

# Gluon radiation by heavy quarks

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**Abstract.** The gluon emission cross section of a heavy quark colliding a light parton from the plasma is computed in pQCD at leading order. It is possible to compute the complete energy dependence of the result and therefore to assess the range of applicability in energy of standard high-energy approximation.

## 1. Introduction

It is generally assumed that in ultrarelativistic heavy ion collisions a plasma of quarks and gluons is produced, and then quickly expands and hadronizes. The main objective of present experiments at ultrarelativistic heavy ion colliders is to study this plasma. The study of the plasma properties is difficult because the initial momentum distribution of the light quarks is not known and that before being detected the system passes the chiral/confinement phase transition in which hadrons are formed. Hadrons interact on the way to the detectors what modifies their spectrum. Therefore soft light hadrons do not carry information on the early stage of the plasma.

The situation is much better for heavy quarks (c and b) which are created in hard processes. Here perturbative QCD allows for the calculation of the production cross sections (in contradistinction to light quarks) and these cross sections have also been measured. Also the details of the chiral/confinement phase transition are less important than for light quarks because, due to its mass, the momentum of the heavy quark determines the momentum of the open charm hadrons. In addition, the momentum distribution at production and at the transition is very different from that expected if the heavy quarks are in thermal equilibrium with the plasma of light quarks and gluons. Therefore the modification of the initial momentum distribution by the interaction of the heavy quarks with the plasma carries information on the plasma properties.

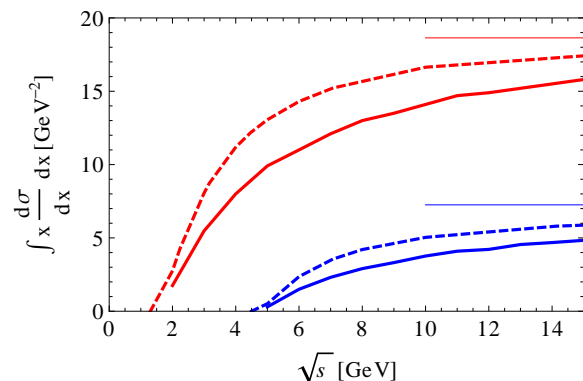
The interaction of the heavy quark with the plasma has two parts, elastic collisions and radiative collisions. For the first a model was developed [1] in which the cross section of the elementary interactions are calculated by perturbative QCD with a running coupling constant and an infrared behavior adjusted so as to match hard thermal loop calculations. Embedding these cross sections in the hydrodynamical description of the expanding plasma of Heinz and Kolb it was shown that the collisional energy loss underpredicts the measured energy loss of heavy mesons at large momenta as well as their elliptic flow by roughly a factor of two.

The present work extends our pQCD calculation toward the calculation of the radiative energy loss, following the Gunion-Bertsch approach to light quark radiation phenomenology in high-energy collisions [2]. We compute the gluon emission cross section of a heavy quark (instead

of a light quark in [2]) colliding a light parton from the plasma. The heavy quark is supposed to be relativistic but with an intermediate energy so that coherence effects are not dominant. We keep track of the full energy dependence of the result in order to assess the range of applicability in energy of the standard high-energy approximation. The latter has the advantage of leading to very compact expression for the various integrated cross sections and also of making more transparent the discussion of physical phenomenons. We discuss in particular the occurrence of the dead cone effect.

## 2. Energy dependence

The quantity  $\int x (d\sigma/dx) dx$ , which is related to the radiated energy loss  $dE_{\text{rad}}/dz$  normalized to the density of scatterers  $\rho$  and the quark energy  $E_Q$ , was recently computed for a heavy quark traversing a hot QCD medium [3]. The result is shown in Fig. 1 for a charm quark  $m_c = 1.3$  GeV and a bottom quark  $m_b = 4.5$  GeV.



**Figure 1.** Energy dependence of  $\int x (d\sigma/dx) dx$  for charm (upper curves) and bottom (lower curves). Solid lines show the full energy dependence and dashed lines correspond to an approximation where the influence of finite energy is taken into account only in phase space boundaries. Thin straight lines show the asymptotic results. The screening scale is taken as  $\mu = 0.4$  GeV.

It can be seen that at respectively  $E_c \sim 10$  GeV and  $E_b \sim 40$  GeV, 50% of the respective asymptotic behaviors are reached. This indicates first that the finite energy constraint is effective in the high-energy range of relevance nowadays, say  $E_Q \approx p_T \sim 10 - 100$  GeV, especially for the  $b$  sector. A simplified model in which the fully differential cross section is approximated by its high-energy limit ( $s \rightarrow \infty$ ) but phase space limitation is taken into account upon integration is capable of explaining typically half of the deviation between the full result and the asymptotic limit.

A second consequence of the above observation is that high-energy phenomenons are at least indicative of the physics at work in the range  $10 - 100$  GeV. The high-energy limit expressions can thus serve as basis for the discussion of the full result.

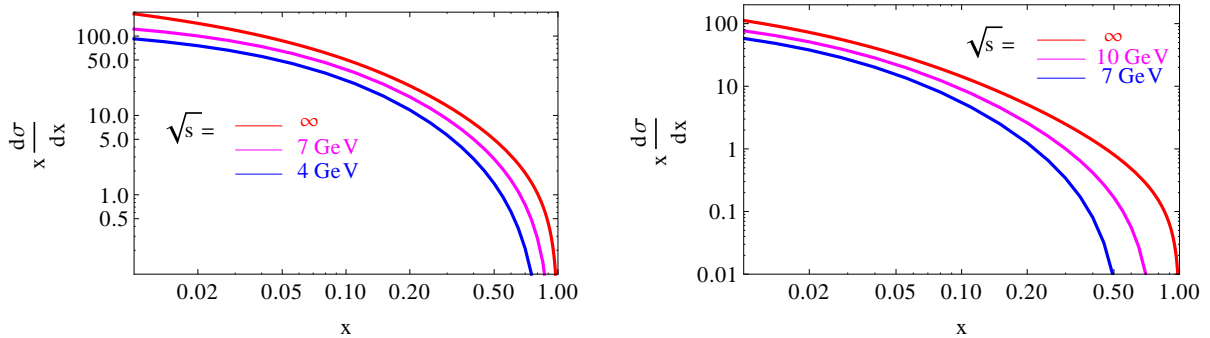
The asymptotic behavior roughly scales as

$$\int x \frac{d\sigma}{dx} dx \propto \frac{1}{\mu m_Q}$$

giving a suppression of heavy quark radiation  $\propto 1/m_Q$  but also showing a sensitivity to the details of screening. This behavior results from contributions from both below  $x_M = \mu/m_Q$  ('hard' collisions) and above ('soft' collisions) to the  $x$ -integral.

The fractional energy loss spectrum  $x d\sigma/dx$  is displayed in Fig. 2 for charm (left) and bottom (right). The top curve on each plot is the asymptotic result. It approaches the finite energy results best in the intermediate  $x$ -range, slightly below  $x_M = \mu/m_Q$ , which is also the range of relevance for quenching phenomenology [4].

Below we give some details on the origin of radiation in the high-energy limit.



**Figure 2.** Charm (left) and bottom (right) fractional energy loss spectrum,  $x d\sigma/dx$ , for various  $\sqrt{s}$ , together with the high-energy approximation  $\sqrt{s} \rightarrow \infty$ .  $\mu = 0.4$  GeV.

### 3. High-energy approximation

Let us first explain how the high-energy approximation is realized. We consider the collision of a heavy quark  $Q$  onto a massless parton  $q$  with the production of a gluon  $g$ :

$$Q(P) + q(q) \rightarrow Q(P') + q(q') + g(k)$$

where 4-momenta are indicated between brackets. The heavy quark momentum may be parameterized as

$$P = p + \frac{m_Q^2}{s - m_Q^2} q$$

where  $p$  is a second light-like vector, in addition to  $q$  the light-like momentum of the initial massless quark, and chosen such that  $2p \cdot q = s - m_Q^2$ . The gluon momentum may be written as

$$k = xp + yq + \vec{k}_\perp,$$

where  $x$  is the light-cone momentum fraction already encountered and  $\vec{k}_\perp$  the transverse momentum of the gluon which is a second quantity of interest for radiation.  $y \equiv y(x, \vec{k}_\perp)$  is fixed by imposing  $k^2 = 0$ . The last relevant momentum is the momentum transfer to the light quark  $\ell = q - q'$  and we will also write  $\ell^2 = t$ .

At large  $s$  the fully differential cross section factorizes

$$\frac{d\sigma}{dx d^2k_\perp d^2\ell_\perp} \approx \frac{d\sigma_{\text{el}}}{d^2\ell_\perp} \times P_g(x, \vec{k}_\perp, \vec{\ell}_\perp),$$

where high-energy means that  $s - m_Q^2 \gg |t|, \vec{k}_\perp^2$  [3]. Then the  $s$ -dependence disappears from the cross section, be it either differential or integrated. The reason for this simplification comes from  $d\sigma_{\text{el}}/d^2\ell_\perp \propto 1/t^2$  at large  $|t|$  and  $P_g \propto 1/k_\perp^4$  at large  $k_\perp$  making the cross section insensitive to  $s$  at large  $s$ . In return the cross section is sensitive to low momentum transfer and in particular to the Debye screening effect in the plasma. At small  $t$  we use the prescription

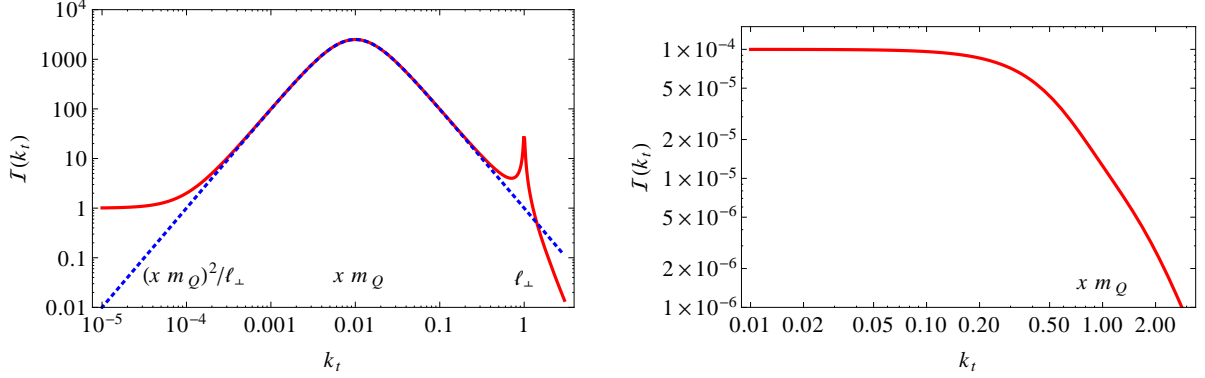
$$\frac{d\sigma_{\text{el}}}{d^2\ell_\perp} \propto \frac{1}{t^2} \rightarrow \frac{1}{(t - \mu^2)^2}$$

with  $\mu$  related to the Debye mass  $m_D$ . This sets a natural scale for  $\ell_\perp \sim \mu$  in the cross sections.

Now

$$P_g \propto \left( \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + x^2 m_Q^2} - \frac{\vec{k}_\perp - \vec{\ell}_\perp}{(\vec{k}_\perp - \vec{\ell}_\perp)^2 + x^2 m_Q^2} \right)^2 \quad (1)$$

and there are different regimes depending on the comparison between  $\ell_\perp$  and  $x m_Q$ . If  $\ell_\perp \gg x m_Q$  we have a hard scattering regime, with a behavior indicated in Fig. 3 (left).



**Figure 3.** Gluon distribution  $P_g$  (averaged over the angle between  $\vec{k}_\perp$  and  $\vec{\ell}_\perp$ ) for  $\ell_\perp = 1$  and  $x m_Q = 0.01$  (hard scattering, left) and  $\ell_\perp = 0.01$  and  $x m_Q = 1$  (soft scattering, right).

In this regime, there is no interference between the two terms in the bracket of eq. (1) at intermediate  $\vec{k}_\perp$  (where the second term is negligible), as can be seen in the plot where the dotted curve is simply the resulting  $P_g$ , eq. (1), when dropping the second term. As a consequence,  $P_g \propto 1/\vec{k}_\perp^2$  in the range  $x m_Q < k_\perp < \ell_\perp$ , and

$$\int P_g d^2 k_\perp \sim \ln \frac{\ell_\perp}{x m_Q}.$$

The suppression of radiation for  $k_\perp < x m_Q$  generates a dead cone. Replacing  $\ell_\perp$  by  $\mu$  the range  $x < x_M \equiv \mu/m_Q$  in  $d\sigma/dx$  is identified.

If  $\ell_\perp \ll x m_Q$  (soft scattering regime) there is a strong interference at all  $\vec{k}_\perp$  as shown in Fig. 3 (right), leading to

$$\int P_g d^2 k_\perp \sim \frac{\ell_\perp^2}{x^2 m_Q^2}.$$

Replacing  $\ell_\perp$  by  $\mu$  leads to the identification of the  $x > x_M$  range in  $d\sigma/dx$ . In this regime, the mass dependence is not reducible to the dead cone phenomenon.

## References

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