

Strangeness Balance in Heavy-ion Collisions

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- Why do we love strangeness
- Properties of strange particles in-medium
- HADES data on strangeness production. Ξ puzzle.
- Minimal statistical model for strangeness

Strangeness is interesting because

- ✓ It is a **tag** on a hadron, saying that it was not in colliding nuclei but is **produced** in the course of collision.
- ✓ **Strange quarks** like **baryons**: $K, \Lambda, \Sigma, \Xi, \Omega, \dots$, **anti-strange quarks** like **mesons** K, \dots
 - strangeness/anti-strangeness **separation** in baryon-rich matter
- ✓ Strangeness is **conserved** in strong interaction
 - Strangeness production **threshold is high**,
 - sensitive to possible **in-medium effect**.
QGP signal? (Rafelski-Mueller conjecture)

Strangeness is difficult because

- ✓ Strangeness **production cross sections** poorly known
(new data from HADES on pp, COSY on pn, ANKA)
- ✓ Limited exp. information about **elementary reactions** among strange particles
- ✓ **Strong couplings** among various strange species. Complicated dynamics

Strange particles in nuclear medium

1. Hyperons $E_Y(p) = \sqrt{m_Y^2 + p^2} \longrightarrow \sqrt{(m + S_Y)^2 + p^2} + V_Y$

potential model

scalar and vector potentials

In relativistic mean-field models S and V originates
from exchanges of scalar and vector mesons

Usually one relates vector potentials to the potential for nucleons $V_Y = \alpha_Y V_N$

where α_Y is deduced from some quark counting rule

Scalar potentials are fixed by the optical potential $U_Y = S_Y + V_Y$,
acting on hyperons in an atomic nucleus

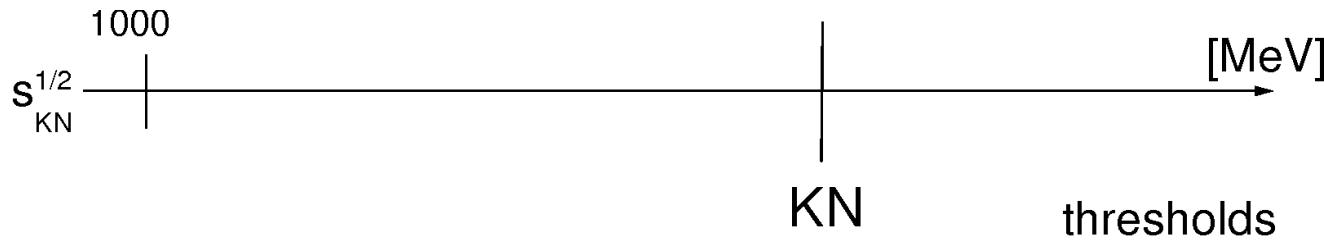
$$U_\Lambda = -27 \text{ MeV} \quad [\text{Hashimoto, Tamura, Prog.Part.Nucl.Phys. 57, 564 (2006)}]$$

$$U_\Sigma = +24 \text{ MeV} \quad [\text{Dabrowski, Phys.Rev.C 60, 025205 (1999)}]$$

$$U_\Xi = -14 \text{ MeV} \quad [\text{Khaustov et al., Phys.Rev.C 61, 054603 (2000)}]$$

Caution: extrapolation of the attractive hyperon potentials in RMF models to higher densities may lead to problems with astrophysical constraints on the neutron star masses!!!

2. anti-strange mesons

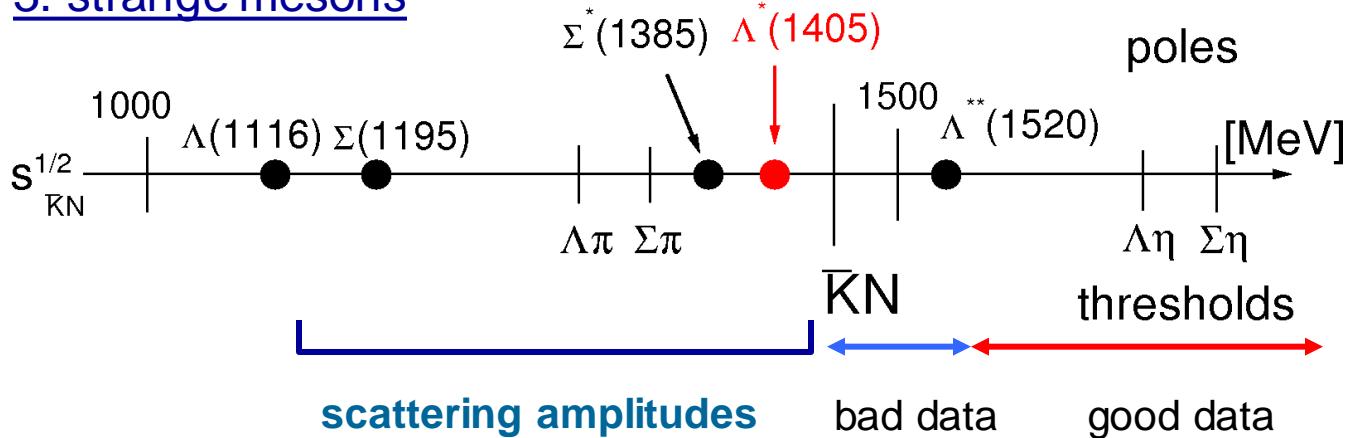


potential model

$$E(p) = \sqrt{m^2 + p^2} \longrightarrow \sqrt{(m + S)^2 + p^2} + V$$

scalar and vector potentials: common prejudice $S < 0$; $V > 0$

3. strange mesons



Exp. $\bar{K}N$ scattering

scattering model including s-, p-, d- partial waves

Coupled-channel scattering:
Weise, Kaiser, ...
Oset, Ramos, ...
Lutz, EEK, ...

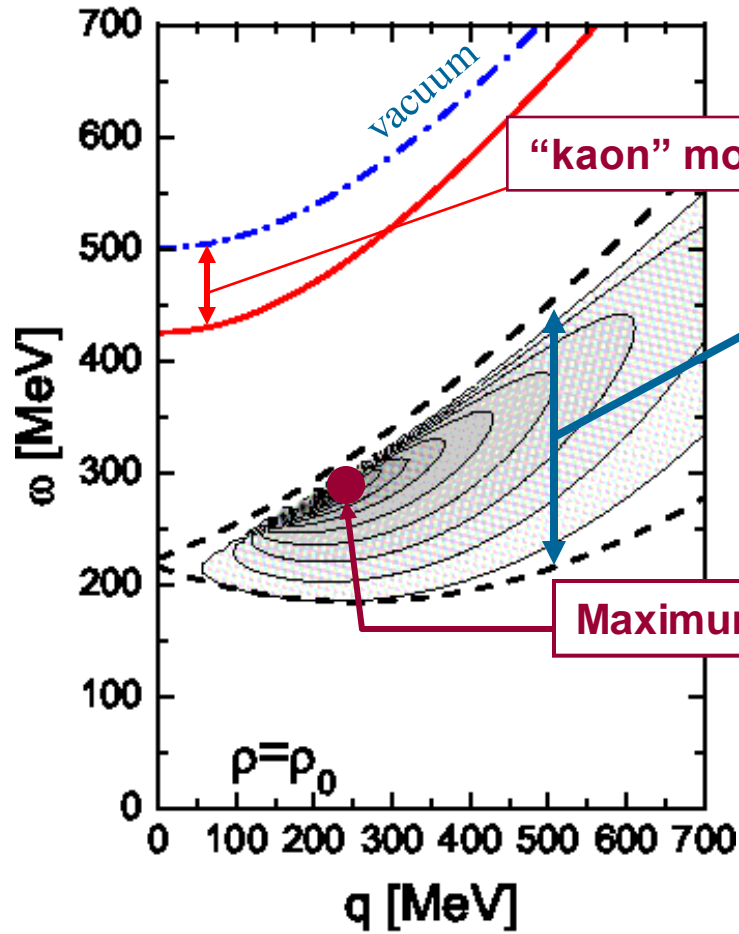
Various partial wave couple in medium

4. K^- in medium

schematic spectral density

EEK, Voskresensky, Kampf NPA588(1995);
 EEK, Voskresensky, PRC60 (1999)

spectral density



“kaon” mode with attractive potential

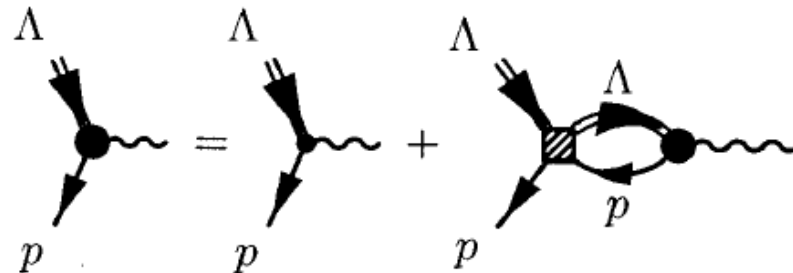
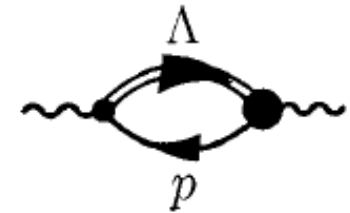
particle-hole modes

$$\omega \sim m_\Lambda - m_N$$

Maximum of the spectral function is shifted to higher energies

short-range correlations

quasi-particle branch



realistic spectral densities

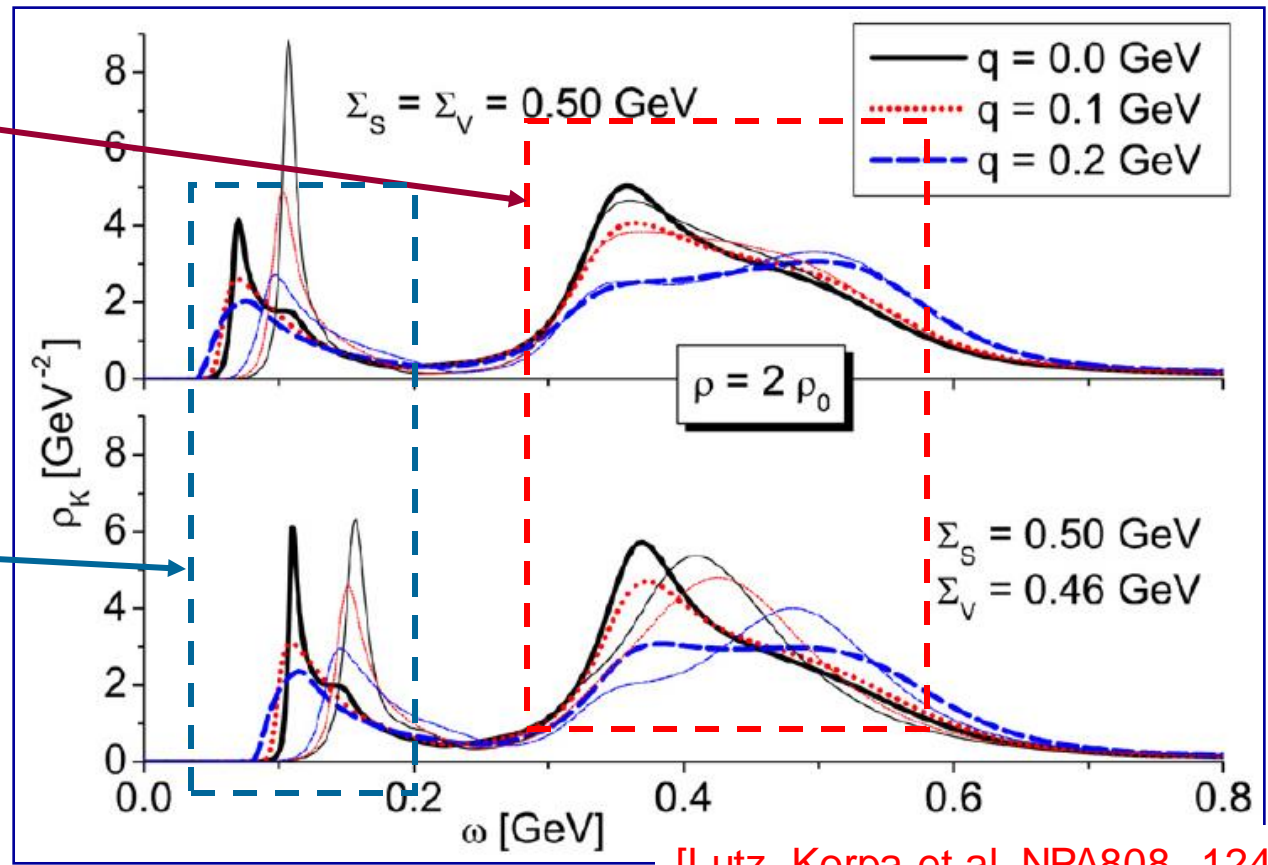
realistic K⁻N interactions +

self-consistent calculations

Oset, Tolos et al; Lutz, Korpa, et al

$\Lambda(1405), \Sigma(1385)$

$\Lambda(1116), \Sigma(1190)$



[Lutz, Korpa et al, NPA808, 124]

Courageous attempts to include spectral function in transport codes by Giessen, Frankfurt, and Nantes groups [Bratkovskaya, Cassing, Aichelin et al]

How to release the in-medium kaons?

fireball break up time $\sim 1/m_\pi$

$$m_K \rightarrow m_K - 75 \text{ MeV} \rho / \rho_0$$

HADES: complete measurement of particles containing strange quarks in Ar+KCl collisions @ 1.76 AGeV

Agakishiev (HADES) PRL 103, 132301 (2009);
Eur. Phys.J. A47 21 (2011)

one experimental set-up for all particles!

We study the relative distributions of strangeness among various hadron species

We are not interested in how strangeness is produced!

We know the final K^+ multiplicity!

$$R_{K^-/K^+} = \frac{N_{K^-}}{N_{K^+}} = 2.5_{-0.9}^{+1.2} \times 10^{-2} \quad R_{\Lambda/K^+} = \frac{N_{\Lambda+\Sigma^0}}{N_{K^+}} = 1.46_{-0.37}^{+0.49}$$

$$R_{\Sigma/K^+}^{(\text{Hades})} = \frac{1}{2} \frac{N_{\Sigma^++\Sigma^-}}{N_{K^+}} = 0.13_{-0.11}^{+0.16} \quad R_{\Xi/\Lambda/K^+} = \frac{N_{\Xi^-}}{N_{\Lambda+\Sigma^0} N_{K^+}} = 0.20_{-0.11}^{+0.16}$$

if $K^+ + K^0_s$ data are used for total strangeness

$$R_{\Sigma/K^+}^{(\text{iso})} = \frac{1}{2} \frac{N_{\Sigma^++\Sigma^-}}{N_{K^+}} = 0.30_{-0.17}^{+0.23}$$

total strangeness is $(1+\eta) K^+$

isospin asymmetry factor $\eta = \frac{A-Z}{Z}$

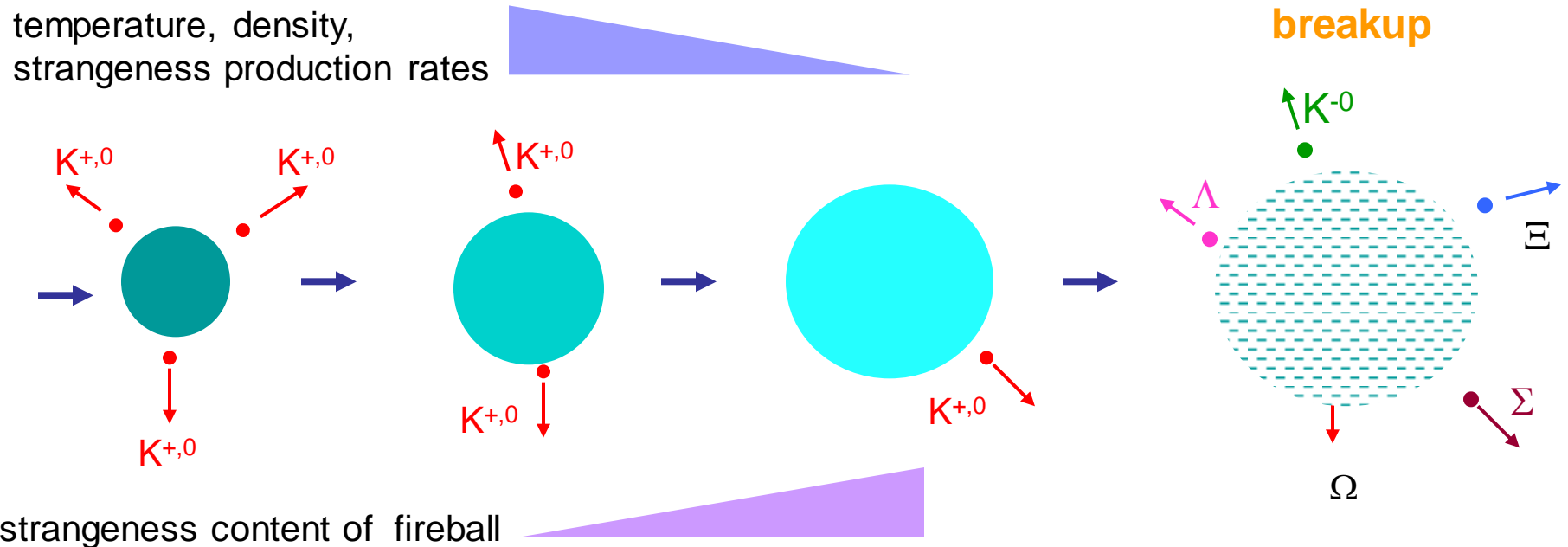
for ArK and ArCl collisions $\eta=1.14$

This number is much bigger than the results of stat. models and transport codes

Minimal statistical model for strange particles:

At SIS energies K^+ and K^0 have long mean free paths and escape the fireball right after their creation in direct reactions.

The fireball have some negative strangeness which is **statistically** distributed among K^- , anti- K^0 , Λ , Σ , Ξ , Ω



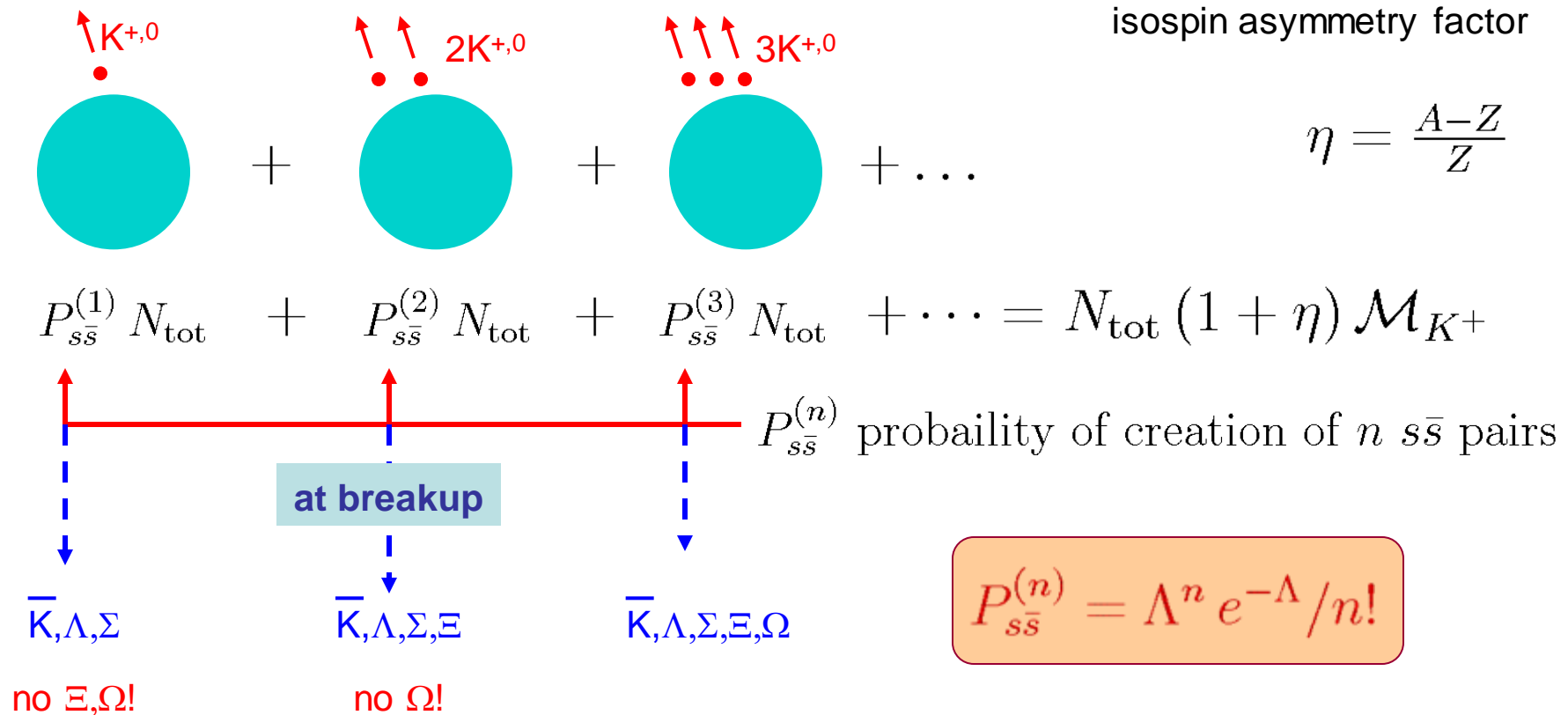
anti-strangeness released = *strangeness accumulated inside* = **strangeness released at breakup**

We know the average kaon multiplicity $\mathcal{M}_{K^+} = (2.8 \pm 0.4) \times 10^{-2}$

Of course kaons are produced not piecewise but as whole entities.

$$\text{events with } K^+ \longrightarrow N_{K^+} = \mathcal{M}_{K^+} \cdot N_{\text{tot}} \longleftarrow \text{total number of events}$$

Multi-kaon event classes:



Λ -- integral probability of the pair production

Let W be the probability of (s bar-s) pair production in a unit of volume and a unit of time, which is a function of local temperature and density.

$$\Lambda = \int_0^{t_{\text{col}}} V(t) \mathcal{W}(\rho(t), T(t)) dt$$

$$V(t) = f(t) V_{\text{fo}}$$

$$t_{\text{col}} = \tau V_{\text{fo}}^{1/3}$$

$$\Lambda = \tau \overline{\mathcal{W}} V_{\text{fo}}^{4/3} = \lambda V_{\text{fo}}^{4/3}$$

V_{fo} freeze-out volume

The value of λ is fixed by the total K^+ multiplicity observed in an inclusive collision.

We denote the multiplicity of K^+ mesons produced in each n -kaon events as:

$$M_{K^+}^{(n)} = \frac{n}{1 + \eta} P_{s\bar{s}}^{(n)}$$

$$\mathcal{M}_{K^+} = \sum_n \langle M_{K^+}^{(n)} \rangle = \frac{1}{1 + \eta} \sum_n n \langle P_{s\bar{s}}^{(n)} \rangle = \frac{\langle \Lambda \rangle}{1 + \eta}$$

$$\langle \dots \rangle = \frac{2}{b_{\text{max}}^2} \int_0^{b_{\text{max}}} db b(\dots) \quad \text{-- averaging over the collision impact parameter}$$

$$r_0 = 1.124 \text{ fm} \quad b_{\text{max}} = 2 r_0 A^{1/3}$$

$$V_{\text{f.o.}}(b) = \frac{2A}{\rho_{B,\text{fo}}} F(b/b_{\text{max}}) \quad \text{overlap function}$$

[Gosset et al, PRC 16, 629 (1977)]

$$\langle V_{\text{f.o.}} \rangle \approx \frac{A}{2 \rho_{B,\text{fo}}}$$

↑ freeze-out density

$$\langle P_{s\bar{s}}^{(1)} \rangle = (1 + \eta) \mathcal{M}_{K^+} \left[1 - (1 + \eta) \zeta^{(2)} \mathcal{M}_{K^+} + \frac{1}{2} \zeta^{(3)} (1 + \eta)^2 \mathcal{M}_{K^+}^2 \right]$$

$$\langle P_{s\bar{s}}^{(2)} \rangle = \frac{1}{2} (1 + \eta)^2 \mathcal{M}_{K^+}^2 \left[\zeta^{(2)} - (1 + \eta) \zeta^{(3)} \mathcal{M}_{K^+} \right]$$

$$\langle P_{s\bar{s}}^{(3)} \rangle = (1 + \eta)^3 \frac{1}{6} \zeta^{(3)} \mathcal{M}_{K^+}^3$$

$$\zeta^{(n)} = \frac{\langle V_{fo}^{\frac{4}{3}n} \rangle}{\langle V_{fo}^{4/3} \rangle^n}$$

$$\zeta^{(1)} = 1, \quad \zeta^{(2)} = 2.51, \quad \zeta^{(3)} = 8.11$$

enhancement factors!!

total strangeness multiplicity $M_S^{(n)} = n P_{s\bar{s}}^{(n)}$

Using the experimental kaon multiplicity $\mathcal{M}_{K^+} = (2.8 \pm 0.4) \times 10^{-2}$ we estimate

$$\frac{\langle M_S^{(2)} \rangle}{(1 + \eta) \mathcal{M}_{K^+}} \simeq 15\%$$

of kaons is produced pairwise

$$\frac{\langle M_S^{(3)} \rangle}{(1 + \eta) \mathcal{M}_{K^+}} \simeq 1\%$$

of kaons is produced triplewise

The statistical probability that strangeness will be released at freeze-out in a hadron of type a with the mass m_a is

$$P_a = z_S^{s_a} V_{\text{fo}} p_a = z_S^{s_a} V_{\text{fo}} \nu_a e^{q_a \frac{\mu_B(t)}{T(t)}} f(m_a, T_{\text{fo}})$$

s_a # of strange quarks in the hadron

ν_a spin-isospin degeneracy factor

q_i baryon charge of the hadron

$$f(m, T) = \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right)$$

baryon chemical potential $\mu_B(t) \simeq -T(t) \ln \left\{ 4[f(m_N, T) + 4f(m_\Delta, T)] / \rho_B(t) \right\}$

z_S is a *normalization factor* which could be related to a probability of one s -quark to find itself in a hadron a

This factor follows from the requirement that **the sum of probabilities** of production of different strange species and their combinations, which are allowed in the finale state, **is equal to one**.

This factor depends on **how many strange quarks are produced**. Hence, it is different in single-, double- and triple-kaon events.

$$P_a^{(n)} = z_S^{(n)s_a} V_{\text{fo}} p_a$$

single-kaon event: $n = 1$ only \bar{K} , Λ and Σ can be in the final state

$$P_{\bar{K}}^{(1)} + P_{\Lambda}^{(1)} + P_{\Sigma}^{(1)} = 1 = z_S^{(1)} V_{fo} (p_{\bar{K}} + p_{\Lambda} + p_{\Sigma})$$

multiplicity of \bar{K} , Λ , Σ $M_a^{(1)} = g_a P_{s\bar{s}}^{(1)} P_a^{(1)} = g_a P_{s\bar{s}}^{(1)} z_S^{(1)} V_{fo} p_a$

isospin factor $\xrightarrow{\quad\quad\quad} \uparrow$

double-kaon event: $n = 2$ $\bar{K}\bar{K}$, $\bar{K}\Lambda$, $\bar{K}\Sigma$, $\Lambda\Lambda$, $\Lambda\Sigma$, $\Sigma\Sigma$ and Ξ can be in the final state

$$(P_{\bar{K}}^{(2)} + P_{\Lambda}^{(2)} + P_{\Sigma}^{(2)})^2 + P_{\Xi}^{(2)} = 1$$

$$z_S^{(2)2} V_{fo}^2 (p_{\bar{K}} + p_{\Lambda} + p_{\Sigma})^2 + z_S^{(2)2} V_{fo} p_{\Xi} = 1$$

multiplicity of \bar{K} , Λ , Σ $M_a^{(2)} = g_a 2 P_{s\bar{s}}^{(2)} P_a^{(2)} (P_{\bar{K}}^{(2)} + P_{\Lambda}^{(2)} + P_{\Sigma}^{(2)})$

multiplicity of Ξ $M_{\Xi}^{(2)} = g_{\Xi} P_{s\bar{s}}^{(2)} P_{\Xi}^{(2)}$

particle ratios:

We included leading and next-to-leading contributions

$$\begin{aligned} R_{K^-/K^+} &= \eta \frac{\langle M_{\bar{K}}^{(1)} + M_{\bar{K}}^{(2)} \rangle}{(1 + \eta) \mathcal{M}_{K^+}} = \frac{\eta p_{\bar{K}}}{p_{\bar{K}} + p_{\Lambda} + p_{\Sigma}} Y_1 \\ R_{\Lambda/K^+} &= \frac{1}{\mathcal{M}_{K^+}} \left\langle M_{\Lambda}^{(1)} + M_{\Lambda}^{(2)} + \eta \frac{M_{\Sigma}^{(1)} + M_{\Sigma}^{(2)}}{\eta^2 + \eta + 1} \right\rangle = (1 + \eta) \frac{\left[p_{\Lambda} + \frac{\eta p_{\Sigma}}{\eta^2 + \eta + 1} \right]}{p_{\bar{K}} + p_{\Lambda} + p_{\Sigma}} Y_1 \\ R_{\Sigma/K^+} &= \frac{\eta^2 + 1}{2(\eta^2 + \eta + 1)} \frac{\langle M_{\Sigma}^{(1)} + M_{\Sigma}^{(2)} \rangle}{\mathcal{M}_{K^+}} = \frac{(\eta^2 + 1)(\eta + 1)}{2(\eta^2 + \eta + 1)} \frac{p_{\Sigma}}{p_{\bar{K}} + p_{\Lambda} + p_{\Sigma}} Y_1 \\ R_{\Xi/\Lambda/K^+} &= \frac{\frac{\eta}{1+\eta} \langle (M_{\Xi}^{(2)} + M_{\Xi}^{(3)}) \rangle}{\langle M_{\Lambda}^{(1)} + M_{\Lambda}^{(2)} + \eta \frac{M_{\Sigma}^{(1)} + M_{\Sigma}^{(2)}}{\eta^2 + \eta + 1} \rangle \mathcal{M}_{K^+}} = \eta \frac{p_{\Xi}/(p_{\bar{K}} + p_{\Lambda} + p_{\Sigma})}{\langle V_{fo} \rangle \left(p_{\Lambda} + \frac{\eta p_{\Sigma}}{\eta^2 + \eta + 1} \right)} Y_2 \end{aligned}$$

in **blue** the standard results; in **red** corrections

$$Y_1 = 1 - \frac{(1 + \eta) \mathcal{M}_{K^+} p_{\Xi}}{(p_{\bar{K}} + p_{\Lambda} + p_{\Sigma})^2} \frac{\langle V_{fo}^{5/3} \rangle}{\langle V_{fo}^{4/3} \rangle^2} \quad \text{small correction } < 5\%$$

$$Y_2 = \frac{1}{2} \tilde{\zeta}^{(2)} = \frac{1}{2} \frac{\langle V_{fo}^{5/3} \rangle}{\langle V_{fo}^{4/3} \rangle^2} \langle V_{fo} \rangle \simeq 0.52 \quad \text{strong suppression!}$$

$\Xi/\Lambda/K$ ratio is sensitive to the fireball freeze-out volume

Ratios as functions of the freeze-out temperature

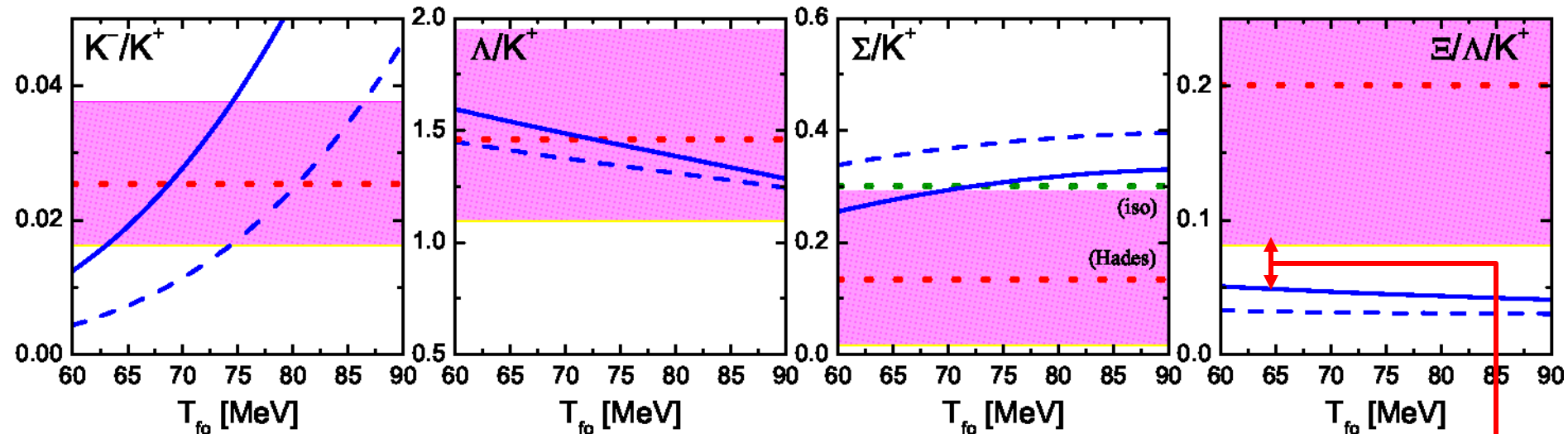
parameters of the model: $\rho_{B,fo} = 0.6 \rho_0$

potential models for strange particles in medium

potentials for nucleons Δ s:

$$S_N \simeq S_\Delta \simeq -190 \text{ MeV} \rho_B / \rho_0$$

$$V_N \simeq V_\Delta \simeq +130 \text{ MeV} \rho_B / \rho_0$$



- results with in-medium potentials
- - - results with vacuum masses

best fit for K^- , Λ ratios: $T_{f.o.} = 69 \text{ MeV}$

double strangeness
suppression Y_2

Inclusion of potentials

improves the temperature match
for K and Λ ratios,

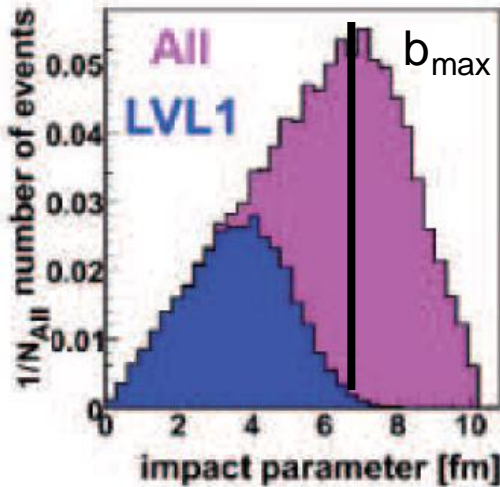
improves Σ ratio (repulsive potential),
increases Ξ ratio (not strong enough)

HADES trigger effect

LVL1 trigger

HADES counts only the events with MUL>16

[Schade, PhD thesis2010]



trigger function

$$T_{\text{LVL1}}(b) = \begin{cases} b, & b < 3.9 \text{ fm} \\ 3.6 e^{-0.27 \left(\frac{b}{1\text{fm}} - 3.75 \right)^2}, & b \geq 3.9 \text{ fm} \end{cases}$$

$$\langle V_{\text{f.o.}} \rangle_{\text{LVL1}} = \frac{2\pi \int_0^{b_{\text{max}}} db T_{\text{LVL1}}(b) V_{\text{f.o.}}(b)}{2\pi \int_0^{b_{\text{max}}} db T_{\text{LVL1}}(b)} = 1.77 \langle V_{\text{f.o.}} \rangle$$

$$R_{\Xi/\Lambda/K^+} \propto \frac{1}{\langle V_{\text{fo}} \rangle}$$

$$R_{\Omega/(\Lambda K^-)/K^+} \propto \frac{1}{\langle V_{\text{fo}} \rangle^2}$$

$$R_{\Omega/\Xi/K^+} \propto \frac{1}{\langle V_{\text{fo}} \rangle}$$

ratio	exp. value	inclusive	triggered
$\Xi/\Lambda/K^+$	$0.20^{+0.16}_{-0.11}$	0.047	0.026

Triggering can effect the ratios with multi strange particles

1. in medium potential and freeze-out density

A more attractive Ξ in-medium potential? We would need $U_{\Xi} < -120 \text{ MeV}$ to increase the ratio $\Xi^{-}/\Lambda/K^{+}$ up to the lowest end of the empirical error bar.

Such a strong attraction exceeding the nucleon optical potential is unrealistic. It would imply that Ξ baryon is bound in nucleus stronger than two Λ s,

$$2(m_{\Lambda} + U_L) - (m_{\Xi} + m_N + U_{\Xi} + U_N) \sim 100 \text{ MeV} > 0.$$

This would influence the description of doubly strange hypernuclei

The leading order analysis of hyperon and nucleon mass shifts in nuclear matter using the **chiral perturbation theory** [Savage, Wise, PRD 53, 349 (1996)] shows that the Ξ shift is much smaller than nucleon and Λ shifts.

Recent analyses [Polinder, Haidenbauer, Meissner, PLB 653, 29 (2007),
Gasparyan, Haidenbauer, Hanhart, arXiv:1111.0513]
support the relative smallness of ΞN scattering lengths.

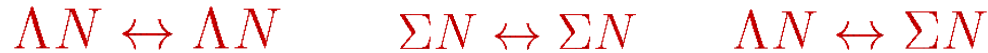
We can take somewhat larger freeze-out density: $\rho_{B,fo} = 0.7 \rho_0$

$$R_{\Xi/\Lambda/K^+} = 0.026 \longrightarrow 0.028$$

2. Non-equilibrium effects

The main assumption of our model is that the strange subsystem is in **thermal equilibrium** with a non-strange subsystem and that strange particles are in **chemical equilibrium** with each other.

For Λ and Σ



$$\sigma \sim 80 - 25 \text{ mb}$$

for relative momenta p_T to $2p_T$

$p_T \sim 300 \text{ MeV}$ is the thermal momentum for $T=70 \text{ MeV}$

For \bar{K}



For Ξ

ΞN interaction is expected to be smaller than ΛN and ΣN interactions

$$\sigma(\Xi^- p \rightarrow \Xi^- p) \sim 15 \text{ mb} \quad \sigma(\Xi^- p \rightarrow \Lambda \Lambda) \lesssim 10 \text{ mb} \quad \sigma(\Xi^0 p \rightarrow \Xi^0 p) \lesssim 15 \text{ mb}$$

[Polinder, Haidenbauer, Meissner, PLB 653, 29 (2007)]

Scattering of Ξ s on pions for nearly isospin symmetrical matter is **considerably weaker** than the πN scattering (very narrow $\Xi^*(1532)$ resonance, not broad $\Delta(1232)$)

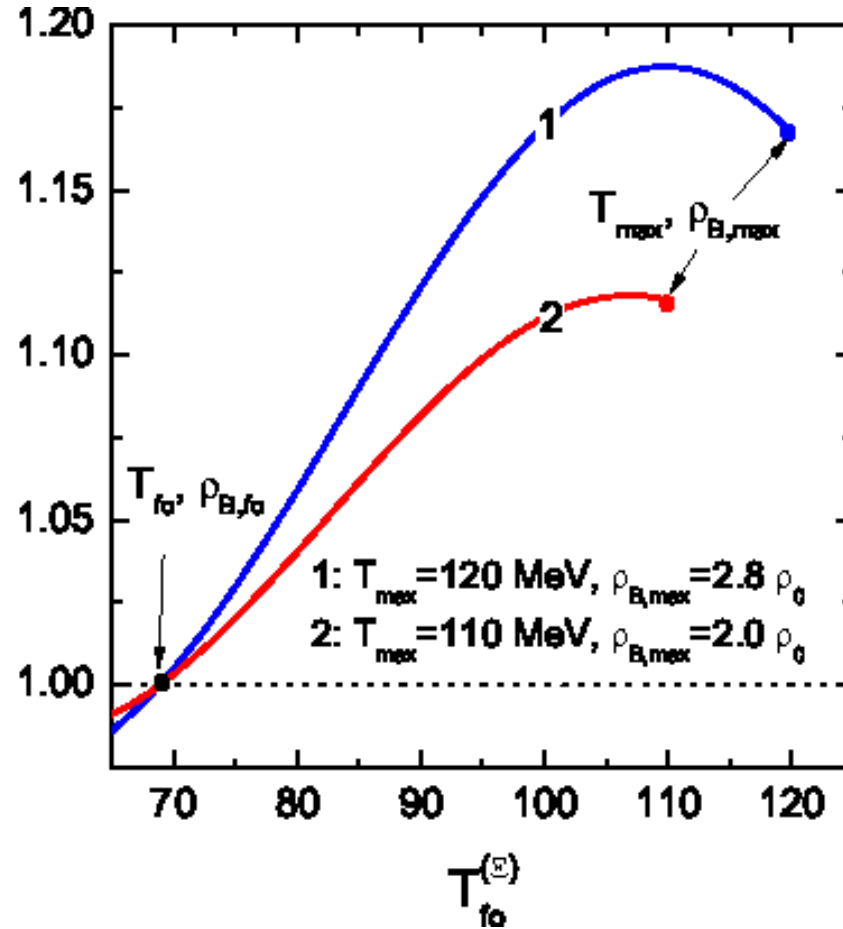
Ξ baryons are presumably weakly coupled to the non-strange system

Earlier freezeout!

$$R_{\Xi/\Lambda/K^+}^{\text{non-eq.}} \sim \frac{p_{\Xi}[\tilde{T}_{\text{fo}}]}{(p_{\Lambda} + \frac{1}{3}p_{\Sigma})(p_{\bar{K}} + p_{\Lambda} + p_{\Sigma})[T_{\text{fo}}]} Y_2 \quad \tilde{T}_{\text{fo}} > T_{\text{fo}}$$

increase of the ratio

$$h_{\Xi} = \frac{R_{\Xi/\Lambda/K^+}^{\text{non-eq.}}}{R_{\Xi/\Lambda/K^+}}$$



The enhancement is too small! We need at least factor 5!

3. Direct reactions

To get any substantial increase in the number of Ξ 's we have to assume that these baryons are **not absorbed after being produced** and their number is determined by the rate of **direct production reactions**, as, for example, for dileptons.

However, this raises a new question:
whether there are sufficiently strong sources of Ξ baryons and enough time t ?

Where do Ξ baryons come from?

strangeness creation reactions: $\bar{K}N \rightarrow K\Xi - 380 \text{ MeV}$ $N_{K^-} \ll N_{\Lambda, \Sigma}$
 $\pi\Sigma \rightarrow K\Xi - 480 \text{ MeV}$ *very exothermic,*
 $\pi\Lambda \rightarrow K\Xi - 560 \text{ MeV}$ *very inefficient*

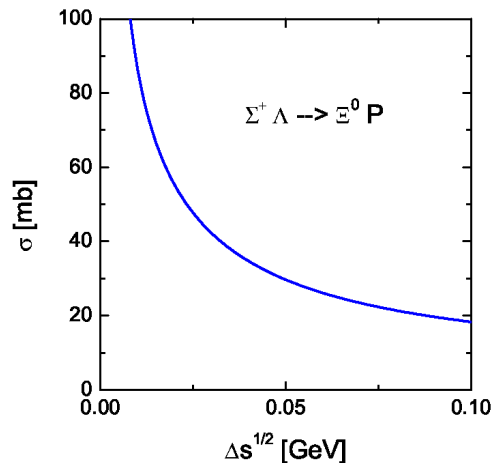
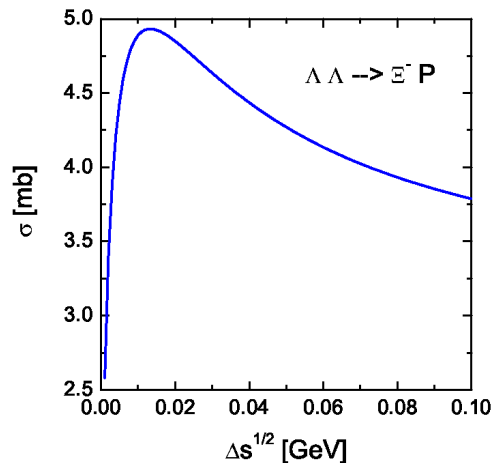
strangeness recombination reactions:

ss quarks are strongly bound in Ξ !

anti-kaon induced reactions $\bar{K}\Lambda \rightarrow \Xi\pi + 154 \text{ MeV}$ $\sigma \sim 10 \text{ mb}$
 $\bar{K}\Sigma \rightarrow \Xi\pi + 232 \text{ MeV}$ [Li,Ko NPA712, 110 (2002)]

double-hyperon processes $\Lambda\Lambda \rightarrow \Xi N - 26 \text{ MeV}$ can be more efficient since
 $\Lambda\Sigma \rightarrow \Xi N + 52 \text{ MeV}$ $N_{K^-} \ll N_{\Lambda, \Sigma}$
 $\Sigma\Sigma \rightarrow \Xi N + 130 \text{ MeV}$ [Tomasik, E.K., arXiv:1112.1437]

[Polinder, Haidenbauer, Meissner, PLB 653, 29 (2007)]



[Li,Chen,Ko,Lee 1204.1327]
 calculated the same cross sections in Born approximation [much larger σ] and implemented in transport code.

→ increased Ξ production

Conclusions:

Strangeness is interesting and complicated!
We need “complete strangeness measurement not only kaons, hyperons but also multi-strange baryons and phi’s!

- HADES data show the problems with the strangeness balance:
too few Σ baryons and too many Ξ are observed.
- Isospin corrections could help to understand Σ yield.
- With an inclusion of in-medium potentials yield we can describe K^-/K^+ , Λ/K^+ , and Σ/K^+ ;
- $\Xi/\Lambda/K^+$ ratio cannot be described. Suppression of the ratio calculated in the statistical model is due to explicit strangeness conservation in each collision and HADES event trigger!

Ξ out of chemical equilibrium! Production via direct reaction!

- The main source of Ξ 's is strangeness recombination reactions.
- Double-hyperon processes can be very important.
- Anti-kaon induced reactions can be strongly enhanced if
the attractive kaon potential is included