Systematic Properties of the Tsallis Distribution: Energy Dependence of Parameters.

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Abstract. Changes in the transverse momentum distributions with beam energy are studied using the Tsallis distribution as a parameterization. The dependence of the Tsallis parameters q, T and the volume on beam energy is determined. The Tsallis parameter q shows a weak but clear increase with beam energy with the highest value being approximately 1.15. The Tsallis temperature and volume are consistent with being independent of beam energy within experimental uncertainties.

In the analysis of new data, a Tsallis-like distribution gives excellent fits to the transverse momentum distributions as shown by the STAR [1] and PHENIX [2] collaborations at RHIC and by the ALICE [3], ATLAS [4] and CMS [5] collaborations at the LHC. In this talk we review the parameterization used by these groups and propose a slightly different one which leads to a more consistent interpretation and has the bonus of being thermodynamically consistent [7].

In the framework of Tsallis statistics [6, 8, 9, 10, 11] the entropy, S, the particle number, N, the energy E and the pressure P are given by corresponding integrals over the Tsallis distribution:

$$f \equiv \left[1 + (q - 1)\frac{E - \mu}{T}\right]^{-\frac{1}{q-1}}. \tag{1}$$

where T and μ are the temperature and the chemical potential, q is a new variable often refereed to as Tsallis parameter. The relevant thermodynamic quantities are given by (see e.g. [11]

$$S = -gV \int \frac{d^3p}{(2\pi)^3}\left[f^q \ln_q f - f\right], \tag{2}$$

$$N = gV \int \frac{d^3p}{(2\pi)^3} f^q, \tag{3}$$

$$E = gV \int \frac{d^3p}{(2\pi)^3}\sqrt{p^2 + m^2} f^q, \tag{4}$$

$$P = g \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E} f^q. \tag{5}$$

V is the volume, g is the degeneracy factor. The short-hand notation

$$\ln_q(x) \equiv \frac{x^{1-q} - 1}{1 - q}, \tag{6}$$

often referred to as q-logarithm has been used. These expressions are thermodynamically consistent, e.g. it can be shown [11] that consistency relations of the type

$$N = V \left.\frac{\partial P}{\partial \mu}\right|_T ,$$

(7)

and

$$\epsilon + P = Ts + \mu n$$

(8)

(where n, s, ϵ refer to the densities of the corresponding quantities) are indeed satisfied.

Following from Eq. (3), the momentum distribution is given by,

$$\frac{d^3N}{d^3p} = \frac{gV}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-q/(q-1)} ,$$

(9)

or, expressed in terms of transverse momentum, p_T, transverse mass, $m_T \equiv \sqrt{p_T^2 + m^2}$, and rapidity y

$$\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2} \left[1 + (q-1)\frac{m_T \cosh y - \mu}{T}\right]^{-q/(q-1)} .$$

(10)

At mid-rapidity, $y = 0$, and for zero chemical potential, as is relevant at the LHC, this reduces to

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .$$

(11)

In the limit where the parameter q goes to 1 it is well-known that this reduces the standard Boltzmann distribution. The parameterization given in Eq. (10) is close to the one used in [1, 2, 3, 4, 5]:

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left[1 + \frac{m_T - m_0}{nC}\right]^{-n} ,$$

(12)

where n (not to be confused with the particle density) and C are fit parameters and has been discussed in detail in Ref. [7]. At mid-rapidity $y = 0$ and zero chemical potential, they have a similar dependence on the transverse momentum as Eq. (11) apart from an additional factor m_T on the right-hand side. However, the inclusion of the rest massis not in agreement with the Tsallis distribution as it breaks m_T scaling which is present in Eq. (10) but not in Eq. (12). The inclusion of the factor m_T leads to a more consistent interpretation of the variables q and T.

A very good description of transverse momenta distributions at RHIC has also been obtained in Refs [12, 13] on the basis of a coalescence model where the Tsallis distribution is used for quarks. Tsallis fits have also been considered in Ref. [14, 15, 16] but with a different power law leading to smaller values of the Tsallis parameter q.

Interesting results were obtained in Refs. [17, 18] where spectra for identified particles were analyzed and the resulting values for the parameters q and T were considered.

It has been shown that, at lower energies it is possible to describe transverse momentum spectra in $p - p$ collisions with Boltzmann distributions provided one takes into account the decays of resonances [19]. This is not in contradiction with the results presented here as the transverse momentum distributions become more and more Boltzmann like at lower energies as can be seen in the values of q which are closer to one as the energy is decreased.

The energy dependence in $p - p$ collisions can be determined by studying data for charged particles at beam energies of 0.54 [20], 0.9, 2.36 and 7 TeV [3, 4, 5]. These involve distributions

summed over charged particles. The fits were performed using a sum of three Tsallis distributions, the first one for π^+, the second one for K^+ and the third one for protons p. The relative weights between these were simply determined by the corresponding degeneracy factors, i.e. 1 for for π^+ and K^+ and 2 for protons. The fit was taken at mid-rapidity and for $\mu = 0$ using the following expression:

$$\frac{1}{2\pi p_T} \frac{d^2 N(\text{charged particles})}{dp_T dy}\bigg|_{y=0} = \frac{2V}{(2\pi)^3} \sum_{i=1}^{3} g_i m_{T,i} \left[1 + (q-1)\frac{m_{T,i}}{T}\right]^{-\frac{q}{q-1}}, \qquad (13)$$

where $i = (\pi^+, K^+, p)$ and $g_{\pi^+} = 1$, $g_{K^+} = 1$ and $g_p = 2$. The factor 2 in front of the right hand side of this equation takes into account the contribution of the antiparticles $(\pi^-, K^-, \bar{p})$. A comparison with charged particle distributions is shown in Fig. 1. It is to be noted that the Tsallis distribution presented above also gives an excellent description of transverse momentum distributions in $p - Pb$ collisions at all pseudorapidity intervals obtained by the ALICE collaboration [21]. The Tsallis parameters q and T needed to describe the transverse momentum distributions of charged particles are shown in Fig. 2. The value of qt has a tendency to increase slowly with increasing energy [7] while no clear energy dependence for T can be discerned.

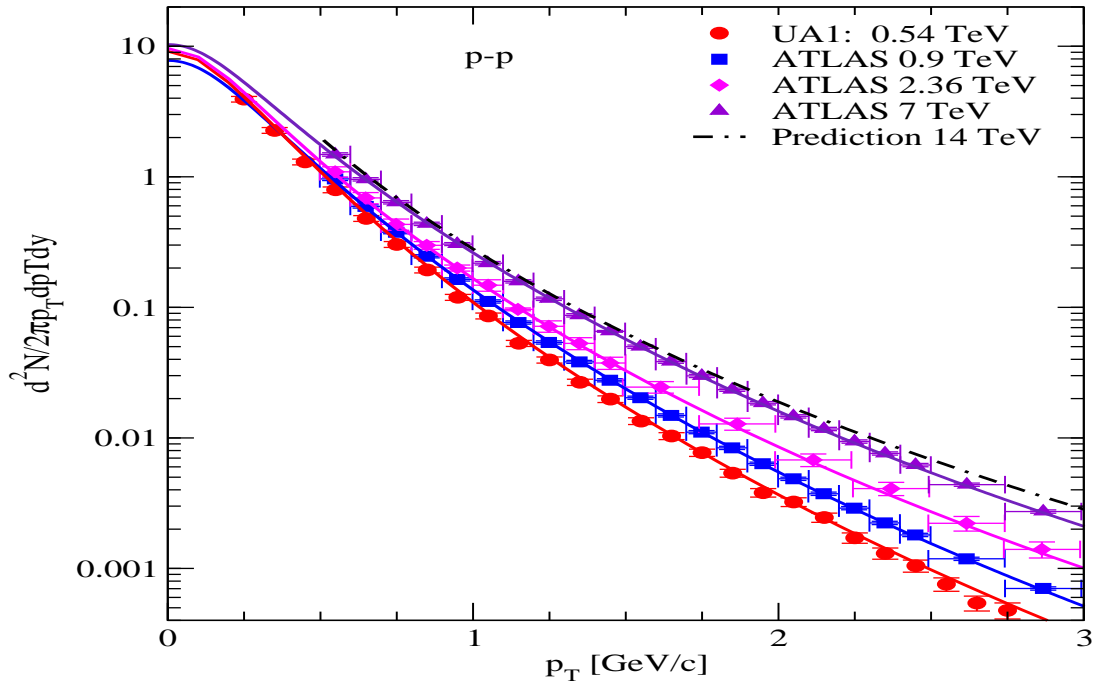In conclusion, the Tsallis distribution leads to excellent fits to the transverse momentum

Figure 1. Fits to transverse momentum distributions of charged particles [4, 20] using the Tsallis distribution.

distributions in high energy $p - p$ and $p - Pb$ collisions. By comparing results from UA1 [20] to results obtained at the LHC [4, 5] it has been possible to extract the parameters q, T and V for a wide range of energies [7]. A consistent picture emerges from a comparison of fits using the Tsallis distribution in high energy $p - p$ collisions.
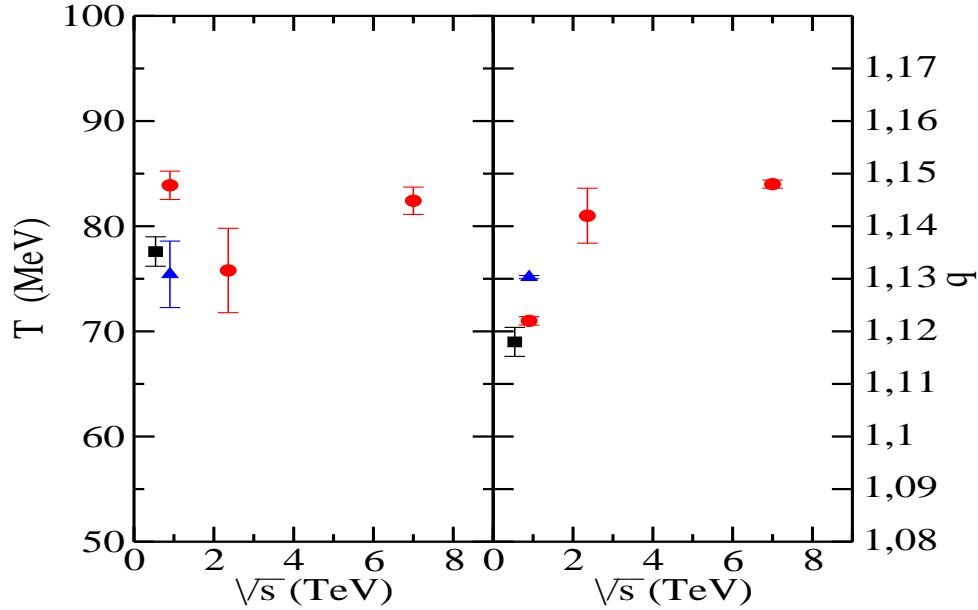
Figure 2. Values of the Tsallis parameters T (left) and q (right), as a function of beam energy, obtained from fits to the transverse momentum distributions of charged particles as described in the text. The square point is from [20], the round points are from [4] while the triangle point is from [3].

References

[1] Abelev B I *et al* 2007 *Phys. Rev. C* **75** 064901
[2] Adare A *et al* 2010 *Phys. Rev. C* **83** 052004; 2011 *Phys. Rev. C* **83** 064903
[3] Aamodt K *et al* 2011 *Eur. Phys. J. C* **71** 1655; 2010 *Phys. Lett. B* **693** 53
[4] Aad G *et al* 2011 *New J. Phys.* **13** 053033
[5] Khachatryan V *et al* 2010 *Phys. Rev. Lett.* **105** 022002
[6] Tsallis C 1988 *J. Statist. Phys.* **52** 479
[7] Cleymans J, Lykasov G I, Parvan A S, Sorin A S, Teryaev O V, Worku D 2013 *Phys. Lett. B* **723** 351
[8] Tsallis C, Mendes R S, Plastino A R 1998 *Physica A* **261** 534
[9] Conroy J M, Miller H G and A. R. Plastino A R 2010 *Phys. Lett. A* **374** 4581
[10] Cleymans J and Worku D 2012 *J. Phys. G* **39** 025006
[11] Cleymans J and Worku D 2012 *Eur. Phys. J. A* **48** 160
[12] Ürmössy K, Biró T S 2010 *Phys. Lett. B* **689** 14
[13] Ürmössy K, Biró T S 2009 *J. Phys. G* **36** 064044
[14] Wong C-Y, Wilk G 2012 *Acta Physica Polonica* **43** 2047
[15] Wibig T 2010 *J. Phys. G: Nucl. Part. Phys.* **37** 115009
[16] Wibig T and Kurp I 2003 *JHEP* **0312** 039
[17] Marques L, Andrade-II E, Deppman A arXiv:1210.1725[hep-ph]
[18] Sena I, Deppman A 2013 *Eur. Phys. J. A* **49** 17
[19] Becattini F and Passaleva G 2002 *Eur. Phys. J. C* **23** 551
[20] Albajar C *et al* 1990 *Nucl. Phys. B* **335** 261
[21] Abelev B I *et al* 2013 *Phys. Rev. Lett.* **110** 082302