Universal Hadronization Conditions and Strange Hadron Production at LHC and RHIC

presented at SQM 2013, Birmingham, July 25, 2013

Successful description of all available particle yields in Pb–Pb collisions at 2.76 TeV measured by the ALICE experiment as a function of centrality within the framework of non–equilibrium statistical hadronization model shows universal intensive hadronization conditions of the particle source; critical pressure $P \simeq 80\,\mathrm{MeV/fm^3}$, energy density $\varepsilon \simeq 0.5\,\mathrm{GeV/fm^3}$ and entropy density $\sigma \simeq 3.3\,\mathrm{fm^{-3}}$. Use of these conditions reduces the number of non–equilibrium SHM free parameters to two at LHC, source volume dV/dy and strangeness content γ_s. For RHIC, we have to use three parameters due to non-zero chemical potential. At these conditions, hadronization occurs at chemical freeze-out temperature of $T \simeq 140\,\mathrm{MeV}$ with $\gamma_q \simeq 1.6$. Saturation of the specific strangeness content of the fireball, $s/S \to 0.03$, shows saturation already for small systems at LHC energy suggesting a chemically equilibrated QGP as a source of hadrons.

(see MP, J. Rafelski, http://arxiv.org/abs/arXiv:1303.0913,
and MP, J. Letessier, V. Petracek, J. Rafelski, http://arxiv.org/abs/arXiv:1303.2098)

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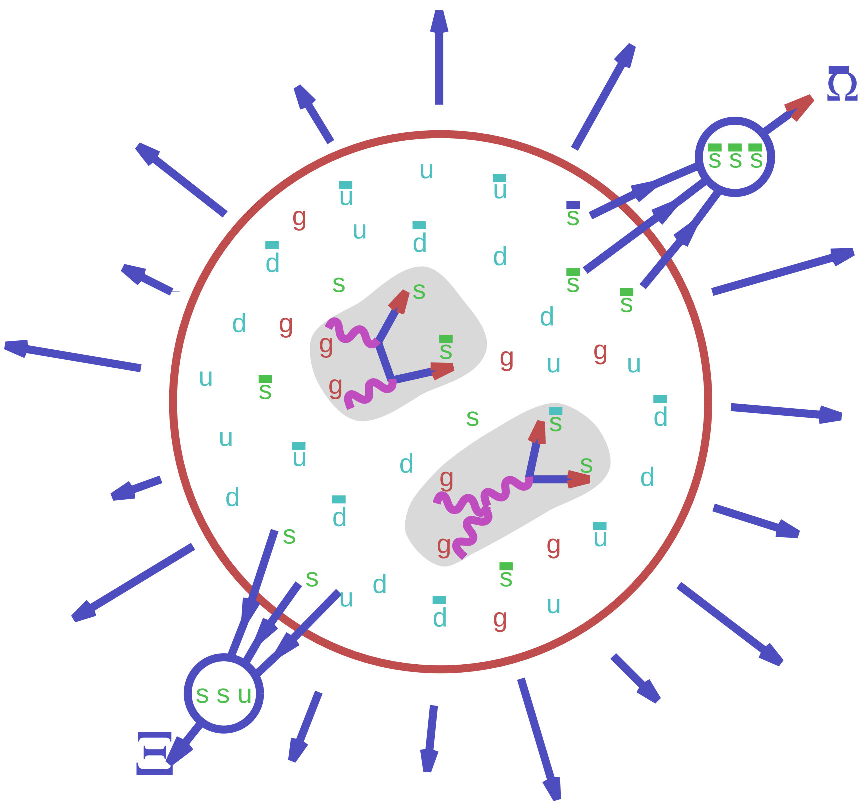
Outline

- **Hadronization of QGP**
- **SHM describes heavy ion collision at all energies**
- **Centrality dependent hadron yields results**
- **Statistical parameters of hadronizing fireball**
- **Bulk properties and hadronization conditions**
- **Strangeness Production in QGP**

Hadronization of Quark-Gluon Plasma

Recombinant quark hadronization weighted by phase space has the following consequences:

- **Enhanced yields of multi-flavored (strange, charm, bottom) anti-baryons**
- **$\Rightarrow$ Unexpected baryon to meson yield ratios.**



1. $GG \to s\bar{s}$ **(thermal gluons collide)**
   $GG \to c\bar{c}$ **(initial parton collision)**
   $GG \to b\bar{b}$ **(initial parton collision)**
   gluon dominated reactions
2. **RECOMBINATION of pre-formed** $s,\ \bar{s},\ c,\ \bar{c},\ b,\ \bar{b}$ **quarks**

Formation of complex rarely produced multi flavor (exotic) (anti)particles enabled by coalescence between $s,\ \bar{s},\ c,\ \bar{c},\ b,\ \bar{b}$ quarks made in different microscopic reactions; this is a signature of deconfinement.

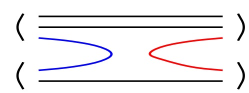
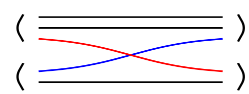
FERMI STATISTICAL HADRONIZATION MODEL (Fermi-SHM) applies these principles in combination with phase space dominance

SHM with QUARK CHEMISTRY

Quarks combine into hadrons – we count the valance quark content.
Number relates to the content in the QGP phase:

1) all agree that $B = 3(q^{QGP} - \bar{q}^{QGP}) = 3(q^h - \bar{q}^h)$ is conserved;
2) Net strangeness remains zero $s - \bar{s} = 0$;
3) No additional charm pair production in hadronization;
4) Similarly one argues the number of strange quark pairs is nearly preserved;
5) By extension can the number of light quark pairs be preserved?

Preserving net baryon number, net strangeness, and number of pairs we independently keep: $s, \overline{s}, q, \overline{q} \to$ unchanged SO WE NEED 4 CHEMICAL PARAMETERS.

γ_i	**controls overall abundance of quark $(i = q, s)$ pairs**	**Absolute chemical equilibrium**	**HG production** 
λ_i	**$=e^{\mu_i/T}$ controls difference between strange and light quarks $(i = q, s)$**	**Relative chemical equilibrium**	**HG exchange** 

See Physics Reports 1986 Koch, Müller, JR

Boltzmann gas: $\gamma \equiv \frac{\rho(T,\mu)}{\rho^{\rm eq}(T,\mu)}$ absolute chemical equilibrium: $\gamma \to 1$,

across phase boundary (QGP→hadrons): $\gamma_s^{\rm QGP}\rho_{\rm eq}^{\rm QGP}V^{\rm QGP} = \gamma_s^{\rm h}\rho_{\rm eq}^{\rm h}V^{\rm h}$

Three 'classic' choices for SHM Fits

1. Chemical Equilibrium ($\gamma_q = \gamma_s = 1$)

- **Strange and light quarks reach equilibrium during hadronization**
- **Normally not possible in computation since model not perfect and incomplete hadron mass spectrum would cause modifications. Given model-perfect hadron mass spectrum this approach means that we have not made QGP and that hadrons cooked slowly to equilibrate all flavors.**

2. Chemical Semi-Equilibrium ($\gamma_q = 1, \gamma_s \neq 1$)

- **Perfect hadron mass spectrum, model acknowledges the time dependent strangeness cooking (since RHIC, agreement on $\gamma_s \neq 1$)**
- **Often $\gamma_s < 1$ is required since it must approach equilibrium from below in hadron phase. Hadrons made fast, strangeness not equilibrated.**

3. Chemical Non-Equilibrium ($\gamma_q \neq 1, \gamma_s \neq 1$)

- **QGP was formed. Neither strange and light quarks do not reach equilibrium during hadronization before free-streaming sets.**
- **IF fit near $\gamma_q = \gamma_s = 1$ we would have evidence of chemical equilibration.**

Keep in mind: Sudden hadronization $\Rightarrow$ QGP quark content imprinted in measured hadron abundances.

Statistical Hadronization Method and Fits of Hadron Yields

Full analysis of experimental hadron yield results requires a significant numerical effort in order to allow for resonances, particle widths, full decay trees, isospin multiplet sub-states.

Kraków-Tucson (W. Broniowski, W. Florkowski)
(and SHARE 2 +Montreal (S.Y. Jeon) and
SHARE with Charm (not yet released))
collaboration produced a public package

SHARE= Statistical HAdronization with REsonances
(current version SHAREv2.2)

G. Torrieri, J. Letessier, J. Rafelski, et al, Comp. Phys. Com. 167, 229 (2005); Comp. Phys. Com. 175, 635 (2006), and

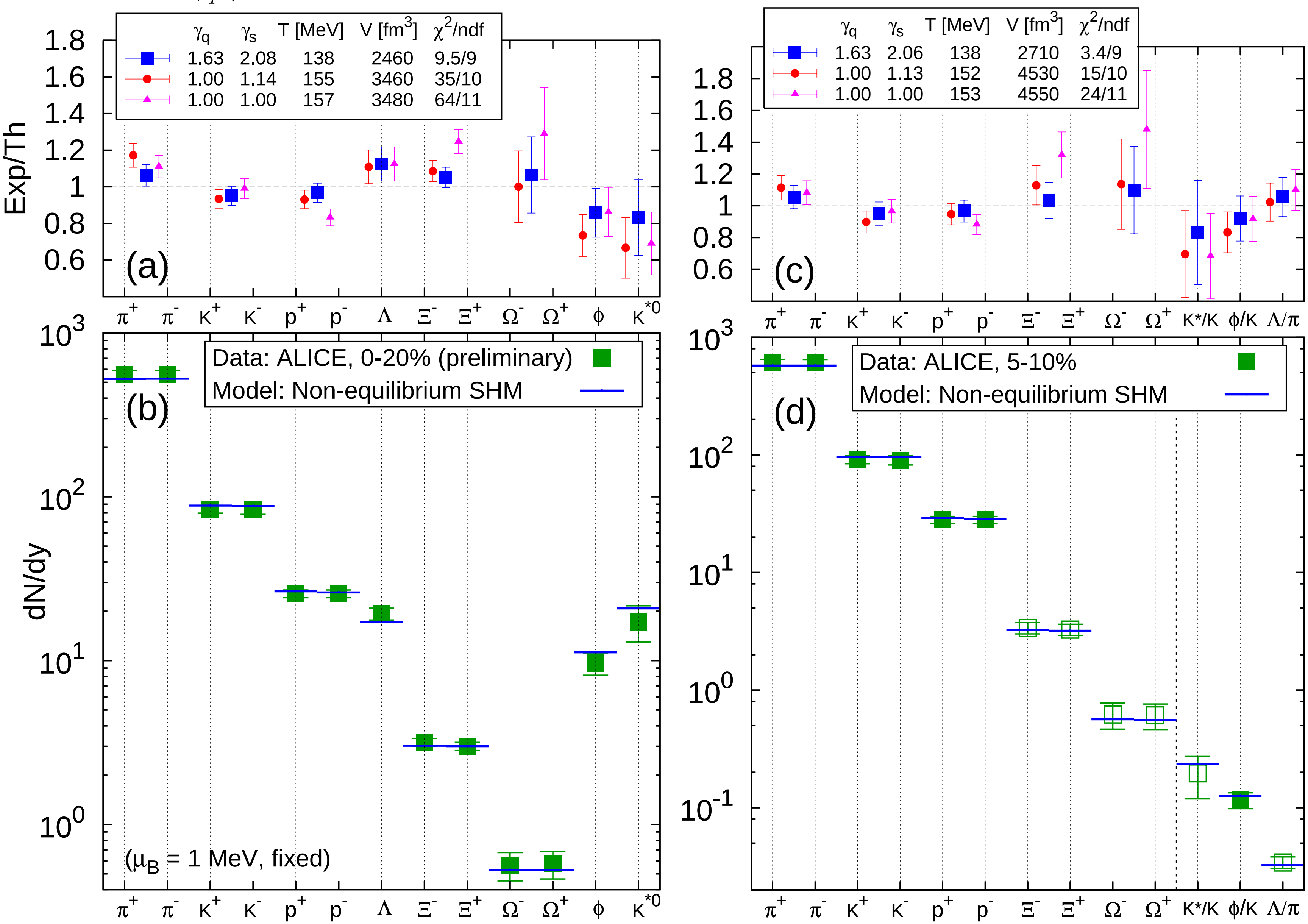
SHARE with CHARM in 2013

M. Petran, et al, Manuscript in Preparation Main objective: include hadrons from charm hadron decay in hadron multiplicities. Charm yields from SHM charm hadronization.

Our main edges on competition:
a) Full chemical non-equilibrium possible in fits.
b) Output produces also physical properties of particle source.
c) Fit can include physical properties of the source as input.
d) Particle fluctuations can be fitted.

SHM FITS LHC Pb–Pb DATA

SHM with $\gamma_q \neq 1$ fits all measured hadrons.

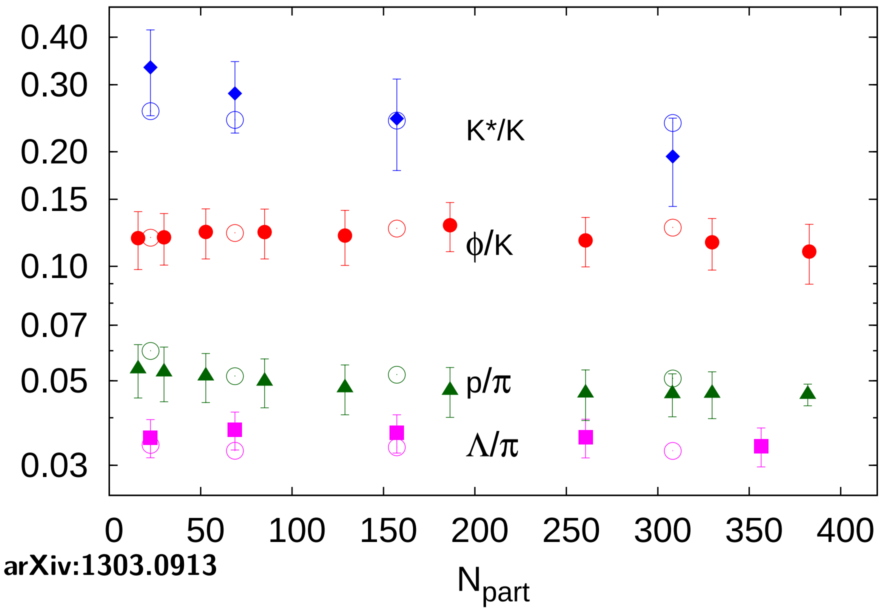


prelim data wide centrality 0–20% **5–10% final π,K,p data**

Solving incompatible particle binning – at LHC

Centrality bins combined

- **Four wide centrality bins**
- **K*/K predicted more constant by other particles**
- **Fit to p and π $\rightarrow$ p/$\pi \simeq 0.05$**

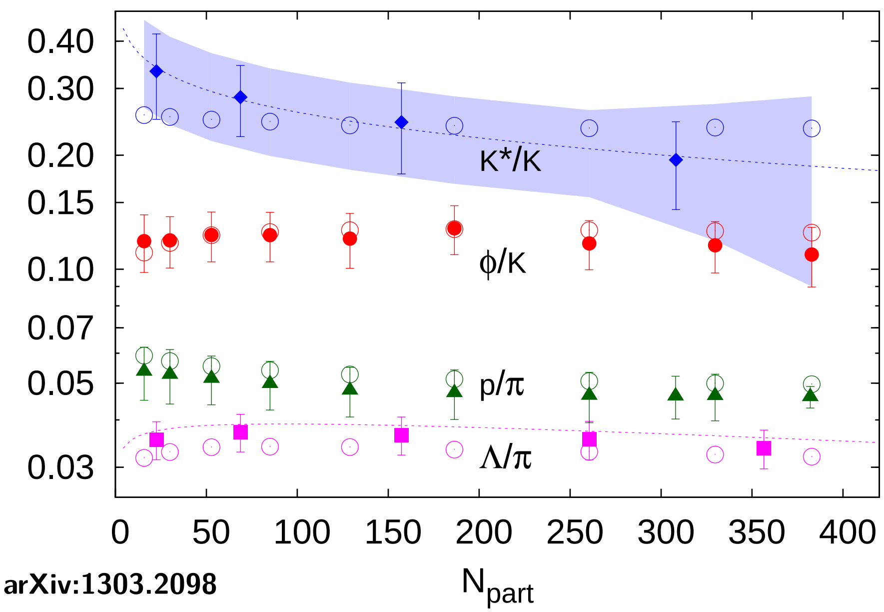


arXiv:1303.0913

Use interpolated data

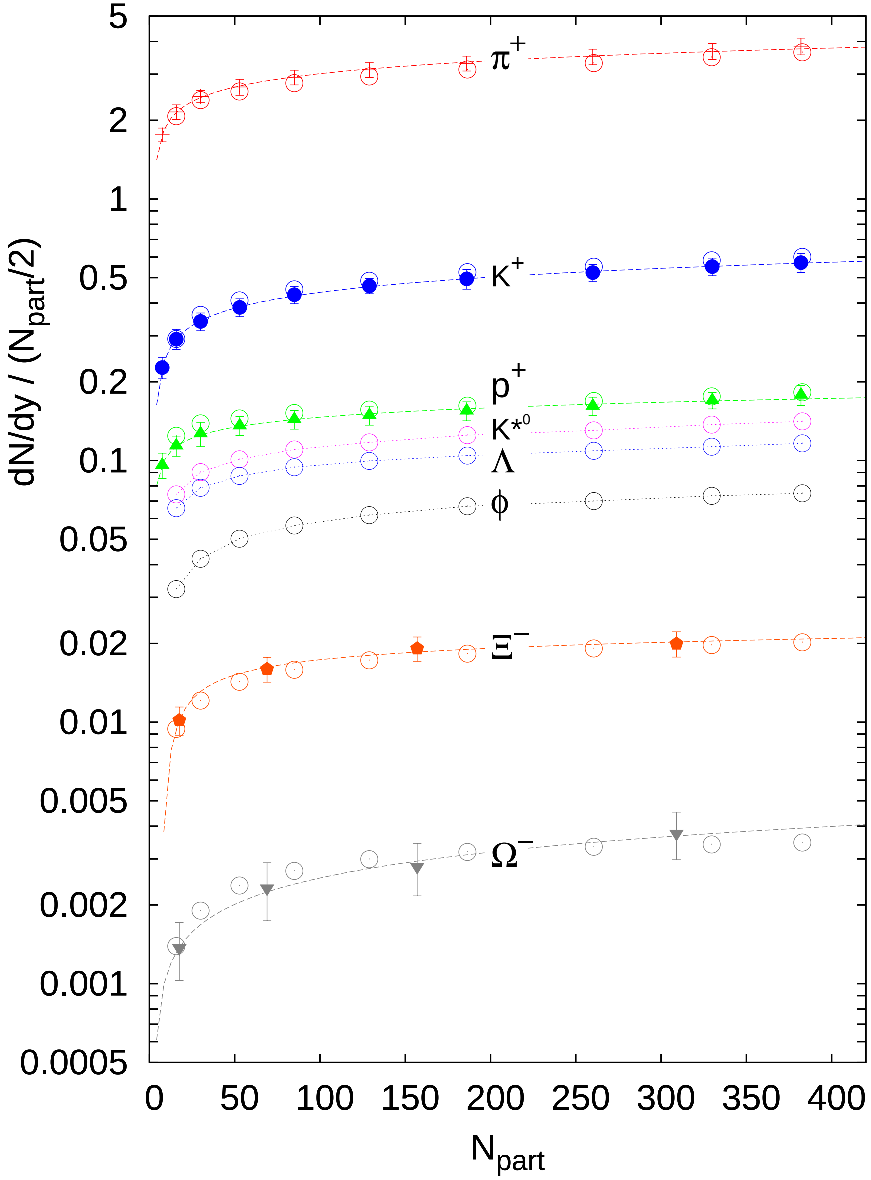
- **Nine fine centrality bins**
- **K*/K error band accounts for extrapolation error**

Fits come out very similar, Λ/π largest ($\lesssim 1.2$ s.d.) discrepancy in both approaches



arXiv:1303.2098

Interpolation method



- **Different hadron species, same interpolation**

$$\frac{dN\left(N_{\rm part}\right)}{dy} = a\, N_{\rm part}^{b} + c \qquad (1)$$

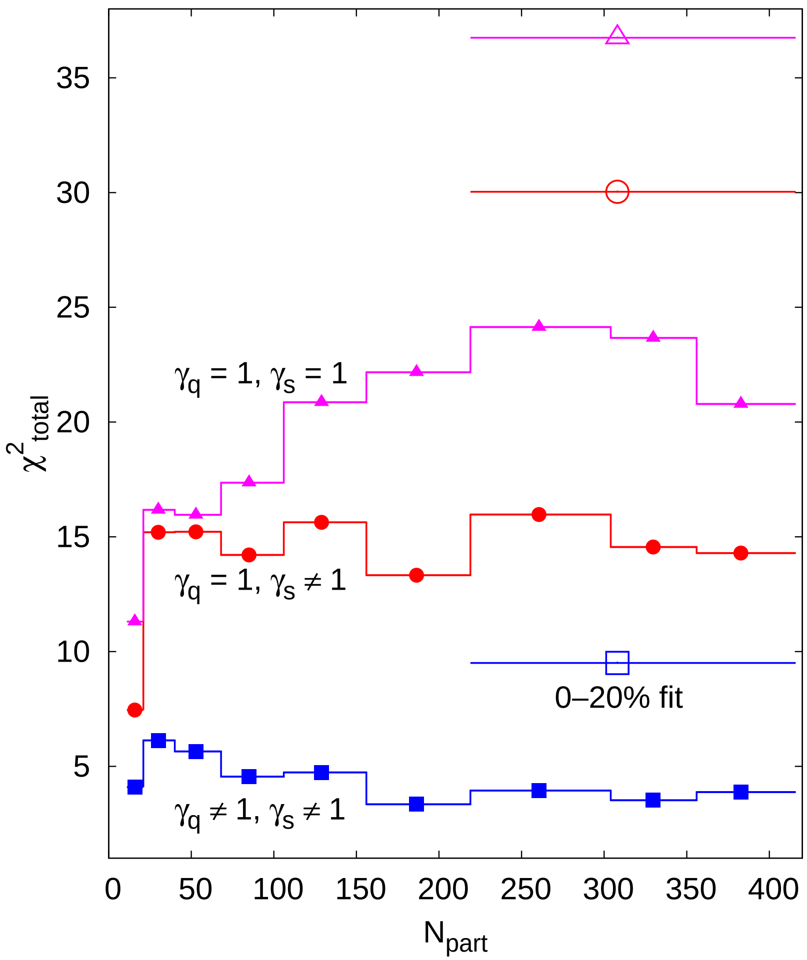
- **Common power-law,** $1.09 \leq b \leq 1.19$**, (except** Ω**:** $b \simeq 1.3$**, more data will resolve this)**
- **Data well described by our interpolation**
- **data point: solid symbols with error (where missing, data from ratios). Lines: interpolation, open symbols – SHM fit**

FIT QUALITY:

Chemical non-equilibrium SHM works at all centralities

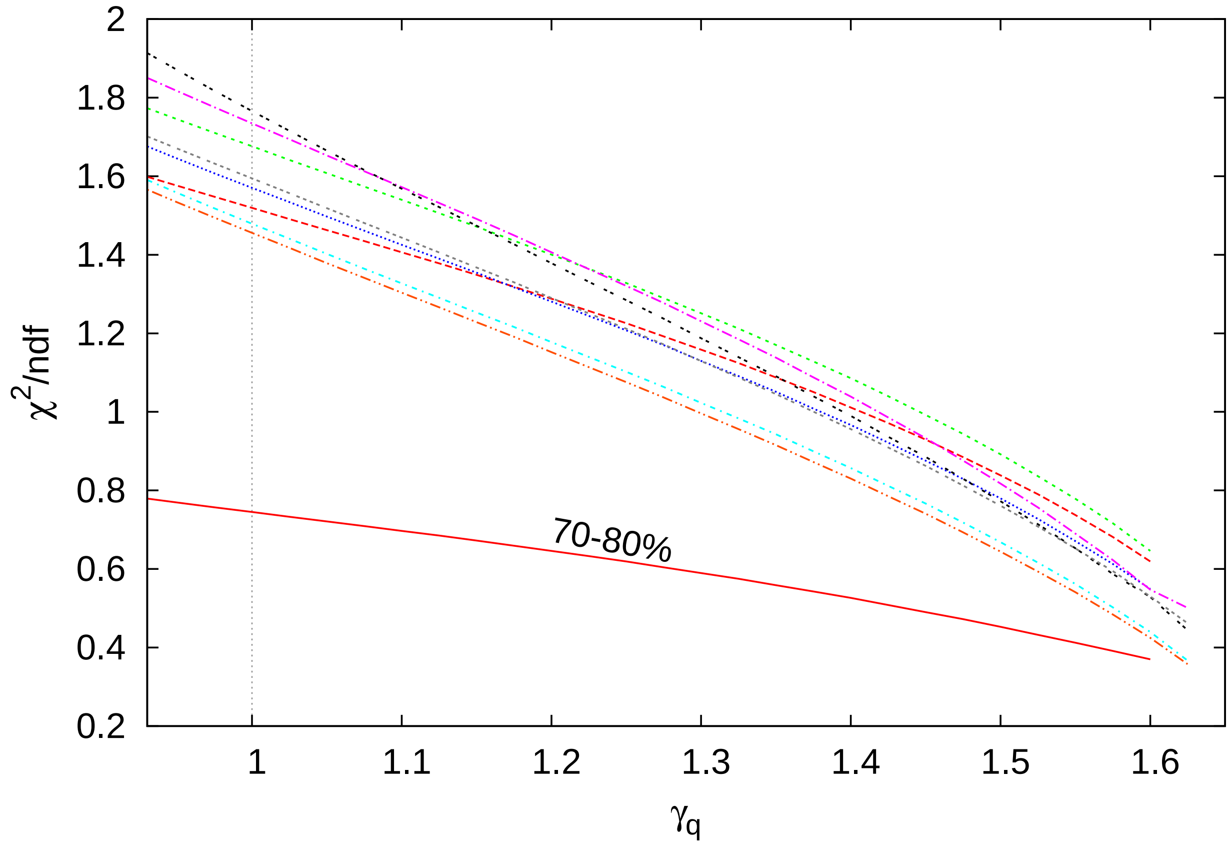
Non-equilibrium

- $\chi^2/\mathbf{ndf} \simeq 4.5/9 = 0.5$,
- **constant across centrality**
- **improvement by factor of 3 resp. 5 comparing to $\gamma_q = 1$**
- **Only in peripheral collisions $\gamma_q \simeq 1$ maybe possible,** low χ^2 may be an artifact of larger experimental errors.



LIGHT QUARKS u, d OUT OF EQUILIBRIUM

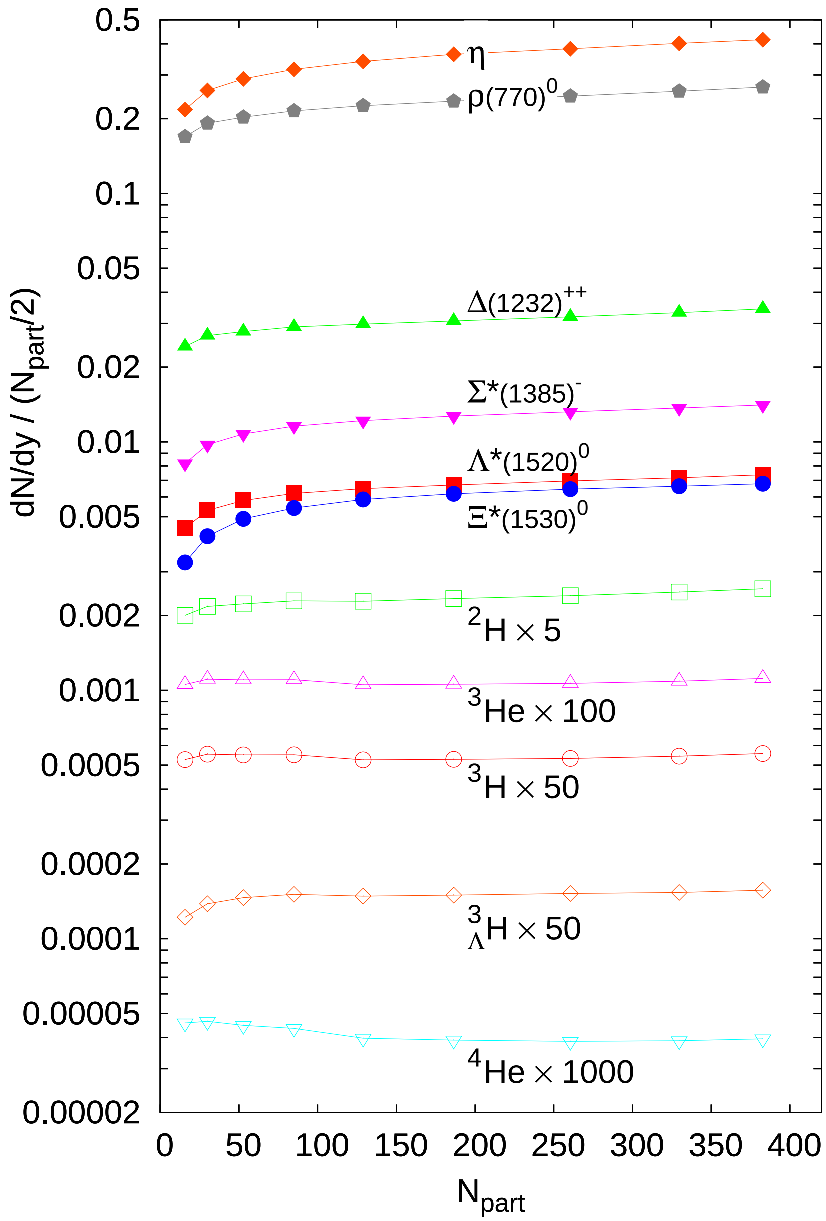
χ^2 profile of γ_q



- $\gamma_q = 1$ – **no special meaning for SHM**
- **Special role of** $\gamma_q \to \gamma_q^{\rm crit} \equiv e^{m_\pi/2T} \simeq 1.61$ – **B-E condensation of** π^0
- χ^2 **in most peripheral bin may allow equilibrium**

PREDICTED PARTICLE YIELDS

- **Unobserved hadrons predicted based on fit parameters**
- **Baseline for production rate of (anti-)nuclei from SHM**

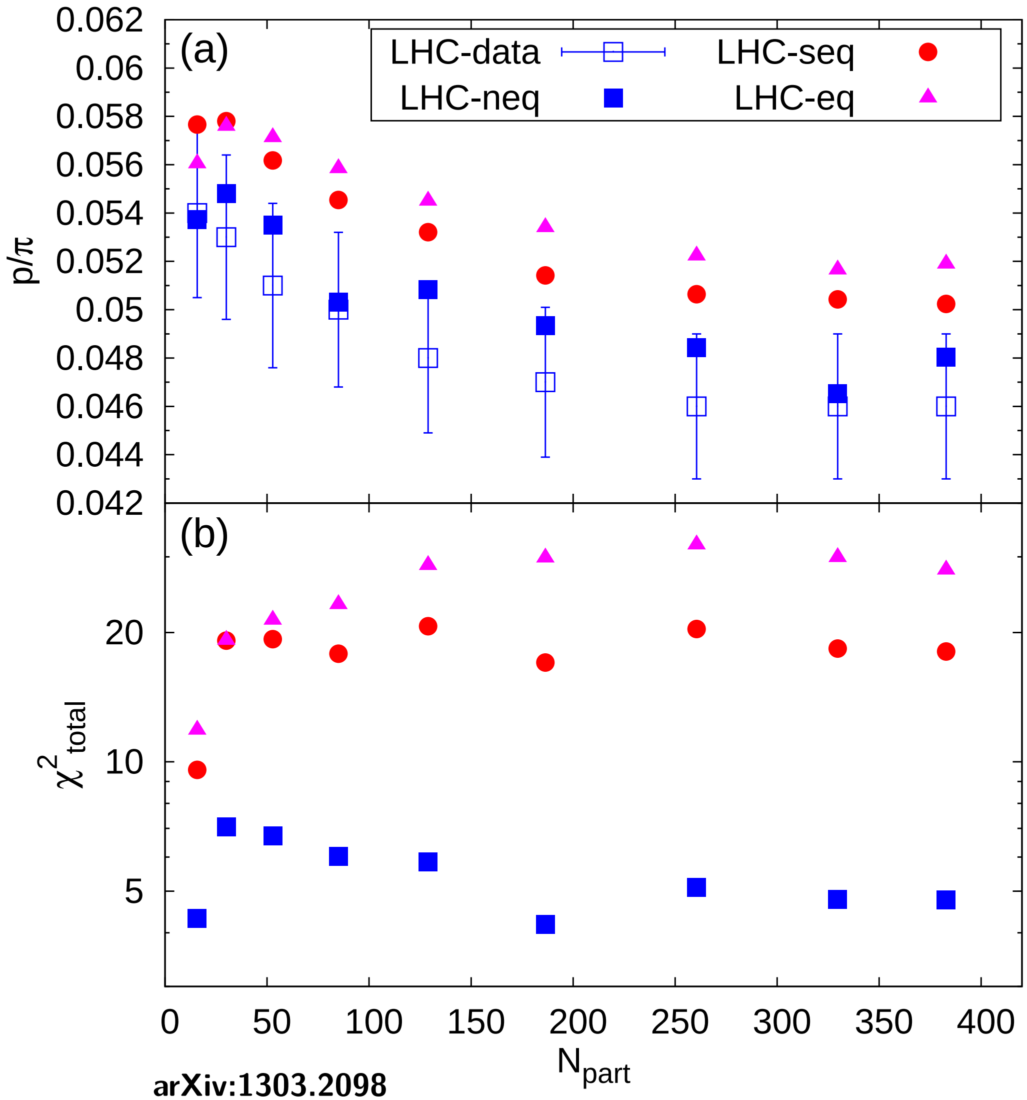


NONSTRANGE BARYON TO MESON RATIO

$$\frac{\mathrm{baryon}(qqq)}{\mathrm{meson}(q\overline{q})} \propto$$
$$\propto \frac{\gamma_q^3}{\gamma_q^2} F(T, m_b, m_m)$$

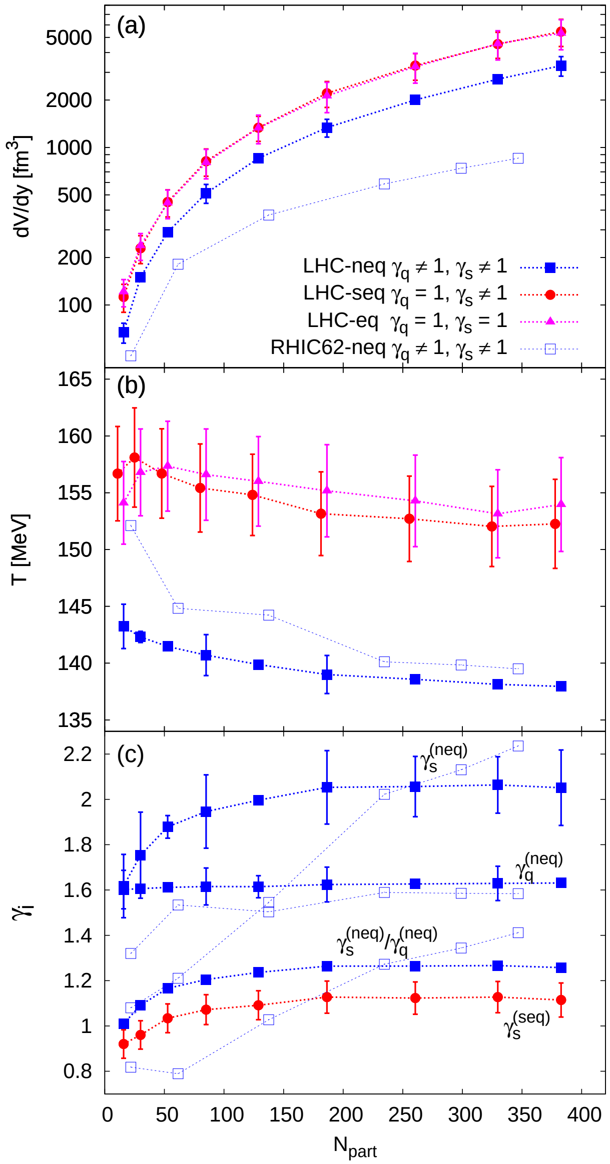
Note slow 15% rise of p/π for peripheral collisions. This slow centrality dependence of p/π data clarifies the role of annihilation afterburner, see e.g. Iu.A. Karpenko, et al, Phys. Rev. C 87, 024914 (2013) [arXiv:1204.5351]

Post hadronization processes, while explaining protons, create new inconsistencies with multi-strange baryons, namely Ξ, see, e.g. J. Steinheimer, et al., Phys. Rev. Lett. 110, 042501 (2013) [arXiv:1203.5302]



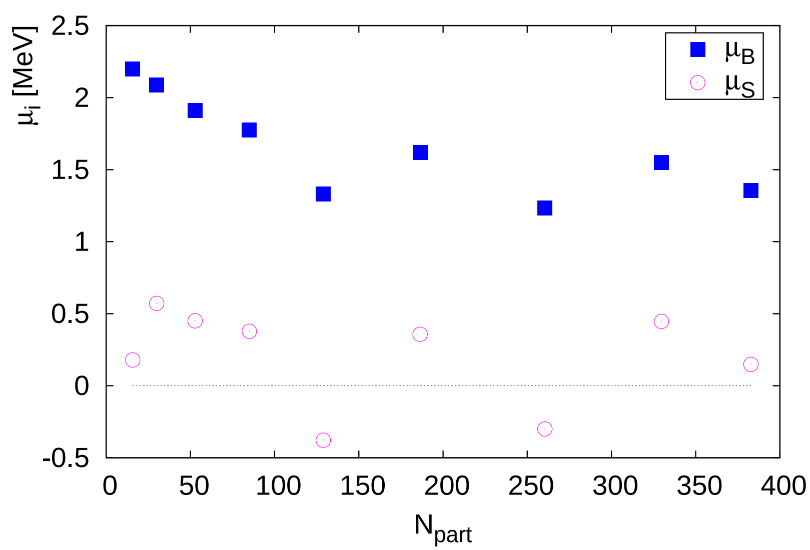
arXiv:1303.2098

Comparison: SHM PARAMETERS: RHIC—LHC



- **dV/dy 4 times bigger than RHIC-62**
- $T_{LHC} \simeq T_{RHIC} \simeq 140 - 145$ **MeV**
- **always** $\gamma_q \neq 1$**, typically** $\gamma_q = 1.6$
- $\gamma_s \simeq 2$**, constant for** $N_{part} \geq 100$ **— clear difference to RHIC**

- **at LHC** $\mu_B \to 0$**. To estimate magnitude:** $(\langle b \rangle - \langle \bar{b} \rangle)/N_{part} = 0.0054 \pm 1\%$ $\to$ **smoother** μ_B **and** μ_S

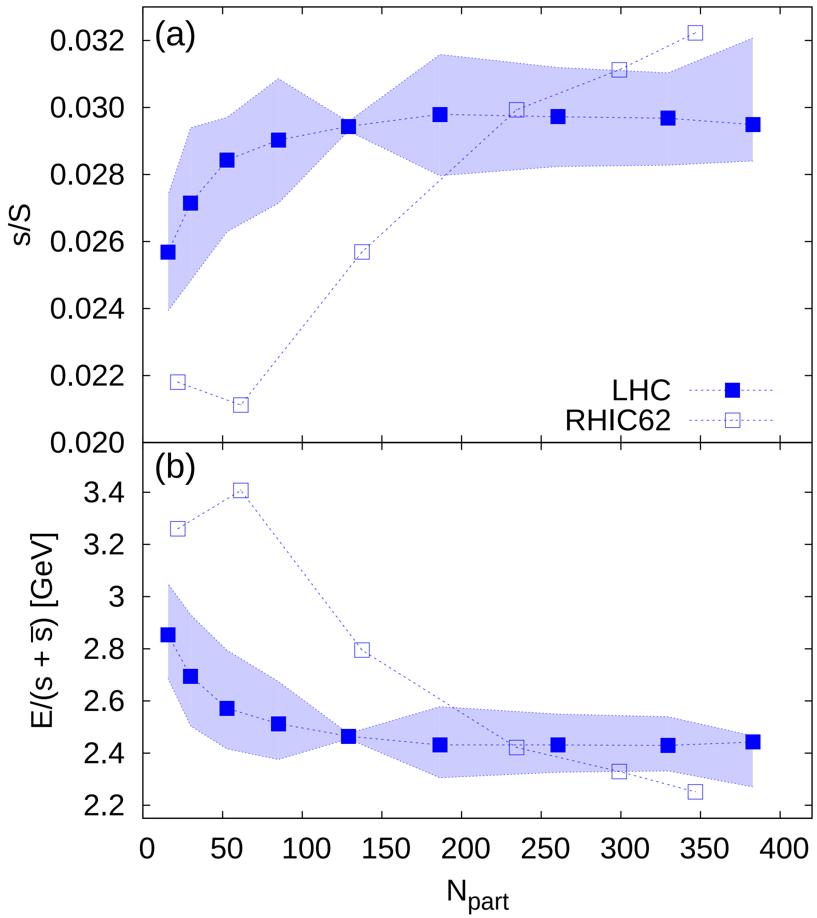


THERMAL PARAMETERS → PHYSICAL PROPERTIES

Centrality	0–20%	0–5%	5–10%	10–20%	20–0%	30–40%	40–50%	50–60%	60–70%	70–80%
$\langle N_{\rm part}\rangle$	308	382.8	329.7	260.5	186.4	128.9	85.0	52.8	30.0	15.8
$dV/dy\,[\rm fm^3]$	2463±6	3304±469	2715±81	2003±47	1337±173	853.9±5.9	512.2±70.1	289.4±5.5	149.8±5.0	66.9±9.7
T [MeV]	138.3±0.0	138.0±0.0	138.1±0.0	138.6±0.0	139.0±1.7	139.9±0.0	140.7±1.8	141.5±0.0	142.3±0.5	143.2±2.0
γ_q	1.63±0.00	1.63±0.00	1.63±0.08	1.63±0.00	1.62±0.08	1.62±0.05	1.62±0.08	1.61±0.00	1.61±0.00	1.60±0.09
γ_s	2.08±0.00	2.05±0.17	2.06±0.13	2.06±0.13	2.05±0.16	2.00±0.01	1.95±0.16	1.88±0.05	1.75±0.19	1.62±0.14
$\chi^2_{\rm total}/\rm ndf$	9.51/9	3.87/9	3.52/9	3.94/9	3.35/9	4.73/9	4.55/9	5.65/9	6.13/9	4.09/9
$\varepsilon\,[\rm GeV/fm^3]$	0.462	0.453	0.457	0.467	0.476	0.487	0.505	0.516	0.521	0.527
$P\,[\rm MeV/fm^3]$	78.5	77.1	77.7	79.1	80.5	82.3	85.1	86.8	87.9	89.2
$\sigma\,[\rm fm^{-3}]$	3.20	3.14	3.17	3.23	3.28	3.36	3.46	3.53	3.56	3.60
s/S	0.0299	0.0295	0.0297	0.0297	0.0298	0.0294	0.0290	0.0284	0.0272	0.0257
$S_{\rm LHC}/S_{\rm RHIC}$	3.07	3.23	3.10	2.93	2.75	2.56	2.33	2.06	1.74	1.27

Comparative Analysis of RHIC and LHC Data:

s-enhancement



Panel (a): strangeness per entropy s/S content of the fireball at LHC2760 (filled squares) and at RHIC62 (open squares) as a function of centrality;. Colored bands represent uncertainty based on γ_s uncertainty. Main difference RHIC-LHC: volume-like result for LHC much earlier compared to RHIC. Indication of higher specific strangeness content in most central RHIC collisions. at LHC, longer lived fireball, which is equilibrated QGP at smaller N_{part}.

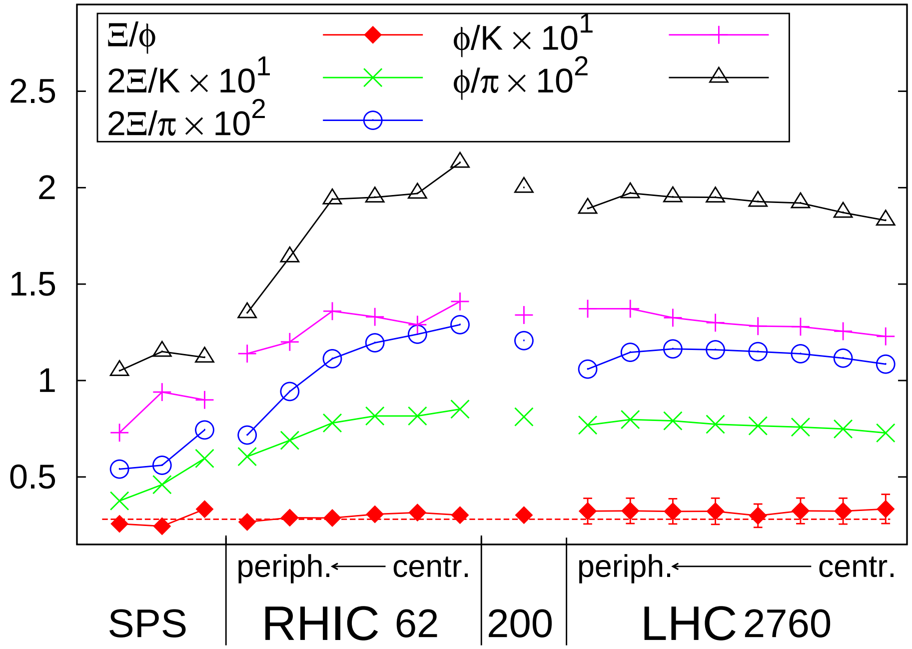
Panel (b): the thermal energy cost to make a strange–anti-strange quark pair. Shows transit from pp-like peripheral process to thermal QGP process. Two strangeness production mechanisms compete, at LHC, gluon fusion energetically more effective.

M. Petran, et al “Hadron production and QGP Hadronization in Pb-Pb collisions at LHC,” arXiv:1303.2098 .

First indication of Universal Hadronization

All world data (SPS,RHIC,LHC) yield same ratio

$$\frac{\Xi}{\phi} \equiv \sqrt{\frac{\overline{\Xi}(\bar{s}\bar{s}\bar{d})^{+}\Xi(ssd)^{-}}{\phi(s\bar{s})\phi(s\bar{s})}} = \gamma_q f(T) = 0.277 \pm 10\%,$$



- **By taking the product of particle and antiparticle, we eliminate baryo-chemical potential $\mu_{\rm B}$ as well as strange chemical potential $\mu_{\rm S}$.**
- **We also eliminate the strange quark phase space occupancy γ_s, because the strange and anti-strange quark content in the numerator and denominator is the same.**
- **The overall normalization V is eliminated by the fact that we have the same number of hadrons in the ratio numerator and denominator.**

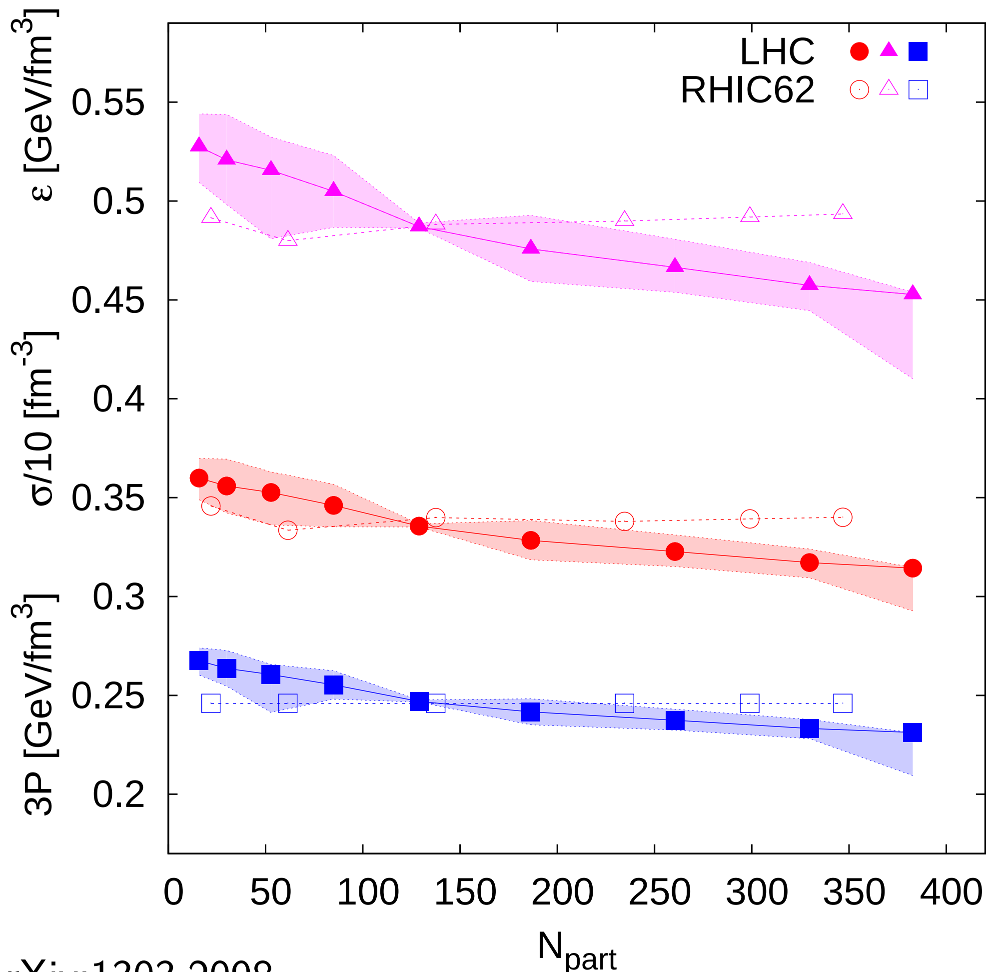
discussion in: M. Petran and J. Rafelski, "Multistrange Particle Production and the Statistical Hadronization Model," Phys. Rev. C 82, 011901 (2010)

At LHC, most ratios constant → chemical equilibration of QGP.

UNIVERSALITY OF FIREBALL PHYSICAL PROPERTIES

- **LHC Compatible with RHIC-62**
- **Error band is γ_s uncertainty**
- **Slight decrease with centrality**
- **Decrease $\rightarrow$ super–cooling**
- **Compare $\varepsilon = 0.5$ GeV/fm^3 $\simeq$ $\simeq 3.3\varepsilon_{\rm nucl}$**

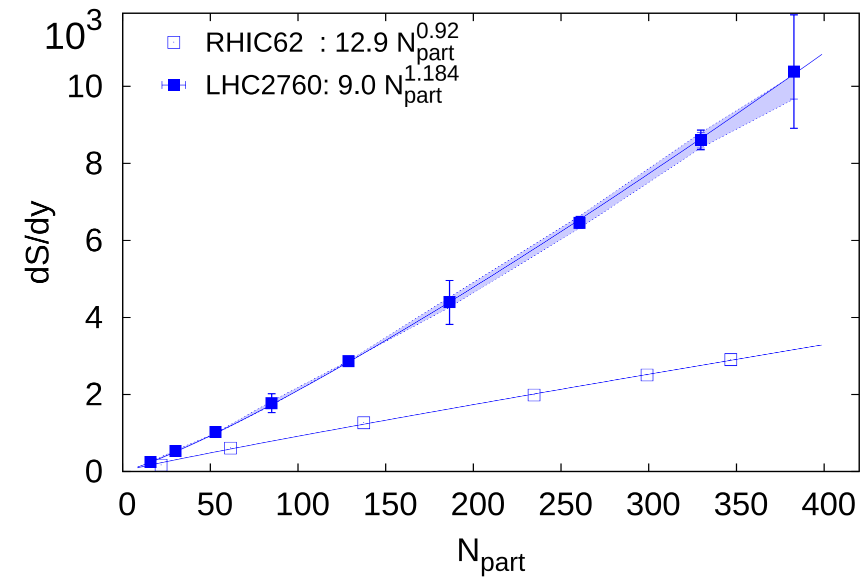
Requiring Universal Hadronization Conditions $\Rightarrow$ only two free non–equilibrium model parameters – dV/dy, γ_s



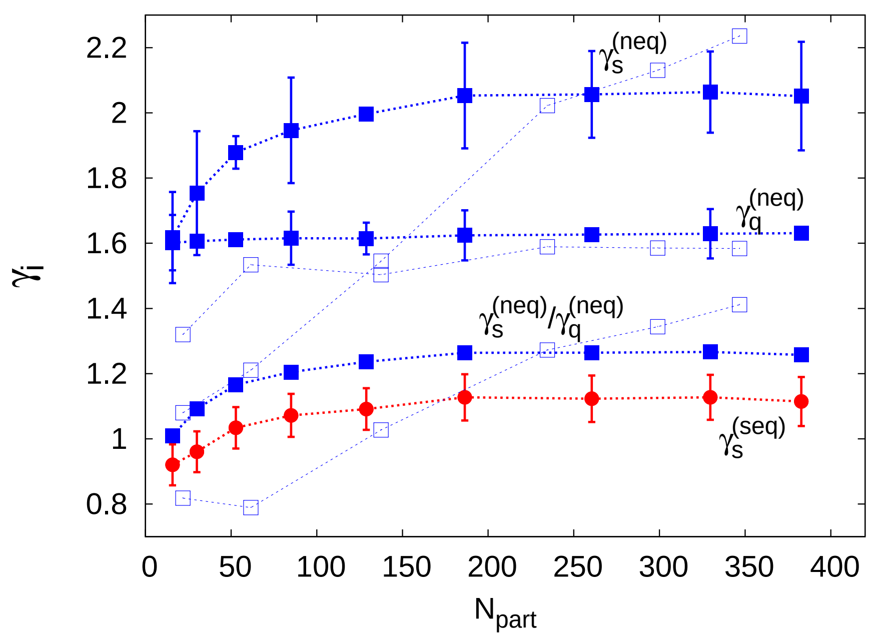
arXiv:1303.2098

UNIVERSALITY OF HADRONIZATION: RHIC—LHC differences

- **VOLUME**
  Size related to initial stage
- **Almost linear centrality dependence at RHIC-62**
- **Faster rise at LHC: additional entropy production? charm?**

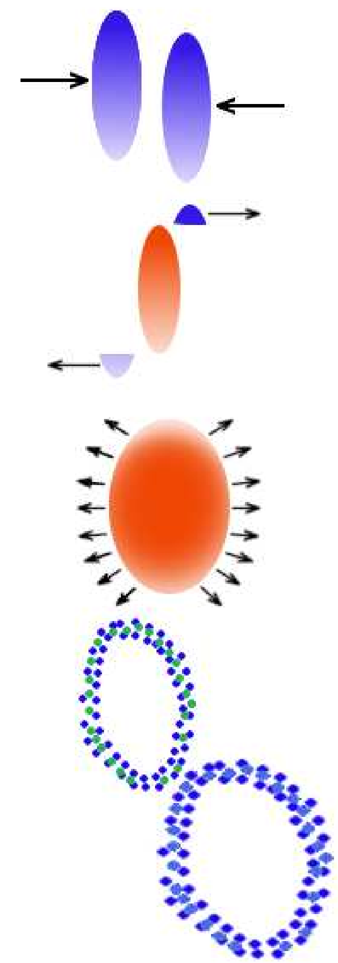


- **QUARK ABUNDANCE** $= \gamma_i$
  For most central, RHIC and LHC similar.
- $\gamma_q = 1.6$ **always similar**
- **Different centrality dependence**
- **LHC saturates strangeness early**
- **RHIC more strangeness rich**



STRANGENESS production at LHC

Goal: $\boxed{s\bar{s} = 300}$ pairs in strange hadrons. Where from?



- Partons in nuclei collide, fragment — hard parton processes, $\sim 100\ s\bar{s}$ pairs (similar mechanism for $c\bar{c}$, $b\bar{b}$)
- Thermalization into a fireball of QGP (entropy production ends)
- Fireball expansion and cooling, the rest of strangeness thermally produced predominantly by gluon fusion ($GG \to s\bar{s}$)
- QGP $\to$ Hadrons: Hadronization
- When T low and V large enough chemical freeze-out – directly at time of hadronization
- Hadrons probably still re-scatter, but chemical abundances do not change
- Kinetic freeze-out – hadrons stop interacting free-streaming into detectors

QGP properties are imprinted in hadron abundances
for the specific scenario we favor:
direct explosive breakup of QGP fireball. 'Sudden hadronization'

No experimental evidence against sudden hadronization.
Post–hadronization afterburners motivated by the effort to make equilibrium hadronization models plausible at LHC.

Strangeness at RHIC and LHC: Strangeness / Entropy

s/S: ratio of the number of active degrees of freedom in QGP,

For IN PLASMA chemical equilibrium :

$$\frac{s^Q}{S^Q} \simeq \frac{1}{4}\frac{n_s}{n_s + n_{\bar{s}} + n_q + n_{\bar{q}} + n_G} = \frac{\frac{g_s}{2\pi^2}T^3(m_s/T)^2K_2(m_s/T)}{(g2\pi^2/45)T^3 + (g_s n_{\rm f}/6)\mu_q^2 T} \simeq \frac{1}{35} = 0.0286$$

with $\mathcal{O}(\alpha_s)$ interaction $s/S \to 1/31 = 0.0323$

CENTRALITY A, and ENERGY DEPENDENCE: $\gamma_s^{\rm Q} \to 1$

Chemical non-equilibrium occupancy of strangeness $\gamma_s^{\rm Q}$

$$\frac{s^Q}{S^Q} = \frac{0.03\gamma_s^{\rm Q}}{0.4\gamma_{\rm G} + 0.1\gamma_s^{\rm Q} + 0.5\gamma_q^{\rm Q} + 0.05\gamma_q^{\rm Q}(\ln\lambda_q)^2} \to 0.03\gamma_s^Q(\tau).$$

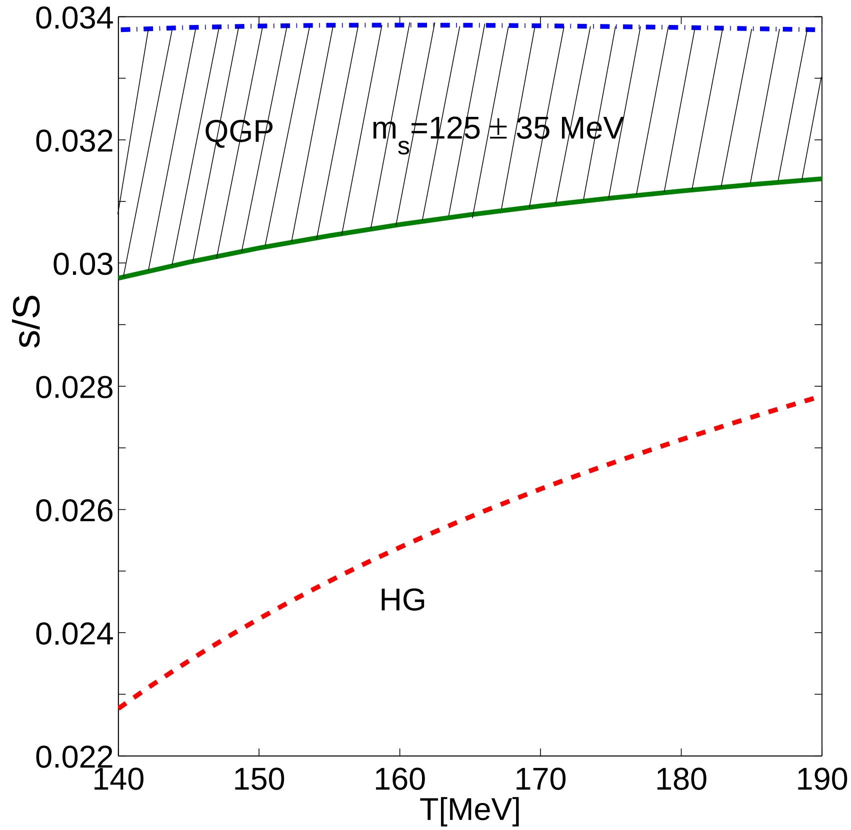
For chemically equilibrated QGP $\gamma_s^Q(\tau) \to 1$

Analysis of experiment: we count all strange/nonstrange hadrons in final state, we extrapolate to unmeasured particle yields and/or kinematic domains, and evaluate resonance contributions and cascading:

$$\frac{s^H}{S^H} \simeq \frac{\text{count of primary strange hadrons}}{(\text{nonstrange} + \text{strange})\ \text{entropy} = 4\,\text{number of primary mesons} + \ldots} \simeq \frac{s^Q}{S^Q}$$

s/S QGP and HG comparison in chemical equilibrium

at a common measured T. This shows how different is the strangeness phase content ('phase contrast')



Strangeness to entropy ratio $s/S(T; \mu_B = 0, \mu_S = 0)$ for the chemically equilibrated QGP (green, solid line for $m_s = 160$ MeV, blue dash-dot line for $m_s = 90$ MeV); and for chemically equilibrated HG (red, dashed).

When counting strangeness we remember that a lot of strangeness is hidden $s\bar{s}$-states η, η', ϕ

I. Kuznetsova and J. Rafelski, "Heavy flavor hadrons in statistical hadronization of strangeness-rich QGP," Eur. Phys. J. C 51, 113 (2007) [hep-ph/0607203].

Strangeness / Entropy during hadronization QGP → hadrons

Model	$dV/dy\,[\mathrm{fm}^{-3}]$	$T\,[\mathrm{MeV}]$	s^{QGP}	s^{HG}	$S^{QGP} \times 10^{-3}$	$S^{HG} \times 10^{-3}$
'non–equilibrium' $\gamma_q \neq 1 \neq \gamma_s$	**3304**	**138**	**617**	**306**	**22.3**	**10.4**
'semi–equilibrium' $\gamma_q = 1 \neq \gamma_s$	**5441**	**152**	**1392**	**314**	**49.3**	**12.2**
'equilibrium' $\gamma_q = 1 = \gamma_s$	**5320**	**154**	**1410**	**295**	**49.9**	**12.3**

QGP values calculated from T, dV/dy, massless quarks; HG values from hadrons fitted to data

- **Sudden hadronization – factor 2 change in both s and S → s/S stays constant**
- **Other scenarios ($\gamma_q = 1$) – factor 4-4.7 change in s and S → s/S drops by 10–17%**

WHAT HAPPENS TO STRANGENESS AND ENTROPY DURING/AT HADRONIZATION ???

Strangeness and Entropy matching → quark masses

We analyze the phase space as a function of the quark masses...

$$\frac{ds}{dy} = \frac{1}{2\pi^2}\frac{T^3}{(\hbar c)^3}\frac{dV}{dy} g_s \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}\left[\left(\frac{n m_s}{T}\right)^2 K_2\left(\frac{n m_s}{T}\right)\right] = 306$$

$$\Rightarrow m_s = 300\,\mathrm{MeV}/c^2$$

With m_s, matching the entropy fixes m_q

$$\frac{dS}{dy} = \frac{4}{2\pi^2}\frac{T^3}{(\hbar c)^3}\frac{dV}{dy} \sum_{\substack{n=1\\ \imath=q,s}}^{\infty} g_\imath \frac{(-1)^{n+1}}{n^4}\left[\left(\frac{n m_\imath}{T}\right)^2 K_2\left(\frac{n m_\imath}{T}\right) + \frac{1}{4}\left(\frac{n m_\imath}{T}\right)^3 K_1\left(\frac{n m_\imath}{T}\right)\right] = 10400$$

$$\Rightarrow m_q = 266\,\mathrm{MeV}/c^2$$

STRANGENESS and ENTROPY in QGP and HG match

when/if quarks acquire masses

of $\mathrm{m_s} = 300$ MeV/c^2 and $\mathrm{m_q} = 266$ MeV/c^2

CONCLUSIONS

- **TO describe hadron production use non-equilibrium SHARE SHM**
- $\gamma_q > 1$ **necessary to describe** p/π **ratios** Ω
- **For good fit good hadron production predictions**
- **LHC SHM parameters very similar to RHIC:**
  differences: bigger volume dV/dy**, strangeness equilibrated for smaller systems –** γ_s
- **Earlier QGP equilibrated for** $N_{\rm part} > 100$ **at LHC**
- **Universal hadronization conditions**
- **Small systematic decrease may indicate super-cooling at LHC**
- **Sudden hadronization works at LHC, non-equilibrium SHM describes all hadron yields at LHC!**

REMEMBER:
We use transverse momentum integrated hadron yields to be independent from matter dynamics.
Volume we fit is dV/dy not HBT volume. Relation is not yet fully established.
Investigate strangeness and entropy discontinuity from QGP to hadron phase.

BACKUP SLIDES

NO PROTON–ANTI-PROTON ANNIHILATION

- **Anti-proton mean free path**

$$L_{\rm event} = \frac{1}{\sigma_{\rm event}\rho_{\rm p}^h}$$

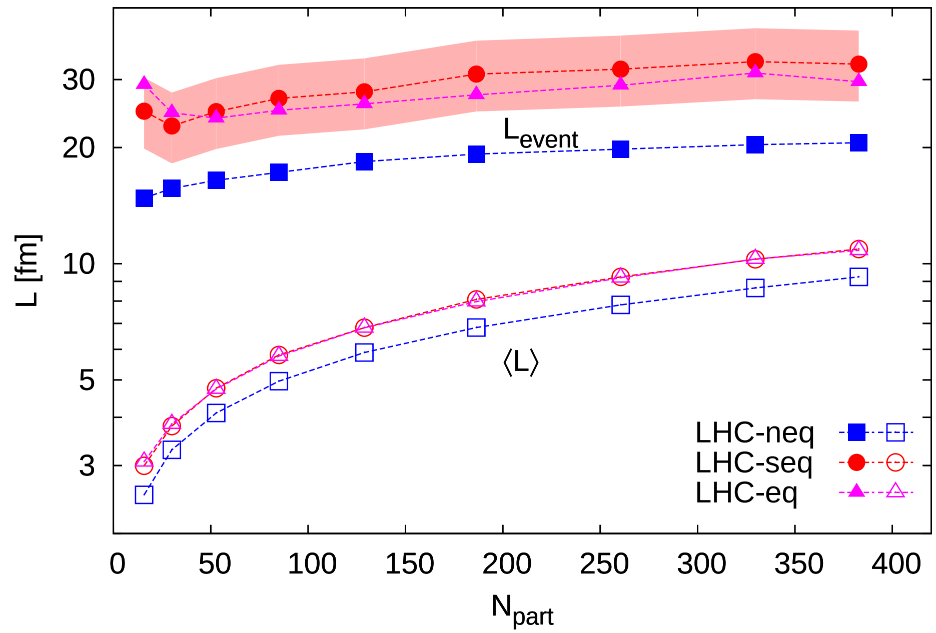
- **Fireball size scale**

$$\langle L\rangle \simeq [(dV/dy)/(4\pi/3)]^{1/3}$$

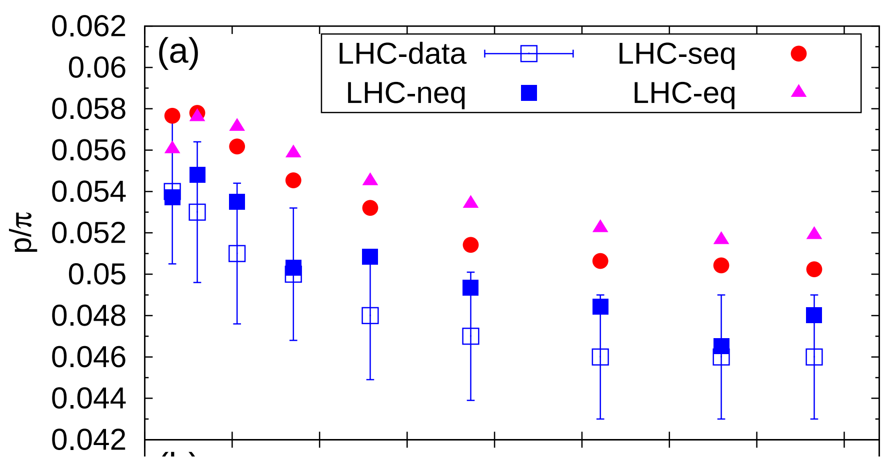
- **Creation reactions equally strong**

$$\rm p + \overline{p} \longleftrightarrow \rho + \omega$$

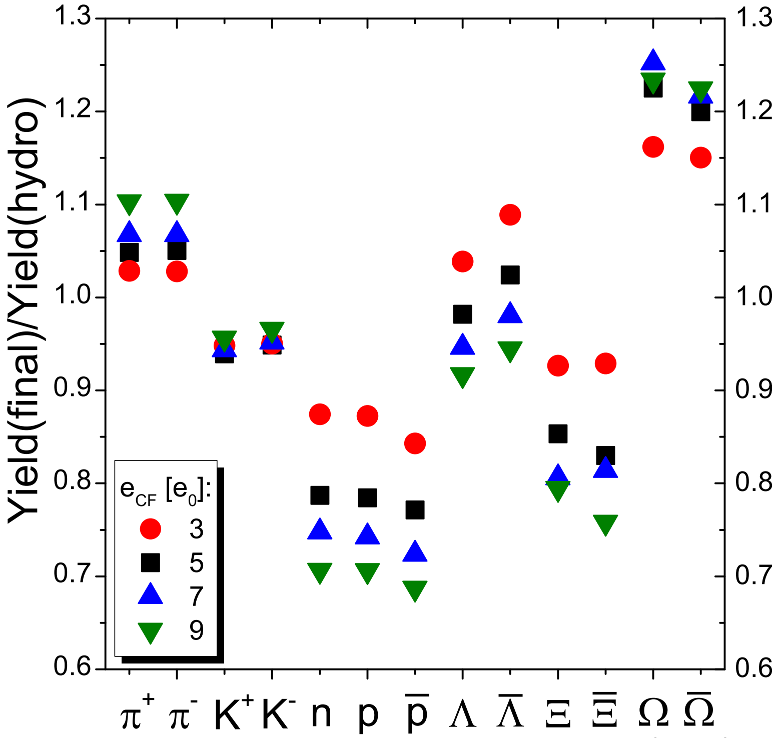
- **No annihilation in peripheral collisions, p/$\pi \simeq 0.05$**



- **Centrality dependence of p/π data in contradiction with Phys. Rev. C 87, 024914 (2013) [arXiv:1204.5351 [nucl-th]]**



POST-HADRONIZATION INTERACTIONS
solving protons creates problems with multistrange baryons



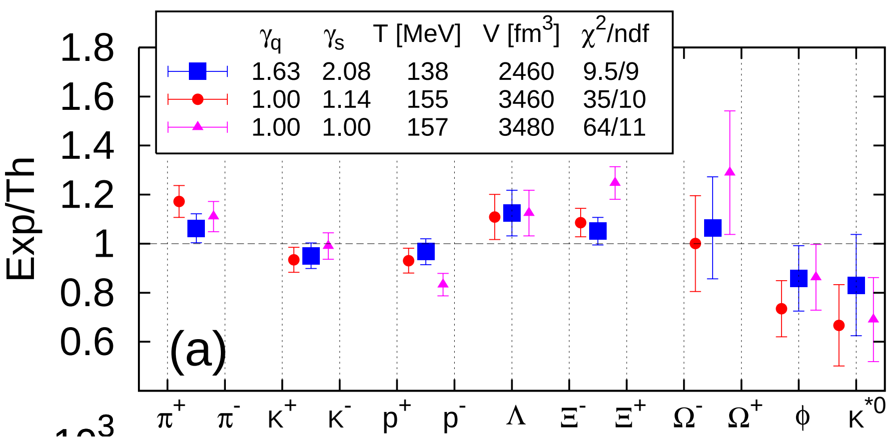
J. Steinheimer, et al., Phys. Rev. Lett. 110, 042501 (2013)

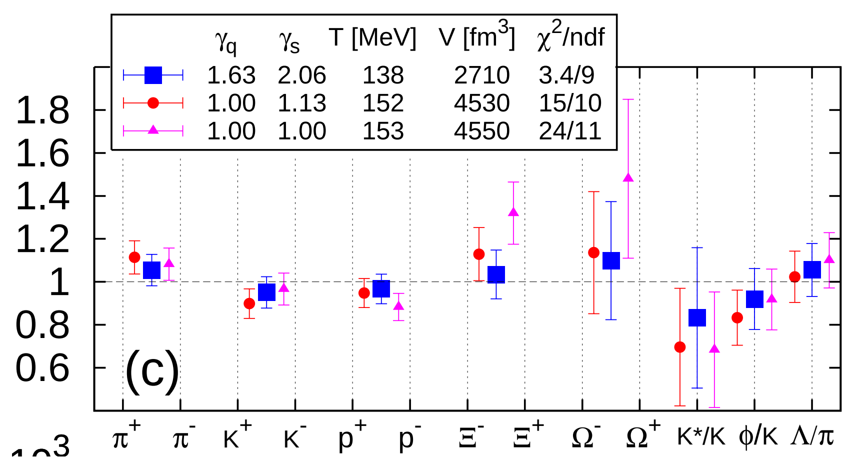
Post-hadronization interactions

- decrease model proton yield to match experiment
- increase model Ω closer to exp.
- but deplete Ξ further from exp.

I.A. Karpenko, et al., (Phys. Rev. C 87, 024914 (2013)) reports p/π to be 3σ from experimental data for 20–30% centrality.

CHALLENGE FOR POST-EQUILIBRIUM-HADRONIZATION INTERACTIONS TO MODEL ALL BARYONS and CENTRALITY DEPENDENCE





Chemistry in terms of SHM parameters incl. non-equilibrium

- γ_q ($\gamma_s, \gamma_c, \ldots$): u, d ($s, c, \ldots$) **quark phase space yield,**

$$\frac{qqq - \text{baryons}}{q\bar{q} - \text{mesons}} \propto \frac{\gamma_q^3}{\gamma_q^2} \cdot \left(\frac{\gamma_s}{\gamma_q}\right)^n$$

- γ_s/γ_q **shifts the yield of strange vs non-strange hadrons:**

$$\frac{\overline{\Lambda}(\bar{u}\bar{d}\bar{s})}{\bar{p}(\bar{u}\bar{u}\bar{d})} \propto \frac{\gamma_s}{\gamma_q}, \qquad \frac{\mathrm{K}^+(u\bar{s})}{\pi^+(u\bar{d})} \propto \frac{\gamma_s}{\gamma_q}, \qquad \frac{\phi}{h} \propto \frac{\gamma_s^2}{\gamma_q^2}, \qquad \frac{\Omega(sss)}{\Lambda(sud)} \propto \frac{\gamma_s^2}{\gamma_q^2},$$

Statistical Meaning of Parameters

Example of NUCLEONS $\gamma_N = \gamma_q^3$**:**

$$\Upsilon_N = \gamma_N e^{\frac{\mu_b}{T}}, \qquad \Upsilon_{\overline{N}} = \gamma_N e^{\frac{-\mu_b}{T}};$$

$$\sigma_N \equiv \mu_b + T \ln \gamma_N, \qquad \sigma_{\overline{N}} \equiv -\mu_b + T \ln \gamma_N$$

Meaning of parameters from e.g. the first law of thermodynamics:

$$dE + P\,dV - T\,dS = \sigma_N\,dN + \sigma_{\overline{N}}\,d\overline{N}$$

$$= \mu_b(dN - d\overline{N}) + T \ln \gamma_N (dN + d\overline{N}).$$

NOTE: For $\gamma_N \to 1$ **the pair terms vanishes, the** μ_b **term remains, it costs** $dE = \mu_B$ **to add to baryon number.**