Shear viscosity of a multi-component hadronic system

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Abstract. The shear viscosity η and entropy density s of a hadron gas with zero baryon number density are calculated using the Chapman-Enskog and virial expansion approaches, respectively. Interactions are included via the K-matrix parametrization of cross sections preserving the unitarity of the S-matrix. In the four component mixture $(\pi - K - N - \eta)$, a total of 57 resonances up to 2 GeV mass are included. Interactions forming resonances reduce the magnitude of η and increase s, both effects serving to progressively reduce η/s as the temperature nears the QCD phase transition temperature.

1. Introduction

Studies of strongly interacting matter at high temperature T have revealed the qualitative behavior of the temperature dependence of the shear viscosity η and its ratio with entropy density s [1, 2]. The ratio η/s decreases with increasing T in the hadronic sector. Perturbative QCD calculations [3] predict an η/s that increases logarithmically with T in the quark-gluon plasma (QGP). This ratio, conjectured to be bound by a limit of $\simeq 1/(4\pi)$ (in units of $\hbar/k_B$) from AdS/CFT calculations [4], likely reaches a minimum around the QCD phase transition temperature T_c [5]. To date, most hydrodynamical model studies of heavy-ion collisions at RHIC/LHC have assumed a constant η/s, whereas some studies have incorporated hadronic cascade models [6] following hydrodynamic evolution to explore the effect of a large η in the hadronic phase. Here, we present results of η/s in a multi-component mixture of hadrons so that its effect on observed collective flow in heavy-ion collisions may be assessed.

2. Formalism

We employ the Chapman-Enskog approach, generalized to include relativistic kinematics, for the calculation of the shear viscosity. In this approach, the off-equilibrium distribution function can be written as $f(x,p) = f^0(x,p)\,[1 + \phi(x,p)]$, where $f^0(x,p)$ is the equilibrium distribution function and $\phi(x,p)$ is the deviation function which contains shear and bulk viscosities as well as heat conductivity [7]. We follow closely the formalism developed in [8] considering only number conserving elastic processes. In the context of hadronic interactions, our recent calculations have been detailed in [9, 10]. The magnitude of η is inversely proportional to the momentum transport cross sections of the various constituent particles in the system. Large cross sections, characteristic of a strongly interacting system, naturally lead to small viscosities. For all but the lightest particles, first principle calculations (or data) of hadronic interactions, particularly those involving massive resonances do not exist. One therefore often uses empirical parameterizations of these hadronic cross sections. To assess the impact of different parameterizations of cross sections on the shear viscosity, we have examined three forms obtained from (i) measured phase shifts (where available) [2], (ii) the Breit-Wigner parametrization [11], and (iii) the $K-$matrix [12] parametrization. Here, we show results ensuing from the $K-$matrix parametrization. Leading contributions to the thermal properties from interactions between hadrons are calculated using the second virial coefficient deduced from two-body phase shifts according to the procedures described in [13, 14, 15].

3. Results

In a pion gas, 8 resonant channels lead to $\pi\pi$ final states [16]. With increasing T, all of these resonances influence how η and s vary with T as exemplified by the two cases: (i) $\pi\pi \to \rho \to \pi\pi$ and (ii) $\pi\pi \to$ All Channels $\to \pi\pi$ (Fig. 1). Relative to the case when only the $\rho-$ resonance is considered (dashed curves), η decreases (due to increased transport cross sections) and s increases (due to the increased number of (spin and isospin) degrees of freedom). The corresponding ratios of η/s are shown in Fig. 2. As $T \to T_c = 155\pm5$ MeV from lattice QCD simulations [17], η/s decreases more rapidly when all channels are included (solid curves) than when only the $\rho-$ channel is considered (dashed curves). The inclusion of several resonances not only causes a reduction in η, it also results in a significant increase in s. Both of these effects render the ratio η/s to become progressively small as $T \to T_c$.

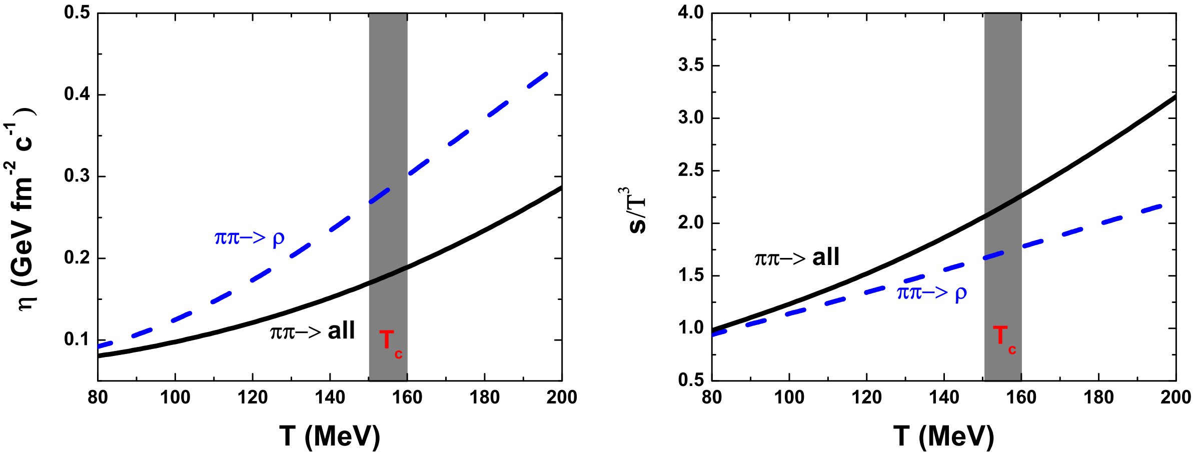

Figure 1. Results of η and s/T^3 for a single component pions gas. Dashed curves correspond to the case when only the $\rho-$resonance is considered. Solid curves show results when all possible resonances are included. The transition temperature, T_c, is indicated by the rectangular box [17].

In the four-component mixture comprising of $\pi - K - N - \eta(548)$, we include the dominant resonances produced in binary elastic interactions among the various constituents. To understand how the inclusion of increasing number of particles (and their associated resonances) determines the magnitudes of η and η/s, we also show results for the two-component mixtures of $\pi - K$ and $\pi - N$, as well as for the three-component mixture of $\pi - K - N$ together with results for the $\pi - K - N - \eta(548)$ mixture. The interaction measure $I = (\epsilon - 3P)/T^4$ calculated using the virial expansion method is compared with the lattice result of the Budapest-Wuppertal (BW) collaboration [17] (left panel of Fig. 3). To be consistent with the ingredients of transport

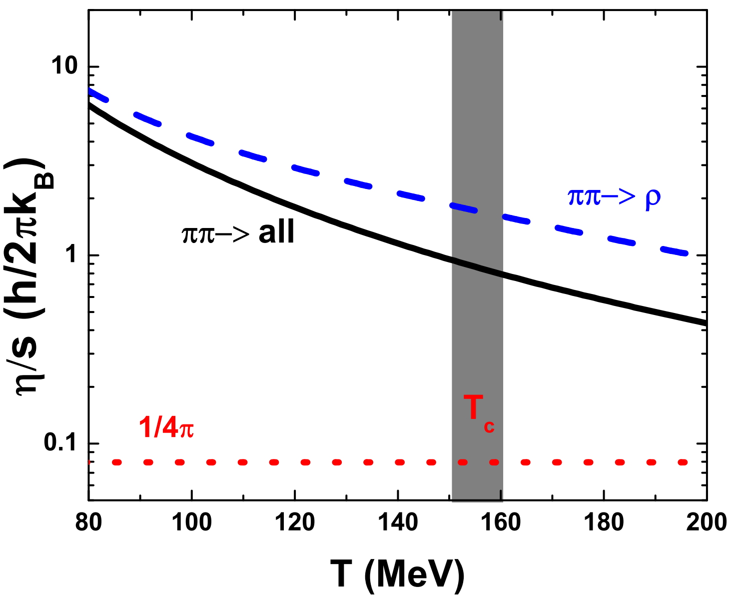

Figure 2. Same as Fig. 1, but for the ratio η/s.

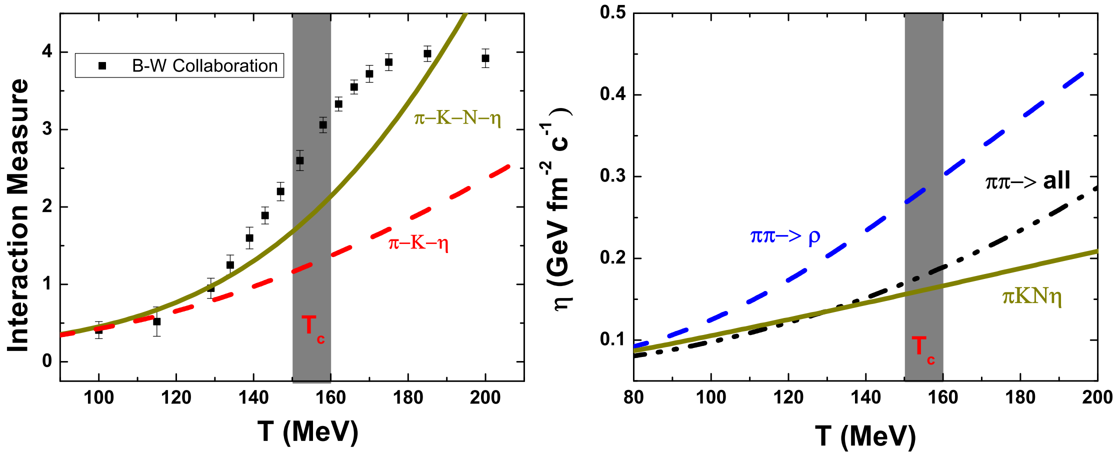

Figure 3. Left panel: Interaction measure I versus temperature T from lattice calculations (data points) with 3-quarks, $m_\pi = 135$ MeV and lattice spacing, $N_t = 8$ (BMW Collaboration). Right panel: Shear viscosity versus temperature. The transition temperature, T_c, is shown by the rectangular box [17].

calculations, only those resonances that are formed in the $\pi - K - \eta - N$ mixture are included in the virial expansion approach. The number of resonances included in the case of the $\pi - K - \eta$ mixture is 21, whereas 57 resonances are included in the $\pi - K - \eta - N$ mixture. The additional resonances present in the four component mixture improve the agreement with the lattice result up to 140 MeV. The inclusion of additional mesons and baryons more massive than realized in the $\pi - K - \eta - N$ mixture is expected to improve agreement with the current lattice results (as supported by results of calculations with all resonances in the PDG book) up to the phase transition temperature T_c [17]. The progressive decreasing in the magnitude of the shear viscosity with increasing temperature as more and more resonances are included is readily apparent from

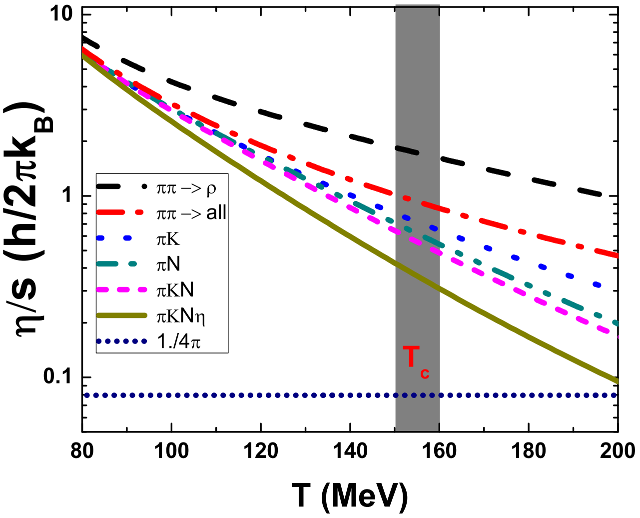

Figure 4. Shear viscosity to entropy density ratio of interacting hadrons. Results are for a single component system (π gas), two component mixtures ($\pi - K$ and $\pi - N$), a three component mixture ($\pi - K - N$) and a four component mixture ($\pi - K - \eta(548) - N$). The horizontal curve at $1/(4\pi)$ is the AdS/CFT result.

Fig. 3. The inclusion of even more resonances than considered in this work is, therefore, likely to reduce the magnitude of η even further as T_c is approached (see also [18]).

The results of η/s are presented in Fig. 4. The role of increasing number of resonances is evident even in the case of a single component pion gas. In addition to decreasing η, resonances increase s. Both these effects serve to decrease η/s with increasing temperature. A similar trend is also observed in the case of binary mixtures as seen from the results for the $\pi - K$ and $\pi - N$ mixtures. Results for the three $(\pi - K - N)$ and four $(\pi - K - \eta(548) - N)$ component systems, highlight the increasing role of the enhanced entropy density in these systems as the heaviest resonances are not very effective in transferring momentum in a direction perpendicular to that of fluid flow. For reference, the AdS/CFT result of $1/(4\pi)$ is also shown in this figure. From the trends seen in these results, we infer that the inclusion of additional mesons and baryons will further decrease η/s as T_c is approached though not to the level of the AdS/CFT result.

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References

[1] Kapusta J I 2010 *Viscous Properties of Strongly Interacting Matter at High Temperature* (Relativistic Nuclear Collisions Vol I) ed R Stock (Landolt-Bornstein New Series) pp 23
[2] Prakash M, Prakash M, Venugopalan R, and Welke R 1993 *Phys. Rep.* **227** 331
[3] Arnold P , Moore G, and Yaffe L G 2000 *JHEP* **0011** 001
[4] Kovtun P K, Son D, and Starinets A O 2005 *Phys. Rev. Lett.* **94** 111601
[5] Csernai L P, Kapusta J I and McLerran L D 2006 *Phys. Rev. Lett.* **97** 152303
[6] Gale C, Jeon S, Schenke B, Tribedy P, and Venugopalan R 2013 *Phys. Rev. Lett.* **110** 012302
[7] Leeuwen W van, Kox A J, and Groot S de 1975 *Physica* **79A** 233
[8] Leeuwen W van 1975 *Physica* **81A** 249
[9] Wiranata A and Prakash M 2012 *Phys. Rev. C* **85** 054908
[10] Wiranata A, Koch V, Prakash M, and Wang X N 2013 *Phys. Rev. C* **88** 044917
[11] Bass S A, Gyulassy M, Stöcker H, and Greiner W 1999 *J Phys. G* **25** R1
[12] Chung S U*et al* 1995 *Ann. Physik* **4** 404
[13] Dashen R, Ma S, and Bernstein H J 1969 *Phys. Rev.* **187** 349
[14] Welke G, Venugopalan R, and Prakash M 1990 *Phys. Lett. B* **245** 137
[15] Venugopalan R and Prakash M 1992 *Nuclear Physics A* **A546** 718
[16] Beringer J *et al* 2012 *Phys. Rev. D* **86** 010001
[17] Borsanyi S *et al* 2010 *JHEP* **1011** 077
[18] Hostler J N, Noronha J, and Greiner C 2009 *Phys.Rev.Lett.* **103** 172302