Appearance of a quark matter phase in hybrid stars

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Neutron star: supernova remnant

SN 1987A in the Large Magellanic Cloud
But we cannot observe any pulsar

The pulsar in the Crab nebula
This explosion was observed more than 900 years ago...
May 1054, a star appeared in the east sky. The size was as same as Jupiter.

The historical record in China “宋史” Chinese people also saw this explosion
Fig. 14. Schematic mass–radius relation showing three stable families of compact stars. The letters A, B, . . . , I refer to critical points (turning points) where a vibrational mode changes stability [45, 69, 70]. The stability (solid lines) or instability (dotted lines) of the three lowest-lying modes ($n = 0, 1, 2$) is depicted by the numbers. Higher modes are stable. See text for more details.
Schematic image of QCD Phase diagram

- **Critical End Point**
- **Hadron Phase**
- **u, d** Quark Gluon Plasma
- **Hadron-quark mixed phase**
- **Hyperon**
- **Hyperon-quark mixed phase**
- **Quark Gluon Plasma**
- **Neutron star?**

- **T**
- **μ**
- **strangeness**
Inner structures of the neutron star…

Inner structures strongly depend on EOS of the matter

“Core”

Quark matter ?

Hyperon matter ?

“Density discontinuity”

Maxwell Construction

“Mixed Phase”

“Hybrid star”

Hadron matter

4
Implication to a hybrid star

“Gibbs conditions”

\[
P_{\text{quark}} = P_{\text{Hadron}}
\]

\[
T_{\text{quark}} = T_{\text{Hadron}}
\]

\[
\mu_{\text{B}}^{\text{quark}} = \mu_{\text{B}}^{\text{Hadron}}
\]

\[
\mu_{\text{e}}^{\text{Hadron}} = \mu_{\text{e}}^{\text{quark}}
\]

FIG. 2. Similar to Fig. 1 but for slightly less massive star. Mixed crystalline phase now extends to star’s center. Radius is 12.3 km.

Uniform (nucleon) drop rod slab tube bubble

Uniform (quark) 

Coulomb energy

\[ \varepsilon_S + \varepsilon_C = 2\varepsilon_C \]

They didn’t solve the Poisson equation


with screening effect

solve Poisson equation with linear approximation

Maxwell construction picture” ・・・〇
Quark Phase

\[ \Omega_{Q} = \int d^{3}r \left[ \frac{3\pi^{2}}{4} \left( 1 + \frac{2\alpha_{c}}{3\pi} \right) \rho_{u}^{4} \right] - \mu_{u}\rho_{u} \]

interaction : One Gluon Exchange

\[ \Omega_{d} = \int d^{3}r \left[ \frac{3\pi^{2}}{4} \left( 1 + \frac{2\alpha_{c}}{3\pi} \right) \rho_{d}^{4} \right] - \mu_{d}\rho_{d} \]

\[ \Omega_{s} = \int d^{3}r \left[ \epsilon_{s}(\rho_{s}) - \mu_{s}\rho_{s} + B \right] \quad B \cdots \text{bag constant} \]

\[ \Omega_{I} = \Omega_{u} + \Omega_{d} + \Omega_{s} \]

Hadron (Nucleon) Phase

\[ \Omega_{n} = \int d^{3}r \left[ \frac{3}{10m} \left( 3\pi^{2} \right)^{2} \rho_{n}^{5} \right] - \mu_{n}(\rho_{p}, \rho_{n}) \rho_{n} + \epsilon_{\text{pot}}(\rho_{p}, \rho_{n}) \]

\[ \Omega_{p} = \int d^{3}r \left[ \frac{3}{10m} \left( 3\pi^{2} \right)^{2} \rho_{p}^{5} \right] - \mu_{p}(\rho_{p}, \rho_{n}) - V\rho_{p} \]

\[ \Omega_{H} = \Omega_{n} + \Omega_{p} \]

Electron : Phase I & Phase II

\[ \Omega_{\text{em}} = \int d^{3}r \left[ -\frac{1}{8\pi e^{2}}(\nabla V)^{2} - \frac{(V - \mu_{e})^{4}}{12\pi^{2}} \right] \]

\[ V(r) = -\int d^{3}r' \frac{Q_{i}\rho_{i}(r')}{|r - r'|} \]

\[ E_{V} = \frac{1}{2} \int d^{3}r d^{3}r' \frac{Q_{i}\rho_{i}(r)Q_{j}\rho_{j}(r')}{|r - r'|} \]

interaction : effective potential to reproduce the nuclear matter saturation property

We can get "equation of motion" from

\[ \frac{\partial \Omega_{\text{tot}}}{\partial \rho_{i}} = 0 \]

\[ \mu_{i} = \frac{\partial \mathcal{E}}{\partial \rho_{i}} - Q_{i}V \]

gauge invariant form
Equations of motion

● **Quark phase**

\[ \mu_u = \left( 1 + \frac{2\alpha_c}{3\pi} \right) \pi \frac{2}{3} \rho_u^{\frac{1}{3}} - \frac{2}{3} V \]

\[ \mu_d = \left( 1 + \frac{2\alpha_c}{3\pi} \right) \pi \frac{2}{3} \rho_d^{\frac{1}{3}} + \frac{1}{3} V \]

\[ \mu_s = \epsilon_s + \frac{2\alpha_c}{3\pi} \left[ p_{Fs} - 3 \frac{m_{Fs}^2}{\epsilon_{Fs}} \ln \left( \frac{\epsilon_{Fs} + p_{Fs}}{m_s} \right) \right] + \frac{1}{3} V \]

● **Nucleon phase**

\[ \mu_n = \frac{p_{Fn}^2}{2m} + \frac{2S_0 (\rho_n - \rho_p)}{\rho_0} + \epsilon_{\text{bind}} + \frac{K_0}{6} \left( \frac{\rho_n + \rho_p}{\rho_0} - 1 \right)^2 \]

\[ + \frac{K_0}{9} \left( \frac{\rho_n + \rho_p}{\rho_0} - 1 \right) + 2C_{\text{sat}} \frac{\rho_n + \rho_p}{\rho_0} - C_{\text{sat}} \]

\[ \mu_p = \mu_n - \frac{p_{Fn}^2}{2m} + \frac{p_{Fp}^2}{2m} - 4S_0 (\rho_B - 2\rho_p) \]

\[ \mu_e = \left( 3\pi^2 \rho_e \right)^{\frac{1}{3}} + V \]

Poisson equation

\[ \nabla^2 V = 4\pi e^2 \left[ \left( \frac{2}{3} \rho_u - \frac{1}{3} \rho_d - \frac{1}{3} \rho_s \right) \theta(R - r) + \rho_p \theta(r - R) - \rho_e \right] \]

**Chemical equilibrium**

\[ \mu_u - \mu_s + \mu_e = 0 \]

\[ \mu_d = \mu_s \]

\[ \mu_n (\equiv \mu_B) = \mu_p + \mu_e \]

\[ \mu_n = \mu_u + 2\mu_d \]

\[ \mu_p = 2\mu_u + \mu_d \]

**Quark phase**

**Nucleon phase**

**Quark & nucleon boundary**

**With Gibbs conditions**

\[ \rho_i \text{ is the function of } V \]

\[ V \text{ is the function of } \rho_i \]

Poisson equation become highly nonlinear equation. With screening effect, it asks for rearrangement of \( \rho_i \).
"Finite size effects"

- Screening effect
- Surface tension

Lattice QCD (finite temperature) $10 \sim 100$ [MeV/fm$^2$]

Kajantie et al NPB357 (1991)693
Huang et al PRD42(1990)2864

$\sigma=40$ MeV/fm$^2$
Our EOS ⇒ Tolman-Oppenheimer-Volkoff (TOV) equation

Implication to hybrid stars

T.E., PRC83, 068801 (2011)
Mixed phase

Radius 12.6 km

$\sigma = 40$

$1.68 M_\odot$

Radius 13.2 km

$\sigma = 40$

screening

$1.68 M_\odot$

uniform quark

uniform hadron

$R, R_W [\text{fm}]$

$r [\text{km}]$

$0$ $2$ $4$ $6$ $8$

$0$ $10$ $20$ $30$
About 1000 pulsars are observed…

Neutron stars (hybrid stars) have many physical phenomena

- Glitch phenomena
- Cooling problem
- Strong magnetic field: $10^{12} \text{ G} \sim 10^{15} \text{ G (magneters)}$
- Maximum mass: $\sim 2.1M_\odot$
- Other…

Rotation
Including the rotation effect

Stationary rigid rotation (Uniform rotation)
Axially symmetric with respect to the spin axis
Perfect fluid


Our EOS
Rotating Neutron Star (RNS)
<table>
<thead>
<tr>
<th>Name</th>
<th>Spin [Hz]</th>
<th>Mass/M ☉</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0024-7204H</td>
<td>312</td>
<td>1.41 ± 0.08</td>
</tr>
<tr>
<td>J0437-4715</td>
<td>174</td>
<td>1.76 ± 0.20</td>
</tr>
<tr>
<td>J0514-4002A</td>
<td>126</td>
<td>&lt; 1.52</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>288</td>
<td>1.26 ± 0.14</td>
</tr>
<tr>
<td>J1012+5307</td>
<td>190</td>
<td>1.64 ± 0.22</td>
</tr>
<tr>
<td>J1713+0747</td>
<td>219</td>
<td>1.53 ± 0.08</td>
</tr>
<tr>
<td>4U1608-52</td>
<td>619</td>
<td>1.70 ± 0.40</td>
</tr>
<tr>
<td>J1748-2446I</td>
<td>105</td>
<td>1.85 ± 0.05</td>
</tr>
<tr>
<td>SAXJ1808.4-3658</td>
<td>401</td>
<td>1.40 ± 0.20</td>
</tr>
<tr>
<td>J1824-2452C</td>
<td>240</td>
<td>&lt; 1.37</td>
</tr>
<tr>
<td>B1855+09</td>
<td>187</td>
<td>1.58 ± 0.13</td>
</tr>
<tr>
<td>J1903+0327</td>
<td>465</td>
<td>1.67 ± 0.01</td>
</tr>
<tr>
<td>J1909-3744</td>
<td>339</td>
<td>1.44 ± 0.02</td>
</tr>
<tr>
<td>J1911-5958A</td>
<td>306</td>
<td>1.40 ± 0.16</td>
</tr>
<tr>
<td>J2019+2425</td>
<td>254</td>
<td>&lt; 1.51</td>
</tr>
</tbody>
</table>

Masses of neutron stars with millisecond periods.
Including rotation: EOS (with screening)
### Neutron star periods and radii

<table>
<thead>
<tr>
<th>Name</th>
<th>Spin [Hz]</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>4U1608-52</td>
<td>619</td>
<td>11.5 ± 2.0</td>
</tr>
<tr>
<td>SAXJ1808.4-3658</td>
<td>401</td>
<td>10.0 ± 4.0</td>
</tr>
</tbody>
</table>

\[ Rp \quad \text{and} \quad R_{\text{eq}} \]
Mixed Phase

Quark phase

Hadron phase

Magnetic field

Strong magnetic field $\sim 10^{12}$ G
Magnetars $\sim 10^{15}$ G
The origin of magnetic field unknown...

spin-polarization of nuclear matter $\Rightarrow$ many calculations in 1970s,
But negative results...
J. Dabrowski et al. PRC 17 (1978) 1516

$\Rightarrow$ spin-polarization of liquid $^3$He
“favorable”

cf. Dynamo effect

How about quark matter?
spin-polarization $\Rightarrow$ may be possible
T. Tatsumi PLB 489 (2000) 280
Quark matter would exist or not?

cf. Dynamo effect
Summary

- Inner structures of the star strongly depend on EOSs
- EOSs confront observations.
- “Rotation” restricts EOSs of the matter.

Future plans:
- Rotation effects on inner structures of the star
- Magnetic fields are needed for our EOS
- Strong magnetic fields – what is the origin?

Thank you for your attention.