

# The elliptic flow and the shear viscosity of the QGP within a kinetic approach

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**Abstract.** We use a relativistic transport approach to study the role of a temperature dependent shear viscosity to entropy density ratio,  $\eta/s(T)$ , on the build-up of the elliptic flow,  $v_2$ . The recent results from  $\sqrt{s_{NN}} = 62.4\text{GeV}$  at RHIC up to 2.76 TeV at LHC have shown an intriguing property of the  $v_2(p_T)$ , which appears to be nearly invariant with energy. We show that in our transport approach this surprising behavior can be described by a particular temperature dependence of  $\eta/s(T)$ , typical of matter that undergoes a phase transition or a cross-over, with a rise and fall and the minimum close to critical temperature  $T_c$ .

## 1. Introduction

The experimental results from RHIC and recently from LHC have provided a lot of information about the physics of quark-gluon plasma (QGP). In particular in these experiments has been shown that the elliptic flow  $v_2 = \langle \cos(2\varphi_p) \rangle = \langle (p_x^2 - p_y^2)/(p_x^2 + p_y^2) \rangle$ , which is a measure of the momentum anisotropy of the emitted particle, is the largest ever seen in uRHICs [1, 2]. On the other hand theoretical calculations within viscous hydrodynamics [3] or transport approach [4, 5, 6] have shown that this large value of  $v_2$  is consistent with a very low shear viscosity to entropy density ratio  $\eta/s$  close to the conjectured lower bound for a strongly interacting system in the limit of infinite coupling,  $\eta/s = 1/4\pi$  [7]. However recent measurements of higher harmonics  $v_n = \langle \cos(n\varphi_p) \rangle$  with  $n > 2$  seems to confirm a small value of  $\eta/s \sim 1 - 3$  [8, 9]. In itself a small value of  $\eta/s \sim 1/4\pi$  it is not an evidence of the creation of a QGP phase but rather a phenomenological estimation of its temperature dependence which could give us more information of how the matter created in these collisions undergoes a phase transition [10, 11]. On the other hand, recent results of both the STAR Collaboration at RHIC [12] and the ALICE at LHC [2] have shown a surprising property of the  $v_2(p_T)$  that it appears invariant in a very wide colliding energy range of  $62.4\text{ GeV} \leq \sqrt{s_{NN}} \leq 2.76\text{ TeV}$ . It is important to explain the invariance of  $v_2(p_T)$  and how it is related to  $\eta/s$ . We show that the invariance of  $v_2(p_T)$  in the energy range  $62.4\text{ GeV} \leq \sqrt{s_{NN}} \leq 2.76\text{ TeV}$  is caused by a fall and rise of the  $\eta/s(T)$  as one would expect if the created matter undergoes a phase transition.

## 2. Relativistic Boltzmann Transport approach at fixed $\eta/s$

In this section we introduce a Relativistic Boltzmann Transport (RBT) approach at fixed  $\eta/s$  ratio. It is inspired by the success of the hydrodynamical approach that has shown the key role played by the  $\eta/s$ . Within this approach it is possible to make a direct comparison to viscous hydrodynamics because instead of focusing on specific microscopic details we fix the cross section in order to have the wanted  $\eta/s$  and it provides a tool to directly estimate the viscosity of the plasma in a wider range of  $\eta/s$  and  $p_T$ .

To solve the RBT with fixed  $\eta/s(T)$ , we determine locally in space and time the total cross section  $\sigma_{tot}$  according to the Chapman-Enskog theory. For a pQCD inspired cross section,  $d\sigma/dt \sim \alpha_s^2/(t - m_D^2)^2$ , typically used in parton cascade approaches[4, 13, 5], this gives:

$$\eta/s = \frac{1}{15} \langle p \rangle \tau_\eta = \frac{1}{15} \frac{\langle p \rangle}{g(a) \sigma_{tot} \rho}, \quad (1)$$

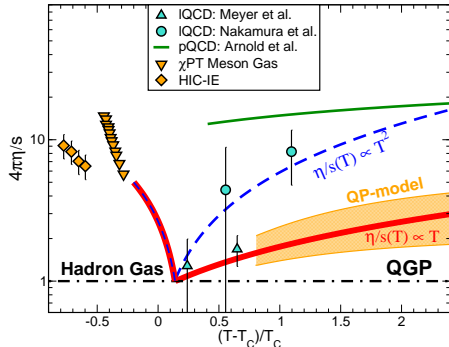
where  $a = m_D/2T$ , with  $m_D$  being the screening mass regulating the angular dependence of the cross section  $\sigma_{tot}$ . The  $g(a)$  function is the proper function accounting for the pertinent relaxation time  $\tau_\eta^{-1} = g(a) \sigma_{tot} \rho$  associated to the shear transport coefficient and it is given by  $g(a) = 50^{-1} \int dy y^6 [(y^2 + 3^{-1})K_3(2y) - yK_2(2y)] h(a^2 y^{-2})$  with  $K_n$ -s being the Bessel functions and the function  $h$  relating the transport cross section to the total cross section  $\sigma_{tr}(s) = \sigma_{tot} h(m_D^2/s)$  and  $h(\zeta) = 4\zeta(1 + \zeta)[(2\zeta + 1)\ln(1 + 1/\zeta) - 2]$ . As shown in Ref. [14] the eq.(1) correctly describes the relation between  $\eta/s \leftrightarrow T, \sigma(\theta), \rho$  of the system. This approach is similar to the one [15], where it is shown to converge to viscous hydrodynamic in 1+1D.

In the calculations shown in the next section we have used longitudinal boost invariant initial conditions. The initial  $dN/d\eta$  have been chosen in order to reproduce the final  $dN_{ch}/d\eta(b)$  at mid rapidity as observed in the experiments at RHIC and LHC energies. In coordinate space the partons are initially distributed according to the Glauber model. In the momentum space the distribution is thermal up to  $p_T = 2 \text{ GeV}$  and at larger  $p_T$  we include the spectrum of non-quenched minijets according to standard NLO-pQCD calculations. We have used the relation  $(\tau A_T)^{-1} dN_{ch}/d\eta \propto T^3$  to determine how the maximum temperature,  $T_{m0}$  scales with the collision energy. The initial time,  $\tau_0$ , is fixed by uncertainty relation between the initial average thermal energy and the initial time  $T_{m0} \tau_0 \approx 1$ . As also done in hydrodynamics we fix  $T_{m0} = 340 \text{ MeV}$  and  $\tau_0 = 0.6 \text{ fm}/c$  for  $Au + Au$  at  $\sqrt{s} = 200 \text{ GeV}$ . which implies  $T_{m0} = 290 \text{ MeV}$  and  $560 \text{ MeV}$  respectively for  $\sqrt{s} = 62.4 \text{ GeV}$  and  $2.76 \text{ TeV}$ . The temperature profile scales with the energy density as  $T(\vec{r}) = T_{m0}(\epsilon(\vec{r})/\epsilon(0))^{1/4}$ .

## 3. The $v_2(p_T)$ at different beam energies and $\eta/s(T)$

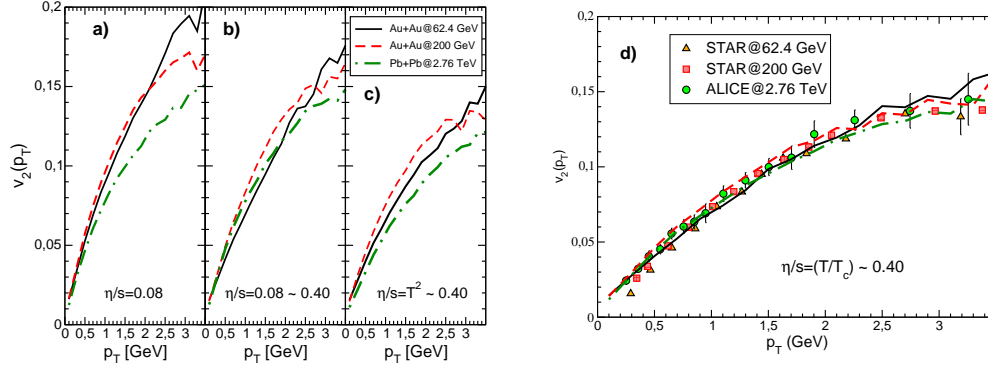
In this section we discuss a connection between the different regions of  $\eta/s(T)$  phase diagram in Fig.1 and the value of the  $v_2(p_T)$  at different beam energies and how it could be related to the intriguing invariance of  $v_2(p_T)$  with the collision energies as observed in the experiments [2, 12]. From a theoretical point of view there are several indications that  $\eta/s$  should have a typical behavior of phase transition with a 'U' shape. In Fig.1 it is shown the  $\eta/s(T)$  used in our calculations. To have a match with the estimates of  $\eta/s$  in the chiral perturbation theory for a meson gas [16, 17], shown by down-triangles in Fig. 1, we have considered an  $\eta/s$  increasing linearly at low temperature  $T < T_0 = 1.2T_c$  which also determines kinetic freeze out (f.o.). We however consider an  $\eta/s$  increasing up to 0.40 at  $T = 0.8T_c$  a value which is also comparable to the estimate of  $\eta/s$  extrapolated from heavy-ion collisions at intermediate energies (HIC-IE diamonds in Fig. 1)[18, 19]. On the other hand at higher temperature lattice QCD (lQCD) calculations have shown large error bars for  $\eta/s$  and it is not possible to infer a clear temperature dependence in the QGP phase. Therefore for  $T > 1.2T_c$  we have considered two cases: one with a linear dependence  $4\pi\eta/s = T/T_0$  (red solid line in fig.1) in agreement with lQCD data of Ref.

[20] (up-triangles) and quasi-particle model prediction (orange band) where  $\eta/s \sim T^{1-1.5}$  [21]. The other one with a quadratic dependence  $4\pi\eta/s = 3.64(T/T_0 - 1) + (T/T_0)^2$  (blue dashed



**Figure 1.** Different temperature dependent parametrizations for  $\eta/s$ . The orange area takes into account the quasi-particle model predictions for  $\eta/s$  [21]. The different lines indicate different T dependencies assumed in the simulation of heavy-ion collision while with the green line it is shown the result for the pQCD calculation [22]. Symbols are as in the legend. See the text for more details.

line) resembling the IQCD in quenched approximation in Ref. [23] as given also in [24]. We also consider a common case of a constant  $\eta/s$  at its conjectured minimum value  $1/4\pi$ . In Fig. 2(a-d) show the differential elliptic flow  $v_2(p_T)$  for the three different beam energies at RHIC and LHC at the same centrality 10 – 20%. In Fig. 2a the results are shown for  $\eta/s = 1/4\pi$  all over the evolution of the system. As clearly shown in such a case it is not possible to predict an invariance  $v_2(p_T)$  in the range of collision energies explored, it breaks at the LHC energy (dot-dashed line) with a breaking up of about 20%. In this case where  $\eta/s = const$ , the  $v_2(p_T)$  at LHC energies is smaller than that at lower energies and this is an implicit effect coming from the different initial  $p_T$  distributions. At LHC energies the initial spectrum is flatter than that at lower energies and for fixed  $\eta/s$  it produces a smaller  $v_2(p_T)$ , for details see [25]. In Fig. 2b we study the effect of a temperature dependent  $\eta/s(T)$  with an  $\eta/s$  increasing in the cross over region and constant at  $1/4\pi$  in the QGP phase. Comparing Fig. 2a and 2b we can see that the effect of an increasing  $\eta/s$  at lower temperature is to reduce the elliptic flow at lower collision energy, the  $v_2(p_T)$  becomes more sensitive to the value of  $\eta/s$  at low temperature. This different behavior is determined by the different initial temperature and consequent lifetime of the fireball. In fact at RHIC energies such a lifetime is about 4 – 6 fm/c while at LHC it is about 10 fm/c. Therefore at RHIC the elliptic flow has not enough time to fully develop in the QGP phase, while at LHC the lifetime is long enough to let the  $v_2$  develop almost completely in the QGP phase and it becomes sensitive to the value of  $\eta/s$  in the QGP phase, similar result has been found in Ref. [26]. From this reasoning one has the hint that an invariant  $v_2(p_T)$  can be caused by a specific T-dependence of  $\eta/s$  that balances the suppression due to the viscosity above and below  $T_c$  where a minimum in  $\eta/s$  should occur. In Figs. 2c and 2d we have shown the effect of a temperature dependence of  $\eta/s$  for  $T > T_C$  in the QGP phase. Comparing Fig. 2c and 2d we see that the rapidly increasing  $\eta/s(T)$  affects more the system created at LHC and this generates again a larger splitting of the  $v_2(p_T)$  among the different energies. This means that also a strong T-dependence in the QGP phase is in contrast with the observed  $v_2(p_T)$  invariance. In Fig. 2d it is shown how it is possible to reproduce an almost perfect invariance (within a 5%) with the collision energy if we consider a linear temperature dependence  $4\pi\eta/s = T/T_0$ , according to the solid red line in Fig. 1. In Fig. 2d the symbols are the experimental results for the  $v_2$  [4] measured at RHIC and LHC energy, data taken from [2, 12]. However, we notice that the initial conditions are such that the  $p_T$  experimental spectra are fairly well reproduced for all the cases considered except for the case of  $\eta/s \sim T^2$  at LHC energy. In such a case the distribution of mini jets cannot be discarded and the final  $p_T$  spectra remain too flat if the  $\eta/s(T) \sim T^2$  and one would exclude this case.



**Figure 2.**  $v_2(p_T)$  at mid rapidity for 10% – 20% collision centrality. The solid line, the dashed line and the dot dashed line refer to:  $Au+Au$  at  $\sqrt{s} = 62.4$  GeV and  $\sqrt{s} = 200$  GeV and  $Pb+Pb$  at  $\sqrt{s} = 2.76$  TeV, respectively for different behavior of  $\eta/s(T)$  as indicated in the labels.

#### 4. Conclusions

We have studied within a transport approach the effect of a temperature dependent  $\eta/s$  on the generation of the  $v_2(p_T)$  from RHIC to LHC collision energies. We find that depending on the range of energies explored the suppression of the  $v_2(p_T)$  due to the viscosity of the medium has different contamination coming from hadronic phase or QGP phase. We have shown that it is possible to have a nearly invariant  $v_2(p_T)$  if  $\eta/s$  has a 'fall and rise' temperature behavior typical of a phase transition.

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