Systematic Properties of the Tsallis Distribution: Energy Dependence of Parameters

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Work done in collaboration with:

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Outline

Transverse Momentum Distribution

STAR collaboration, B.I. Abelev at al.
arXiv: nucl-ex/0607033; Phys. Rev. **C75**, 064901 (2007)
PHENIX collaboration, A. Adare et al.
arXiv: 1102.0753 [nucl-ex]; Phys. Rev. **C83**, 064903 (2011)
ALICE collaboration, K. Aamodt et al.
arXiv: 1101.4110 [hep-ex]; Eur. Phys. J. **C71**, 1655 (2011)
CMS collaboration, V. Khachatryan et al.
arXiv: 1102.4282 [hep-ex]; JHEP **05**, 064 (2011)
ATLAS collaboration, G. Aad et al.
arXiv: 1012.5104 [hep-ex]; New J. Phys. **13** (2011) 053033.

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = p_\mathrm{T} \times \frac{\mathrm{d}N}{\mathrm{d}y} \frac{(n-1)(n-2)}{nT(nT + m_0(n-2))} \left(1 + \frac{m_\mathrm{T} - m_0}{nT}\right)^{-n}$$

What is the connection with the Tsallis distribution?
Also, the physical significance of the parameters n and T has never been discussed by STAR, PHENIX, ALICE, ATLAS, CMS.

In the grand canonical ensemble the particle number, energy density and pressure are given by

$$
\begin{aligned}
N &= gV \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E-\mu}{T}\right), \\
\epsilon &= g \int \frac{d^3p}{(2\pi)^3} E \exp\left(-\frac{E-\mu}{T}\right), \\
P &= g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \exp\left(-\frac{E-\mu}{T}\right),
\end{aligned}
$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor.

In particular, the particle number is:

$$E\frac{d^3N}{d^3p} = \frac{gVE}{(2\pi)^3}\, e^{-\frac{E-\mu}{T}},$$
$$\frac{d^2N}{m_T dm_T dy} = \frac{gVm_T\cosh y}{(2\pi)^2}\, e^{-\frac{m_T\cosh y-\mu}{T}},$$

at mid-rapidity, $y = 0$ and zero chemical potential this becomes

$$\left.\frac{d^2N}{m_T dm_T dy}\right|_{y=0} = \frac{gVm_T}{(2\pi)^2}\, e^{-\frac{m_T}{T}}$$

m_T scaling! Hopeless!

Tsallis Distribution

Possible generalization of Boltzmann-Gibbs statistics

Constantino Tsallis
Rio de Janeiro, TBPF
J. Stat. Phys. 52 (1988) 479-487

Citations: 1 389
Citations in HEP: 513

Entropy: Tsallis vs Boltzmann

The Boltzmann entropy is given by

$$S^B = -g \sum_i \left[f_i \ln f_i - f_i \right) \right] , \tag{1}$$

The Tsallis entropy is given by

$$S^B_T = -g \sum_i \left[f_i^q \ln_q f_i - f_i \right) \right] , \tag{2}$$

which uses

$$\ln_q(x) \equiv \frac{x^{1-q} - 1}{1-q} , \tag{3}$$

often referred to as q-logarithm.
By maximizing the entropy one obtains expressions for particle density, energy density and pressure.

For high energy physics a consistent form of Tsallis statistics for the particle number, energy density and pressure is given by

$$
\begin{aligned}
N &= gV\int \frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,\\
\epsilon &= g\int \frac{d^3p}{(2\pi)^3}E\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,\\
P &= g\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .
\end{aligned}
$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. The Tsallis distribution introduces a new parameter q which for transverse momentum spectra is always close to 1.

Thermodynamic consistency

$$dE = -pdV + TdS + \mu dN$$

Inserting $E = \epsilon V$, $S = sV$ and $N = nV$ leads to

$$d\epsilon = Tds + \mu dn$$

$$dP = nd\mu + sdT$$

In particular

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied for the Tsallis distribution.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,$$

which, in terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, becomes (at mid-rapidity for μ = 0)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} ,$$

J.C. and D. Worku, J. Phys. **G39** (2012) 025006; arXiv:1203.4343[hep-ph].

Comparison of the Tsallis form with the STAR, PHENIX, ALICE, ATLAS, CMS distributions

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)},$$

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = p_\mathrm{T}\times\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left[1+\frac{m_\mathrm{T}-m_0}{nT}\right]^{-n}$$

For the comparison use the following substitution:

$$n \to \frac{q}{q-1}$$

$$nT \to \frac{T+m_0(q-1)}{q-1}$$

After this substitution one obtains

$$\frac{d^2N}{dp_T\,dy} = p_T \frac{\mathrm{d}N}{\mathrm{d}y} \frac{(n-1)(n-2)}{nT(nT+m_0(n-2))} \left[\frac{T}{T+m_0(q-1)}\right]^{-q/(q-1)} \left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .$$

To be compared with

$$\frac{\mathrm{d}^2N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .$$

Apart from several constant factors, which can be absorbed in the volume V, only a factor of m_T differs! However, m_0 shouldn't appear as it destroys m_T scaling. The inclusion of the factor m_T leads to a more consistent interpretation of the variables q and T.

Interpretation of Tsallis Parameter q

G. Wilk and Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770

$$\left[1+(q-1)\frac{E-\mu}{T}\right]^{-1/(q-1)} = \int d\left(\frac{1}{T_B}\right) f\left(\frac{1}{T_B}\right) \exp\left(-\frac{E-\mu}{T_B}\right)$$

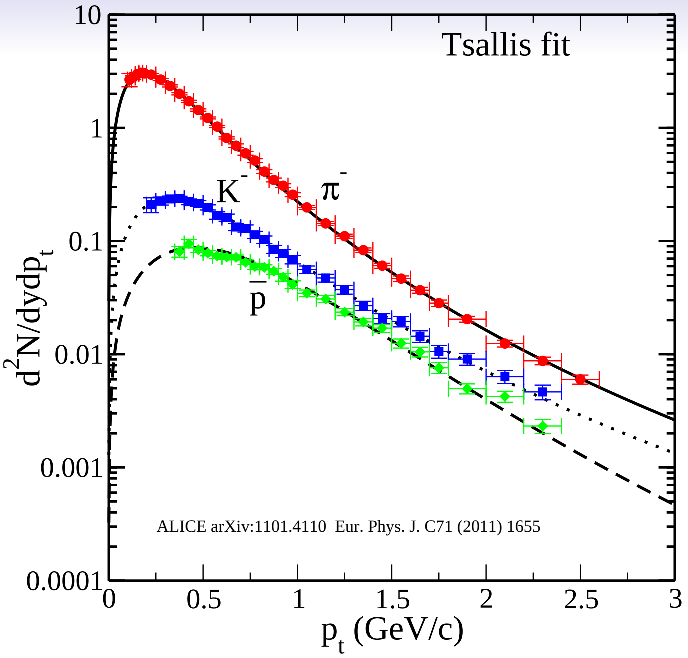
Interpretation of Tsallis Parameter q

G. Wilk and Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770

$$\left\langle \frac{1}{T_B} \right\rangle = \frac{1}{T}$$

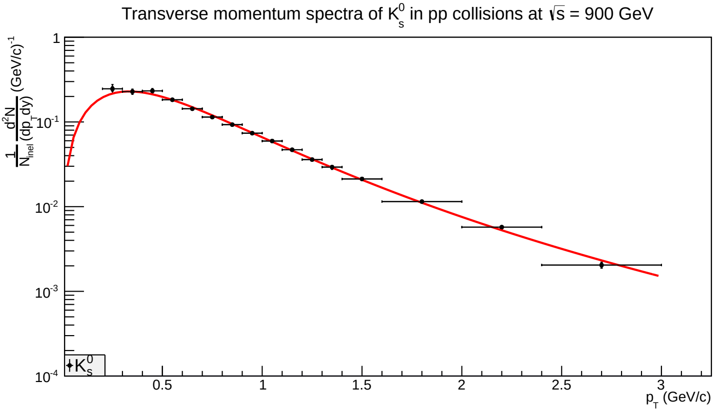
and also

$$\frac{\left\langle \left(\frac{1}{T_B}\right)^2 \right\rangle - \left\langle \frac{1}{T_B} \right\rangle^2}{\left\langle \frac{1}{T_B} \right\rangle^2} = q - 1$$

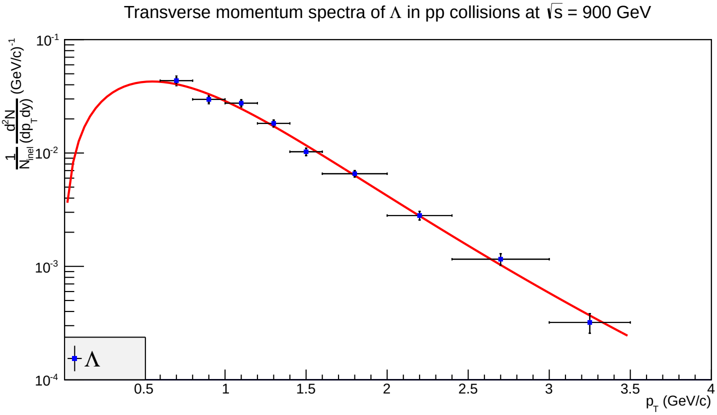


Tsallis fit
10
1
0.1
0.01
0.001
0.0001
0
0.5
1
1.5
2
2.5
3
K-
π-
p̄
d²N/dydp_t
p_t (GeV/c)
ALICE arXiv:1101.4110 Eur. Phys. J. C71 (2011) 1655

Tsallis Distribution p-p

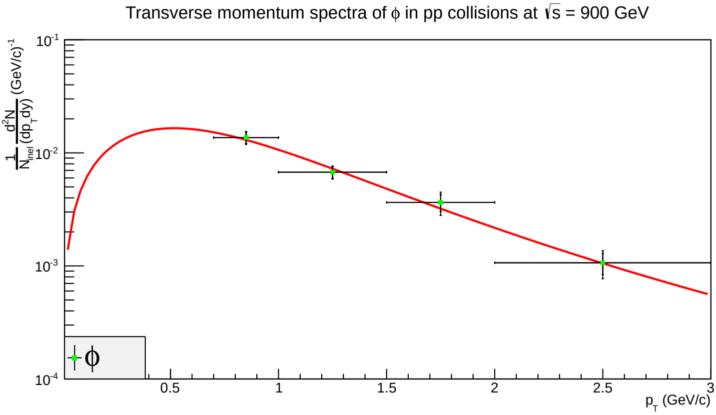


Tsallis Distribution p-p



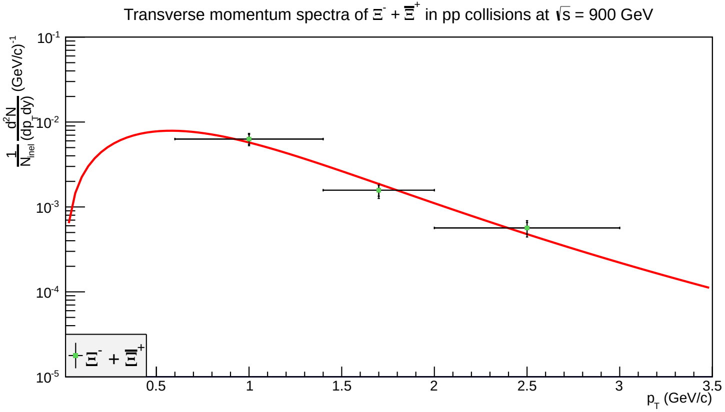
M.D. Azmi

Tsallis Distribution p-p



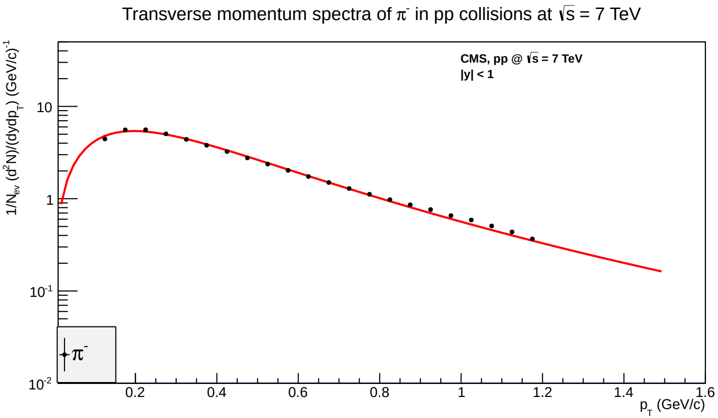
M.D. Azmi

Tsallis Distribution p-p

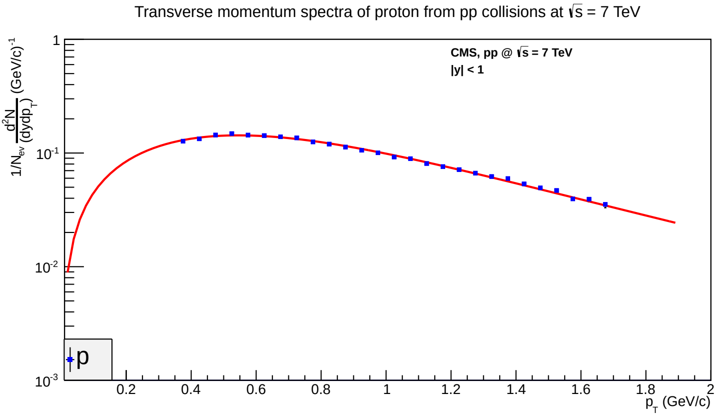


M.D. Azmi

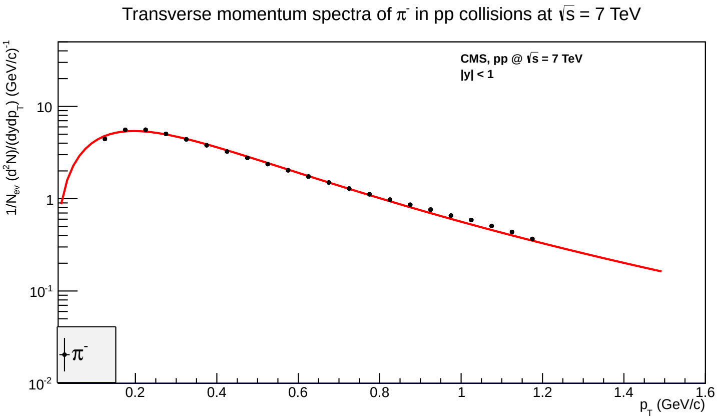
Tsallis Distribution p-p



Tsallis Distribution p-p

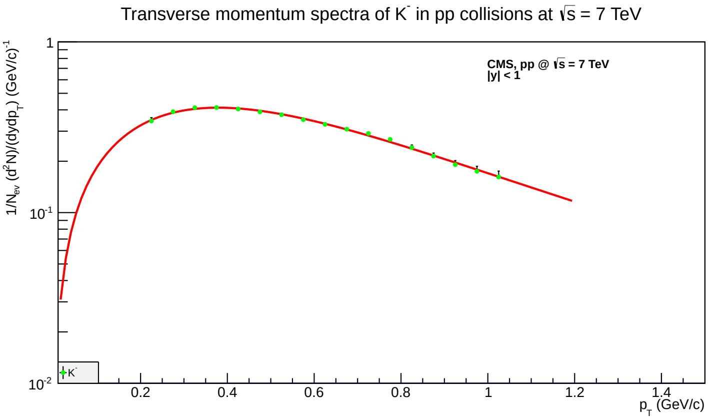


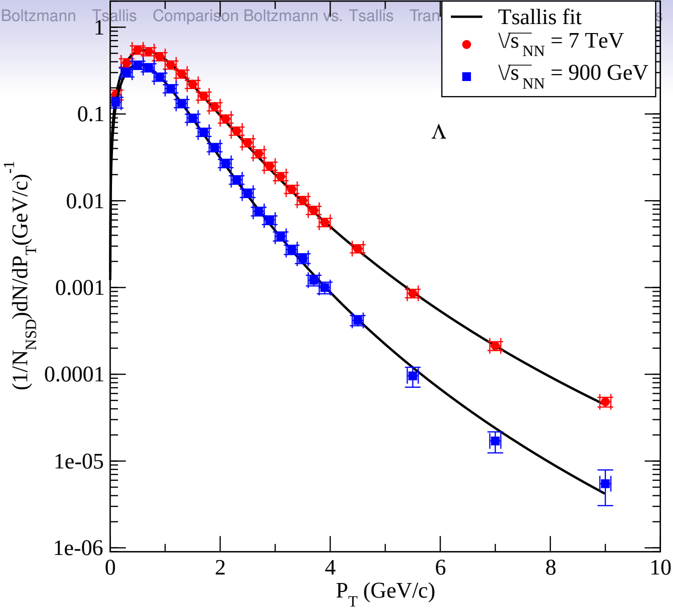
Tsallis Distribution p-p



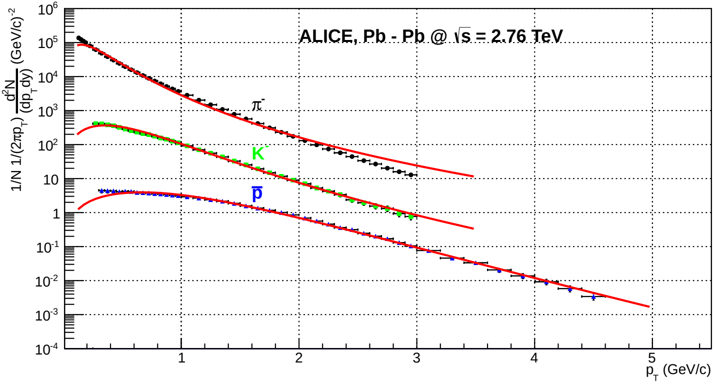
M. Danish Azmi

Tsallis Distribution p-p



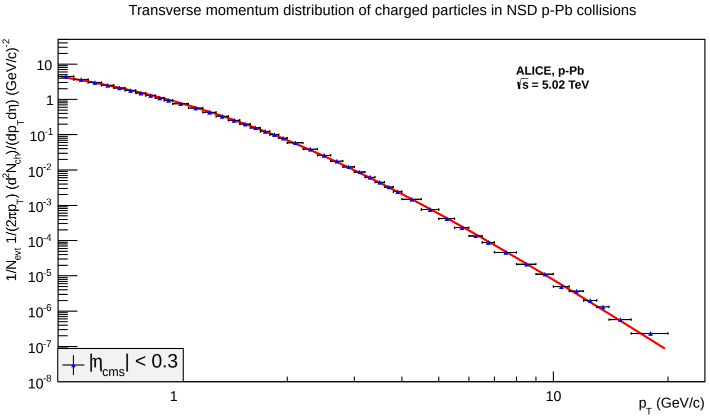

Tsallis fit
√s_NN = 7 TeV
√s_NN = 900 GeV
Λ
(1/N_NSD)dN/dP_T(GeV/c)^-1
P_T (GeV/c)
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
2
4
6
8
10

Tsallis Distribution does not describe Pb-Pb



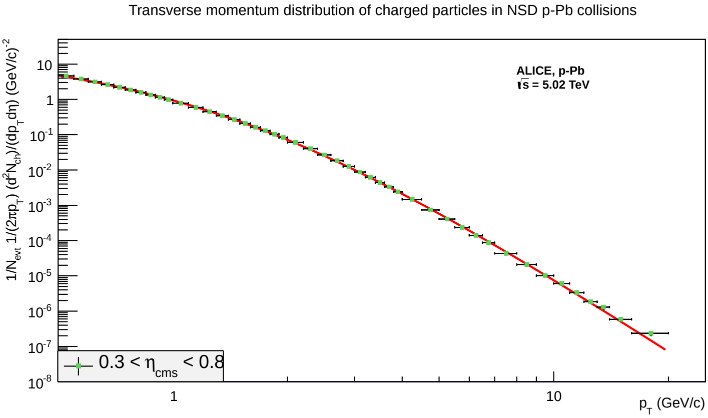
M. Danish Azmi

... but works for p-Pb



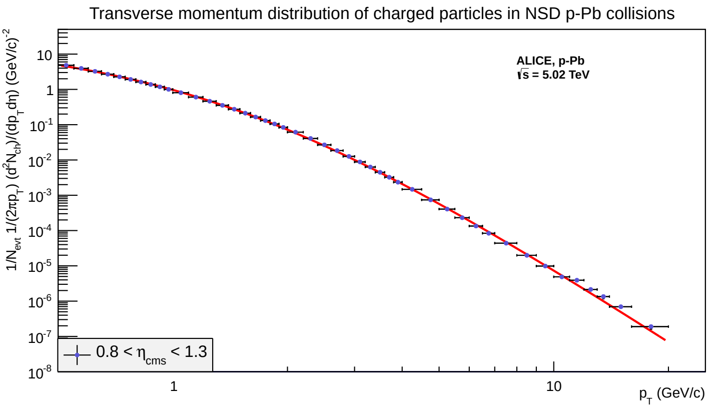
q = 1.140 M. Danish Azmi

... but works for p-Pb at all rapidities

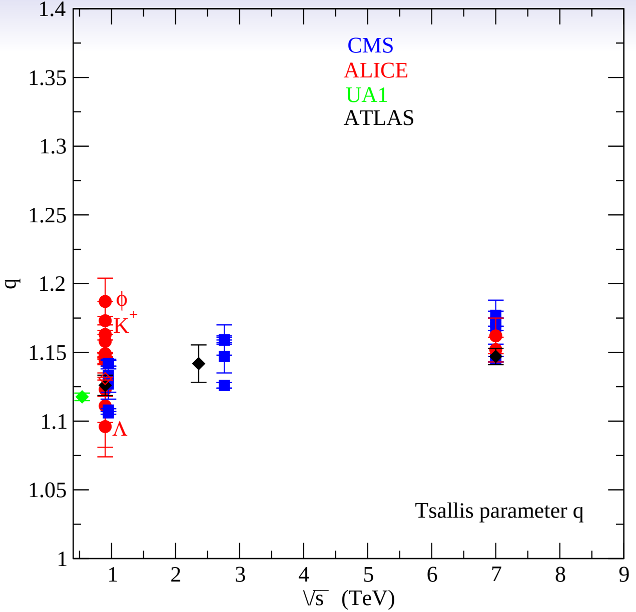


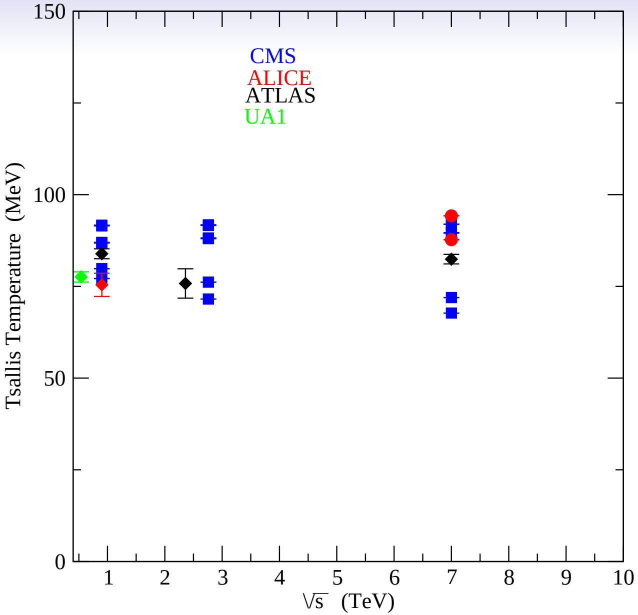
q = 1.139 M. Danish Azmi M. Danish Azmi

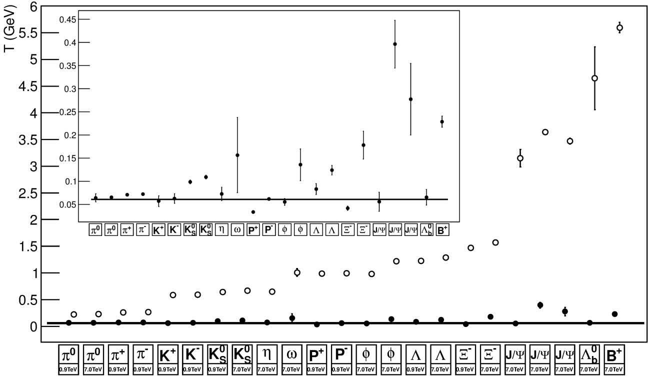
... but works for p-Pb at all rapidities.



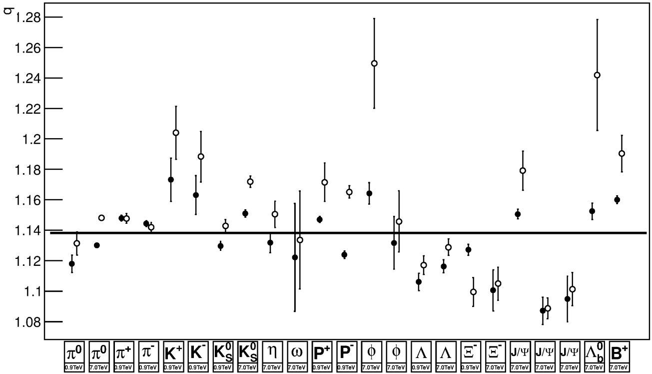
$q = 1.139$ M. Danish Azmi


CMS
ALICE
UA1
ATLAS
q
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1
2
3
4
5
6
7
8
9
√s (TeV)
ϕ
K+
Λ
Tsallis parameter q

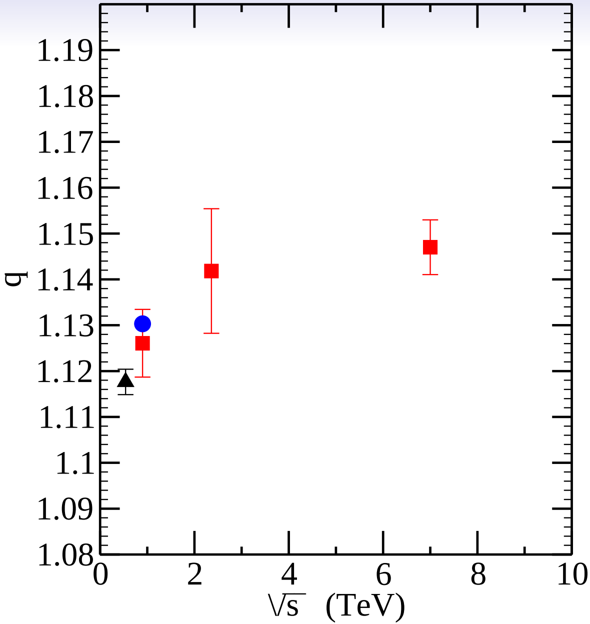

CMS
ALICE
ATLAS
UA1
Tsallis Temperature (MeV)
√s (TeV)
150
100
50
0
1
2
3
4
5
6
7
8
9
10

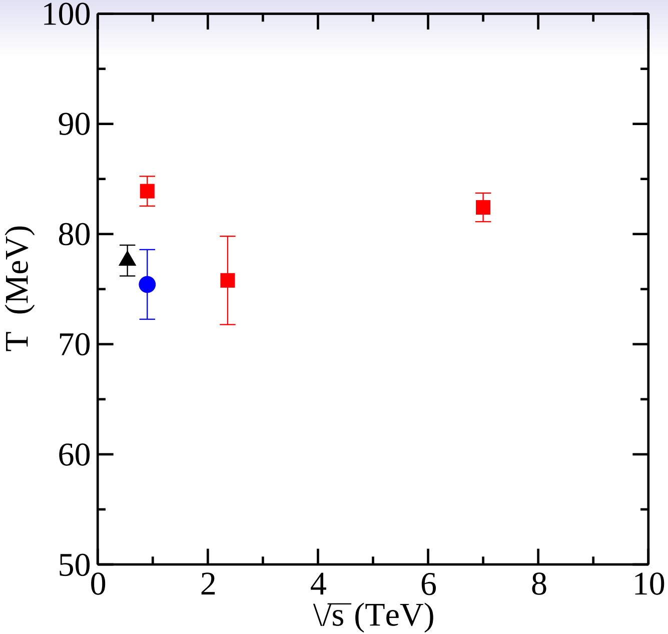


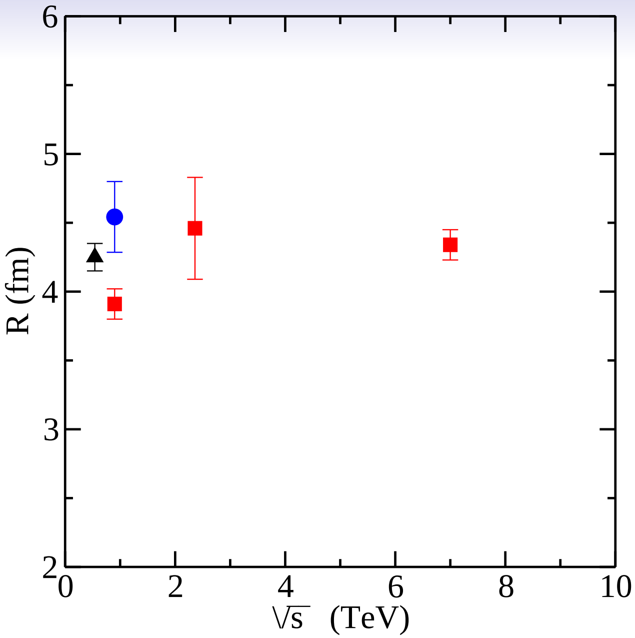
Open symbols effective temperature T obtained using ALICE distribution. Closed symbols use Tsallis distribution.
L. Marques, E. Andrade-II and A. Deppman arXiv:1210.1725 [hep-ph



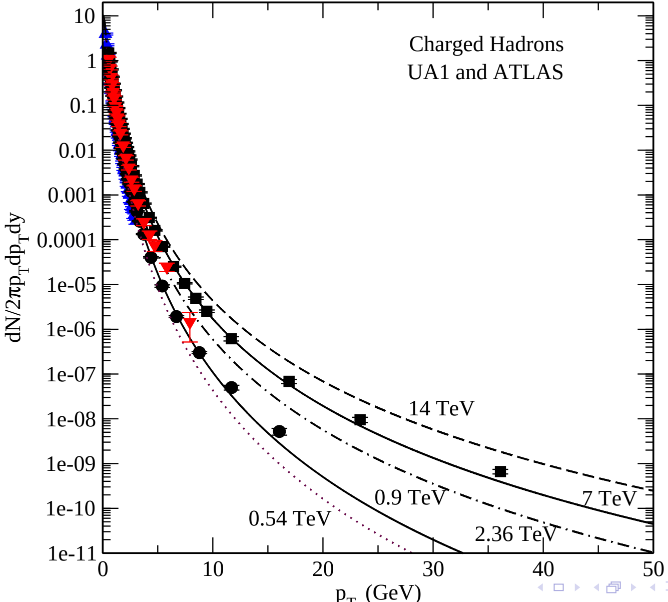
Open symbols: parameter *q* obtained from the ALIICE distribution. Closed symbols use Tsallis distribution.
L. Marques, E. Andrade-II and A. Deppman arXiv:1210.1725 [hep-ph]


q
1.19
1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
1.09
1.08
0
2
4
6
8
10
√s (TeV)


100
90
80
70
60
50
T (MeV)
0
2
4
6
8
10
√s (TeV)



R (fm)
6
5
4
3
2
0
2
4
6
8
10
√s (TeV)


Charged Hadrons
UA1 and ATLAS
14 TeV
7 TeV
0.9 TeV
2.36 TeV
0.54 TeV
dN/2πp_T dp_T dy
p_T (GeV)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
0
10
20
30
40
50

Conclusion:
Use

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}, \qquad (4)$$

instead of

$$\frac{d^2N}{dp_T dy} = p_T \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left[1+\frac{m_{\mathrm{T}}-m_0}{nT}\right]^{-n} \qquad (5)$$