Inverse Magnetic Catalysis Induced by Sphalerons

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Outline

- Magnetic Catalysis
- Motivation
- Our Idea
- Sphaleron Rate
- Outlooks

Magnetic Fields

- $1\ \mathrm{MeV}^2 = 1.44 \times 10^{13}$ Gauss, $m_\pi^2 \sim 2.8 \times 10^{17}$ Gauss
- In heavy ion collisions: 10^{18} to 10^{19} Gauss ($\Delta t \sim 10^{-24} s$)
- Compact stars: 10^{10} to 10^{15} Gauss
- Early Universe: up to 10^{24} Gauss

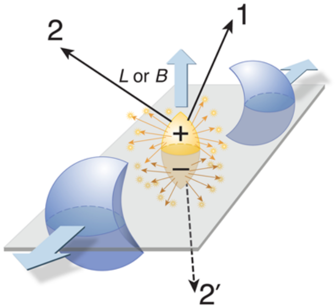


Illustration by Carin Cain

Landau Levels

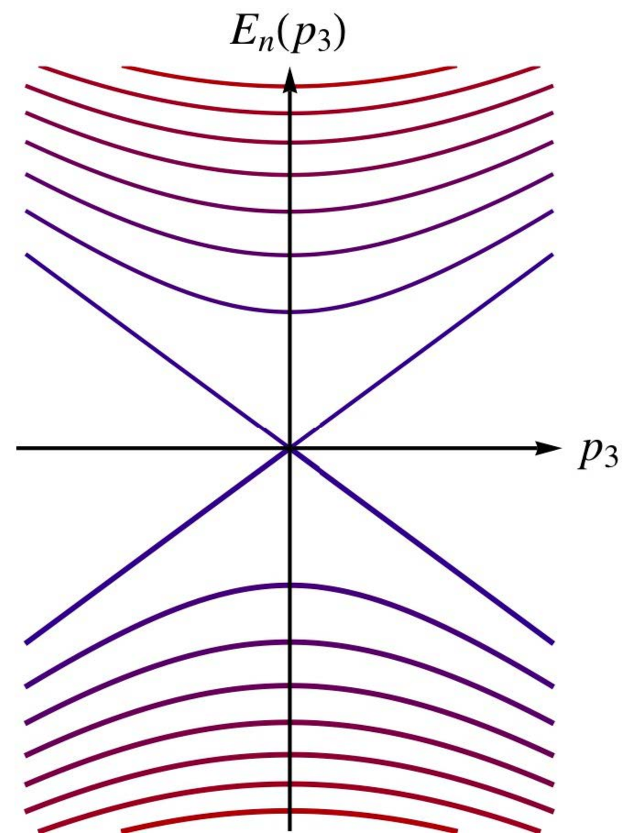
- Fermions in magnetic field

$$\mathcal{L} = \bar{\Psi} i \gamma^{\mu} \mathcal{D}_{\mu} \Psi + \text{intereactions}$$

- Free energy spectrum

$$E_n^{3+1}(p_3) = \pm\sqrt{(2n - 2s_3 + 1)|eB| + p_3^2}$$

where $s_3 = \pm\frac{1}{2}$ and $n = 0, 1, 2, ...(\text{orbital})$

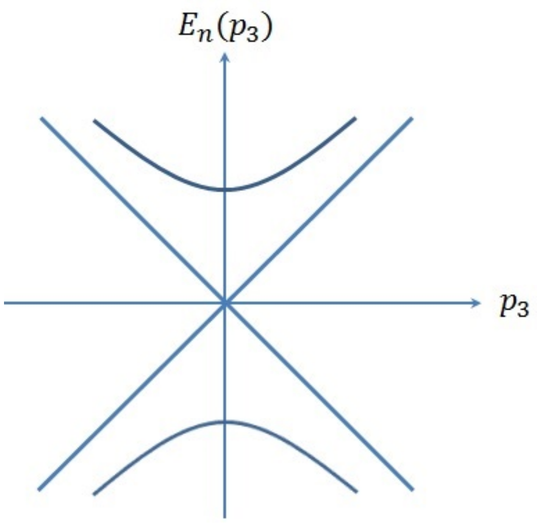


Dimensional Reduction

- Low-energy is due to $n = 0$ Landau level
- Free quark in 3-direction while $B \to \infty$

$$E_0^{3+1}(p_3) = \pm|p_3|$$

where $n = 0$ and $s_3 = -\frac{1}{2}$



- This is $1 + 1$ dimension spectrum
- Propagator looks $1 + 1$ dimension as well

$$S(p_{||}) \approx i \exp\left(-\frac{p_\perp^2}{|eB|}\right) \frac{p_0\gamma_0 - p_3\gamma_3 + m}{p_0^2 - p_3^2 - m^2}(1 - i\gamma^1\gamma^2 \mathrm{sgn}(eB))$$

Essence of Catalysis

- Dimensional reduction

- Schrödinger equation in $1D$ and $2D$ always has a bound state

- Dyson-Schwinger equation for pion(s) has a solution

- Symmetry is broken

- Dynamically generated mass of gluon is: (c.f. Gusynin, Miransky and Shovkovy '94)

$$m_{\rm dyn} \propto g\sqrt{|eB|}$$

MC in QCD

- Toy model

$$\mathcal{L} = \bar{\Psi} i\gamma^\mu \mathcal{D}_\mu \Psi + \frac{G}{2}\left[\left(\bar{\Psi}\Psi\right)^2 + \left(\bar{\Psi} i\gamma^5 \Psi\right)^2\right]$$

- Gap equation (c.f. Miransky and Shovkovy '02)

$$\langle\overline{\Psi}\Psi\rangle = G\,\mathrm{Tr}\left[S(x,x)\right]$$

- Perturbative solution at $G \to 0$

$$\langle\overline{\Psi}\Psi\rangle \approx G\frac{|eB|}{2\pi}$$

Overview of Chiral Symmetry Restoration

NJL and/or PNJL:

- Klevansky and Lemmer ('89)
- Gusynin, Miransky and Shovkovy ('95)
- Menezes et al ('09)
- Fukushima, Ruggieri and Gatto ('10)
- Andersen and Khan ('11)

χPT:

- Shushpanov and Smilga ('97)
- Cohen, McGady and Werbos ('07)
- Agasian and Fedorov ('08)

Large-N QCD:

- Miransky and Shovkovy ('02)

Quark Model:

- Kabat, Lee and Weinberg ('02)

Holographic:

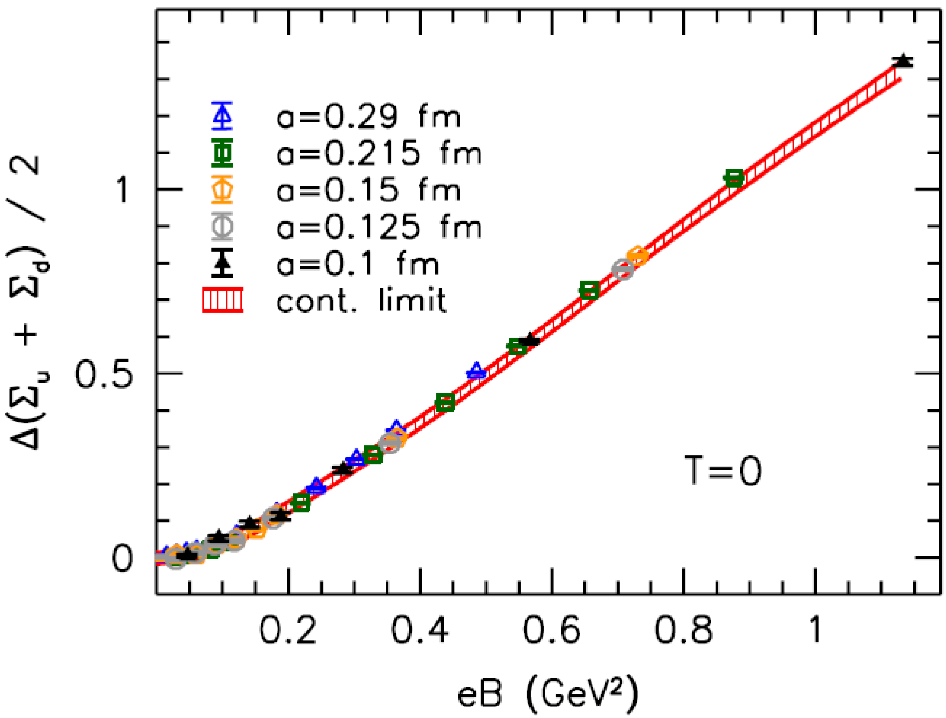
- Preis, Rebhan and Schmitt ('10)
- Callebaut, Dudal and Verschelde ('11)

Lattice:

- Buividovich et al ('08)
- D'Elia, Mukherjee and Sanfilippo ('10)

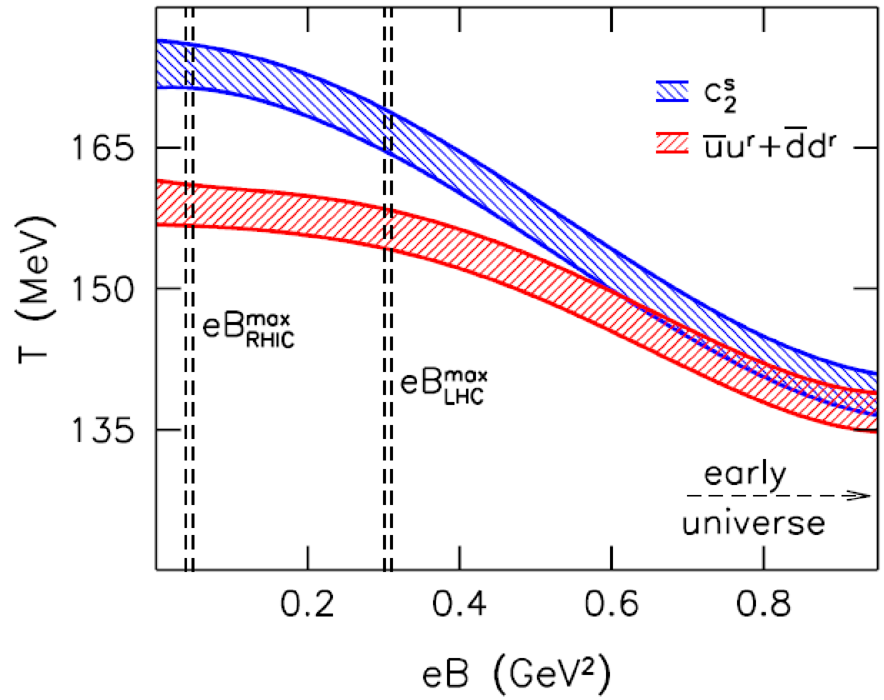
Magnetic Catalysis at Zero Temperature

Linear dependence for large B: (c.f. Gusynin, Miransky and Shovkovy '95)



Motivation

Chiral phase critical temperature investigated as a function of magnetic field: (Bali et al., JHEP 1202, 044, '12)



Approaches to the Problem

- Kenji Fukushima and Yoshimasa Hidaka ('12):

  Not only charged quarks but also neutral mesons are subject to the dimensional reduction.

- Toru Kojo and Nan Su ('12):

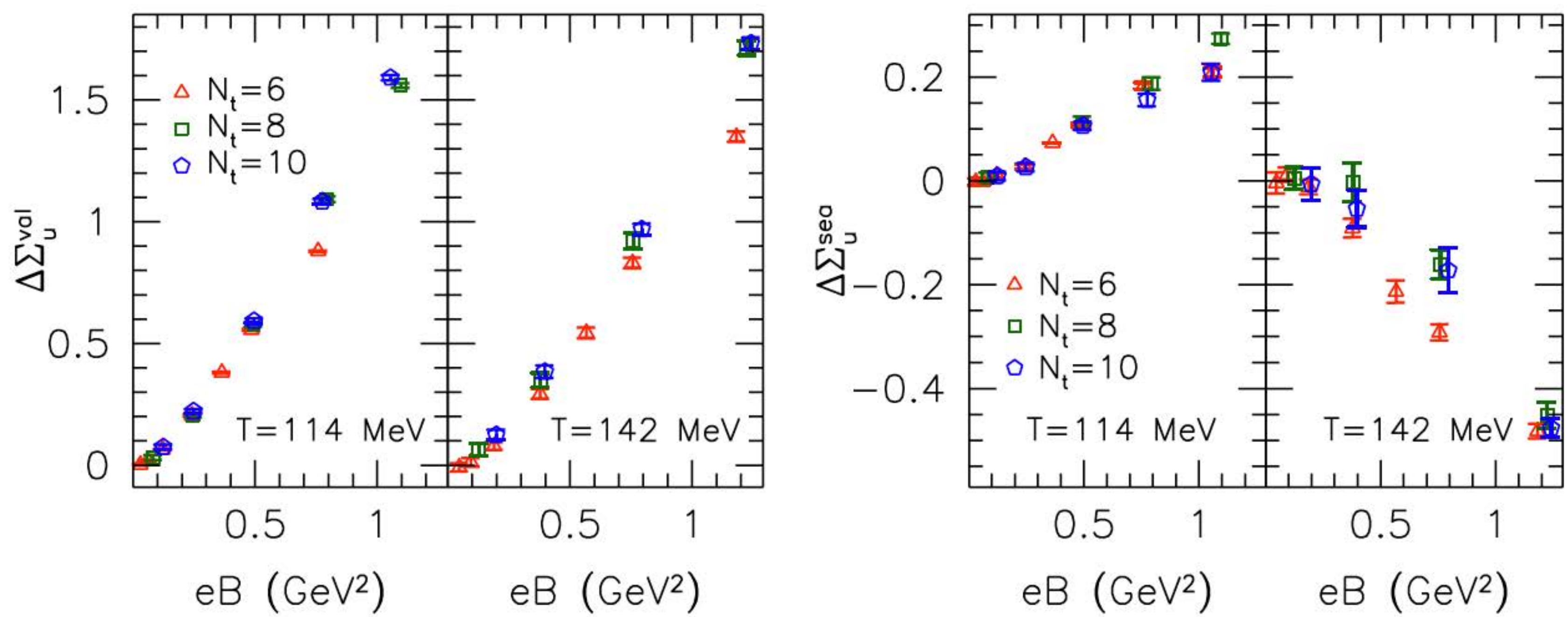
  The quark mass gap behaves as Λ_{QCD}, instead of $\sqrt{|eB|}$.

- Maciej A. Nowak, Mariusz Sadzikowski and Ismail Zahed ('13):

  A small shift in the trivial Polyakov holonomy is magnified by the chiral transition and may account for the anti-catalysis.

Valence vs. Sea Quarks

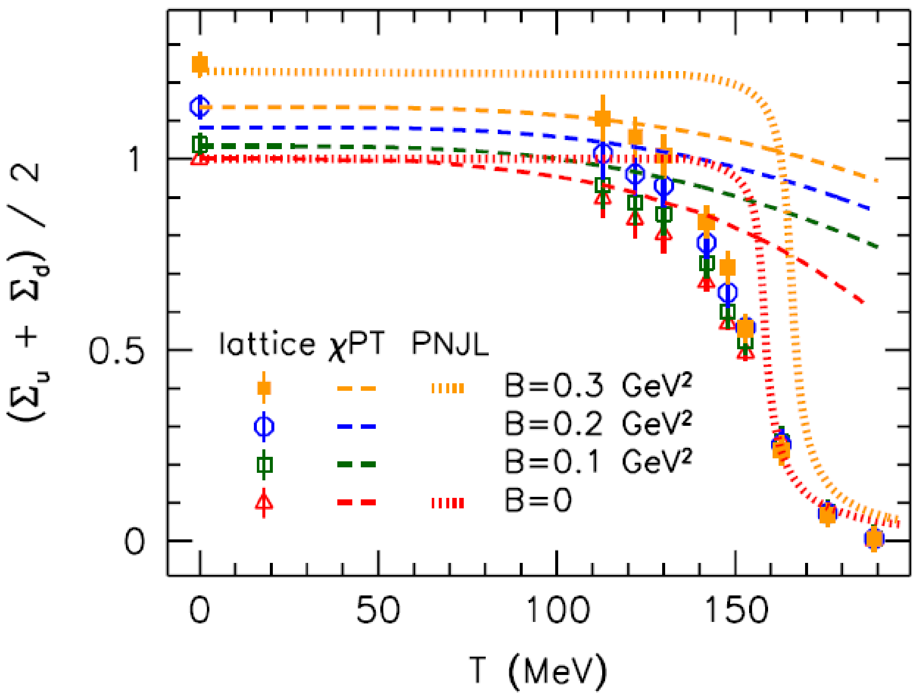
The interaction with sea quarks, which acted by ordering the Polyakov loop, overwhelms the valence enhancement.



(c.f. Bruckmann, Endrodi and Kovacs '13)

Important Clue

χPT (Andersen '12) fit up to $T = 100$ MeV:



Thermal Effect ?

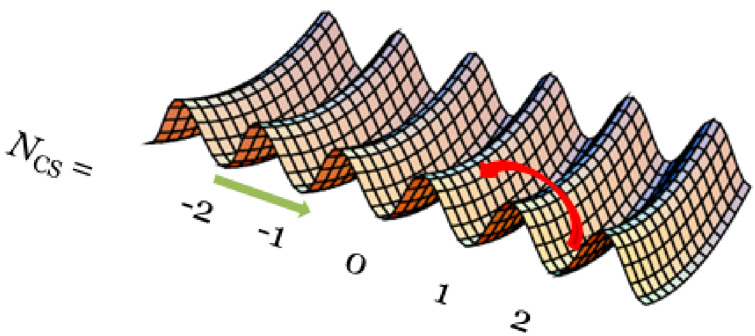
Enhanced by $\mathbf{B}$!

θ Vacuum

By a *large* gauge transformation, the gauge-non-equivalent QCD vacua are connected and therefore become a theta vacuum:

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle$$

The instanton describes the tunneling under the barrier.
The sphaleron corresponds to the falling from the top, generated by thermal fluctuations.
Transitions between vacua require gauge fields with nonzero energy.



Sphalerons Rate and Chiral Quark Density

Index theorem: $\Delta\rho_5 = 2n_f \Delta N_{CS}$

Fluctuation-Dissipation: Sphaleron transition generates ρ_5. Associated free energy F pushes N_{CS} back. (c.f. Khlebnikov and Shaposhnikov '88)

$$\frac{\partial \rho_5}{\partial t} = (4n_f)^2 \frac{\Gamma_{ss}}{T} \frac{\partial F}{\partial \rho_5}$$

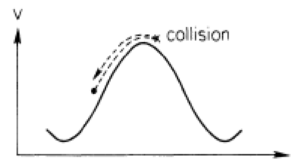


FIG. 4. The system passes over the barrier, collides with the thermal bath, and is knocked back, producing no net transition.

(c.f. Arnold and McLerran '88)

Diffusion rate of topological number: (on thermal fluctuation timescale)

$$\Gamma_{ss} = \frac{(\Delta N_{CS})^2}{Vt} = \int dt \int d^3x \left\langle \frac{g^2}{32\pi^2} F^a_{\mu\nu} \widetilde{F}^a_{\mu\nu}(x) F^a_{\mu\nu} \widetilde{F}^a_{\mu\nu}(0) \right\rangle$$

Local imbalance of chirality: (c.f. Fukushima, Kharzeev and Warringa '08)

$$\rho_5 = \rho_R - \rho_L = \frac{\mu_5^3}{3\pi^2} + \frac{\mu_5 T^2}{3}$$

Note: $\langle \rho_5 \rangle = 0$ but $\langle \rho_5^2 \rangle \neq 0$.

Sphaleron In a Magnetic Field

Energy barrier: $E_B = E_0 - E_{\rm dipole}$

$$E_0 = \frac{4\pi v}{g}\int_0^\infty d\xi \xi^2 \left[\frac{4}{\xi^2}f'^2 + \frac{8}{\xi^4}f^2(1-f)^2 + \frac{1}{2}h'^2 + \frac{1}{\xi^2}h^2(1-f)^2 + \frac{\lambda}{4g^2}(1-h^2)^2\right]$$

Negative dipole moment interaction:

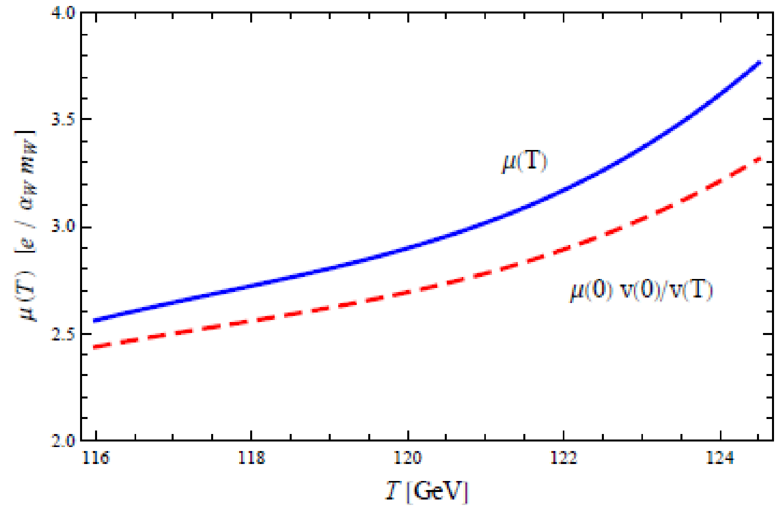
$$E_{\rm dipole} = -\int d^3x\ \mathbf{B}\cdot\mu_T$$

Sphaleron magnetic dipole moment:

$$\mu_T = \frac{2\pi}{3}\frac{g'}{g^3 v}\widehat{\mathbf{B}}\int_0^\infty d\xi\xi^2 h^2(1-f)$$

where $\xi = gvr$ and the boundary conditions hold as:

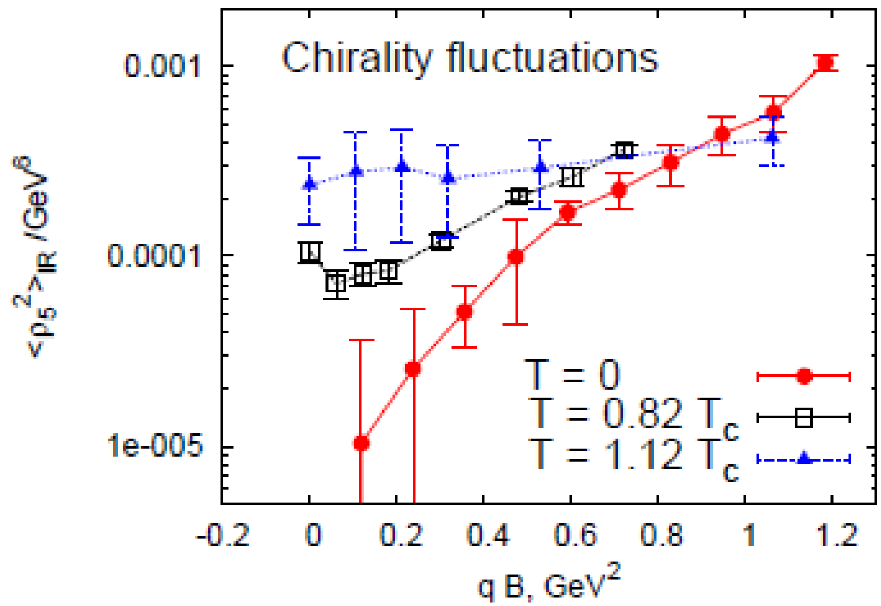
$f(0) = h(0) = 0, \quad f(\infty) = h(\infty) = 1$.



c.f. $SU(2)\times U(1)$ case. (Simone, Nardini, Quiros and Riotto '11)

Evidences from Lattice

Finite expectation value of the chirality square plots as a function of magnetic fields working in $SU(2)$ lattice gauge theory.



(Buividovich, Chernodub, Luschevskaya and Polikarpov '09)

Nonzero Chiral Chemical Potential

Weak coupling: $\Gamma_{ss} = \kappa g^{10} \log(1/g) T^4$ (Bödeker '98)

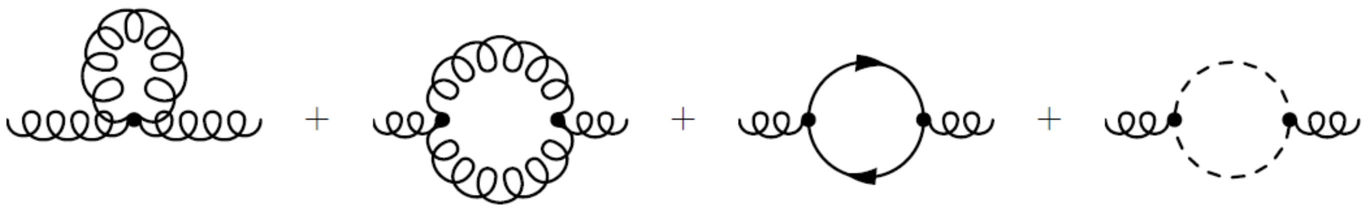
Strong coupling: $\Gamma_{ss} = \frac{(g^2 N_c)^2}{256\pi^3} T^4$ (Son, Starinets '02)

Strong magnetic field: $\Gamma_{ss} = \frac{(g^2 N_c)^2}{384\sqrt{3}\pi^5} eBT^2$ (Basar, Kharzeev '12)

Landau level density ⇨ eB; $1+1$ dimension ⇨ T^2.

$$\text{Local Chirality Fluctuations :} \quad \mu_5 \propto (eB)^{1/2}$$

Naïve Hard-Thermal-Loops Estimation



$$\Delta N_{\rm CS} \sim g^2 \int d^4x\, \mathbf{E}\cdot\mathbf{B} \sim g^2 R^2 (\Delta\mathbf{A})^2 \sim 1 \qquad \Rightarrow\ R \sim (gA)^{-1};$$

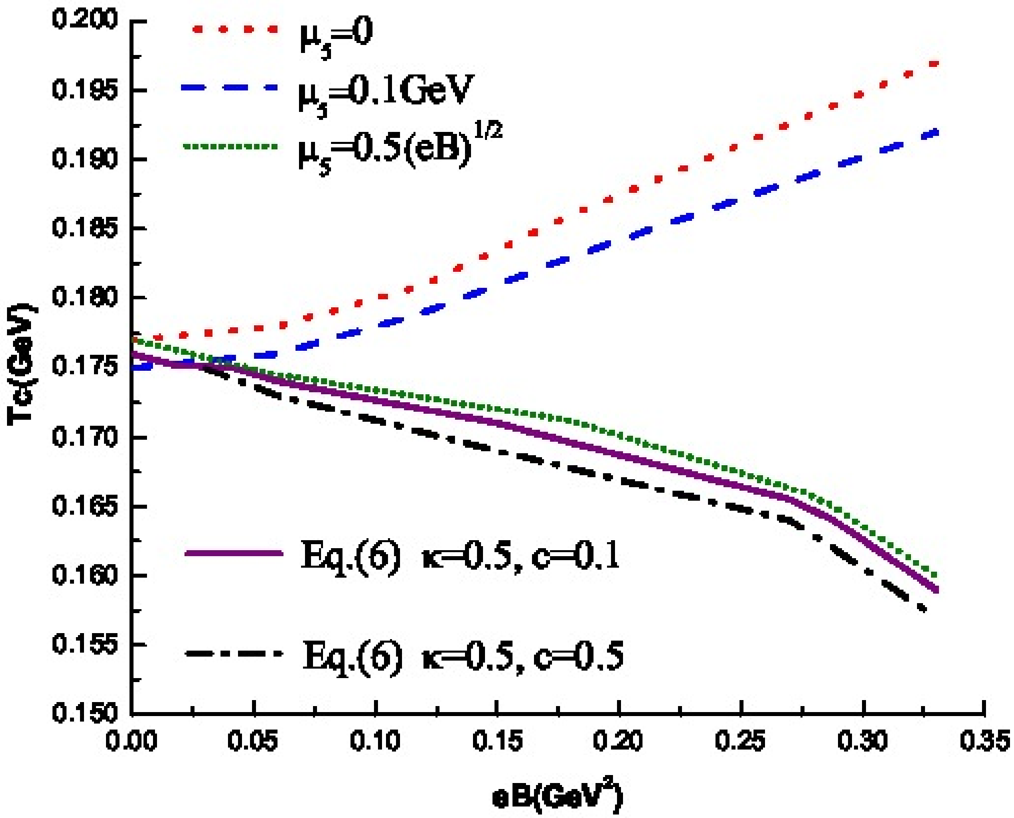
$$\Delta\mathbf{A} \sim gT \qquad \Rightarrow\ \text{time and space scale } t,\ R \sim (g^2T)^{-1}.$$

$$\Gamma_{ss} \sim\ t^{-1}R^{-3} \sim g^8T^4$$ (Arnold and McLerran '88)

The first two diagrams give unchanged $\Gamma_{ss} \sim g^8T^4$, while the third one contributes as: $\Delta A_{(1+1)} \sim g\sqrt{eB}$ and $\Delta A_\perp \sim gT$.

☞ $$\Gamma_{ss}(B,T) \sim\ t^{-1}R_{\shortparallel}^{-1}R_\perp^{-2} \sim g^8 eBT^2$$

Numerical Results



c.f. Eq.(6) of μ_5

$$\mu_5 \simeq \begin{cases} \kappa_1(g)(T + c_1 e^2 B^2/T^3) \\ \text{for} \;\; \sqrt{eB} \lesssim T; \\ \kappa_2(g)(\sqrt{eB} + c_2 T^2/\sqrt{eB}) \\ \text{for} \;\; \sqrt{eB} \gtrsim T, \end{cases}$$

Take home message:
competition between sphaleron and magnetic catalysis.

Conclusions

- Magnetic fields open a new window in the study of the phase diagram of QCD matter.
- As an intrinsically Minkowski quantity, the sphaleron rate is not able to evaluated with Euclidean methods.
- How to calculate sphaleron rate in analytically ?
- Other mechanisms ?
- What is the connection with deconfinement transition ?

Thank You for Your Attention !