

# Landau hydrodynamics for non-central heavy-ion collisions and longitudinal scaling of elliptic flow

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**Abstract.** Landau solution of hydrodynamics is generalized for the non-central high-energy nuclear collisions. The multiparticle production after hydrodynamic expansion from the transversely asymmetric initial state shows elliptic flow formation. Moreover, obtained solution does reproduce the observed longitudinal scaling of the elliptic flow for the different collision energies at LHC and RHIC. It will be argued, that the analyticity of the solution allows us to fit experimental data in order to obtain initial conditions.

## 1. Introduction

The question of particle multiplicity distribution in High-Energy Nuclear Collisions has a long-lasting history. In 1950 Fermi suggests to apply thermodynamics to particle production in high-energy collisions. In 1953 Landau improves the idea with the assumption, that system strongly interacts and particles are produced after the ideal hydrodynamic expansion. And now, this approximate and analytic solution of relativistic hydrodynamics works well not only for multiplicity distribution at LHC and RHIC energies, but with some modifications, it reproduces the experimentally observed longitudinal scaling of elliptic flow.

## 2. Solution of Ideal Hydrodynamics for Particle Production

The solution is inspired by the Landau hydrodynamic model of multi-particle production [1, 2] and [3], where the hydrodynamic equations for the energy-momentum conservation,

$$\partial_\mu T^{\mu\nu} = 0, \tag{1}$$

are solved by separating longitudinal and transverse expansions. It is realistic assumption, as the longitudinal expansion is much faster than transverse and starts with much thinner longitudinal profile. The equation of state of ideal relativistic gas,  $P = e/3$ , and the energy-momentum tensor in usual form,

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - Pg^{\mu\nu}, \tag{2}$$

closes the solvable system of equations.

### 2.1. Longitudinal expansion

The equations of the hydrodynamic longitudinal expansion in 1+1 dimension, along  $z$  axis reads as:

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0z}}{\partial z} = 0, \quad \frac{\partial T^{0z}}{\partial t} + \frac{\partial T^{zz}}{\partial z} = 0. \quad (3)$$

Solution of the equations of hydrodynamics starts by transforming relativistic velocity field components to rapidity terms, as:  $u^0 = \cosh y$ ,  $u^z = \sinh y$ , and the final solution for energy density,  $e(y_+, y_-)$ , and rapidity,  $y(y_+, y_-)$ , reads as [4]:

$$e(y_+, y_-) = e_0 \exp[-4/3(y_+ + y_- - \sqrt{y_+ y_-})], \quad (4)$$

$$y(y_+, y_-) = (y_+ - y_-)/2, \quad (5)$$

while  $z = t \tanh y$ . The above solution of 1+1-dimensional relativistic hydrodynamics equation is connected to the solution of transverse expansion, in order to obtain multiplicities of produced particles for different rapidities.

### 2.2. Transverse expansion

The transverse expansion of hydrodynamic eq. 1 in polar coordinates reads as:

$$\frac{\partial T^{0r}}{\partial t} + \frac{\partial T^{rr}}{\partial r} = 0. \quad (6)$$

Inserting energy-momentum tensor expressions to the above equation and using ideal gas equation of state,  $P = e/3$ , one gets:

$$4e(u^0)^2 \frac{\partial v_r}{\partial t} + 4e(u^0)^2 \frac{\partial v_r^2}{\partial r} + \frac{\partial e}{\partial r} = 0. \quad (7)$$

$$t_{FO} = 2 \cosh y \sqrt{\frac{2aR_\phi}{(1 - f(R_\phi))}}, \quad (8)$$

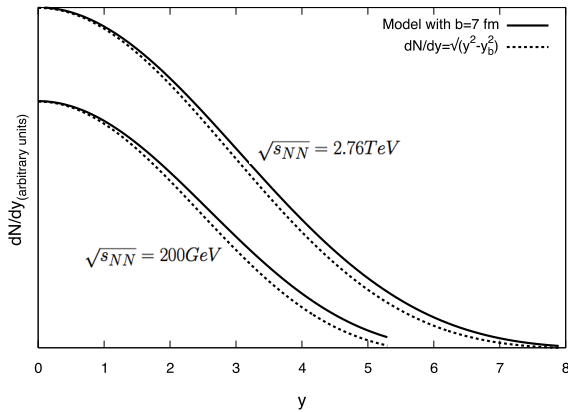
where new function,  $f(R_\phi) = e(r = R_\phi)/e(r = 0)$ , is introduced as a fraction of energy density at the edge of the system with respect to the energy density at the center. The transverse and longitudinal solutions are matched at the time  $t = t_{FO}$ , starting the freeze-out stage, where particles stream freely to the detectors. Knowing, that  $dS = s u^0 dz$  at a given time within element  $dz$  and entropy density,  $s = ce^{3/4}$ , we express entropy change over rapidity from the energy density formula (4), as:

$$\frac{dN}{dy} \propto \frac{dS}{dy} = ce_0^{3/4} \exp[-(y_+ + y_- - \sqrt{y_+ y_-})] \frac{t_{FO}}{\cosh y}. \quad (9)$$

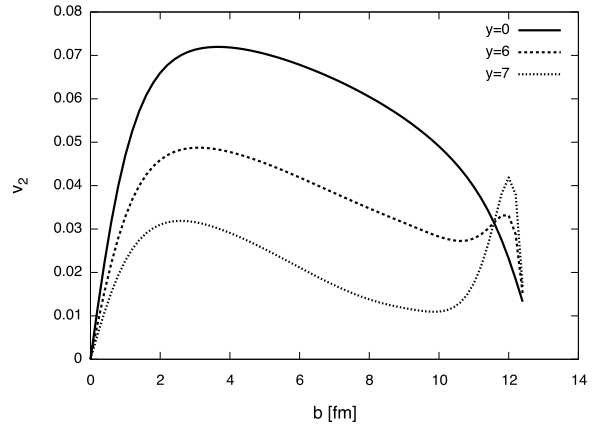
The transformation back to the  $(t, z)$  coordinates is:  $y_+ = \ln((t + z)/\Delta)$ ,  $y_- = \ln((t - z)/\Delta)$ , while  $\Delta$  is the initial thickness of the system in the beam direction,  $z$ . Also,  $\Delta$  is the initial condition after which equation of state assumed to be valid and evolution equations (3) are applied. Inserting the solution for the FO time equation (8) into the entropy equation above and assuming that the number of produced particles is directly proportional to the entropy,  $dN \propto dS$ , one can obtain the number of particles for different rapidities at a fixed angle  $\phi$ . In order to find particle distribution  $d^2N/dy d\phi$  one needs to find  $f(R_\phi)$  and  $\Delta$ , what corresponds to the initial conditions.

### 2.3. Initial conditions

In order to show the validity of the obtained solution, simple and transparent initial conditions will be used. In this case the widely accepted and analytically simple Wounded Nucleon (WN) model [5] will be used to parametrize initial conditions. It is based on the Woods-Saxon nuclear density parametrization [6], as follows:  $\rho_A(\mathbf{r}) = \frac{\rho_0}{1 + \exp(\frac{r-R_A}{d})}$ , which is continuous and can be connected to the Landau equations straightforwardly. The main requirement for the initial conditions and the new function  $f(R_\phi)$  is that for the central collision case,  $b = 0$ , the result must be equal to the original Landau one. The density of wounded nucleons in the transverse



**Figure 1.** Rapidity distribution of produced particles in central collisions (full lines) and in peripheral collisions with impact parameter  $b = 7 \text{ fm}$  (dashed line) for LHC and RHIC energies



**Figure 2.** Elliptic flow,  $v_2$ , dependence on the impact parameter,  $b$ , for different rapidities.

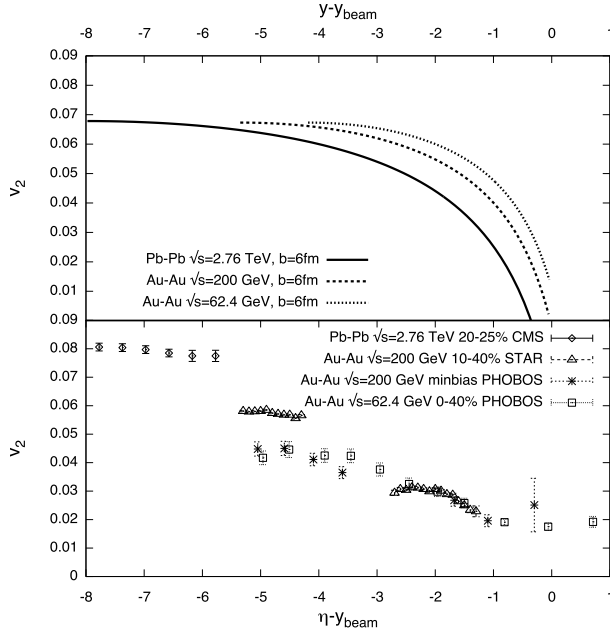
plane and in polar coordinates,  $(r, \phi)$ , can be obtained by:

$$n_{WN}(r, \phi) = T_A(r, \phi) \left[ 1 - \left( 1 - \frac{\sigma T_B(r, \phi)}{B} \right)^B \right] + T_B(r, \phi) \left[ 1 - \left( 1 - \frac{\sigma T_A(r, \phi)}{A} \right)^A \right].$$

The thickness functions are expressed, in usual way:  $T_A(r, \phi) = T_A(x - b/2, y) = \int dz \rho_A(\mathbf{r})$ , using Woods-Saxon parametrization with  $R_A = 1.12A^{1/3} - 0.86A^{-1/3}$  [fm],  $d = 0.54$  [fm] and  $n_0 = 0.17 \text{ fm}^{-3}$ . Now assuming, that energy density is proportional to the Wounded Nucleon density:  $e(r, \phi; b) \propto n_{WN}(r, \phi; b)$ , the function  $f(R_\phi)$  can be obtained. It is by definition, the ratio of energy density at the edge of the system with energy density at the center, for the fixed impact parameter  $b$  and reads as:

$$f(R_\phi) = \frac{n_{WN}(R_\phi, \phi; b) - \min(n_{WN}(R_\phi, \phi; b))}{n_{WN}(0, 0; b)}. \quad (10)$$

The radius of the system,  $R_\phi$ , is dependent on the angle  $\phi$  and is obtained from the geometry on how two circles overlap, as:  $R_\phi^2 + R_\phi b \cos \phi + \frac{b^2}{4} - R_A^2 = 0$ . The term  $\min(n_{WN}(R_\phi, \phi; b))$  is a minimal density at the edge of the system and is used in order to have the original Landau solution for  $R_\phi = R_A$ , so that  $f(R_A) = 0$ . In the case of Woods-Saxon, the density at the edge of the nuclei at  $r = R_A$  is not zero, so the minimal value is subtracted. Now the acceleration term in (7) is the same in central collision and in Landau, but for peripheral collisions acceleration does depend on the angle  $\phi$ . Finally, to calculate the elliptic flow one should merge equations (8), (9)



**Figure 3.** Elliptic flow,  $v_2$ , dependence on the rapidity, shifted by the beam rapidity,  $y_{beam} = \ln(\sqrt{s_{NN}}/m_N)$ , for: Pb-Pb at impact parameter  $b = 6\text{fm}$  collisions at LHC energy  $\sqrt{s_{NN}} = 2.76\text{TeV}$  (full-line), Au-Au at  $b = 6\text{fm}$  at  $\sqrt{s_{NN}} = 200\text{GeV}$  (dashed-line) and at  $\sqrt{s_{NN}} = 62.4\text{GeV}$  (dotted-line) reactions used at RHIC. Data are from [7], [8] and [9]

and the definition of the elliptic flow:  $v_2(y) = \frac{\int d\phi (dN/d\phi dy) \cos(2\phi)}{\int d\phi (dN/d\phi dy)}$ , with the initial energy density gradient  $f(R_\phi)$  and initial longitudinal thickness  $\Delta(\phi)$ . For the peripheral collisions initial thickness is expressed, as  $\Delta(\phi) = \kappa_\phi R_A / \gamma$ , where  $\kappa_\phi = \sqrt{n_{WN}(R_\phi, \phi; b) / \max(n_{WN}(R_\phi, \phi; b))}$ . The following means, that longitudinal expansion (3) starts with azimuthally asymmetric initial thickness, which is wider, where initial nuclear density is higher. The term  $\max(n_{WN}(R_\phi, \phi; b))$  is used to have no effect of the modification for central collision case,  $\kappa_\phi(b = 0) = 1$ . Obtained results are shown in figure (3) in comparison with LHC and RHIC experiments.

### 3. Conclusions

Presented analytical solution for the non-central heavy ion collisions reproduces experimentally observed elliptic flow formation and its longitudinal scaling. Moreover, using experimental data of produced charged particles for different rapidities in Event-on-Event basis,  $d^2N/d\eta d\phi$ , it will be possible to find the initial energy density gradient function in transverse plane,  $f(R_\phi)$ , and the initial thickness,  $\Delta(\phi)$ .

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### References

- [1] Landau L D 1953 *Izv. Akad. Nauk SSSR* **17** 51
- [2] Belenkij S Z and Landau L D 1956 *Nuovo Cim. Suppl.* **3S10** 15
- [3] Tamosiunas K 2011 *Eur. Phys. J. A* **47** 121 (*Preprint DOI 10.1140/epja/i2011-11121-5*)
- [4] Wong C Y 2008 *Phys. Rev. C* **78** 054902 (*Preprint arXiv:0808.1294*)
- [5] A Bialas M B and Czyz W 1976 *Nucl. Phys. B* **111** 461
- [6] Woods R and Saxon D 1954 *Phys. Rev.* **95** 577
- [7] CMS collaboration 2013 *Phys. Rev. C* **87** 014902 (*Preprint arXiv:1204.1409*)
- [8] Abelev B I *et al.* 2008 *Phys. Rev. C* **77** 54901
- [9] Alver B *et al.* (PHOBOS) 2007 *Phys. Rev. Lett.* **98** 242302