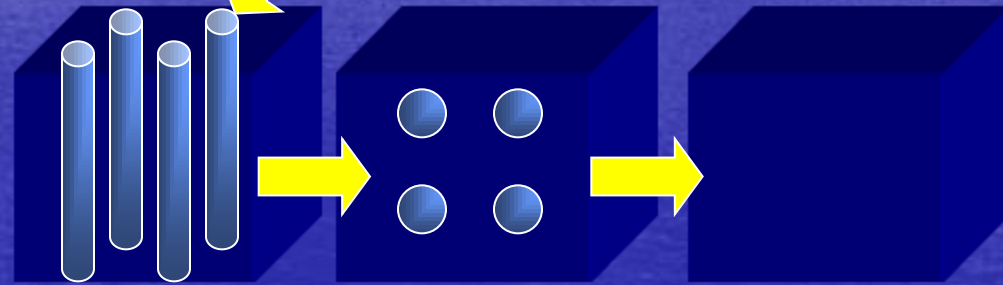
slab



Surface & Coulomb energy

$$\epsilon_S + \epsilon_C$$

$$\epsilon_S = 2\epsilon_C$$



tube

bubble

Uniform (quark)

They didn't solve the Poisson equation

Voskresensky ,Yasuhiro and Tatsumi, PLB541(2002)93 ; NPA723(2003)291

with screening effect  
 solve Poisson equation with linear approximation



“Maxwell construction picture” ...○

$$\Omega_{\text{tot}} = \Omega_{\text{Q}} + \Omega_{\text{H}} + \Omega_{\text{surface}} + \Omega_e + E_V$$

## Quark Phase

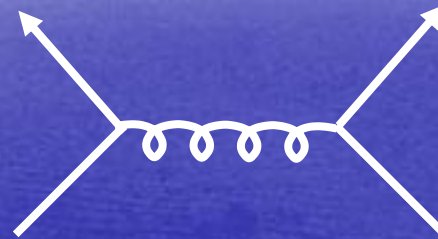
$$\Omega_u = \int d^3r \left[ \frac{3\pi^{\frac{2}{3}}}{4} \left( 1 + \frac{2\alpha_c}{3\pi} \right) \rho_u^{\frac{4}{3}} - \mu_u \rho_u \right]$$

$$\Omega_d = \int d^3r \left[ \frac{3\pi^{\frac{2}{3}}}{4} \left( 1 + \frac{2\alpha_c}{3\pi} \right) \rho_d^{\frac{4}{3}} - \mu_d \rho_d \right]$$

$$\Omega_s = \int d^3r [\epsilon_s(\rho_s) - \mu_s \rho_s + B] \quad B \dots \text{bag constant}$$

$$\Omega_{\text{I}} = \Omega_u + \Omega_d + \Omega_s$$

interaction : One Gluon Exchange



## Hadron (Nucleon) Phase

$$\Omega_n = \int d^3r \left[ \frac{3}{10m} (3\pi^2)^{\frac{2}{3}} \rho_n^{\frac{5}{3}} - \mu_n (\rho_p, \rho_n) \rho_n + \epsilon_{\text{pot}} (\rho_p, \rho_n) \right]$$

$$\Omega_p = \int d^3r \left[ \frac{3}{10m} (3\pi^2)^{\frac{2}{3}} \rho_p^{\frac{5}{3}} - \mu_p (\rho_p, \rho_n) - V \rho_p \right]$$

$$\Omega_{\text{II}} = \Omega_n + \Omega_p$$

interaction : effective potential to reproduce the nuclear matter saturation property

## Electron : Phase I & Phase II

$$\Omega_{\text{em}} = \int d^3r \left[ -\frac{1}{8\pi e^2} (\nabla V)^2 - \frac{(V - \mu_e)^4}{12\pi^2} \right]$$

$$V(r) = - \int d^3r' \frac{Q_i \rho_i(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$$E_V = \frac{1}{2} \int d^3r d^3r' \frac{Q_i \rho_i(\mathbf{r}) Q_j \rho_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

We can get “equation of motion” from

$$\frac{\delta \Omega_{\text{tot}}}{\delta \varphi_i} = 0 \quad \longrightarrow \quad \mu_i = \frac{\delta \mathcal{E}}{\delta \varphi_i} - \underline{Q_i V}$$

gauge invariant form



## ● quark phase

$$\mu_u = \left(1 + \frac{2\alpha_c}{3\pi}\right) \pi^{\frac{2}{3}} \rho_u^{\frac{1}{3}} - \frac{2}{3}V$$

$$\mu_d = \left(1 + \frac{2\alpha_c}{3\pi}\right) \pi^{\frac{2}{3}} \rho_d^{\frac{1}{3}} + \frac{1}{3}V$$

$$\mu_s = \epsilon_s + \frac{2\alpha_c}{3\pi} \left[ p_{Fs} - 3 \frac{m_{Fs}^2}{\epsilon_{Fs}} \ln \left( \frac{\epsilon_{Fs} + p_{Fs}}{m_s} \right) \right] + \frac{1}{3}V$$

## ● nucleon phase

$$\mu_n = \frac{p_{Fn}^2}{2m} + \frac{2S_0(\rho_n - \rho_p)}{\rho_0} + \epsilon_{\text{bind}} + \frac{K_0}{6} \left( \frac{\rho_n + \rho_p}{\rho_0} - 1 \right)^2 + \frac{K_0}{9} \left( \frac{\rho_n + \rho_p}{\rho_0} - 1 \right) + 2C_{\text{sat}} \frac{\rho_n + \rho_p}{\rho_0} - C_{\text{sat}}$$

$$\mu_p = \mu_n - \frac{p_{Fn}^2}{2m} + \frac{p_{Fp}^2}{2m} - \frac{4S_0(\rho_B - 2\rho_p)}{\rho_0} - V$$

$$\mu_e = (3\pi^2 \rho_e)^{\frac{1}{3}} + V$$

Poisson equation

$$\nabla^2 V = 4\pi e^2 \left[ \left( \frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s \right) \theta(R-r) + \rho_p \theta(r-R) - \rho_e \right]$$

## Chemical equilibrium

$$\mu_u - \mu_s + \mu_e = 0$$

$$\mu_d = \mu_s$$

$$\mu_n (\equiv \mu_B) = \mu_p + \mu_e$$

Quark phase

Nucleon phase

$$\mu_n = \mu_u + 2\mu_d$$

$$\mu_p = 2\mu_u + \mu_d$$

Quark & nucleon boundary

## With Gibbs conditions

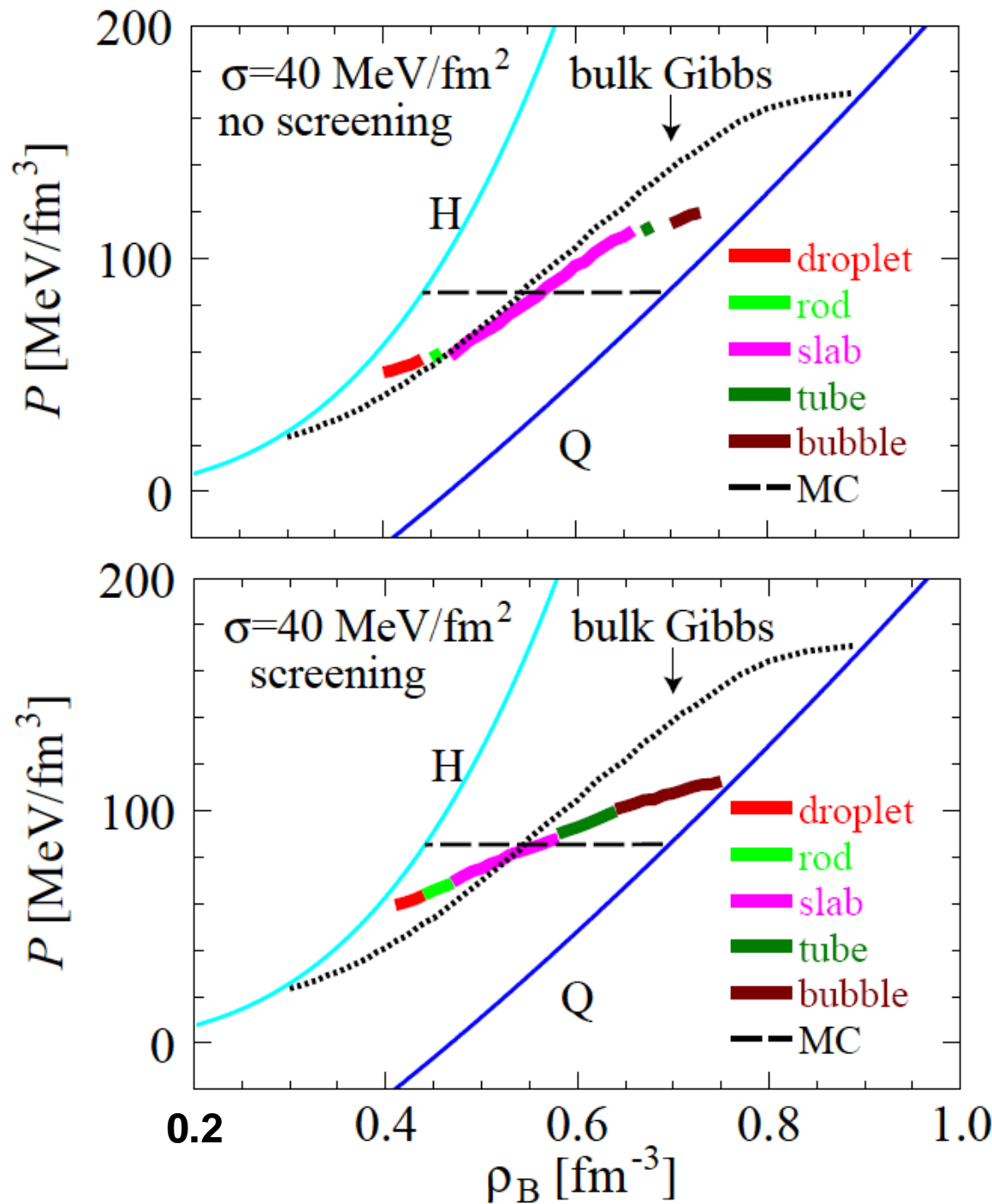
$\rho_i$  is the function of  $V$   
 $V$  is the function of  $\rho_i$

Poisson equation become highly nonlinear equation. With screening effect, it asks for rearrangement of  $\rho_i$ .

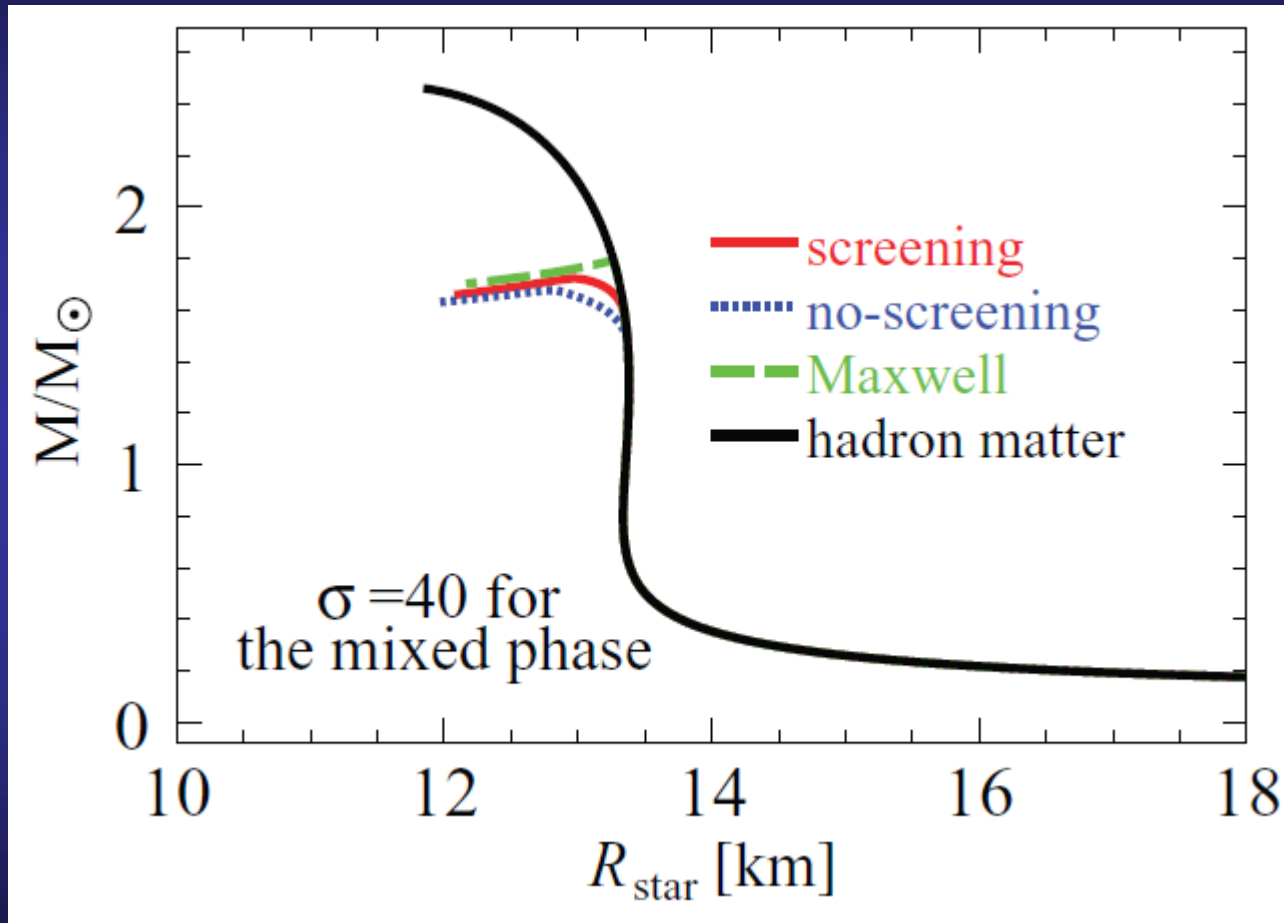
“Finite size effects”

- Screening effect
- Surface tension

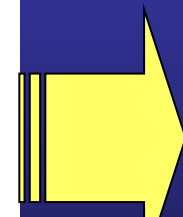
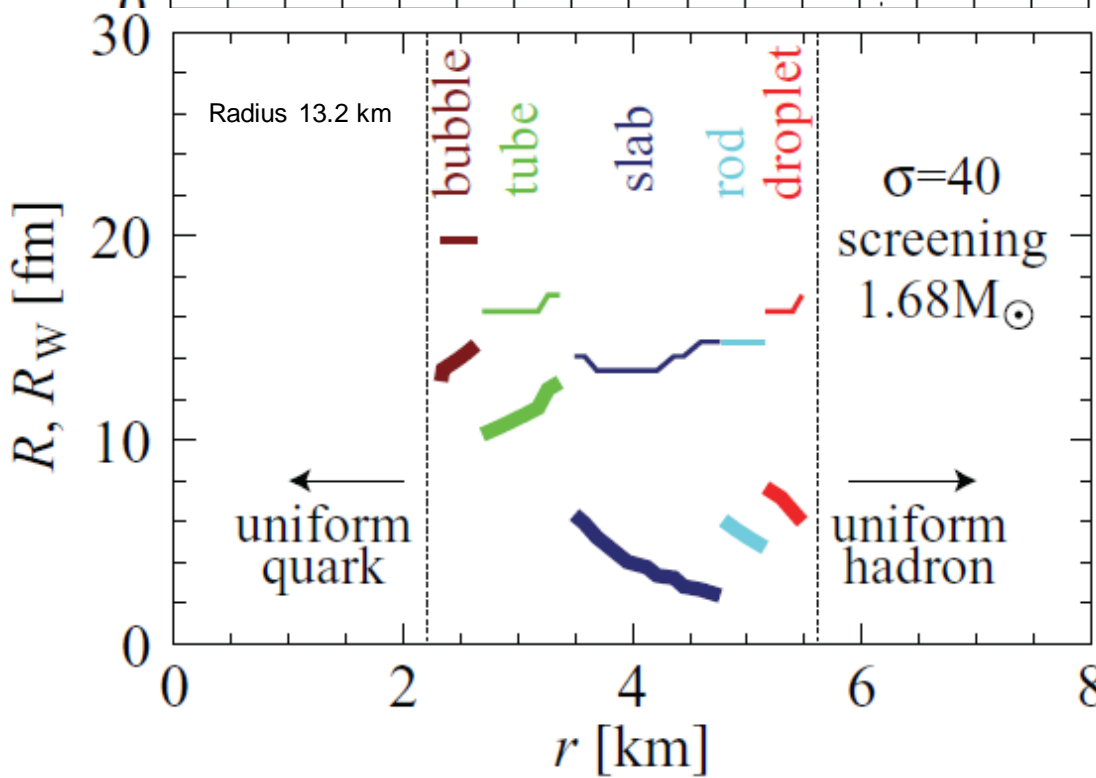
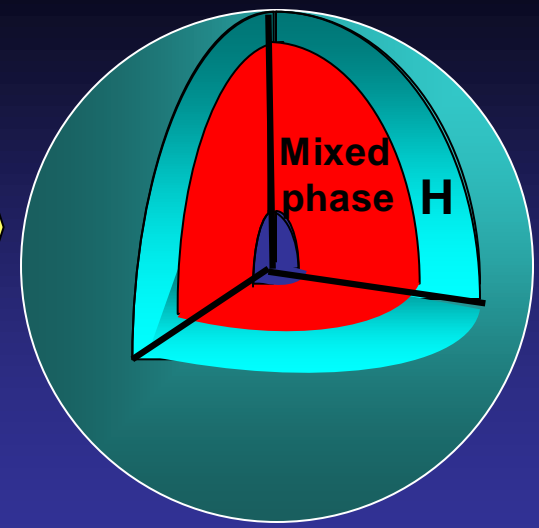
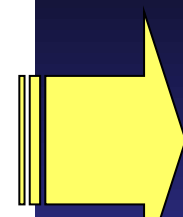
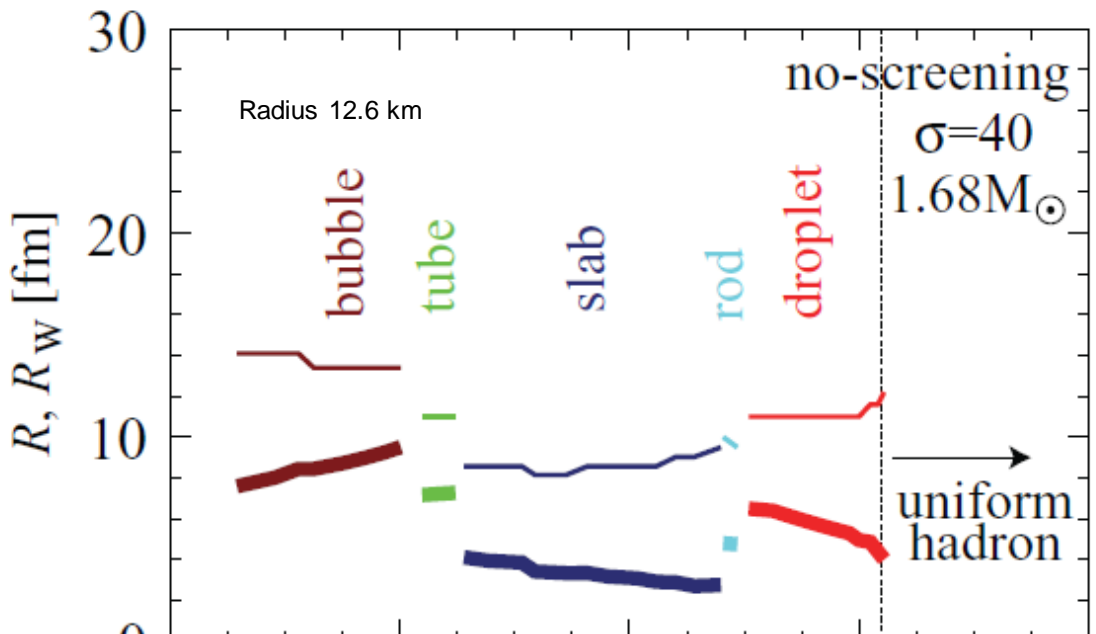
Lattice QCD (finite temperature)  
10 ~ 100 [MeV/fm<sup>2</sup>]  
Kajantie et al NPB357 (1991)693  
Huang et al PRD42(1990)2864



Our EOS  $\Rightarrow$  Tolman-Oppenheimer-Volkoff (TOV) equation

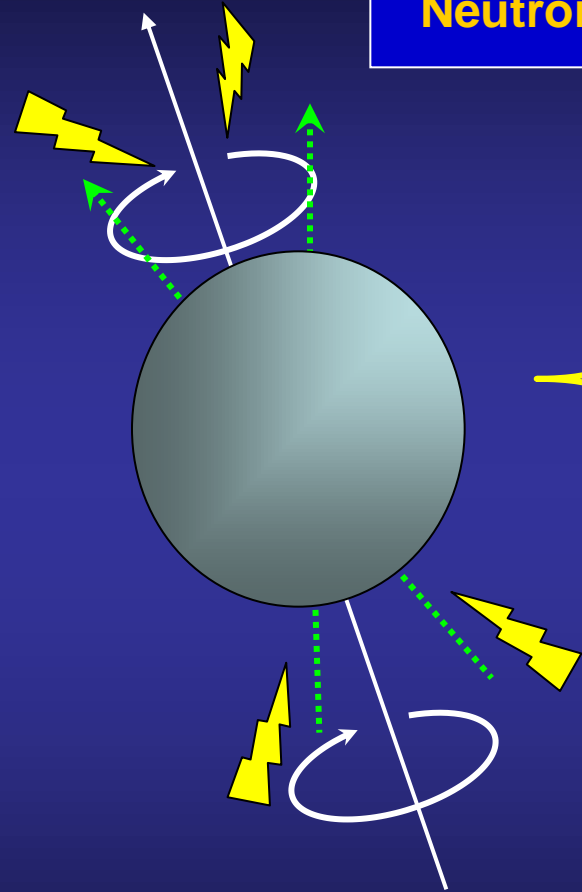


T.E., PRC83, 068801 (2011)



About 1000 pulsars are observed...

Neutron stars (hybrid stars) have many physical phenomena



Glitch phenomena

Cooling problem

Strong magnetic field

Maximum mass

Othres...

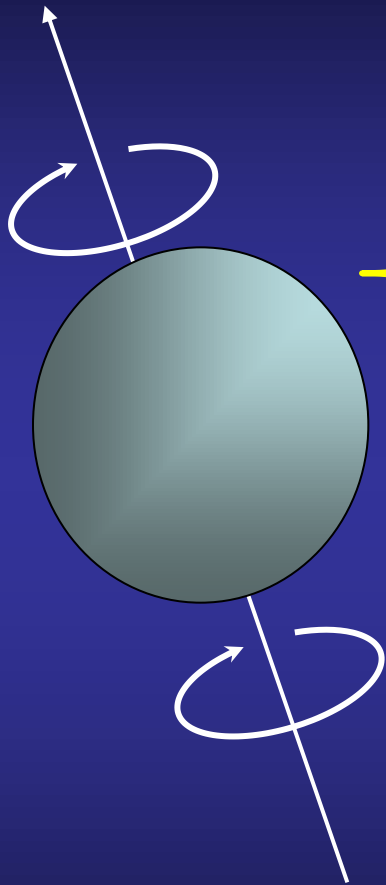
Rotation



$10^{12} \text{ G} \sim 10^{15} \text{ G}$  (magneters)

$\sim 2.1 M_{\odot}$

## Including the rotation effect



**Approximation**

**Stationary rigid rotation (Uniform rotation)**

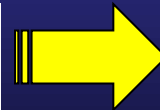
**Axially symmetric with respect to the spin axis**

**Perfect fluid**

**Review of stationary rotation in General Relativity  
: Stergioulas, (2003)**

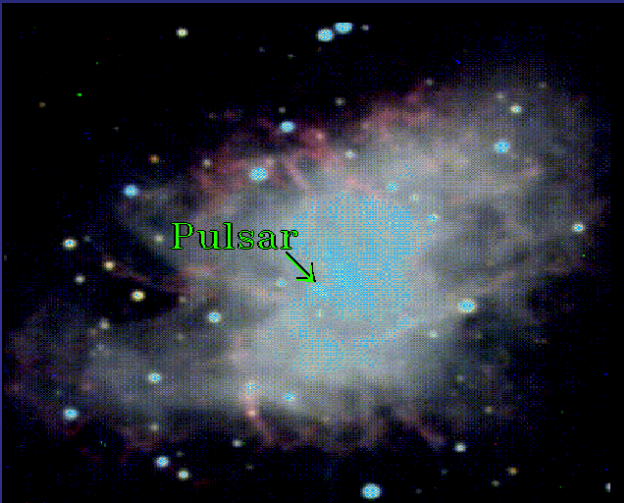
A. Kurkela et al. arXiv:1006.4062[astro-ph.HE]

**Our EOS**



**Rotating Neutron Star (RNS)**

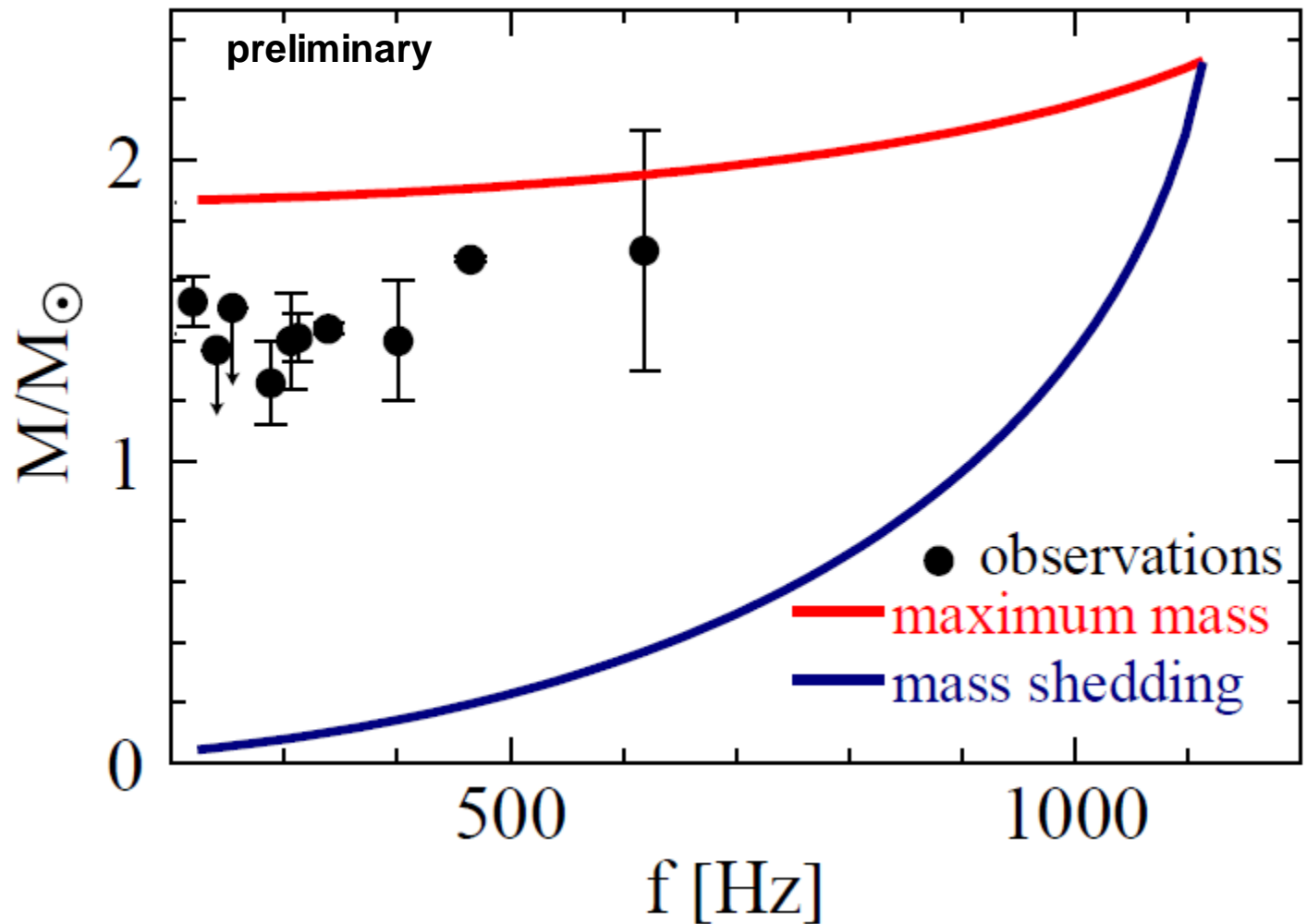
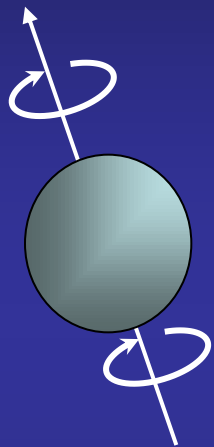
## Observations



Name	Spin [Hz]	Mass/ $M_{\odot}$
J0024-7204H	312	$1.41 \pm 0.08$
J0437-4715	174	$1.76 \pm 0.20$
J0514-4002A	126	$< 1.52$
J0751+1807	288	$1.26 \pm 0.14$
J1012+5307	190	$1.64 \pm 0.22$
J1713+0747	219	$1.53 \pm 0.08$
4U1608-52	619	$1.70 \pm 0.40$
J1748-2446I	105	$1.85 \pm 0.05$
SAXJ1808.4-3658	401	$1.40 \pm 0.20$
J1824-2452C	240	$< 1.37$
B1855+09	187	$1.58 \pm 0.13$
J1903+0327	465	$1.67 \pm 0.01$
J1909-3744	339	$1.44 \pm 0.02$
J1911-5958A	306	$1.40 \pm 0.16$
J2019+2425	254	$< 1.51$

**Masses of neutron stars with millisecond periods**

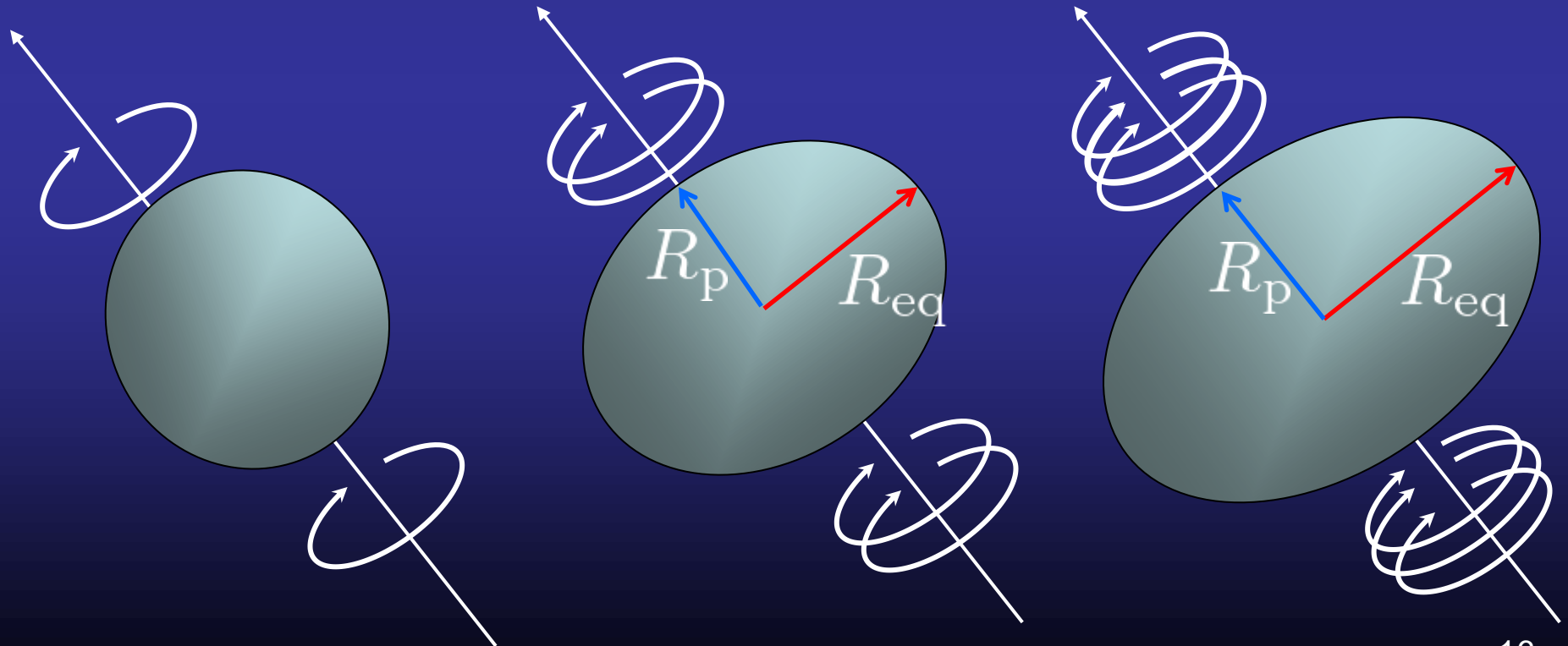
# Including rotation: EOS(with screening)

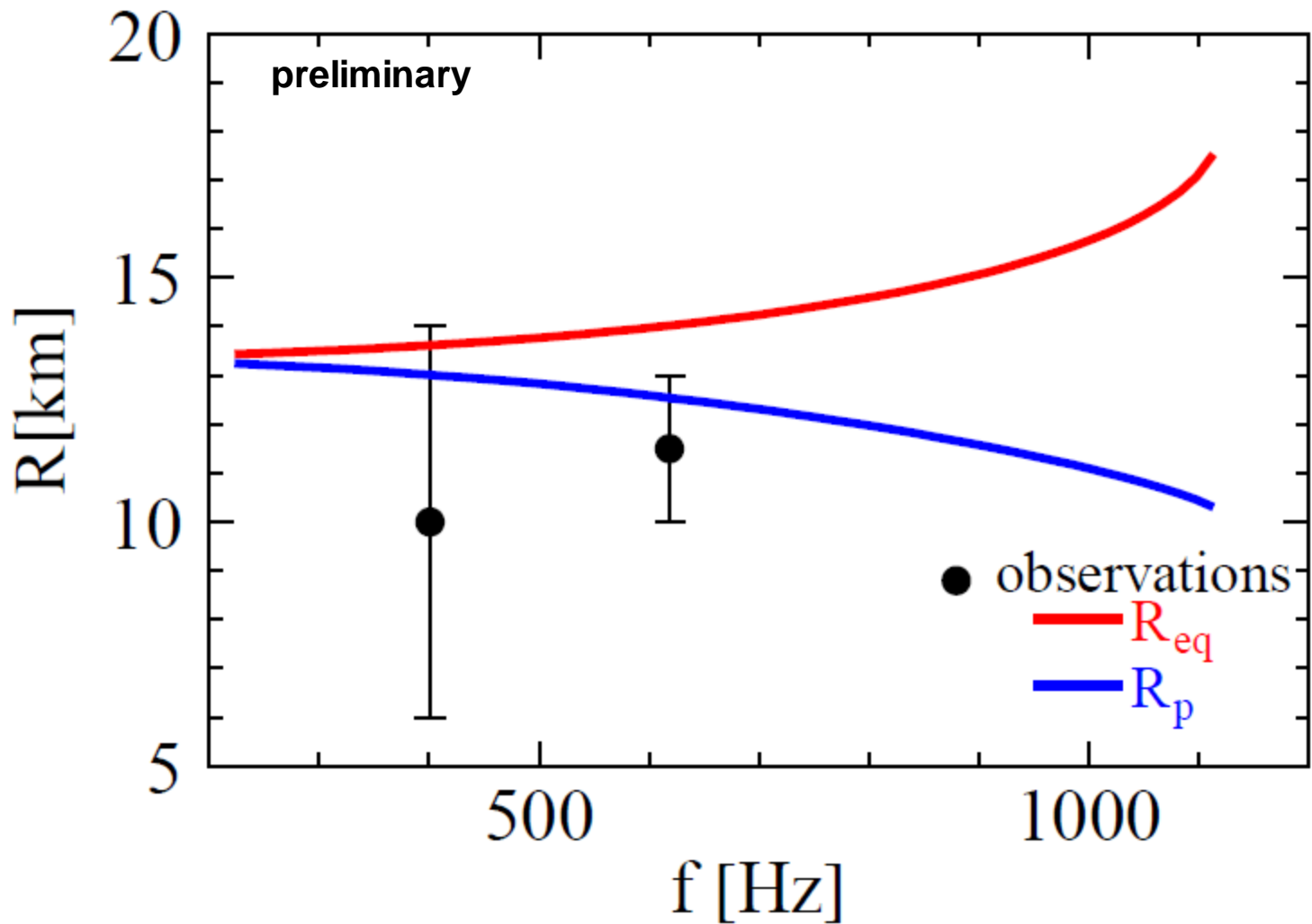
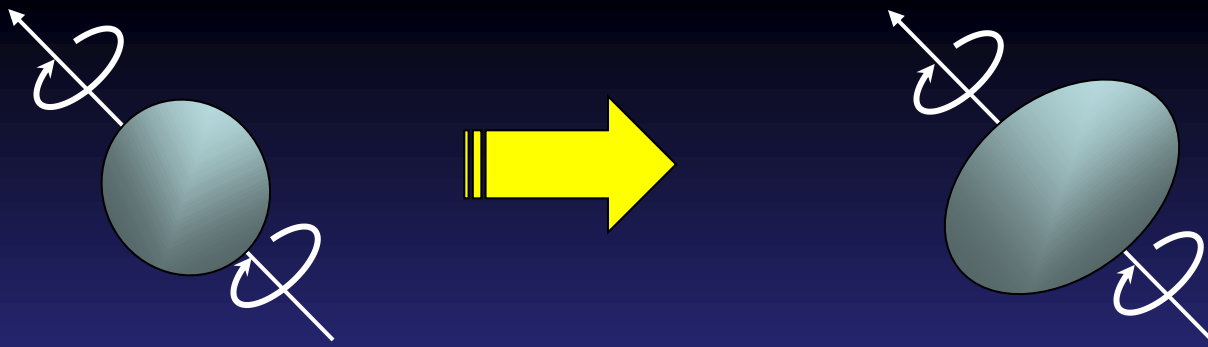


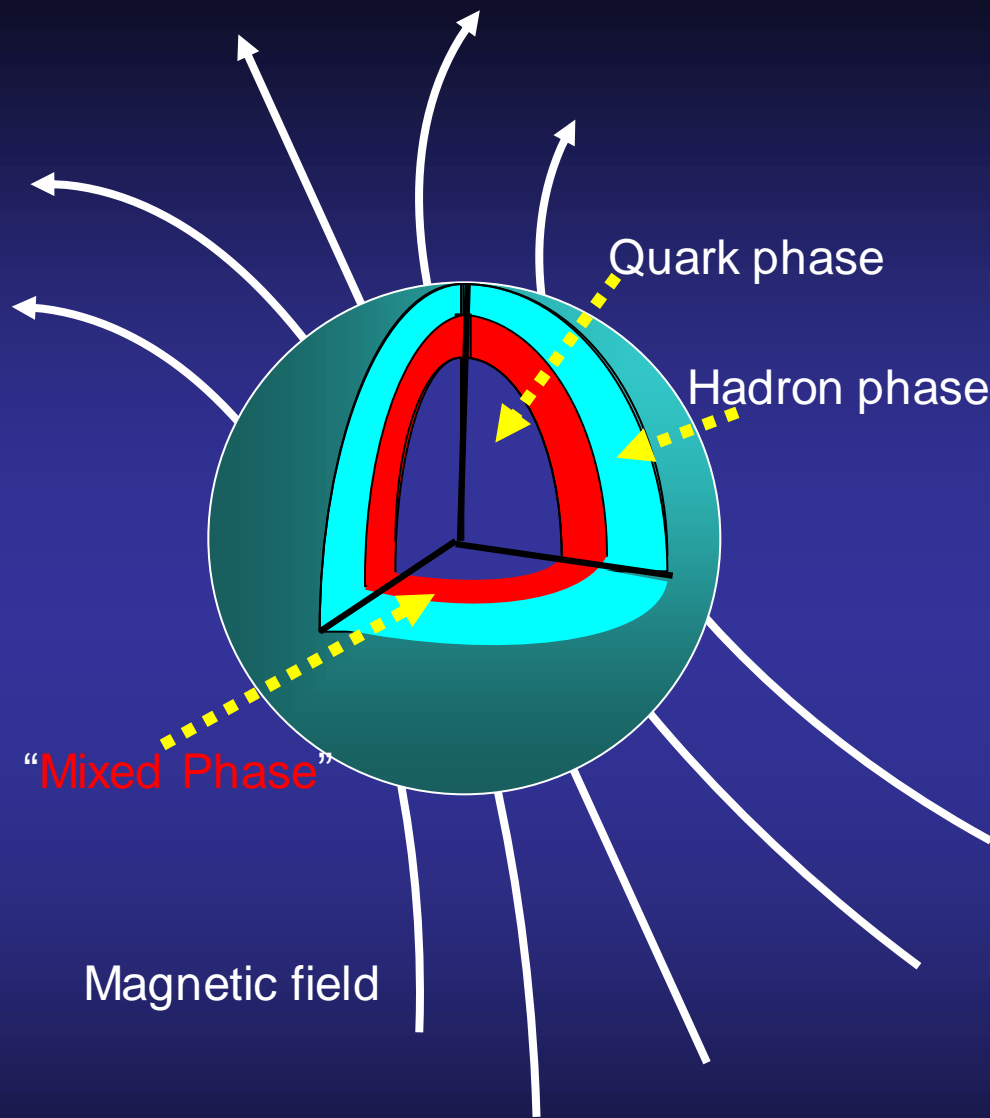


Name	Spin [Hz]	Radius
4U1608-52	619	$11.5 \pm 2.0$
SAXJ1808.4-3658	401	$10.0 \pm 4.0$

**Neutron star periods and radii**







Strong magnetic field  $\sim 10^{12}$  G  
 Magnetars  $\sim 10^{15}$  G  
**The origin of magnetic field  
 unknown...**

spin-polarization of nuclear matter  
 $\Rightarrow$  many calculations in 1970s,  
**But negative results...**  
 cf. J.M. Pearson et al. PRL24(1970)325  
 J. Dabrowski et al. PRC17(1978)1516

$\Rightarrow$  spin-polarization of liquid  $^3\text{He}$   
 “favorable”  
 M. Takano, **T. E.**, R. Kimura  
 and M. Yamada, PTP109(2003)213

How about quark matter?  
 spin-polarization  $\Rightarrow$  may be possible  
 T. Tatsumi PLB489(2000)280  
**Quark matter would exist or not?**

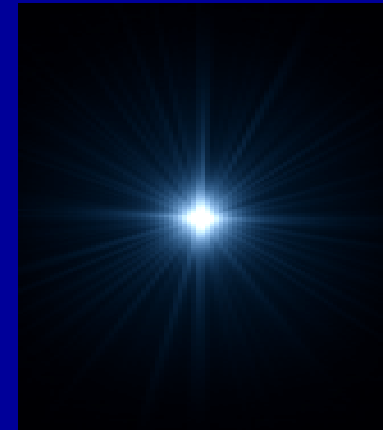
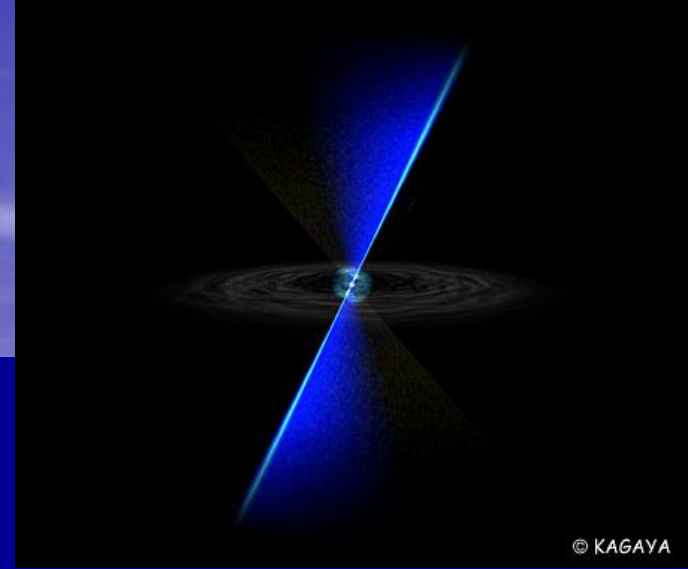
cf. Dynamo effect

## Summary

- Inner structures of the star strongly depend on EOSs
- EOSs confront observations.
- “Rotation” restricts EOSs of the matter.

Future plans ;

- Rotation effects on inner structures of the star
- Magnetic fields are needed for our EOS
- Strong magnetic fields – what is the origin ?



Thank you for your attention.