

Quarks, Flow and Temperature in Spectra

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Abstract. Strangeness in Quark Matter 2013 Theory Review

1. Introduction

A good theory summary should not consist of a direct enumeration of talks, rather a critical reflection, possibly a synthesis of different viewpoints is expected. Here we follow this concept: after identifying the key fundamental questions in the research field on quark matter five competing views will be presented as duelling alternatives. The physics behind experimental findings may not only differ from, but sometimes even contradict one another, as well as mathematical model approaches can be based on diametrically opposite philosophies. We shall exhibit a few simple theoretical exercises in order to illuminate these alternatives. By its very nature this summary is subjective.

In our view there are three fundamental questions about high energy matter: i) the degree of stochasticity in the phenomena we experience, ii) the particle – wave (particle gas – strong field) duality in the description of the acting objects, and iii) the reliability of the conjectured initial state without any direct experimental evidence. "The Lord does not play dice" - surmised Albert Einstein, but quite often it looks like that: we lack information even then, when physically it would not be prohibited to measure it. We are blind to part of the phase space by the design of our detectors (we hope only to an unimportant part). We are trying to model and understand *average* outcomes from hundreds of millions of physically independent events by statistical and continuum physics calculations and we do not have much exclusive information event by event. Furthermore if we had, we possibly could not digest it. We are talking about (point) particles, but calculate with (plane wave) fields. We model initial and intermediate state fluctuations by guessing and expect to see an emerging pattern on the statistics of outcomes. But what are our choices when doing theoretical calculations? What are viable alternatives? What has been our progress lately in answering these fundamental questions? In this review we try to collect some partial answers.

The main "idea-wise" duels occur as follows:

- (i) thermalization: exponential vs power-law p_T spectra
- (ii) hydrodynamics: initial state vs eos-driven flow features
- (iii) hadronization: phase space vs QCD matrix element dominance
- (iv) Quark Matter eos: lattice vs dual gravity methods
- (v) nuclear medium: cross sections vs mean fields
- (vi) simulation: particle vs field based models

In the this review we plan to discuss many of these dichotomies.

Last but not least, before starting to review theoretical contributions to this conference, let us recall Helen Caines' introductory talk about experiments: we have gathered more information from experiments, seeing more size, more time and more gradients in heavy ion collisions. From the overview of several different beam energies we learn approximate relations between yield and rapidity range,

$$\frac{dN_{\text{ch}}}{d\eta} \propto \ln \sqrt{s} \sim y_{\text{beam}}, \quad (1)$$

between volume and charged particle rapidity yields

$$V \approx 730\text{fm}^3 + 2.545 \frac{dN_{\text{ch}}}{d\eta}, \quad (2)$$

and between time duration and charged particles per rapidity

$$\mathcal{T} \approx 0.875 \left(\frac{dN_{\text{ch}}}{d\eta} \right)^{1/3}. \quad (3)$$

These formulas are approximate fits to the figure she showed[1].

2. Thermalization

The competition between thermal and non-thermal views of the intermediate "fireball" state of high-energy heavy ion collisions is reflected in the hunt for exponential spectra. Besides that any continuous spectrum can have a straight line fit on the logarithmic plot in a short enough interval, claiming thermal phenomena must be based on a thermal state, on having (at least local) equilibrium in the energy distribution of the parts of the surmised physical system. Therefore the theoretical study of possible ways of thermalization, of reaching such a state, has key importance.

Either studying the emergence or assuming the existence of a thermal state, thermal and hydrodynamical models are numerous. Just citing some of the titles from the program: "Thermalization of massive partons in anisotropic medium (Florkowski), First 3 seconds (Rafelski), Thermal model (Stachel), Strangeness balance in HIC (Kolomeitsev), Systematic Properties of the Tsallis Distribution (Cleymans), Thermalization through Hagedorn States (Beitel), Relativistic distribution function... (Grossi)," already shows the wealth and divergence among theoretical approaches. While some say that "the fireball never thermalized, but was born in a thermal state", such a statement also requires explicit, quantitative calculations[2, 3, 4, 5].

Since the hydrodynamical approach assumes local thermal equilibrium, these two theoretical tools should be treated together. We summarize the very essence of the relativistic hydro- and thermodynamics as follows.

The formulas are based on the Local Conservation of Noether Currents (LCNC),

$$\partial_{\mu} J^{\mu a} = 0. \quad (4)$$

Here $J^{\mu a}$ is a Lorentz four-vector indexed by μ ranging over all spacetime directions, while a stands generally for any further collection of indices related to the independent generators of infinitesimal symmetry transformations. More specifically the essential LCNC-s used in our theories are related to conserved charges, like baryon (B), strangeness (S) or electric charge (Q); to spacetime shifts (ν) and rotations and Lorentz-boosts (hyperbolic rotations with $\nu\rho$ antisymmetric index pairs):

$$\begin{aligned}\partial_\mu J^{\mu(B,S,Q)} &= 0, \\ \partial_\mu T^{\mu\nu} &= 0, \\ \partial_\mu M^{\mu\nu\rho} &= 0.\end{aligned}\tag{5}$$

In building up hydrodynamics it is essential to explore the connections between these currents. Chemistry deals with several types of charges (q_i^a):

$$J^{\mu a} = \sum_{i=+,-} q_i^a u_{(i)}^\mu\tag{6}$$

denote conductive currents, carried by the common stream, u^μ , or by several streams, $u_{(i)}^\mu$ in multicomponent fluids. There can be further directions in transport, too. Conduction or radiation¹ can be directed also transverse to the matter charge flow: This is most prominent in the energy-momentum. Its symmetric descriptive tensor can be splitted to several terms aligned and orthogonal to a fiducial flow velocity vector, u^μ :

$$T^{\mu\nu} = P^\mu u^\nu + P^\nu u^\mu + \mathfrak{T}^{\mu\nu}\tag{7}$$

with $P^\mu = eu^\mu + q^\mu$. Finally when polarisation related phenomena are considered then the angular momentum and spin density tensor has to be taken into account:

$$M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho} + S^{\mu\nu\rho}\tag{8}$$

is anti-symmetric in all its index pairs.

Expectations and hopes by using hydrodynamics in the description of quark (and sometimes hadronic) matter include

- Determination of Properties of Matter (equation of state and transport coefficients),
- Comparison with Lattice QCD,
- Determination of Initial State just after the Collision
- Key for the early dynamics...

By its nature the majority of our efforts in relativistic hydrodynamics and in the underlying kinetic theory approaches are numerical. A characteristic example of how far such efforts can go and how strong the visualising power of the numerical results can be was presented by Joannis Bouras showing the evolution of Mach cones in BAMP simulations[6, 7].

On the other hand theoretical efforts to improve the classical hydrodynamical calculations do not weaken[8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. In his review talk at this conference Takeshi Kodama presented a picture of a magnifying glass revealing that in reality the flowing matter is not smooth, but very much fluctuating. This coarse graining problem opens the hydrodynamics

¹ Conduction may be viewed as phonon radiation, its speed is also limited by that of the light.

at its ultraviolet end, at small sizes towards possible improvements. At the infrared end, because of missing infinitely large reservoirs even in the heaviest ion collision, the thermodynamical treatment has to be modified by finite size effects[18, 19].

By this point we have to clarify the connection between relativistic hydrodynamics and thermodynamics. In fact, after considering the entropy four-current, S^μ , thermodynamics follows from the above described general structure of the basic system of equations. During a generic process dissipation occurs, closed systems show a tendency of approaching an equilibrium state. The net and local production of entropy is non-negative:

$$\partial_\mu S^\mu + \lambda_a \partial_\mu J^{\mu,a} \geq 0. \quad (9)$$

Here all LCNC-s (locally conserved Noether currents) have to be taken into account, each with its own Lagrange multiplier, λ_a . Classically the energy-momentum tensor plays a prominent role, its Lagrange-multiplier, $\lambda_\nu = \beta_\nu$, describes a local, but moving thermometer. Its invariant length represents a Doppler-shifted local temperature² [20].

The thermodynamic Gibbs potential analogue is a four-vector, an integral of the above (in a standing body it becomes p/T the pressure over the temperature):

$$S^\mu + \lambda_a J^{\mu,a} = \Phi^\mu \quad (10)$$

is the first theorem of thermodynamics. From eqs.(9) and (10) follows the Gibbs-Duhem relation (valid in and out of local equilibrium):

$$\partial_\mu \Phi^\mu \geq J^{\mu,a} \partial_\mu \lambda_a \quad (11)$$

Finally linear transport coefficients, η_{ab} , are defined as positive semi-definite in order to ensure the inequality (9):

$$\partial^\mu \lambda_a = -\eta_{ab} J^{\mu,b}. \quad (12)$$

In higher order hydrodynamics the evolution (relaxation) of such coefficients is also calculated.

However, is this the truth? Do we really produce smooth, flowing and relativistic continua in high-energy experiments? Is there also a definite, sharp-valued thermodynamic temperature? Or we experience pseudo-thermalization and pseudo-flow? Some hints towards that this might be the case dates back to 1975, when it was shown that simple acceleration may produce a thermal feeling, it may lead to radiation spectra appearing as the black-body radiation [2, 3, 4, 5].

The physical reason is simple: a moving monochromatic source radiating with a single frequency, ω , is seen shifted according to the relativistic Doppler-effect:

$$\omega' = \omega \sqrt{\frac{1 - v(\tau)}{1 + v(\tau)}}. \quad (13)$$

If the source accelerates, its momentaneous velocity, $v(\tau)$ depends on the proper time along its trajectory. With a constant deceleration, $v(\tau) = v(0) - g\tau$. The spectral analysis Fourier transforms the ever changing phase by the changing Doppler factor obtaining an intensity distribution according to

$$I(\Omega) \propto \left| \int e^{i \int \omega'(\tau') d\tau'} e^{-i\Omega\tau} d\tau \right|^2. \quad (14)$$

This intensity divided by $\hbar\Omega$ delivers a photon number yield per invariant phase space element; observing only the high energy part of spectra also massive (but light) particles might show

² With the relative velocity between the local flow and the thermometer.

similar effects. Finally it is just a mathematical fact that the integral in eq.(14) is proportional to³

$$I(\Omega) \propto \frac{1}{e^{2\pi\Omega/g} - 1}. \quad (15)$$

By this $T = g/2\pi$ is the Unruh temperature in Planck units. For a velocity change in the order of 1 (lightspeed) in a time traversing about 1 fm, it is really in the order of 150 MeV. By this we are talking about extreme accelerations.

In our experiments, however, we cannot suppose monochromatic radiators or movements with forever constant acceleration. Towards obtaining semiclassical photon spectra from more realistic scenarios work is in progress[21]. As a preliminary a few characteristic photon rapidity distributions are shown in Figure 1 at different transverse momenta, k_T , of the photon. While long deceleration times lead to a flat rapidity distribution, like the Bjorken-flow scenario, pictures produced using short times rather resemble those obtained by the Landau-flow scenario. Not only a pseudo-temperature, but also a pseudo-flow occurs in these patterns.

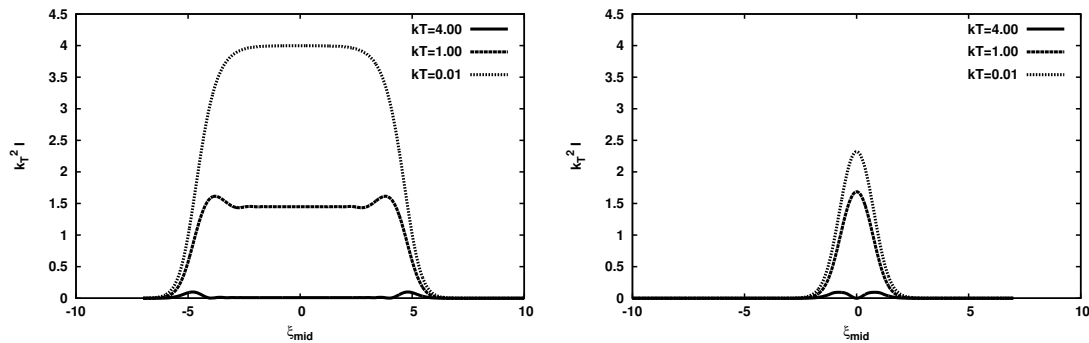


Figure 1. Numerically calculated differential photon rapidity distribution from a classical point charge moving straight with constant deceleration for a finite time. Left for long, right for short deceleration window.

Hydrodynamics is well and alive among the theories in our field embracing a list of intriguing questions to be clarified. A list of talks, like Effects of Jets in the Flow Observables (Takahashi), Thermal Equilibrium and ... Initial Condition (Kodama), Landau hydrodynamics... (Tamosiunas), Shear viscosity of hadrons... (Wiranata), Turbulence, Vorticity and Lambda Polarization (Csernai), Hydrodynamic models of particle production (Bozek), QGP viscosity and the flow ... (Song), Event-by-event correlation... (Molnar), Elliptic Flow from CGC (Scradina), Dynamical freeze-out in event-by-event hydro... (Huovinen), more than proves it.

3. Hadronization

The main question of the hadronization process is since years whether its result is phase space dominated (and in this case statistical descriptions are relevant) or elementary QCD matrix elements play the main role (and in this case only a microdynamical approach prevails). Theoretically both factors can be studied.

We had talks among others on Monte Carlo (Werner), Strongly interacting parton-hadron matter... (Bratkovskaya), Gluon radiation by heavy quarks (Gousset), Diffusion of Non-Gaussianity ... (Kitazawa), Strangeness baryon to meson ratio (Flores), on the Excluded-

³ It is derived by using a new integration variable, z , which satisfies $\omega'(\tau) = dz/d\tau$.

Volume Model (Tiwari), on Interpretation of strange hadron production at LHC (Petran), and on Chemical freezeout via HRG (Chatterjee).

Is a "thermal model" or a "statistical model" is a good approximation to the complicated quantum (chromo-) dynamics? By seeking an answer it can be helpful to factorize our search into several steps. Such a factorizing of the "thermo" concept should include i) finding of a scaling variable which unifies different mass hadron p_{\perp} -spectra, ii) testing the quark coalescence hypothesis, in case baryon and meson branches differ after the above step, iii) finding out the unique functional form on the quark (parton) level describing all hadron spectra, iv) testing trends with binary scaling, participant number, rapidity window, etc. v) and only in a final step interpreting the parameters in physical terms (as on/off-equilibrium, finite/infinite size, quantum/classical, phase space/matrix element dominated). This defactorization, shown on a simple example what we did on RHIC data[22], involves step by step the following questions:

- (i) Is it $f(p_T, m) = f(E_v(p_t, m) - \mu(m)) = f(X)$?
- (ii) Is there a quark scaling? Is it $f_h(X) = f_q^n(X/n)$ for $n = 2, 3$?
- (iii) Do we see an (X-)energy distribution? $f(X) \sim (1 + a\beta X)^{-1/a} \rightarrow e^{-\beta X}$
- (iv) Are there trends with $(N_{part}, N_{bin}, P(N), \sqrt{s}, \Delta\eta \dots)$?
- (v) How to interpret the parameters $\beta, v, a = (q - 1), \dots$?

Also thermal cosmology was introduced by Johann Rafelski. We cite his remark: "When old people make a new theory, they know all about assumptions, approximations, implied or explicit. But young people, who learn it from a textbook, believe that this were the TRUTH". We agree, scientists always should be careful.

4. Quarks off and on Lattices

We heard a number of talks about Quarkonia and Energy Loss, like Heavy Quark Energy Loss (Horowitz), ... Boltzmann vs Langevin (Das), Towards... QGP (Berrehrh), Heavy vs light flavor energy loss... (Uphoff), Jet quenching and Heavy Quarks (Renk), and about thermal behavior of quark matter, such as Free energy vs internal energy potential... (Lee), Quantum and semiclassical... (Katz), ... finite magnetic mass... (Djordjevic). One of the most interesting questions was whether AdS/CFT does well? From Thorsten Trenk's review the answer is "not quite", more precisely "clear no for both light and heavy quarks! AdS techniques predict too much suppression at LHC when tuned to RHIC and extrapolated."

So it remains to study quark matter with the more traditional lattice regularization of QCD. Comparisons between Lattice QCD results with other (non-AdS) models were presented in talks about high-density quark matter (Torrieri), Sphalerons (Chao), flavor hierarchy (Bluhm), the PNJL model (Yamazaki) or the original Nambu-Jona-Lasinio model for SU(3)_f (Marty). Full blood lattice results were reported in talks about First Principles Calculation (Allton), Role of fluctuations in detecting the QCD phase transition (Redlich), and "ab initio Lattice QCD calculations" (Schmidt), while alternative approaches as a talk on Holographic descriptions of dense quark matter (Kumar) or on chiral fluid dynamics (Herald) tried to keep balance.

Most of these approaches were presented in a high-temperature context. Therefore it is the right place to demonstrate that contrary to a wide-spread false belief, high-T *does not mean* the unrestricted applicability of perturbative QCD (pQCD). We may consider the following simple example. Since pQCD relies on high Q^2 physics, we consider the thermal distribution of Q^2 in an ideal Boltzmann gas of massless partons:

$$P(Q^2) = \frac{\iiint dE_1 dE_2 d\cos\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)} \delta(Q^2 - 2E_1 E_2 (1 - \cos\theta))}{\iiint dE_1 dE_2 d\cos\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)}}. \quad (16)$$

This integrals can be analytically solved, the result contains some Bessel K-functions:

$$P(Q^2) = \frac{1}{64T^2} \left(\frac{Q^3}{T^3} K_1 \left(\frac{Q}{T} \right) + 2 \left(\frac{Q^2}{T^2} \right) K_2 \left(\frac{Q}{T} \right) \right) \quad (17)$$

This Boltzmann-Gibbs Q^2 distribution is shown in Figure 2 (left).

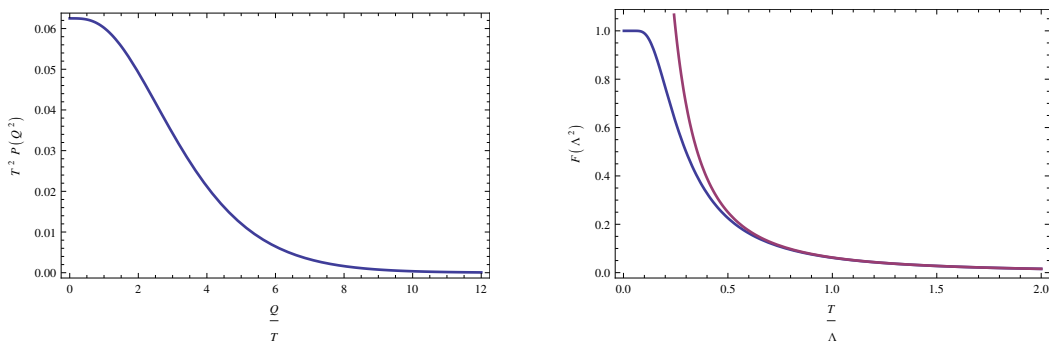


Figure 2. Thermal probability to have Q^2 four-momentum squared (left) and the integrated probability of being soft as a function of temperature (right) for a pair of massless partons.

While the width of the distribution does scale with T , the maximum probability is at $Q = 0$ at any temperature. At arbitrary high temperature non-perturbative effects prevail, only their effect may diminish in some selected (infrared safe) observables.

Regarding an order parameter of Non-Perturbativity, the thermal expectation value of "being soft" may be defined as follows:

$$F(\Lambda^2) := \langle \theta(\Lambda^2 - Q^2) \rangle = \int_0^{\Lambda^2} P(Q^2) dQ^2. \quad (18)$$

This definition selects out the integrated distribution function, pictured in Figure 2 (right). The upper line belongs to the perturbative approximation yielding $\Lambda^2/16T^2$. While the full result is saturated until about $T \approx \Lambda/6$, which can be considered as $T_c \approx 167$ MeV, this approximation starts to be really good only at $T \approx \Lambda$. Hadronic effects may therefore well prevail until $T \approx 1$ GeV. Between $T_1 \approx 1/6$ GeV and $T_2 \approx 1$ GeV both pQCD and non-pQCD worlds reign.

5. Highlights

Finally let us summarize theory highlights of this meeting. In our view either new theories or new aspects, or new proposals for measurable effects, perhaps new calculations on hard old problems are worth considering as highlights, those which give or promise new insights into (collective or individual, strange or less-strange) hadronic physics.

Towards a possibly new theory align efforts to derive the EoS, the transport coefficients, heat capacities (acting on the Tsallis parameter via the formula $q=1+1/C$), the distribution of fluctuations. Expectations and hopes are related to the determination of properties of elementary matter, to comparison of phenomenology based findings with Lattice QCD calculational results, to the very determination of an initial state just after the collision, and to a deepened theoretical understanding of the birth of hadrons.

A new aspect connected with a new proposal was presented by Becattini and Csernai[23, 24] about the Λ -polarisation, enriching the hydrodynamical approach with vorticity and spin effects.

New calculations seem to bring new conclusions: different confinement temperature for the strange than for the non-strange sector – as it has been cited in Markus Bleicher’s opening talk on theory news. Certainly a new insight was expressed in the analysis of Klaus Werner about the EPOS simulation, namely that Flow is Everywhere... Another, relatively new proposal was mediated by Krzysztof Redlich: Fluctuations carry the most important message about quark matter and about the phase transition. He found the so far most beautiful order parameter, jumping from one value to another really sharply at T_c , in form of a sophisticated ratio of susceptibilities. The proposal by Giorgio Torrieri to seek for quarkyonic signals in the v_2 coefficient against p_T plot while keeping the fluctuations was also a novelty.

Finally other topics, like the astronuclear approach in Alford’s review taught us that some theories do not survive observations. This clears ground for optimism.

In summary, quoting a commercial text discovered on a bottle of cider, just replacing the word "apple" by theory, and the word "ice" by (experimental) "data" and "Bulmers original cider" by "Our original quark matter", comes as follows: *We use big theories, small theories, juicy theories and bittersweet theories to make the well-balanced and medium-sweet flavor of our original quark matter that you know and love. Enjoy poured over data for ultimate refreshment.*

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