Turbulence, Vorticity and Lambda Polarization

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Abstract. Present highest energy heavy ion experiments show the dominance of the fluid dynamical processes. The possibility of new collective phenomena, rotation and turbulence occurs, which were known only in macroscopic systems up to now. These cand be detected with a new type of polarization measurements.

In recent years studying heavy ion collisions at LHC energies in a high resolution 3+1D fluid dynamical model we observed that the flow leads to collective rotation [1] in peripheral events arising from the initial angular momentum [2].

This rotation leads to a change of the flow pattern, and the antiflow, as characterized by the v_1 parameter, weakens and peaks at angles, which point more forward rapidities.

The v_1 becomes very small at high, LHC energies, less than one per cent, thus its measurement is not easy. Still the trend that the antiflow peak moves to more forward rapidities is apparent in experiments.

At high energies the role of random fluctuations is significant, and these random fluctuations are becoming strong, exceeding the magnitude of the collective v_1 flow component. Flow harmonics were identified up to high multipolarities, having a maximum amplitude at v_3, in central heavy ion collisions. In these collisions due to symmetry reasons all collective flow harmonics should have vanished. This in itself indicates the low viscosity of the underlying matter, which enables large random fluctuations.

The stability of the flow implies a given amount of viscosity. This stability against turbulence is characterized by the dimensionless, Reynolds number: $Re = l_1 u_1/\nu$, where $\nu = \eta/\rho$ is the kinetic viscosity, η is the shear viscosity, and ρ is the mass (or energy) density, u_1 is the characteristic flow velocity, and l_1 is the characteristic length scale of the flow pattern.

Kovtun, Son and Starinets have shown in a recent work that in certain field theories, that are dual to black branes in higher space-time dimensions, the ratio $\eta/s = 1/4\pi$ [3]. Interestingly, this bound is obeyed by $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills theory in the large N_c limit. Subsequently in another recent work with [4] it was pointed out that η/s has a minimum at the critical point of the QGP - Hadronic Matter phase transition. At pressures below the critical pressure there is a discontinuity, and η/s increases, and at pressures above it there is a broad smooth increase again from the minimum.

Apart of the fluctuatios from low viscosity, the phase transition leads to critical fluctuations also. These fluctuations, and their multipolarity spectrum can identify and characterize the phase transition. This is discussed in large number of publications, including [5, 6]. These fluctuations can shed light on the critical properties of the phase transition.

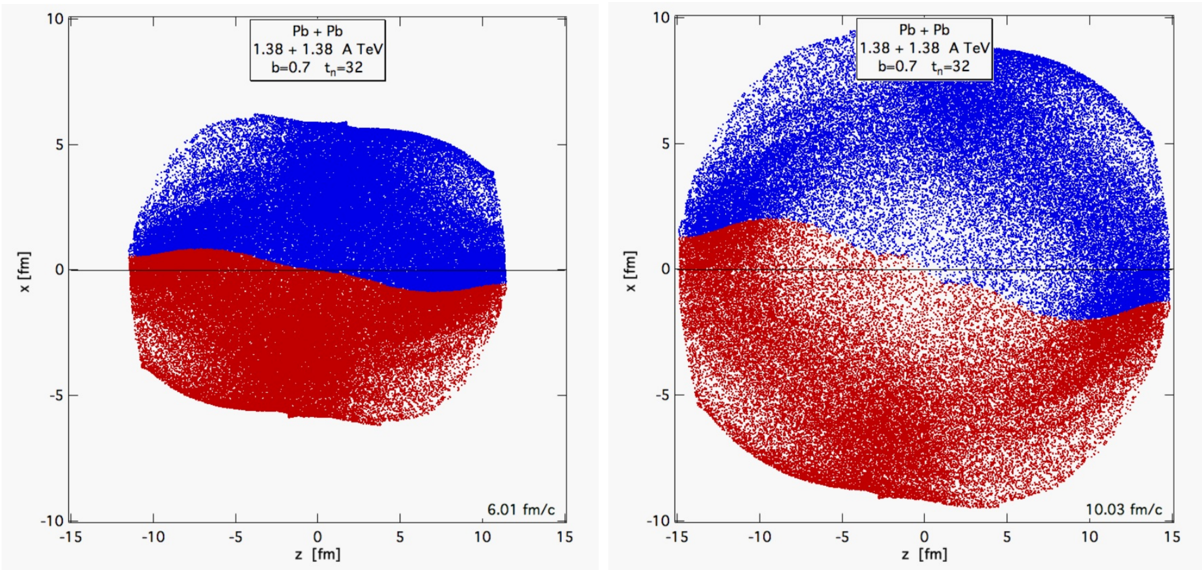

Figure 1. (color online) The position of computational "marker particles" which represents the moton of the net baryon charge in the raction plane for a periheral Pb+Pb reaction. At the initial configuration of the fluid dynamical development the top (projectile side) half of the marker particles were painted red, while the lower (target side) half was painted blue. We can see that with time a non-linear wave develops with increasing amplitude. This is the Kelvin Helmholtz Instability.

A consequence of the low viscosity is turbulence. The most typical type of turbulence is the Kelvin Helmholtz Instability (KHI) which is well known in macroscopic systems, as breaking waves in strong wind or in the air with strong wind shear in different vertical layers. It was observed in low viscosity numerical fluid dynamical model calculations that the KHI may occur in heavy ion collisions under favorable conditions [7]. See Fig. 1. The development of KHI is dependent on several conditions, discussed in ref. [7], but most importantly low viscosity is required. Thus, this phenomenon is a direct measure of the low viscosity.

Turbulent phenomena may occur also from random fluctuations. We have seen that these fluctuations are significant. In ref. [8] vorticity arising from random fluctuations in the transverse plane were studied, and a maximum vorticity amplitude of 0.2 c/fm was found. Due to the fact that this vorticity is a product of random fluctuation in a sample of a large number of collisions the average integrated vorticity should vanish.

On the other hand the vorticity in the reaction plane is due to the initial angular momentum, thus the plane of the rotation is the reaction plane, $[x, z]$ plane, and the rotation axis is the y axis. Due to the high angular momentum, $\approx 10^6\hbar c$, at LHC energies, the vorticity vector is large and pointing into one direction, to the y direction. The amplitude of the vorticity reaches a maximum of 3 c/fm [9], which is more than an order of magnitude larger than the vorticity arising from random fluctuations.

The observation of the collective phenomena is not a trivial problem, although it is practiced for more than three decades. The present emphasis of random fluctuations has extended the evaluation methods to those classes of symmetries, which would not occur in case of collisions without fluctuations.

Without fluctuations, the heavy ion collision is mirror symmetric across the reaction plane, as well as reflection symmetric with respect to the center of mass (c.m.) point. Thus in the earlier, low energy experiments the longitudinal distribution along the z axis was antisymmetric across the c.m. point or with respect to the c.m. point of the rapidity coordinate. From the mirror symmetry across the reaction plane it follows that the distribution as function of the azimuth angle should be symmetric, so only cosine functions can be present in the in azimuthal angle Fourier series.

For random fluctuations these restrictions do not apply. The azimuthal multipole and longitudinal analysis should contain even and odd functions. This does not lead to any problem in exactly central collisions, where global flow patterns do not lead to specific angles.

The precise identification of the c.m. of the observed participant system would still be necessary for "exactly" central collisions, because random azimuthal fluctuations may be correlated with the longitudinal momentum in the c.m. system. Unfortunately this is not done yet in recent experiments, although longitudinal fluctuations of the participant c.m. momentum are not negligible at high beam energies.

Several collective processes in peripheral collision are strongly correlated with the symmetry axes of the collision, such as the reaction plane and the c.m. momentum or rapidity. These include the directed flow, all odd flow harmonics, rotation, and all polarization effects, which are correlated with the reaction symmetries. If these symmetry axes and the c.m. are not identified, then these processes are not measurable or are not measured accurately.

For example if we intend to measure polarization of particles which are arising from the collective rotation in the reaction plane then it is necessary to identify the plane, the direction of the rotation, i.e. which is the projectile and target side, and the position of the rotation axis, i.e. the c.m. momentum of the participants. This is possible to do, as described e.g. in ref. [10]. If this is not done with a given precision than the measurement of these collective phenomena is not possible.

In view of these general comments we discussed the effect of rotation on the directed flow, which turned out very sensitive and subject to significant perturbations from random fluctuations [1]. As the v_1 becomes small ($\sim 1\%$ or less) at RHIC and LHC energies, other measurements were studied which are more sensitive to rotation and turbulence.

The polarization is one of the consequences of rotation and local vorticity, which arises in a stratified shear flow. This can happen due to equipartition of orbital rotation and spin due to local thermalization. The Λ polarization in the participant centre-of-mass frame, as a function of its momentum, depends on the vorticity of the inverse temperature four-vector, $\beta^\mu(x) = (1/T(x))u^\mu(x)$ as shown in ref. [11]. The fact that the peripheral areas of the flow have lower temperatures, leads to an increased thermal vorticity in these areas, thus the thermal vorticity governing polarization is more uniform than the simple vorticity generated just from the velocity field. This also led to the interesting phenomenon that the polarization not necessarily increasing with increasing anglar momentum at high energy collisions, because at the same time the temperature is also strongly increasing.

It is shown that about 8-10% Λ or $\bar{\Lambda}$ polarization can be reached in peripheral heavy ion collisions [12]. This is observable for Λ or $\bar{\Lambda}$ emitted into the $\pm X$ direction in the reaction plane and the both the Λ or $\bar{\Lambda}$ will be polarized into the $-y$ direction (i.e. transverse polarization). This is in contrast to electromagnetic effects where the Λ and $\bar{\Lambda}$ polarizations would be opposite. Of course to perform this measurement the Event by Event identification of the reaction plane, its direction, and the c.m. momentum are necessary. Otherwise the overall integration of all directions, eliminates the effect.

Although early measurements at RHIC were negative, thses were averaged over all azimuth! We propose a selective measurement in the reaction plane (in the $\pm x$ direction) in the event by event (EbE) c.m. frame. Statistical error is much reduced now, so a significant effect is expected

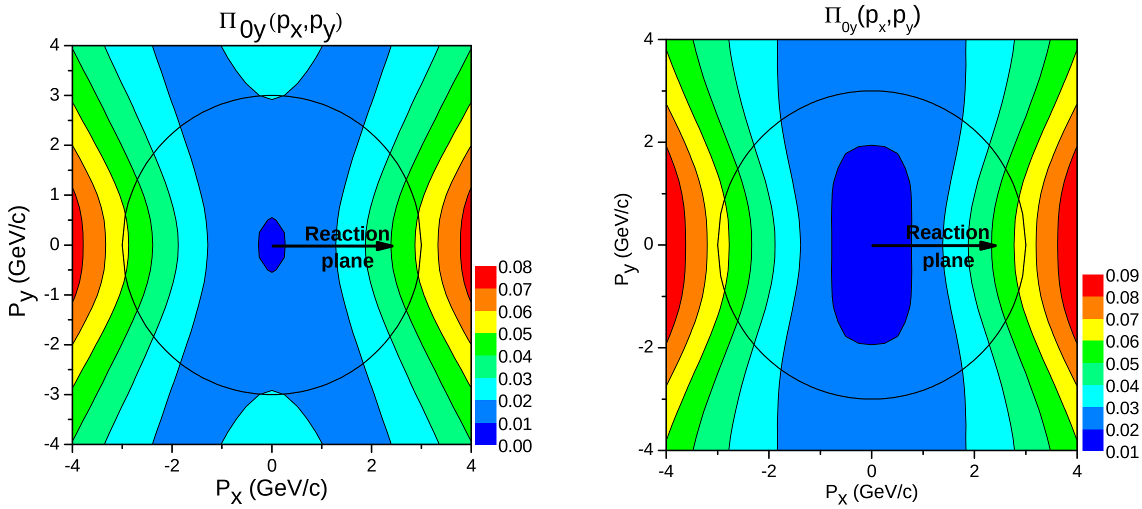

Figure 2. (color online) The polarization of Λ and $\bar{\Lambda}$ due to thermal equipartition with with local vorticity is slightly stronger at RHIC (right) than at LHC (left), due to the much higher temperatures at LHC.

around $p_x \approx 3$ GeV/c. See Fig. 2.

Another sensitive method of studying heavy ion reactions is the Hanbury Brown, and Twiss (HBT) effect. This is widely used for identifying the size of the emitting source, and also the radial expansion speed, which also influences the observed effective radii. Furthermore, with the azimuthal HBT method the shape of the emission ellipsoid can also be determined.

Recently, a Differential HBT (DHBT) method was worked out, which is similarly sensitive to rotation, and has advantages compared to the flow harmonics type of analysis. The estimated signal is of the order of 10% also [13]. As in the previous case this also requires the identification of reaction geometry.

This is not a fundamental problem and at lower energies this was done earlier. Recently the shift of focus towards analysis of fluctuations, diverted attention from this task, but we hope that the experimental teams will pay attention to this task.

In conclusion, we have a good perspective to continue the research along the lines and successes of the earlier research in this field.

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