Heavy Flavor Suppression: Boltzmann vs Langevin

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Abstract. The propagation of heavy flavor through the quark gluon plasma has been treated commonly within the framework of Langevin dynamics, i.e. assuming the heavy flavor momentum transfer is much smaller than the light one. On the other hand a similar suppression factor R_{AA} has been observed experimentally for light and heavy flavors. We present a thorough study of the approximations involved by Langevin equation by mean of a direct comparison with the full collisional integral within the framework of Boltzmann transport equation. We have compared the results obtained in both approaches which can differ substantially for charm quark leading to quite different values extracted for the heavy quark diffusion coefficient. In the case of bottom quark the approximation appears to be quite reasonable.

1. Introduction

The experimental efforts at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies is to create and study the properties of matter called quark gluon plasma (QGP). The heavy flavors, namely, charm and bottom quarks are particularly playing a vital role to serve this purpose. The most common approach to study heavy flavors propagation in QGP is the Fokker-Planck [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] equation that can be realized from the Boltzmann equation which constitutes a significant simplification *i.e.* the heavy flavor momentum transfer is small or the scattering are sufficiently forward peaked. Thus it is a approximation of the full collision term of the Boltzmann equation. Such an approximation is expected to be asymptotically valid for $m/T \rightarrow \infty$, here we present a first study of the validity of such an approximation for b and c quark at temperature typically reached in ultrarelativistic heavy ion collisions at LHC.

2. Relativistic Langevin Equation

The relativistic Langevin equations of motion corresponds to evolve the particles in the phase space according to the following equations [1],

$$dx_i = \frac{p_i}{E} dt,$$
$$dp_i = -\Gamma p_i dt + C_{ij} \rho_j \sqrt{dt} \qquad (1)$$

where Γ and C_{ij} describe the drag force and the stochastic force in terms of independent Gaussian-normal distributed random variables ρ, which obey $< \rho_i \rho_j >= \delta_{ij}$ and $< \rho_i >= 0$, respectively. To study the momentum evolution of charm and bottom quarks, we use the Ito discretization.

3. Relativistic Boltzmann Transport Equation

The relativistic Boltzmann equation can be written as follows [14, 15]:

$$p^{\mu}\partial_{\mu}f(x,p) = \mathcal{C}(x,p) \tag{2}$$

where $\mathcal{C}(x,p)$ is the Boltzmann collision integral, the main ingredient of the cascade codes.

If we define $\omega(p,k)$, the rate of collisions which change the heavy quark momentum from p to $p-k$, then we have [2]

$$\mathcal{C}(x,p) = \int d^3k\,[\omega(p+k,k)f(p+k) - \omega(p,k)f(p)] \tag{3}$$

The first term in the integrand represents the gain of probability through collisions which knock the heavy quark into the volume element, and the second term represents the loss out of that volume element. If we expand $\omega(p+k,k)f(p+k)$ around k,

$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p) + k\frac{\partial}{\partial p}(\omega p) + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial_j}(\omega p) \tag{4}$$

We obtain the Fokker Planck/Langevin Equation. In the following we will discuss a first comparison between the full Boltzmann equation and the Langevin equation coming from Eq. 4.

4. Numerical results and discussion

The basic ingredients required for solving the Langevin equation are the drag and diffusion coefficients and the initial momentum distributions of heavy quarks. We are using $f^c_{t=0} = 4.1 * 10^2/(0.7+0.09p)^{15.44}$ and $f^b_{t=0} = 1/(57.74+p^2)^{5.04}$ as the initial momentum distribution of the charm and bottom quarks respectively. For the sake of comparison, we are solving the Langevin equation in a box where the bulk consists of only gluon at T=0.4 GeV. The elastic collisions of heavy quarks with gluon has been considered within the pQCD framework to calculate the drag and diffusion coefficients. In the present calculation we used $\alpha_s = 0.35$ and the Debye screening mass as $m_D = gT$. It can be mentioned that the experimental data on the nuclear suppression factor and the elliptic flow of nonphotonic electrons provide the best agreement to the experimental data for the spatial diffusion coefficient $D_x = 6/2\pi T$ [12]. Keeping this in mind we enhanced the pQCD cross sections by a factor 2 to match our diffusion coefficients as $D_x = 6/2\pi T$, so that we could extract conclusion of phenomenological interest from our calculation. On the other hand the relativistic Boltzmann equation also solved in a identical environment as the Langevin equation with the enhanced pQCD cross sections by a factor 2, in a Montecarlo cascade [14] based on the stochastic interpretation of the transition rate discussed in Ref [15].

In Fig 1 the ratio of Langevin to Boltzmann spectra for the charm quark has been displayed as a function of momentum at different times to quantify how much the ratio deviates from 1. We started the simulation at $t = 0$ with the same initial distribution for both Lanvevin and Boltzmann equations which leads to a ratio 1. So any deviation from 1 would reflect how much the Langevin deviations differ from the Boltzmann as a approximation. From Fig 1 it is observed that for $t = 4$ fm the deviation of Langevin from Boltzmann is around 25% and for $t = 6$ fm the deviation is around $35\% - 40\%$ at $p = 7$ GeV, which suggests Langevin approach overestimates the interaction considerably due to approximation it involved. we have seen that the effect can be significantly larger or smaller depending on the value of the drag and diffusion coefficients and/or on the angular depndence of the collisions. The same quantity is depicted in Fig 2 as a function momentum at different times for bottom quark. From Fig 2 it is shown that the ratio is almost unity at all the times considered in the manuscript. Therefore, for bottom quark the Langevin approach is really a good approximation of Boltzmann equation.

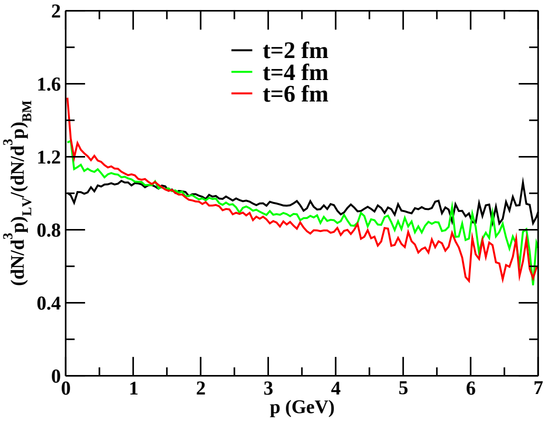

Figure 1. Ratio between the Langevin (LV) and Boltzmann (BM) spectra for charm quark as a function of momentum at different time

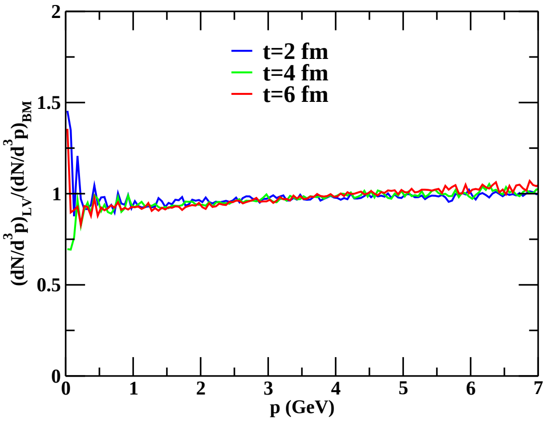

Figure 2. Ratio between the Langevin (LV) and Boltzmann (BM) spectra for bottom quark as a function of momentum at different time

We calculate the nuclear suppression factor, R_{AA}, using our initial $t = 0$ and final $t = t_f$ charm quark distribution as $R_{AA} = \frac{f_{t=f}}{f_{t=0}}$. The nuclear suppression factor, R_{AA}, has been displayed in Fig 3 as a function of momentum from both Langevin and Boltzmann side at $t = 6$ fm. From Fig 3 it is observed that the nuclear suppression factor differ substantially from Langevin to Boltzmann. Since the diffusion coefficient is very important for the phenomenological study, it will be very useful from phenomenological point of view to study how much the diffusion coefficient change from Langevin side to reproduce the same nuclear suppression factor of Boltzmann equation. It is observed that (Fig 3) we need to change (reduced) the diffusion coefficient of Langevin equation by 30% to get similar nuclear suppression factor as of Boltzmann equation, although it is not anyway possible to reproduce the same momentum dependence. Heavy flavor supression within Boltzmann tranport approach can be found in Ref. [16](see also Ref. [17]) .

5. SUMMARY AND CONCLUSIONS

We present a thorough study of the approximations involved by Langevin equation by mean of a direct comparison with the full collisional integral within the framework of Boltzmann

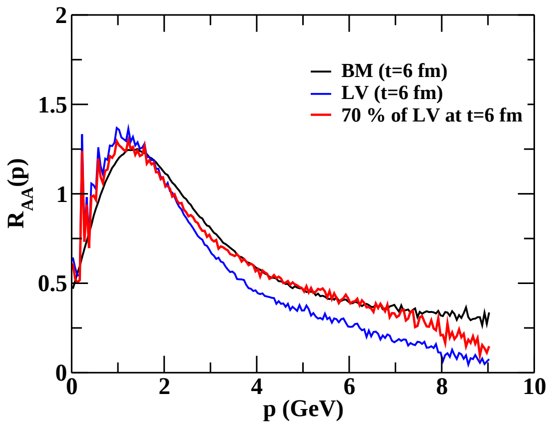

Figure 3. The nuclear suppression factor, R_{AA} as a function of momentum from the Langevin (LV) equation and Boltzmann (BM) equation for charm quark in a box at T=0.4 GeV

transport equation in a box where the bulk consists of only gluon at T=0.4 GeV. We found that the Langevin approach is a good approximation for bottom quark where as for charm quark Langevin approach deviates of about a 30% at intermediate momentum for a time evolution of 5-6 fm/c typical of the QGP created in heavy-ion collisions respect to the spectra calculated from the solution of the full Boltzmann transport equation. It has also been found that to get a similar suppression factor form both the approach we need to reduce the diffusion coefficient of the Langevin approach by around 30%. These results can have significant effects on the heavy ion phenomenology at RHIC and LHC energies. Calculation for a realistic fireball evolution as created in ultra-relativistic heavy-ion collisions is undergoing.

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