

# Dynamical freeze-out criterion in event-by-event hydrodynamics

Hannu Holopainen<sup>1</sup> and Pasi Huovinen<sup>1</sup>

<sup>1</sup> Frankfurt Institute for Advanced Studies, Ruth-Moufang-Straße 1,  
D-60438 Frankfurt am Main, Germany

E-mail: huovinen@fias.uni-frankfurt.de

**Abstract.** In hydrodynamical modelling of ultrarelativistic heavy-ion collisions the freeze-out is typically assumed to take place on a surface of constant temperature or energy density. In this work we apply a dynamical freeze-out criterion, which compares the hydrodynamical expansion rate with the pion scattering rate, to an event-by-event ideal hydrodynamics at the full RHIC collision energy. We present hadron spectra and elliptic flow calculated using (2+1)-dimensional ideal hydrodynamics, and show the differences between constant temperature and dynamical freeze-out criteria. We find that when the freeze-out ratio is fixed to one, the different freeze-out criteria lead to slightly different spectra and  $v_2(p_T)$  in the event-by-event calculations.

## 1. Introduction

In hydrodynamical modelling of heavy-ion collisions the fluid dynamical description is usually assumed to break down, and the momentum distributions to freeze-out, on a thin surface of constant temperature or energy density. However, it has been argued for a while ago that a more physical criterion would be when the expansion rate of the fluid exceeds the scattering rate of the constituents of the fluid [1]. Such a dynamical criterion has been applied to hydrodynamical modelling before [2, 3], but it has not been widely used. In this contribution we study how a dynamical freeze-out criterion affects the final pion and proton distributions and their elliptic anisotropy in event-by-event hydrodynamics. We extend our previous results shown in Ref. [4] by using an equation of state (EoS) for a chemically frozen system.

## 2. Fluid dynamics

Since we want to concentrate on the effects of the freeze-out criterion, we ignore all the complications of viscosity, and use a modified version of the ideal fluid model described in Ref. [5]. The model is boost invariant, and includes finite net baryon density, which, however, does not contribute to pressure. As an EoS we use the *s95p-PCE-v1* parametrisation [6], which contains a chemical freeze-out at temperature  $T_{\text{chem}} = 150$  MeV to reproduce the observed  $\pi/p$  ratio at RHIC [7].

We initialise the system using a Monte Carlo Glauber approach. The entropy density is given as a mixture of binary (75%) and wounded nucleon (25%) profiles. A Monte Carlo Glauber model gives only the positions of the wounded nucleons and binary collisions and we need to distribute entropy around these positions before we can initialise hydrodynamical evolution. Our choice is

a 2-dimensional Gaussian:

$$s(x, y) = \text{const.} \sum_{\text{wn,bc}} \frac{1}{2\pi\Sigma^2} \exp \left[ -\frac{(x-x_i)^2 + (y-y_i)^2}{2\Sigma^2} \right], \quad (1)$$

where  $\Sigma$  is a free parameter controlling the width of the Gaussian. Typical values for this fluctuation size parameter is order of 0.5 fm, and our choice here is 0.8 fm (see Ref. [4]). For net-baryon density a pure wounded nucleon profile is used. To obtain averaged initial state, we evaluate an average of 1000 event-by-event fluctuating initial states, which include the above mentioned smearing by Gaussians. As initial time of the evolution we use  $\tau_0 = 0.6$  fm.

### 3. Dynamical freeze-out criterion

A usual requirement for the validity of fluid dynamics is that the Knudsen number must be much larger than one. Since the ratio of the expansion rate  $\theta$  to the scattering rate  $\Gamma$  is the ratio of an inverse of a macroscopic scale to an inverse of a microscopic scale, it can be identified as a Knudsen number. Thus the requirement that the scattering rate is larger than the expansion rate is equivalent to requiring that the Knudsen number is larger than one. We assume that fluid dynamics is valid until  $K = \theta/\Gamma = 1$ , and use  $K_f = 1$  as the freeze-out criterion in the dynamical case. As a baseline for comparison we use freeze-out at constant temperature of  $T_f = 120$  MeV.

The expansion rate of the system,  $\theta = \partial_\mu u^\mu$ , can be evaluated once the flow velocity is known, but the scattering rate is not given by hydrodynamics, but requires microscopic calculations. Several evaluations of the scattering rate of pions in equilibrated pion gas can be found in the literature [2, 3, 8], but since we are interested in pion scatterings in chemically frozen hadron gas, we calculate the rate ourselves. In semiclassical approximation the scattering rate of pions in kinetically (but not necessarily chemically) equilibrated hadron-resonance gas is given by

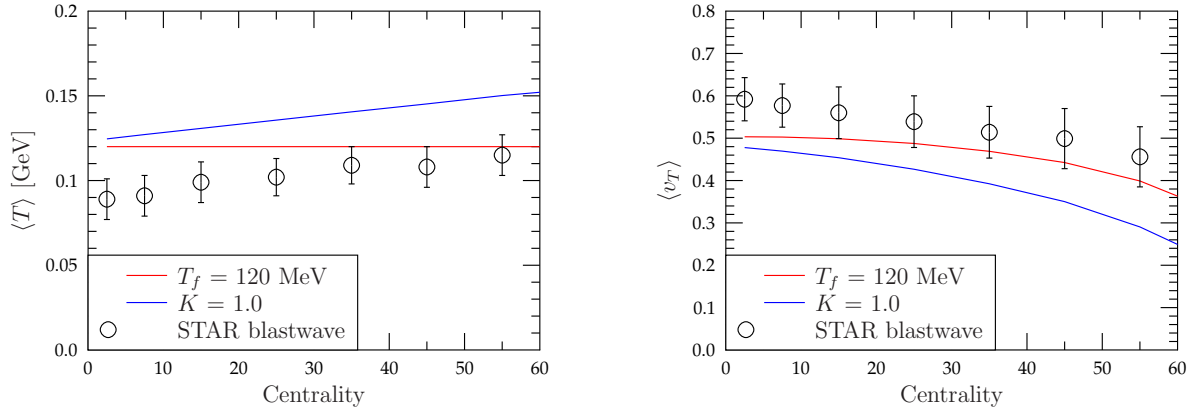
$$\Gamma = \frac{1}{n_\pi(T, \mu_\pi)} \sum_i \int d^3 p_\pi d^3 p_i f_\pi(T, \mu_\pi) f_i(T, \mu_i) v_{\pi i}(s) \sigma_{\pi i}(s), \quad (2)$$

where  $n_\pi$  is the density of pions,  $f_i(T, \mu_i)$  is the thermal distribution function,  $v_{\pi i}$  is the relative velocity,  $\sigma_{\pi i}$  is the cross section for scattering of pion with particle  $i$ , and the summation over  $i$  runs over all particle species included in the EoS. Cross sections are evaluated as in the UrQMD hadron cascade [9], *i.e.* the main contribution comes from resonance formation which is evaluated using the relativistic Breit-Wigner formula. The scattering rates are different for different hadrons, and therefore different hadrons should freeze-out at different times. Implementation of such a sequential freeze-out in fluid dynamics is very difficult, and it has also been argued that a proton requires more than one scattering with pions to change its momentum significantly [10]. To simplify the problem we thus assume that all hadrons freeze-out simultaneously.

Once the constant temperature or constant Knudsen number surface is found, the particle and resonance distributions are calculated using the conventional Cooper-Frye prescription, and the contributions from resonance decays are added to particle distributions.

### 4. Results

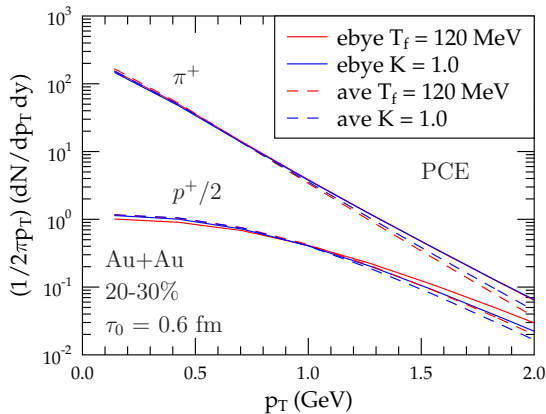
We consider Au+Au collisions at Relativistic Heavy Ion Collider (RHIC) with  $\sqrt{s_{NN}} = 200$  GeV. In Fig. 1 we show the average temperature and flow velocity on the freeze-out surface as function of centrality. The initial state is the average one, and we compare our results to those of the blast-wave fit by the STAR collaboration [11]. Trivially constant temperature freeze-out leads to a centrality independent temperature, whereas constant Knudsen number leads to the hotter surface the more peripheral the collision. As well known, the more central the collision, the larger the flow velocity, but the dependence on centrality is stronger for the dynamical freeze-out criterion. For both criteria the temperature and velocity differs from the blast-wave fits,



**Figure 1.** Average temperature (left) and flow velocity (right) on the freeze-out surface as function of centrality in  $Au + Au$  collisions at  $\sqrt{s_{NN}} = 200$  GeV using freeze-out at constant temperature (red) or at constant Knudsen number (blue). Ideal fluid calculations are compared with the STAR blast-wave fit [11].

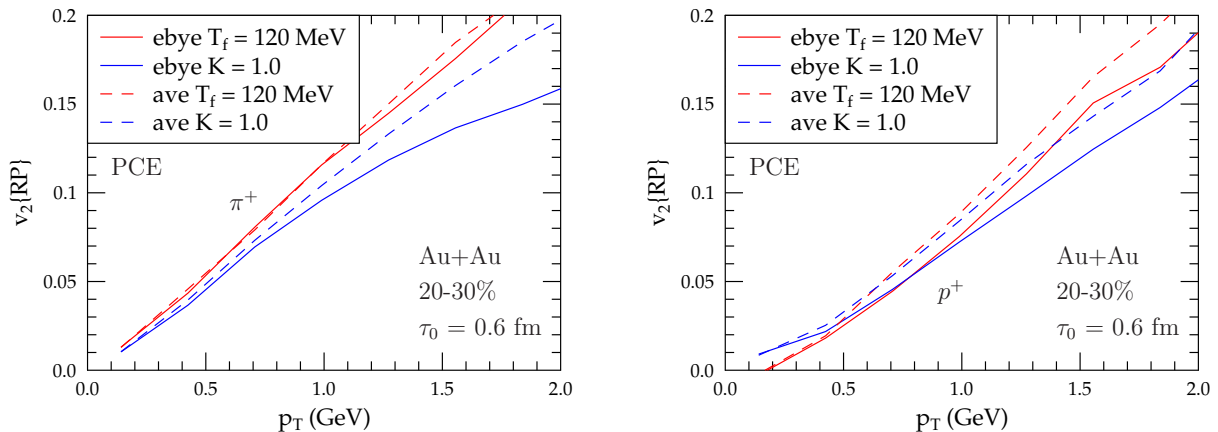
but this is understandable: The shape of the freeze-out surface affects the favoured values of temperature and flow velocity, and fluid dynamical calculation leads to more complicated shape of the freeze-out surface than a simple blast-wave fit. What is interesting although, is that the slopes of the centrality dependence of temperature and flow velocity are similar in the blast-wave fit and in the calculation with dynamical freeze-out criterion.

Fig. 2 depicts the  $p_T$ -spectra of positive pions and protons at 20 – 30% centrality. As well known [5], when all the other parameters are kept unchanged, event-by-event calculation leads to stiffer spectra than the averaged initial state calculation. The constant Knudsen number freeze-out leads to slightly steeper spectra than the constant temperature freeze-out, except for pions in the event-by-event case. Thus, to obtain similar  $p_T$ -distributions slightly larger Knudsen number  $K_f > 1$  might be required for the averaged initial state. On the other hand, since the pion spectrum is independent of the freeze-out criterion in the event-by-event case, a slightly different initial time, or initial shape might be required to reproduce the  $T_f = 120$  MeV result in that case.



**Figure 2.** The  $p_T$ -distributions of positive pions and protons in  $Au + Au$  collisions at  $\sqrt{s_{NN}} = 200$  GeV using freeze-out at constant temperature (red) or at constant Knudsen number (blue). Calculations are done for both averaged (dashed) and fluctuating (solid) initial states.

In Fig. 3 the pion and proton  $p_T$ -differential  $v_2$  are shown in collisions with 20–30% centrality. Since our  $p_T$ -distributions were not tuned to be the same in all the cases, one cannot draw



**Figure 3.** The  $p_T$ -differential elliptic flow of pions (left) and protons (right) in  $Au+Au$  collisions at  $\sqrt{s_{NN}} = 200$  GeV using averaged (dashed) or fluctuating (solid) initial states, and freeze-out at constant  $T = 120$  MeV temperature (red) or at constant Knudsen number  $K = 1$  (blue).

any strong conclusions based on the  $v_2(p_T)$  results shown here. Nevertheless, when constant temperature freeze-out is used, the pion  $v_2(p_T)$  is not sensitive to the event-by-event fluctuations of the initial state [5], but the dynamical criterion creates some sensitivity to fluctuations at  $p_T \gtrsim 1$  GeV. More strikingly, the dynamical criterion reduces pion  $v_2(p_T)$  by  $\sim 10\%$  both in the average initial state and in the event-by-event calculation. For protons the situation is reversed: The fluctuations have now a clear reducing effect independent of the freeze-out criterion, whereas the freeze-out criterion has a weaker and  $p_T$ -dependent effect. At very low  $p_T$  the dynamical criterion increases and at large  $p_T$  decreases the proton  $v_2(p_T)$ .

## 5. Conclusions

We argued that the constant temperature/density freeze-out is an oversimplification, but the effects of the freeze-out criterion on spectra and  $v_2(p_T)$  turned out to be small. The  $\sim 10\%$  reduction in pion  $v_2(p_T)$  is interesting, but it is unknown whether it survives if  $K_f$  is taken as a free parameter, and chosen to lead to the same  $p_T$ -distributions than  $T_f$ .

## Acknowledgments

The work of H.H. was supported by the Extreme Matter Institute (EMMI) and the work of P.H. by BMBF under contract no. 06FY9092.

## References

- [1] Bondorf J P, Garpman S I A and Zimanyi J 1978 *Nucl. Phys. A* **296** 320
- [2] Hung C M and Shuryak E V 1998 *Phys. Rev. C* **57** 1891
- [3] Eskola K J, Niemi H and Ruuskanen P V 2008 *Phys. Rev. C* **77** 044907
- [4] Holopainen H and Huovinen P 2012 *J. Phys. Conf. Ser.* **389** 012018
- [5] Holopainen H, Niemi H and Eskola K J 2011 *Phys. Rev. C* **83** 034901
- [6] Huovinen P and Petreczky P 2010 *Nucl. Phys. A* **837** 26
- [7] Huovinen P 2008 *Eur. Phys. J. A* **37** 121
- [8] Daghig R G and Kapusta J I 2002 *Phys. Rev. D* **65** 064028
- [9] Bass S A *et al.* 1998 *Prog. Part. Nucl. Phys.* **41** 255
- [10] Prakash M, Prakash M, Venugopalan R and Welke G 1993 *Phys. Rept.* **227** 321
- [11] Adams J *et al.* (STAR Collaboration) 2004 *Phys. Rev. Lett.* **92** 112301