Towards the Dynamic Study of On- and Off-Shell Heavy Quarks in the Quark-Gluon-Plasma

SQM 2013

Hamza Berrehrah

Collaborators : E. Bratkovskaya, W. Cassing, P.B. Gossiaux & J. Aichelin

FIAS Frankfurt Institute for Advanced Studies

EMMI

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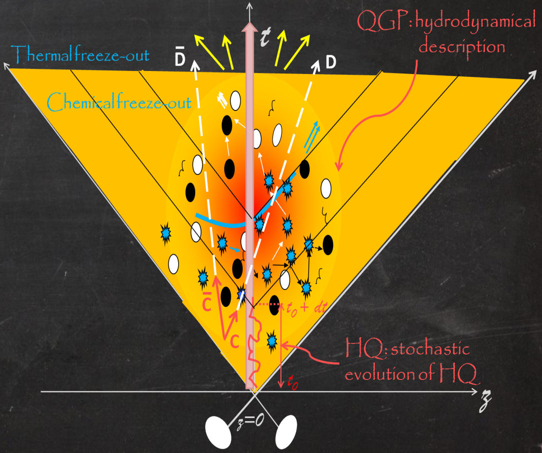
Subatech

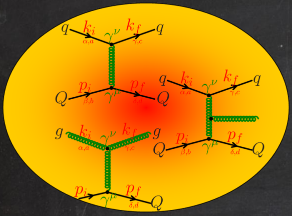
Global Project : HQ dynamics and transport properties in QGP

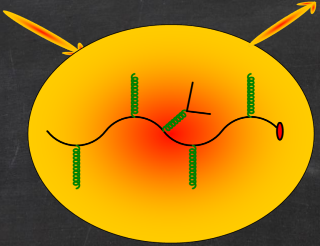
1 Heavy quarks in heavy-ion collisions

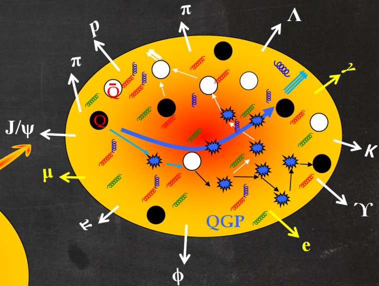
- Large HQ mass $M_{HQ} \gg \Lambda_{QCD}$
- HQ produced at early stage (hard process)
- Ideal probe for the early stage
- Don't come into equilibrium with the medium (interactions not strong enough)



Global Project : HQ dynamics and transport properties in QGP







1. Microscopic Ingredients
   - Elastic cross section : qQ, gQ
   - Inelastic cross section : qQg, gQg, . . .

2. Mesoscopic Quantities
   - Dynamical energy loss
   - Transport coefficients (Viscosity, Drag, Diffusion . . .)

3. Macroscopic Observables
   - PHSD
   - MC@sHQ
   - BAMPS
   - URQMD . . .

   (R_{AA}, v_2, . . .)

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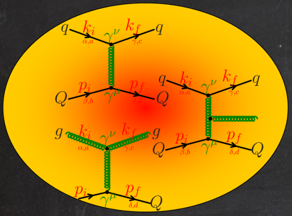

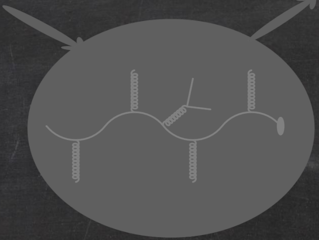

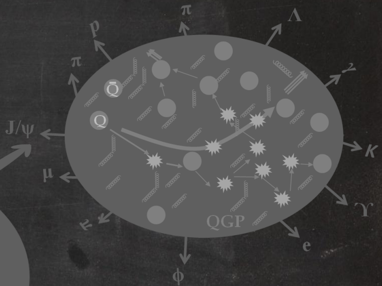

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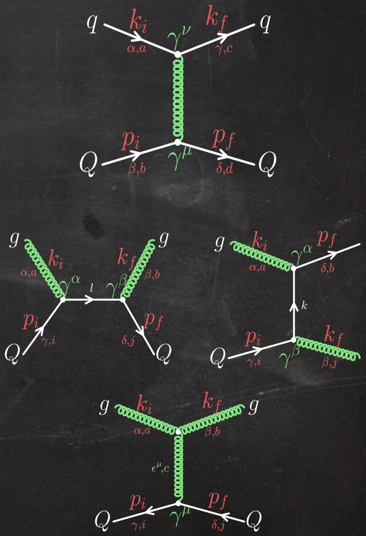
Cross Sections at finite temperature

1 Problems

- Coupling constant α_s (*qqg*, *ggg* and *gggg* vertices)
- Regularization cut-off $(t-\mu)^{-1}$
- Quark and gluon propagators at finite temperature and chemical potential
- *q,Q,g* masses at finite temperature and chemical potential
- Gauge invariance of the amplitudes



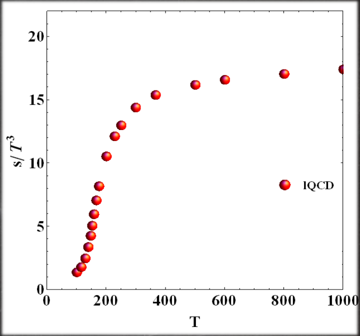
Dynamical Quasi-Particle Model (DQPM)

1 DQPM entropy density

$$s_i^{DQP} = -\int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_i}{\partial T} \left(\Im \ln(-\Delta^{-1}) + \Im \Pi \Re \Delta \right)$$

$$\Delta(p) = \int \frac{d\omega}{2\pi} \frac{A(\omega, \boldsymbol{p})}{p_0 - \omega} \leftrightarrow A(p) = \frac{4\omega\gamma}{(\omega^2 - \boldsymbol{p}^2 - M^2)^2 + 4\gamma^2\omega^2}$$

Peshier, Cassing, PRL 94 (2005) 172301
Cassing, NPA 791 (2007) 365 ; NPA 793 (2007)
lQCD : WB JHEP. 09 (2010) 073 ; 11 (2010) 077



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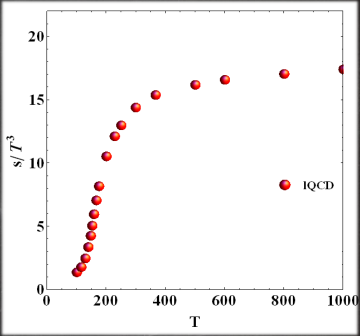

2 DQPM Masses and Widths

$$m_g^2 = \frac{g^2}{6}\left(N_c + \frac{1}{2}N_f\right) T^2 , \qquad m_q^2 = g^2 \frac{N_c^2 - 1}{8N_c} T^2$$

$$\gamma_g = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2} + 1\right) , \quad \gamma_q = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2} + 1\right)$$

3 DQPM Coupling Constant

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left(\lambda^2 (T/T_c - T_s/T_c)^2\right)}$$

4 DQPM Parameters T_c, T_s, λ, c

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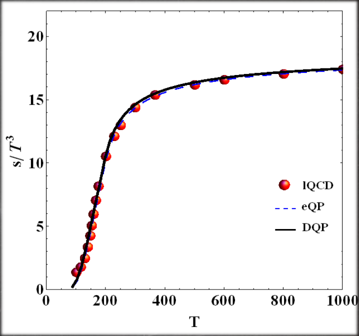

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$T_c = 0.158$ GeV, $T_s = 0.56\,T_c$
$\lambda = 2.42$, $c = 14.4$

4 DQPM Parameters $T_c,\ T_s,\ \lambda,\ c$

Dynamical Quasi-Particle Model (DQPM)

1. DQPM α_s, m and γ

DQPM gives a good description of lQCD results

Peshier, Cassing, PRL 94 (2005) 172301
Peshier, PRD 70 (2004) 034016
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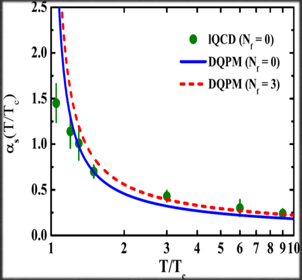

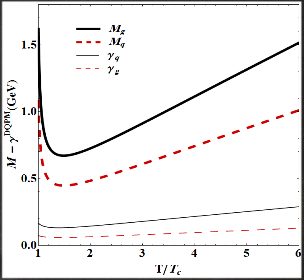

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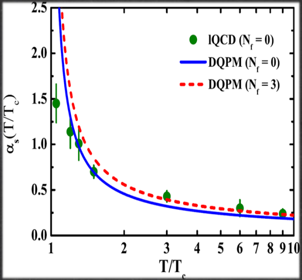
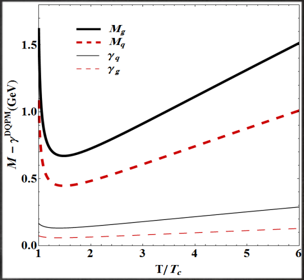
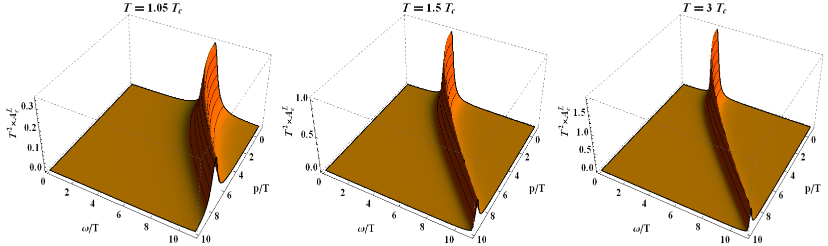
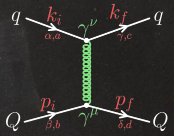




2 DQPM charm $A(\omega, p, T)$



Heavy Quark Elastic Scattering



1. qQ Elastic Scattering :

How to deal with the coupling constant & the infrared (IR) regulator

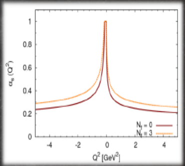
- pQCD (massless g, q)

R. Cutler, D. Sivers Phys. Rev. D 17, 196 (1978), B.L. Combridge Nucl Phys B 151, 429 (1979)

α fix/running, $\alpha(Q^2) = \frac{12\pi}{25\log(Q^2/\Lambda^2)}$, $\sigma^{qQ} = \int_{t_{min}}^{0} \frac{d\sigma}{dt} dt \rightarrow \int_{t_{min}}^{Q_0^2} \frac{d\sigma}{dt} dt$

- HTL (Hard Thermal Loop)

A. Peshier, arXiv :0801.0595 [hep-ph], P.B. Gossiaux, J. Aichelin, Phys.Rev.C78 (2008)



α fix/running, $\frac{\alpha}{t} \rightarrow \frac{\alpha_{eff(t)}}{t - \kappa \tilde{m}_D^2}$, $\tilde{m}_D^2(T) = \frac{N_c}{3}\left(1 + \frac{N_f}{6}\right) 4\pi\alpha_s(-\tilde{m}_D(T^2))T^2$

κ can be fixed to $\kappa \approx 0.2$ by comparing dE/dx to HTL result beyond logarithmic accuracy

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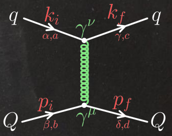

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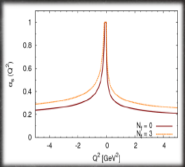

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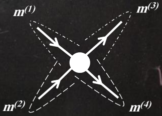
κ can be fixed to $\kappa \approx 0.2$ by comparing dE/dx to HTL result beyond logarithmic accuracy

- **DpQCD (Dressed pQCD)**

  $\alpha^{DQPM}(T)$, DQPM pole mass for g, q, Q and DQPM gluon propagator (finite mass, zero width)

- **IEHTL (Infrared Enhanced HTL)**



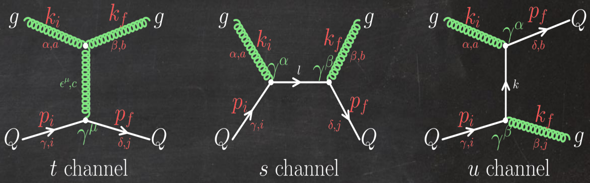
Off-shell kinematical limits, $\alpha^{DQPM}(T)$, $t^{-1} = \left[t - m_g^2 + 2i\gamma_g(p_f^0 - p_i^0)\right]^{-1}$

$$\sigma^{IEHTL}(s,T) = \int \Pi dm^{(i)} \; \sigma^{pQCD}(s, m^{(1)}, m^{(2)}, m^{(3)}, m^{(4)}) \; A_{(1)}(m^{(1)}) \; A_{(2)}(m^{(2)}) \; A_{(3)}(m^{(3)}) \; A_{(4)}(m^{(4)})$$

Heavy Quark Elastic Scattering

1. gQ Elastic Scattering

Additional Problems : 3 gluon vertex, gauge invariance



Heavy Quark Elastic Scattering

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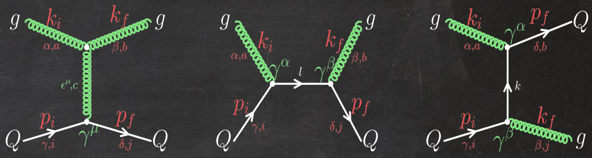


t channel s channel u channel

- **pQCD (massless g, q)**

  Transverse gauge :

  $$\sum_{spins} \varepsilon_i^{\mu} \varepsilon_i^{\star\mu'} = -g^{\mu\mu'} + \frac{2}{s}\left(p_i^{\mu} k_i^{\mu'} + p_i^{\mu'} k_i^{\mu}\right)$$

- **HTL (Hard Thermal Loop)**

  Transverse gauge

  $$G_F^{\mu\nu}(q, m_g) = -i \, \frac{g_{\mu\nu}}{q^2 - \kappa m_D^2}$$

- **DpQCD (Dressed pQCD)**

  Lorentz Covariance : $\sum_{pol,i} \varepsilon_{i,\alpha}\varepsilon_{i,\lambda} = g_{\alpha\lambda} - \frac{k_{i,\alpha}\, k_{i,\lambda}}{(m_g^i)^2}$,

  $$G_F^{\mu\nu}(q, m_g) = -i \, \frac{g_{\mu\nu} - q_\mu q_\nu / m_g^2}{q^2 - m_g^2}, \qquad S_F(p, M_Q) = \frac{\not{p} + M_Q}{p^2 - M_Q^2}$$

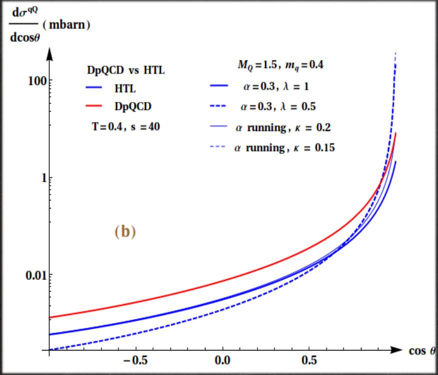
- **IEHTL (Infrared Enhanced HTL)**

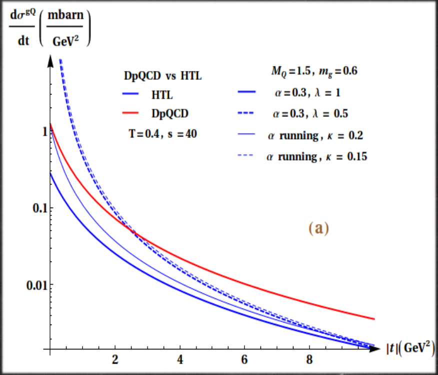
  Lorentz Covariance, $G_F^{\mu\nu}(q, m_g) = -i \, \frac{g_{\mu\nu} - q_\mu q_\nu / m_g^2}{q_0^2 - \mathbf{q}^2 - m_g^2 + i2\gamma_g q_0}$

  $$S_F(p, M_Q) = \frac{\not{p} + M_Q}{p_0^2 - \mathbf{p}^2 - M_Q^2 + i2\gamma_Q p_0}$$

Heavy Quark Elastic Scattering

1 σ_{qQ}, σ_{gQ} : HTL vs DpQCD

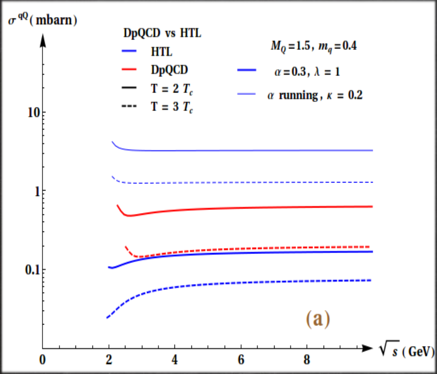


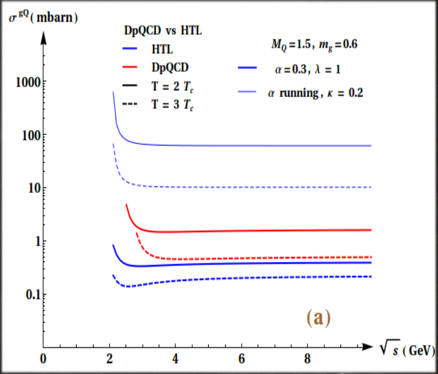


- HTL/DpQCD : Different behaviours for small/large "t"
- σ_{qQ} higher value for running + $\kappa = 0.2$ (differences related to $m_g^{excganged}$)
- σ_{qQ}/σ_{gQ} : Same conclusions with larger values for σ_{gQ} • $\sigma(T\nearrow) < \sigma(T\searrow)$

Heavy Quark Elastic Scattering

1. σ_{qQ}, σ_{gQ} : HTL vs DpQCD





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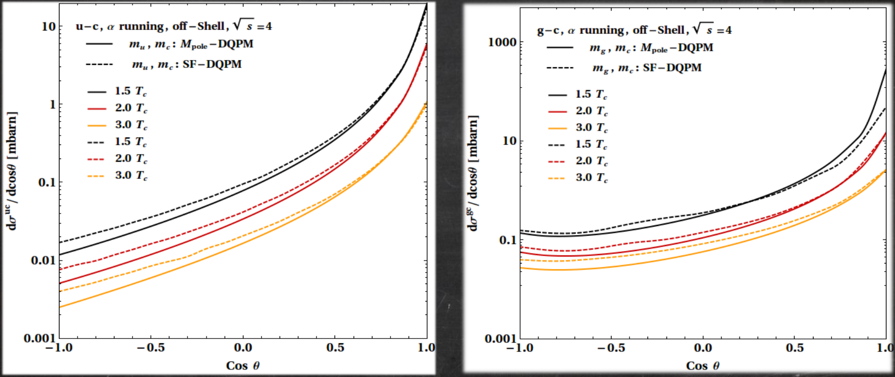
Heavy Quark Elastic Scattering

1. σ_{qQ}, σ_{gQ} : DpQCD (On-Shell) vs IEHTL (Off-Shell)



- Deviation of off-shell curves compared to on-shell ones for large angles θ
- Effect of $A(m, T)$ on σ_{qQ} is negligible (small DQPM parton width)
- $\sigma_{qQ}(T \nearrow) < \sigma_{qQ}(T \searrow)$

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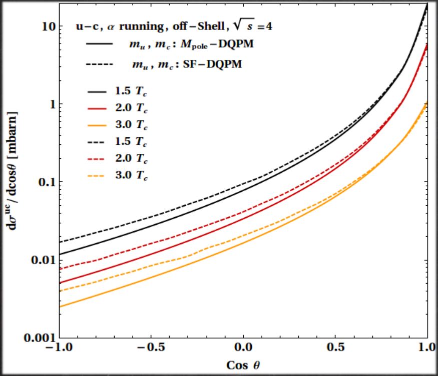

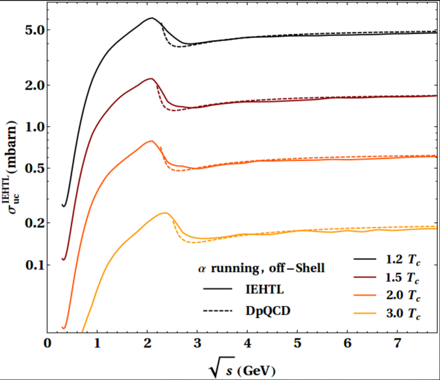

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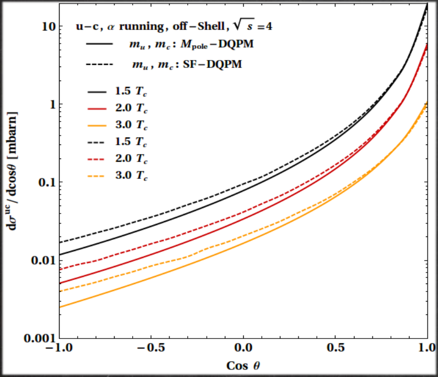

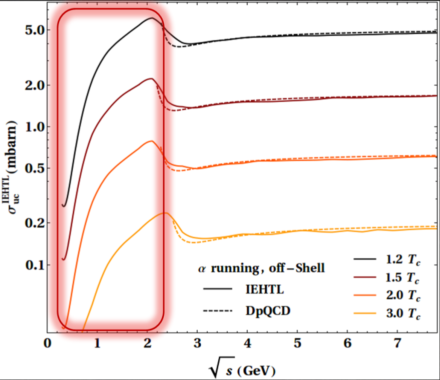

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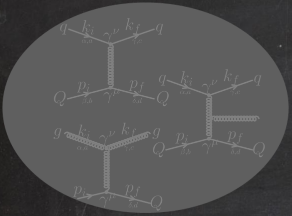
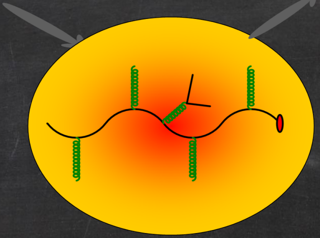
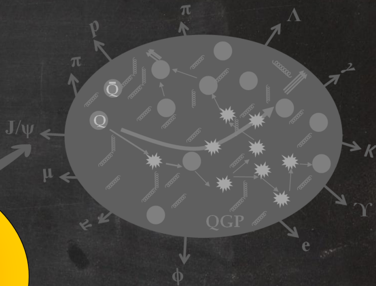
Global Project : HQ dynamics and transport proprieties in QGP







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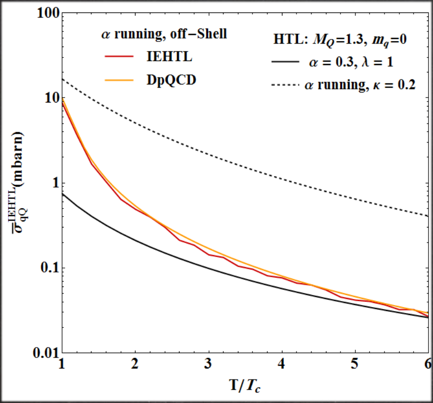
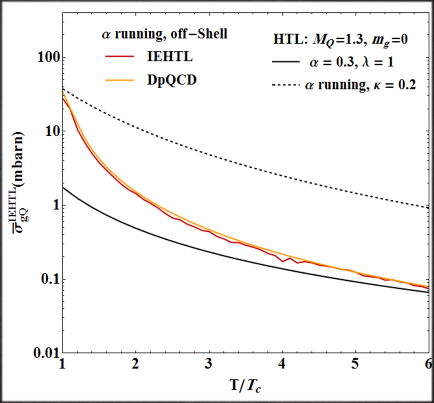
   $(R_{AA}, v_2, \ldots)$

Charm Integrated Elastic Cross Sections

1 qQ Elastic Scattering



2 gQ Elastic Scattering



- Different $T^{-\beta}$ power between HTL ($\beta^{T<1.2T_c} \sim 2$, $\beta^{T>1.2T_c} \sim 1.7$) and DpQCD-IEHTL ($\beta^{T<1.2T_c} \sim 4$, $\beta^{T>1.2T_c} \sim 2$) $\Rightarrow$ different Transport Coefficients
- Effect of $A(m, T)$ on $\bar{\sigma}$ is negligible (small DQPM parton width)

Charm Relaxation Time/Mean Free Path

1 τ_c : On-Shell case

$$(\tau_c^{-1})_{DpQCD} = \sum_{i \in q,\bar{q},g} n_i^{On}(T)\, \sigma_{i\,c}^{DpQCD}(T)$$

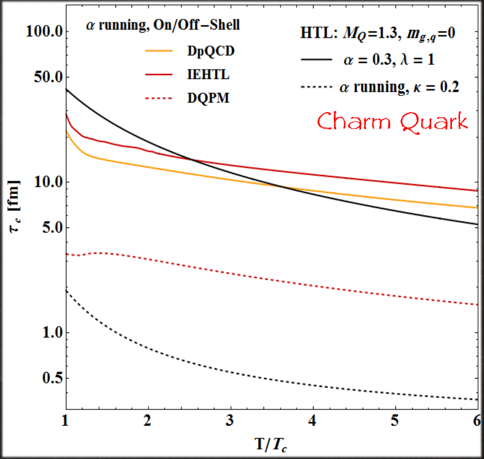
$$\text{with}: n_i^{On}(T) = \int \frac{d^3p}{(2\pi)^3}\, f_i/g_i(p,T,m_i,\mu_i)$$

2 τ_c : Off-Shell case

$$(\tau_c^{-1})_{IEHTL} = \sum_{i \in q,\bar{q},g} n_i^{Off}(T)\, \sigma_{i\,c}^{IEHTL}(T)$$

$$\text{with}: n_i^{Off}(T) = \iint \frac{d^3p}{(2\pi)^3} A_i^{BW}(m_i)dm_i\, f_i/g_i(p,T,m_i,\mu_i)$$

$$(\tau_c^{-1})_{DQPM} = \frac{\hbar c}{\gamma_c(T)}$$



3 $\tau_c \rightleftharpoons \lambda_c : \lambda_c = v_{rel}\tau_c$

More interactions with σ^{HTL} than $\sigma^{DpQCD/IEHTL}$

Charm Shear Viscosity

! η_c : contribution of charm viscosity to the QGP viscosity ($\eta_{QGP}=\eta_g+\eta_q+\eta_Q$)

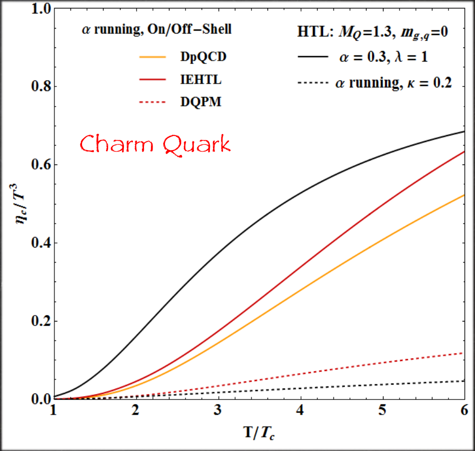
1. η_c : On-Shell case

$$\eta_c^{DpQCD} = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3}\tau_c^{DpQCD}(T) f_Q(p,T,m)\frac{p^4}{E^2}$$

2. η_c : Off-Shell case

$$\eta_c^{IEHTL} = \frac{1}{15T}\int\int \frac{d^3p}{(2\pi)^3} dm\, A_Q^{BW}(m) \times \tau_c^{IEHTL}(T)\, f_Q(p,T,m)\frac{p^4}{E^2}$$

A significant differences between HTL, DQPM and DpQCD/IEHTL for Charm Quark



Charm Elastic Interaction Rates

$$\frac{dN_{coll}[2\to 2]}{dtdV} = \Pi_{i=1}^{4} Tr_{(i)}\, W_{2,2} \times (2\pi)^4 \delta^4(p_1^\mu + p_2^\mu - p_3^\mu - p_4^\mu)\left[f_i(p_1) f_j(p_2)\right]$$

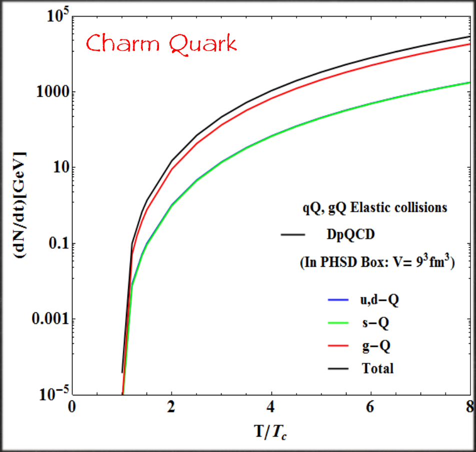
$W_{2,2}$: pQCD transition matrix-element

1. dN/dt : On-Shell case

$$Tr_{(i)} = \int \frac{d^3 p_i}{2E_i}$$

2. dN/dt : Off-Shell case

$$Tr_{(i)} = \int \frac{d^4 p_i}{(2\pi)^4} A_i(p_i)$$



DpQCD/IEHTL : charm quark interacts more with g than q

Summary

1 *qQ, gQ* Elastic Scattering

- HQ collisional scattering calculation following different approaches
- Used a well defined masses and widths determined within DQPM
- DpQCD/IEHTL : relevant evaluation of HQ scattering cross section
- DpQCD/IHETL : fundamental parameters have a well based theoretical determination
- Off-shell HQ collisional scattering cross sections in the sQGP : small effect of finite parton widths but reduced kinematic threshold

H. Berrehrah et al, Collisional processes of on-shell and off-shell heavy quarks in vacuum and in the Quark Gluon Plasma, To be submitted

2 Charm Quark Transport properties

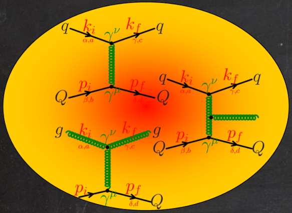
- Phenomenological consequences of using DpQCD/IEHTL cross sections on the HQ collective behaviour
- A significant difference between HTL and DpQCD/IEHTL
- Tow visions : HQ diffuses in a medium with *i*) many massless *q* and *g*, *ii*) few massive/off-shell *q* and *g*

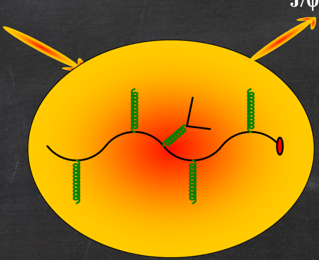
H. Berrehrah et al, Dynamical collisional energy loss and transport coefficients of on-shell and off-shell heavy quarks in vacuum and in the Quark Gluon Plasma, In preparation

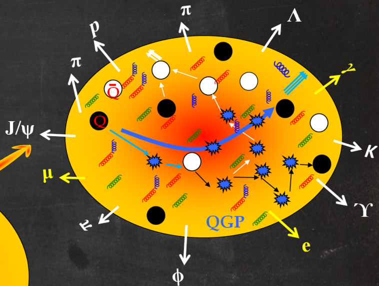
Perspectives

1. *qQ, gQ* Elastic Scattering
   - Improve the description of the 3 gluon vertex at finite temperature
   - Improve the description of the gluon and fermion propagators
   - Improve the description of parton spectral functions at finite temperature/chemical potential/momentum
2. HQ Transport Properties
   - Study the HQ dynamical collisional energy loss using DpQCD/IEHTL cross sections
   - Drag, Diffusion coefficients using DpQCD/IEHTL cross sections
3. HQ Dynamics in the QGP
   - Implement the HQ partonic processes into the PHSD transport approach to study HQ dynamics at SPS, RHIC and LHC

Perspectives : HQ dynamics and transport properties in QGP







1. Microscopic Ingredients
   - Elastic cross section : qQ, gQ
   - Inelastic cross section : qQg, gQg, . . .

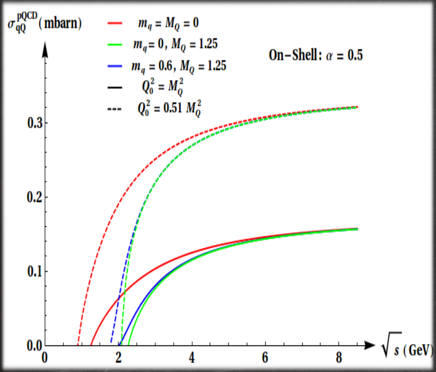
2. Mesoscopic Quantities
   - Dynamical energy loss
   - Transport coefficients (Viscosity, Drag, Diffusion . . .)

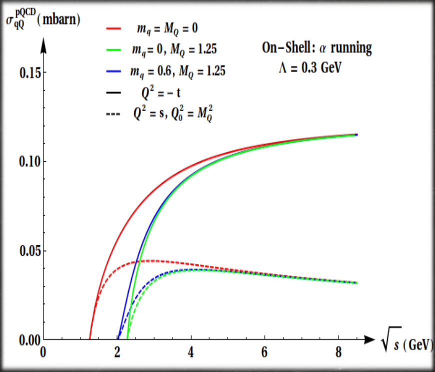
3. Macroscopic Observables
   - PHSD
   - MC@sHQ
   - BAMPS
   - URQMD . . .

   (R_{AA}, v_2, . . .)

Heavy Quark Elastic Scattering

1. σ_{qQ}^{pQCD} : α, IR cut-off, $m_{q,Q}$ effects

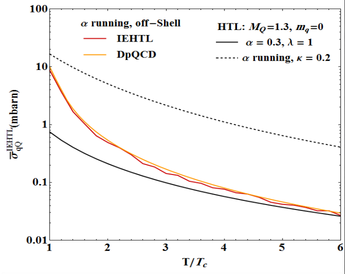


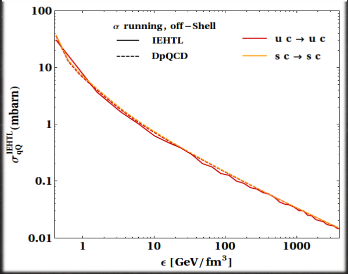


- IR cut-off has a crucial role in determining σ_{qQ}^{pQCD} (Q_0^2 value)
- q mass has no effect on σ_{qQ}^{pQCD} at hight $\sqrt{s}$
- $\sigma(\alpha)$ fix $>$ $\sigma(\alpha)$ running

HQ Integrated Elastic Cross Sections

1 qQ Elastic Scattering





2 gQ Elastic Scattering