TRAINING COURSE ON RADIATION DOSIMETRY:

Interaction of ionising radiation with matter

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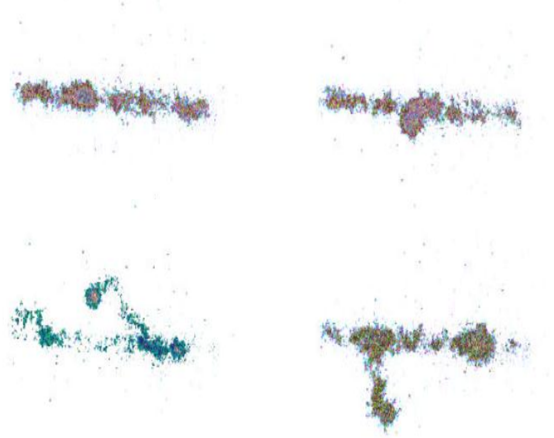
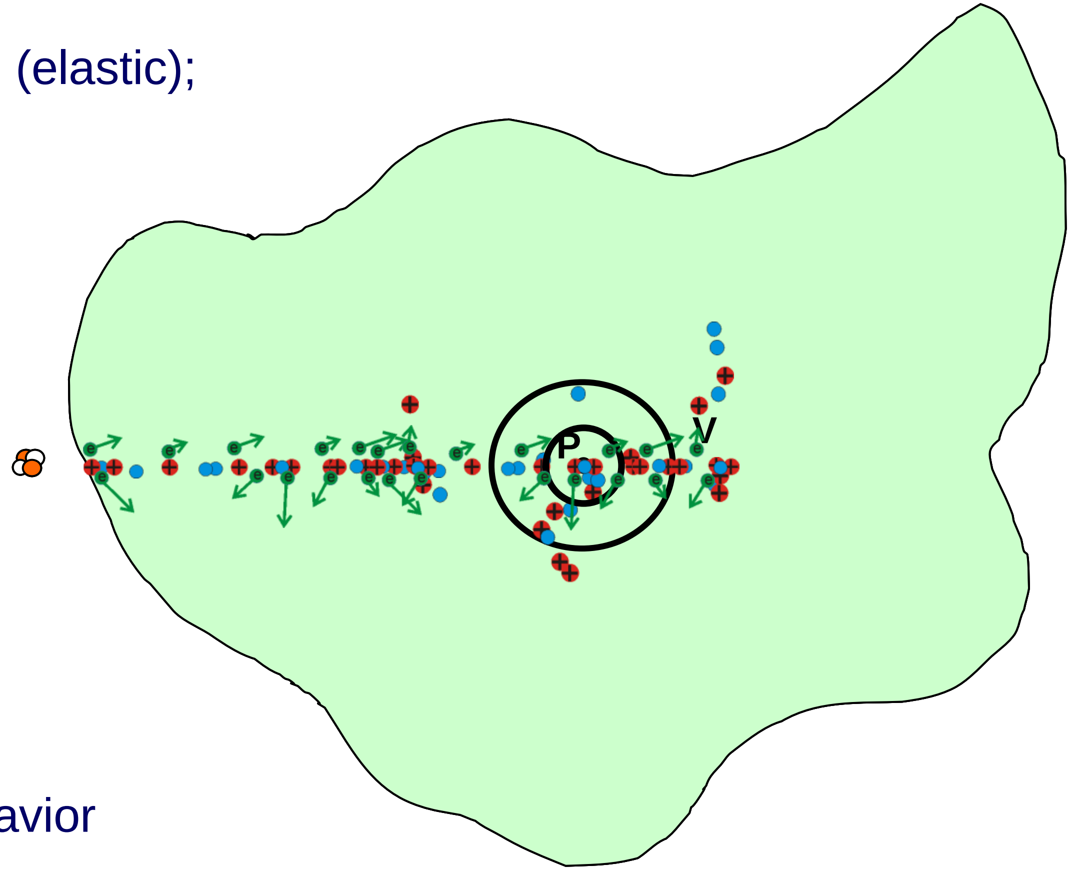
Wed. 21/11/2012, 10:00 – 11:00 am

CHARGED HADRONS

- Ionization and excitation
- Coulomb nuclear scattering (elastic);
- Inelastic reactions.



5 MeV protons



- Stochastic and discrete behavior

Courtesy of P. Colautti, INFN-LNL, Legnaro, Italy

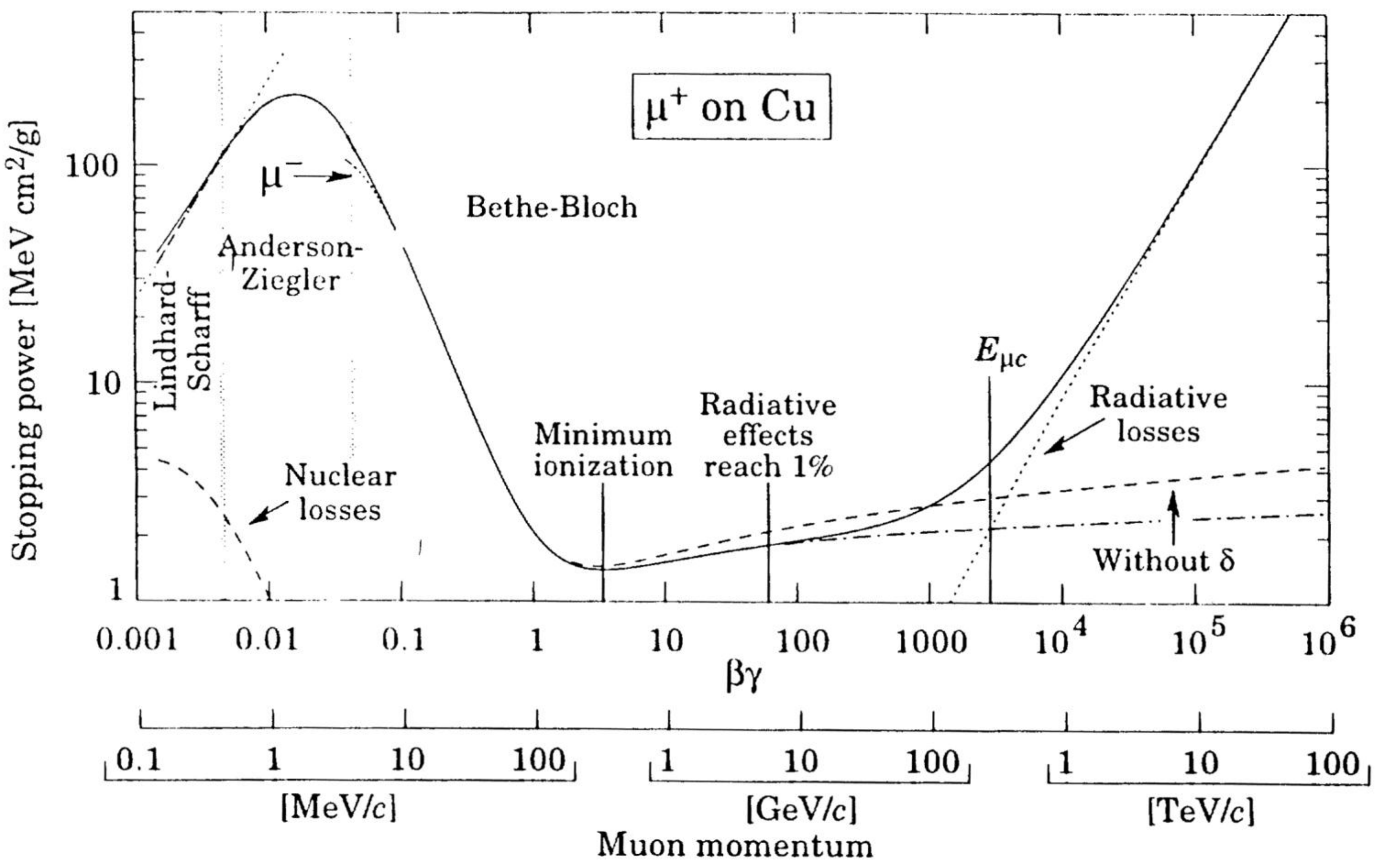
IONIZATION

- Energy loss (Bethe-Bloch formula):

$$-\frac{dE}{dx} = NZk^2 \frac{4\pi e^4 z^2}{m_e v^2}[\ln\frac{2m_e v^2}{I} - \ln(1-\beta^2) - \beta^2 - \frac{\delta}{2}]$$

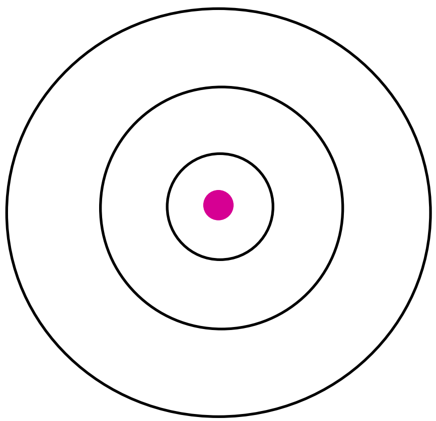
- Mean ionization potential I=(10 eV)Z
  Z>16;
- Mean energy lost per unit path length;
- Continuous slowing-down approximation;

STOPPING POWER

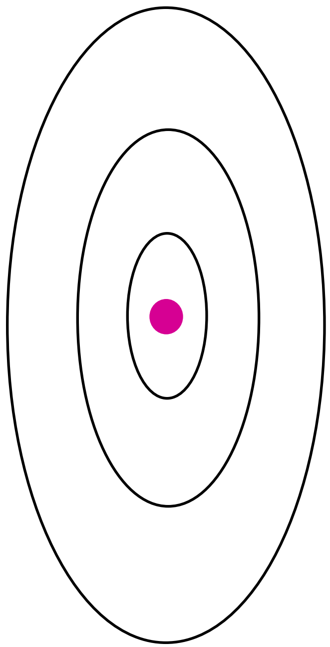


IONIZATION – DENSITY EFFECT

- The particle electric field:



γ=1



- The transverse electric field of a moving particle broadens at relativistic velocities, while the longitudinal one contracts. This reflects in a collision time decrease and in an increase of the impact parameter. The latter reflects in an increase of the interaction cross section.

IONIZATION – DENSITY EFFECT

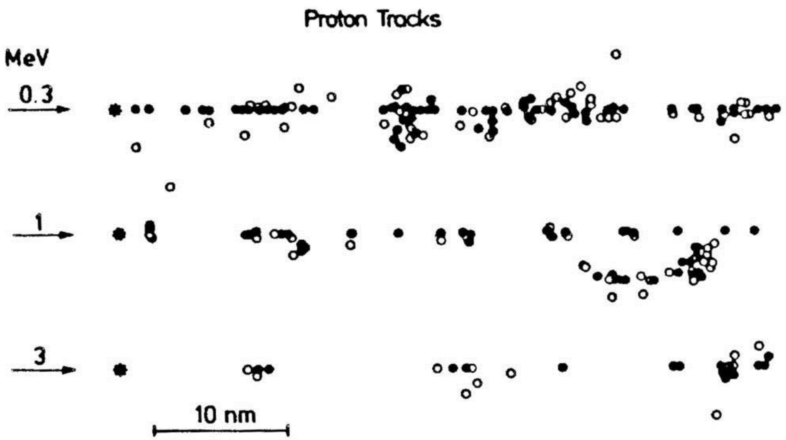
- The electric field of a relativistic particle is higher behind than ahead its direction of motion. There is an asymmetric polarization of nearby atoms. This polarization reduces the effective electric field of the projectile and shields the atoms at higher distances which might alternatively ionized or excited by the broadening of the transverse component of the electric field.

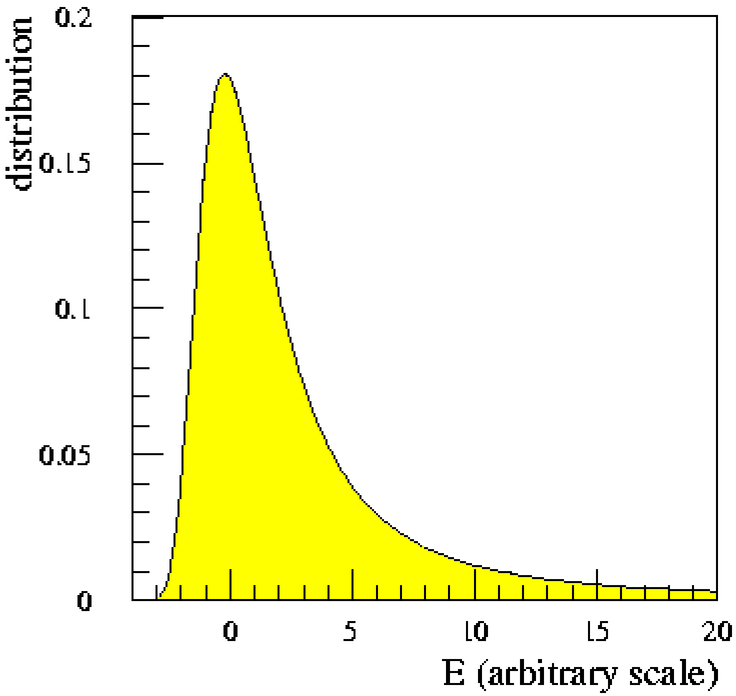
$$-\frac{dE}{dx} = NZk^2 \frac{4\pi e^4 z^2}{m_e v^2} [\ln \frac{2m_e v^2}{I} - \ln(1-\beta^2) - \beta^2 - \frac{\delta}{2}]$$

- The effect is significant when the inter-atomic distance is lower than the impact parameter for soft-collisions, i.e. in condensed materials and high-pressure gases.

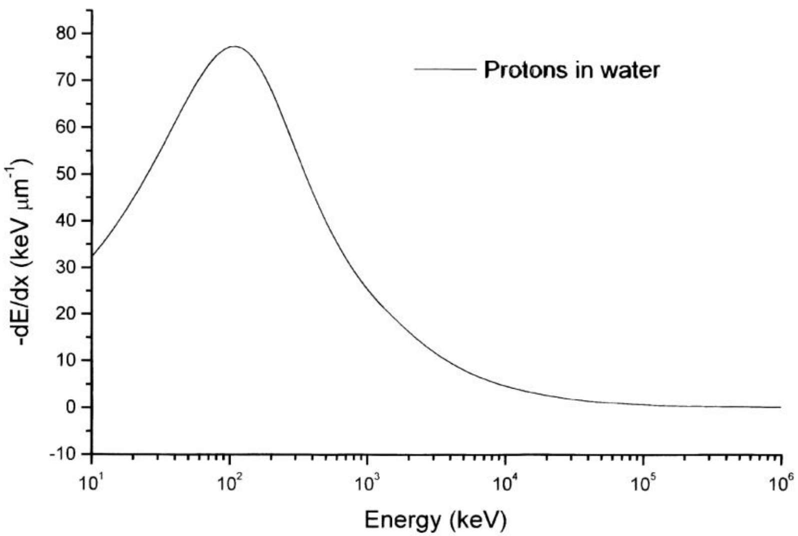
IONIZATION – ENERGY STRAGGLING

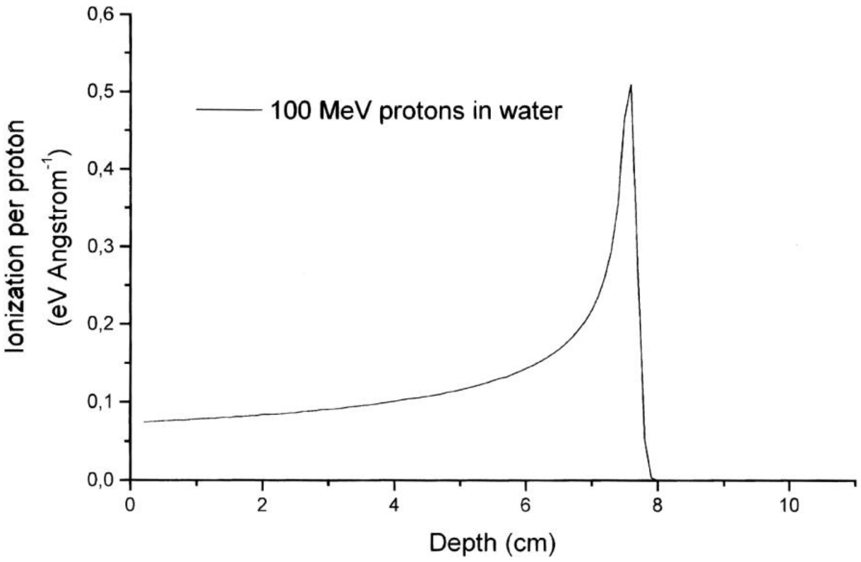
- The Bethe-Bloch formula gives the mean energy lost per unit path length;
- Energy deposition by radiation is discrete and stochastic;
- The stochastic behavior of energy deposition in matter (energy straggling) is described by stochastic distributions such as the Landau-Vavilov distribution.





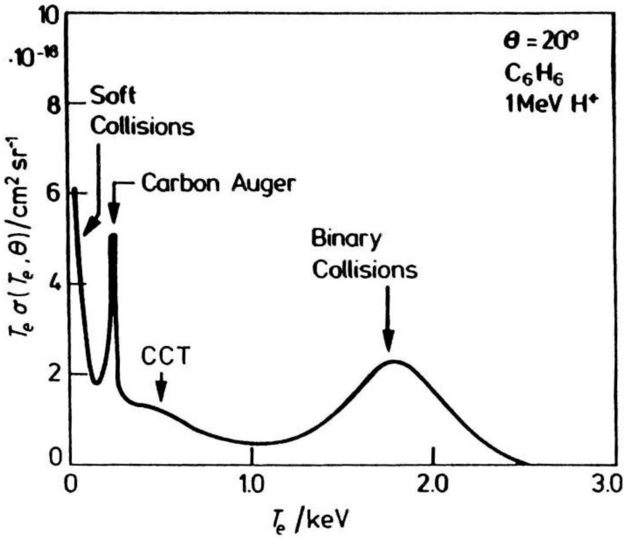
IONIZATION – THE BRAGG PEAK

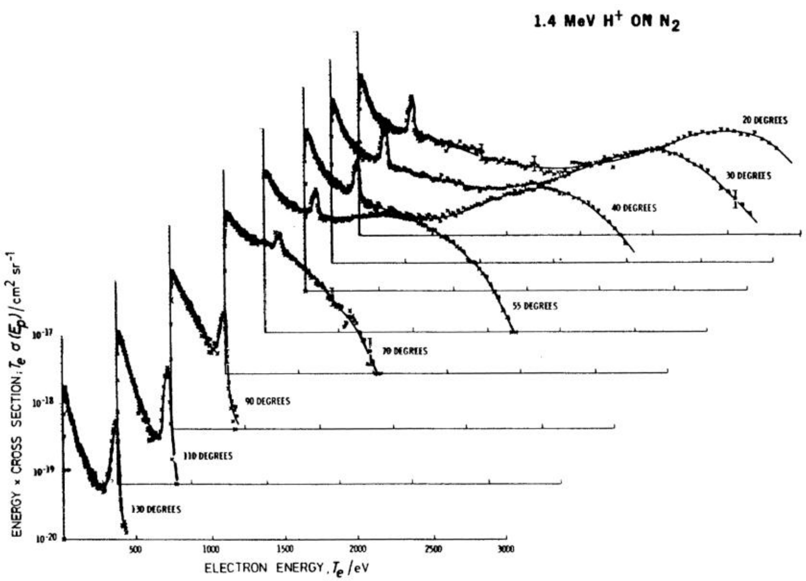




SECONDARY ELECTRONS

12 • • • 4. *Particle Tracks and Energy Deposition*





COULOMB SCATTERING

- By neglecting the screening of atomic electrons;
- the differential x-sec for single Coulomb scattering is described by:

$$\sigma(\Theta)d\Omega = \frac{z^2 Z^2 e^4}{4(4\pi\varepsilon_0)^2 M_0^2 v^4} \frac{1}{\sin^4 \frac{\Theta}{2}} \mathrm{d}\Omega$$

  - ✓ where Θ is the scattering angle in the CM system and M_0 is the reduced mass.

- If the projectile mass is lower than that of the target nucleus $\Theta \cong \theta_{lab}$ and $M_0 \cong M_1$:

$$\sigma(\vartheta)d\Omega = \frac{1}{4} Z^2 z^2 r_e^2 \left(\frac{m_e c}{p\beta}\right)^2 \frac{d\Omega}{\sin^4 \vartheta/2}$$

- for spin zero particles (alphas, pions,…)

$$\sigma(\vartheta)d\Omega = \frac{1}{4} Z^2 z^2 r_e^2 \left(\frac{m_e c}{p\beta}\right)^2 \frac{d\Omega}{\sin^4 \vartheta/2}\left[1-\beta^2 \sin^2 \vartheta/2\right]$$

- for spin one-half particles.
- p is the projectile momentum and

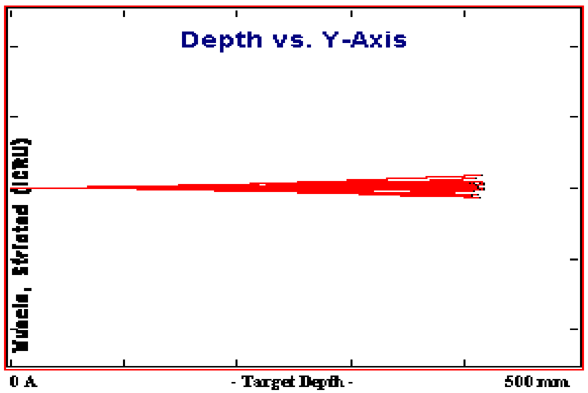
$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$$

COULOMB SCATTERING

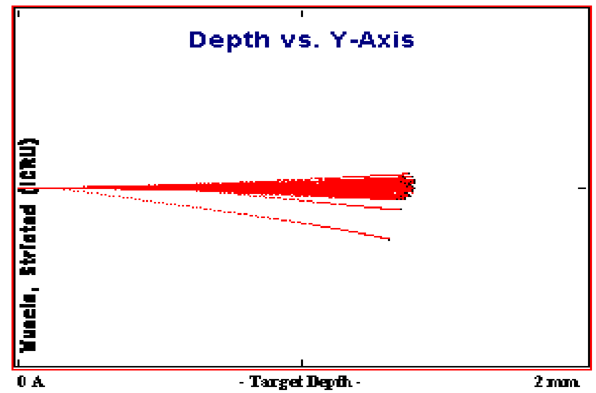
- For electrons (Mott's formula):
  - ✓ valid for high velocities (β≅1) and for light materials (Z≤27);

$$\sigma(\vartheta)d\Omega = \frac{1}{4}Z^2 z^2 r_e^2 \left(\frac{m_e c}{p\beta}\right)^2 \frac{d\Omega}{\sin^4 \vartheta/2}\left[1-\beta^2 \sin^2 \vartheta/2 + \pi\beta\alpha Z\left[1-\sin\vartheta/2\right]\sin\vartheta/2\right]$$

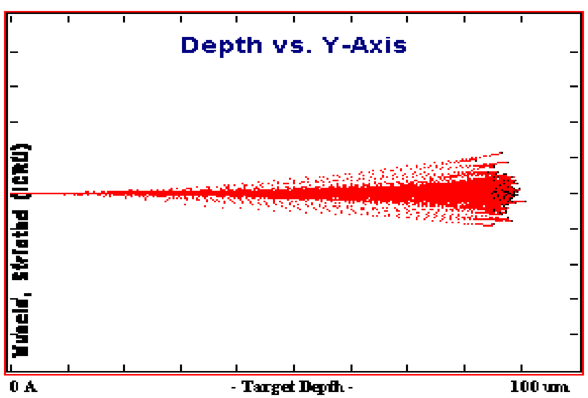
COULOMB SCATTERING



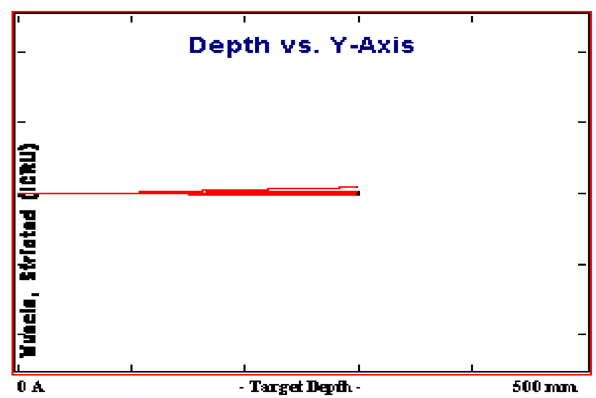
a) 250 MeV protons in soft tissue.



b) 10 MeV protons in soft tissue.

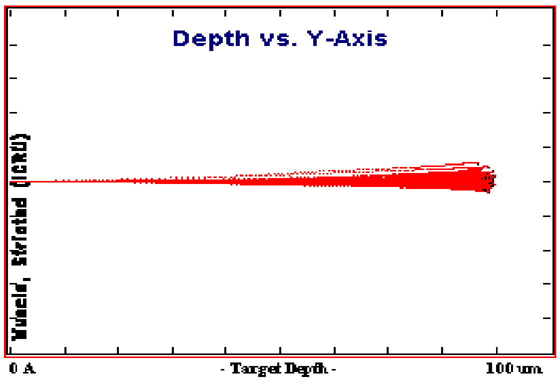


c) 2 MeV protons in soft tissue.

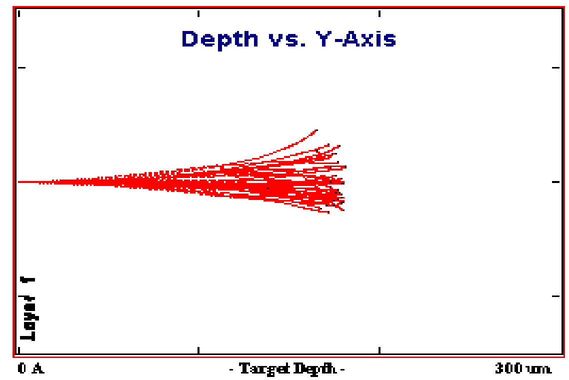


d) 400 MeV/u C ions in soft tissue

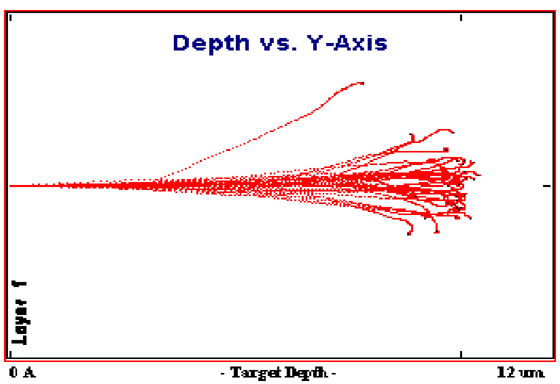
COULOMB SCATTERING



e) 8 MeV alphas in soft tissue.



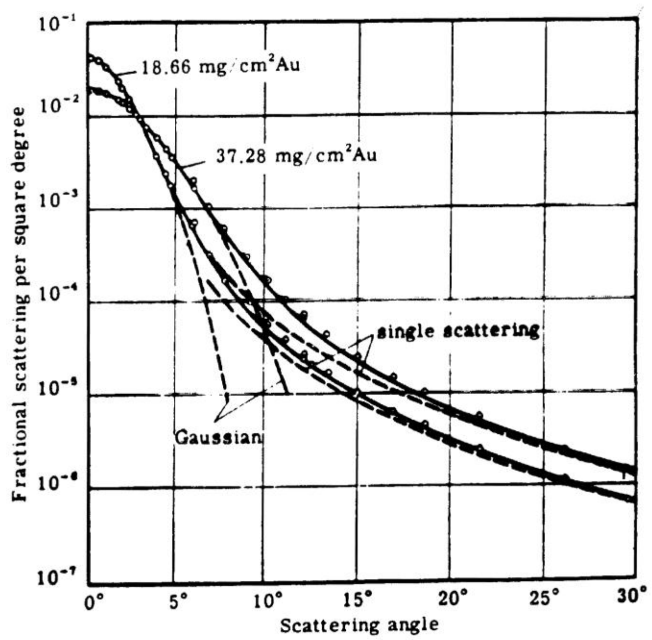
f) 10 MeV protons in Au.



g) 5.57 MeV alphas in Au.

MULTIPLE COULOMB SCATTERING

- In an absorbing medium, a charged particle undergoes a large number of small deflections and a small number of large angle scatterings followed by small deflections.
- Multiple scattering is described by Goudsmit-Saunderson and Moliére.
- Moliere:
  - ✓ valid at small scattering angles (sinθ≅θ) and for a collision number > 20;
  - ✓ described in terms of the screening angle (the minimum angle limiting the scattering event because of the atomic electron screening of nuclei.

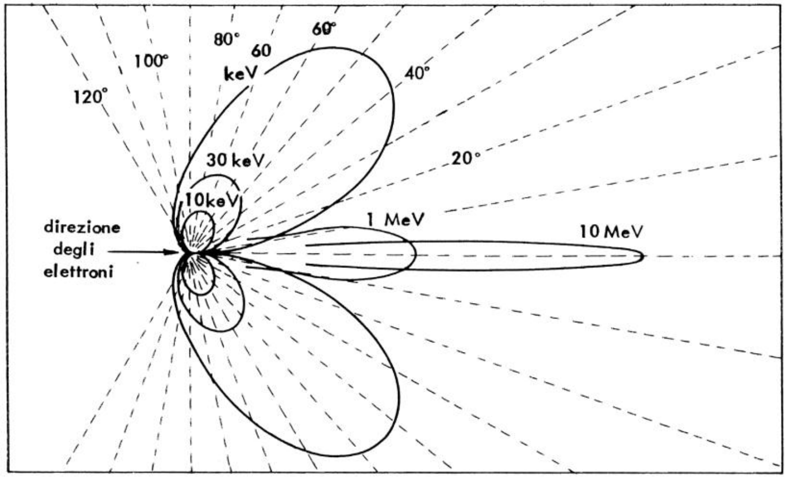


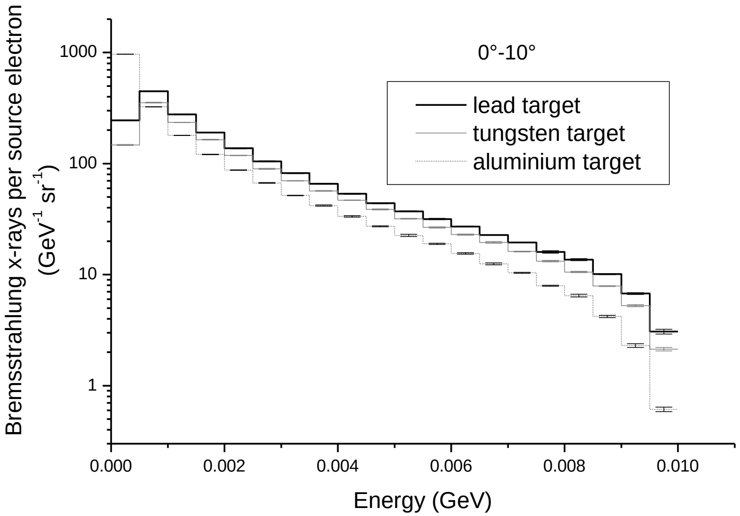
ENERGY LOSS – RADIATIVE COLLISIONS

- Energy loss for electrons:

$$-\frac{dE}{dx} = 4\alpha Z^2 N r_e^2 (T + m_e c^2) \ln\frac{183}{Z^{1/3}} = \frac{4Z^2}{137} N r_e^2 (T + m_e c^2) \ln\frac{183}{Z^{1/3}}$$

- α fine structure constant;

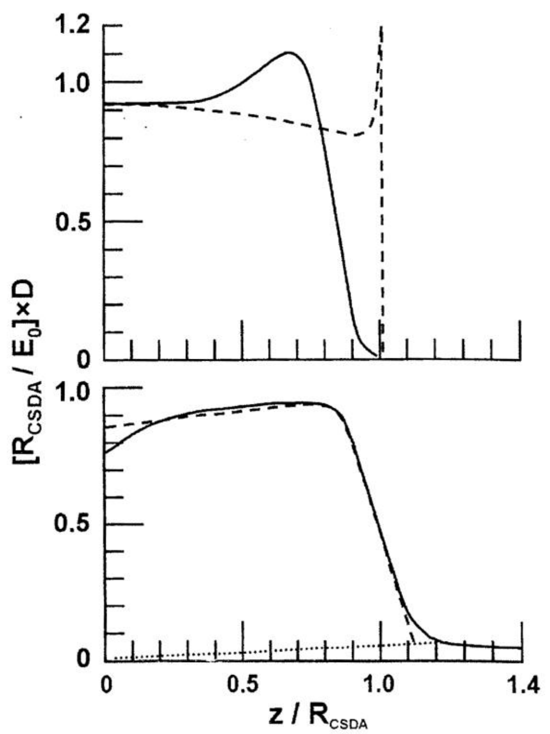




ENERGY LOSS

Figure 13. Scaled absorbed-dose distribution for electrons at initial energy E_0 as a function of depth z in water

Upper: (- - -) pure continuous-slowing-down approximation, (——) multiple angular scattering is taken into account
Lower: (- - -) energy-loss straggling and multiple angular scattering taken into account, (——) catastrophic events included



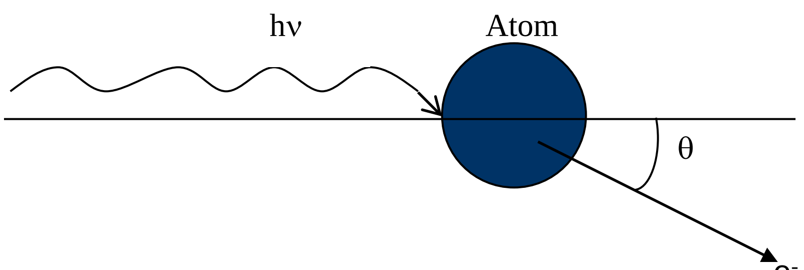
B. Grosswendt, The Physics of Particle Transport: Electrons and Photons, in: The Use of MCNP in Radiation Protection and Dosimetry (1996) ENEA

PHOTONS – PHOTOELECTRIC EFFECT

- Threshold reaction, approximately:

$$E_{th} = 13.6 \frac{(Z-\sigma)^2}{n^2} eV$$

K-shell σ= 1, L-shell σ =5, M-shell σ = 13.
K-shell n = 1, L-shell n= 2, M-shell n= 3.



$T_e = h\nu - B_e$

$$\sigma_k = \phi_0 4\sqrt{2} \frac{Z^5}{137^4} \left(\frac{m_e c^2}{h\nu} \right)^{3.5} \qquad h\nu \ll m_e c^2$$

$$\phi_0 = \frac{8\pi}{3} \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2} \right)^2 = \frac{8\pi}{3} r_e^2 = 6.651 x 10^{-25} cm^2$$

$$\sigma_k = \phi_0 \frac{3}{2} \frac{Z^5}{137^4} \frac{m_e c^2}{h\nu} \qquad h\nu \gg m_e c^2$$

- Characteristic X-rays and Auger electrons are emitted from atomic de-excitation.

PHOTONS – COMPTON EFFECT

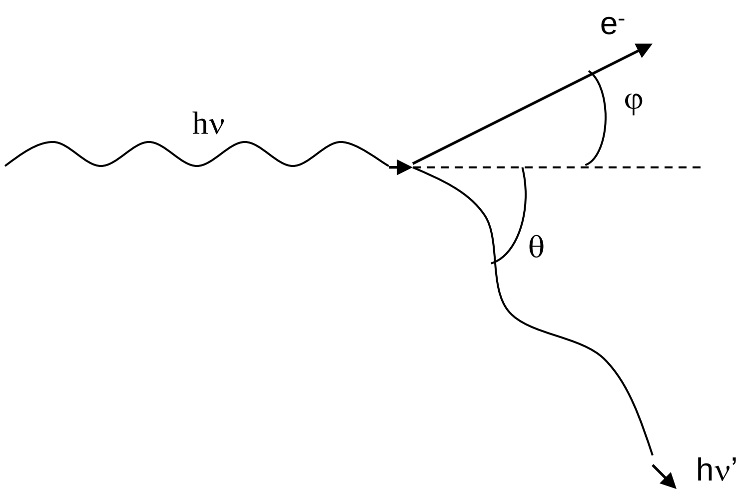
- The Compton effect can be modelled by considering a free electron:

$$h\nu' = \frac{h\nu}{1+\frac{h\nu}{m_e c^2}(1-\cos\vartheta)}$$

$$T_e = h\nu - h\nu' = \frac{h\nu}{1+\frac{m_e c^2}{h\nu(1-\cos\vartheta)}}$$

$$\cot\varphi = \left(1+\frac{h\nu}{m_e c^2}\right)\frac{1-\cos\vartheta}{\sin\vartheta}$$

$$\lambda' - \lambda = \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_e c}(1-\cos\vartheta)$$



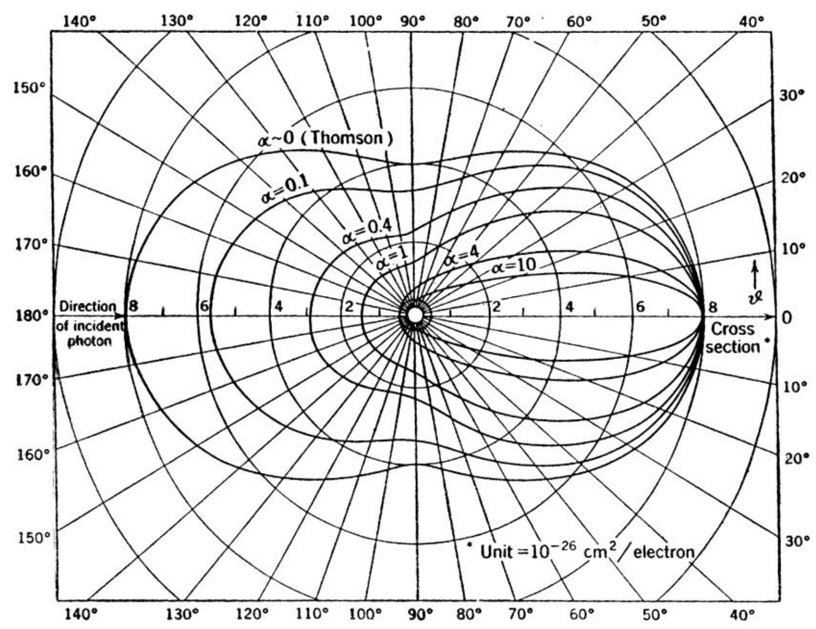
COMPTON EFFECT

Klein-Nishina x-sec:

$$\frac{d\sigma_{KN}(\vartheta)}{d\Omega} = \frac{r_e^2}{2}\left[1 + k(1-\cos\vartheta)\right]^{-2}\left[1 + \cos^2\vartheta + \frac{k^2(1-\cos\vartheta)^2}{1 + k(1-\cos\vartheta)}\right] \quad \mathrm{cm}^2 sr^{-1}$$

$$\mathrm{k} = \frac{\mathrm{h}\nu}{\mathrm{m_e c^2}}$$

- The Compton effect depends on Z



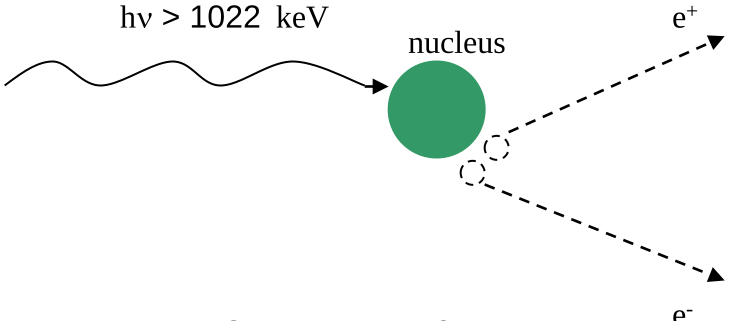
RAYLEIGH SCATTERING

- Predominant at low energies when $h\nu<B$ (electron binding energy). Photons change their direction and the recoil atom absorbs a negligible amount of energy.

$$\sigma_{coh} \propto \frac{Z^{2.5}}{(h\nu)^2}$$

PAIR CREATION

- Threshold energy $2m_ec^2 \cong 1022$ keV;

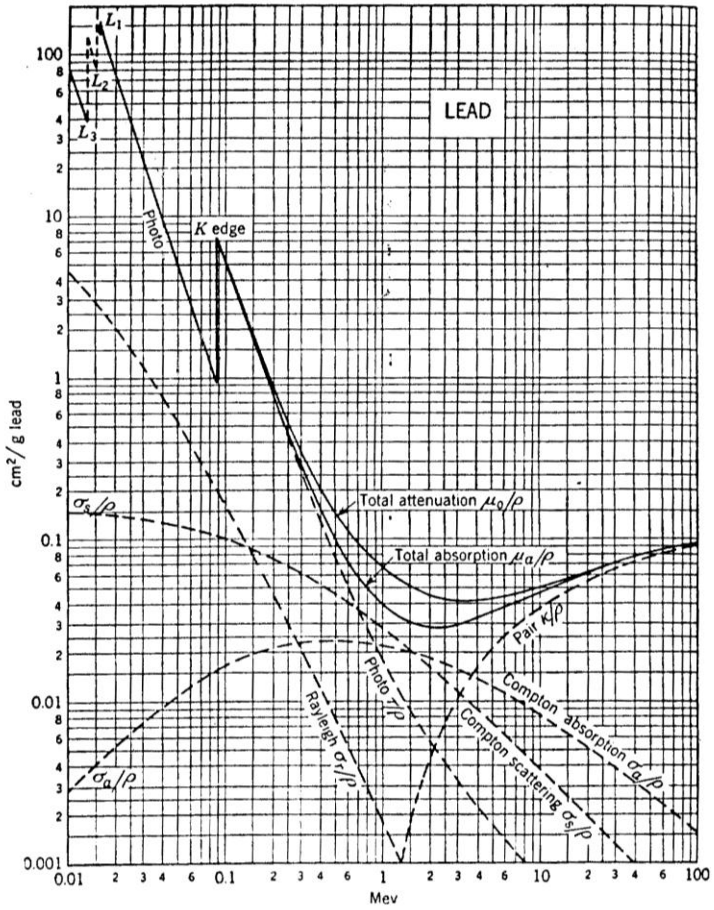


$h\nu = (T_- + m_ec^2) + (T_+ + m_ec^2)$

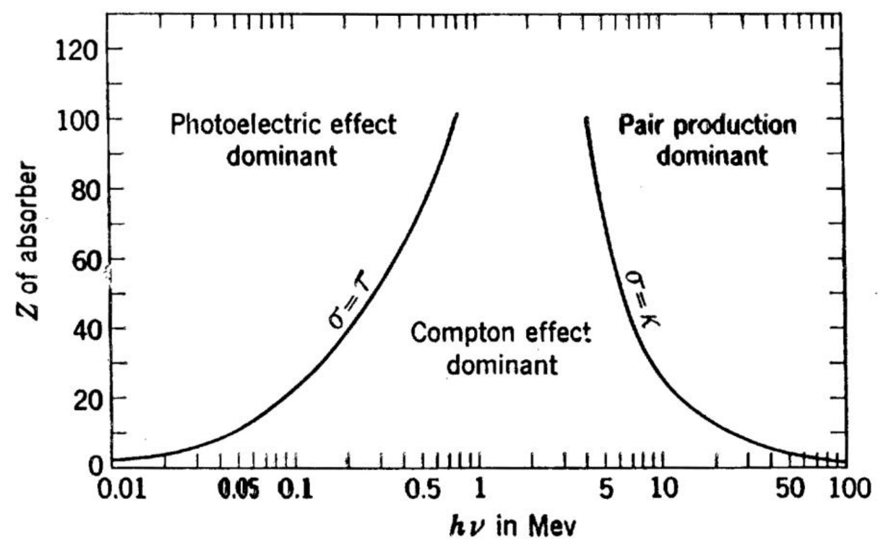
$$\kappa = Z^2 \frac{r_e^2}{137}\left(\frac{28}{9}\ln\frac{2h\nu}{m_ec^2} - \frac{218}{27}\right)$$ $1 << h\nu/m_ec^2 << 1/137\, Z^{-1/3}$

$$\kappa = Z^2 \frac{r_e^2}{137}\left(\frac{28}{9}\ln\frac{183}{Z^{1/3}} - \frac{2}{27}\right)$$ $h\nu/m_ec^2 >> 1/137\, Z^{-1/3}$

PHOTONS



LEAD
K edge
Photo
Total attenuation μ_0/ρ
Total absorption μ_a/ρ
Pair κ/ρ
Compton absorption σ_a/ρ
Compton scattering σ_s/ρ
Rayleigh σ_r/ρ
Photo τ/ρ
σ_s/ρ
σ_a/ρ
cm²/g lead
Mev



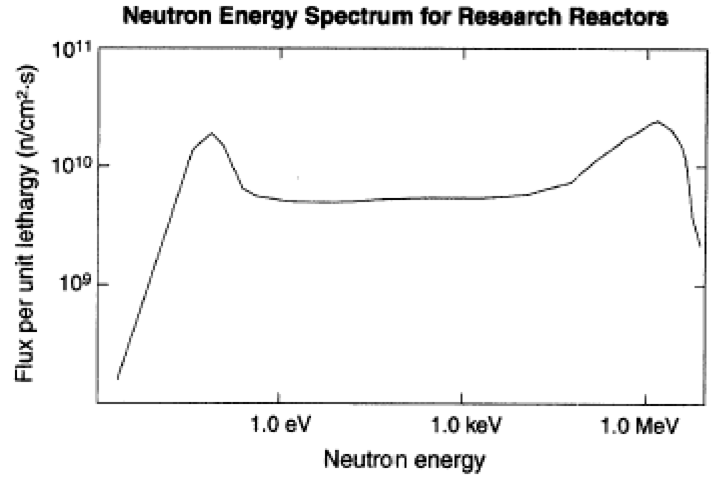
Photoelectric effect dominant
Pair production dominant
Compton effect dominant
$\sigma = \tau$
$\sigma = \kappa$
Z of absorber
$h\nu$ in Mev

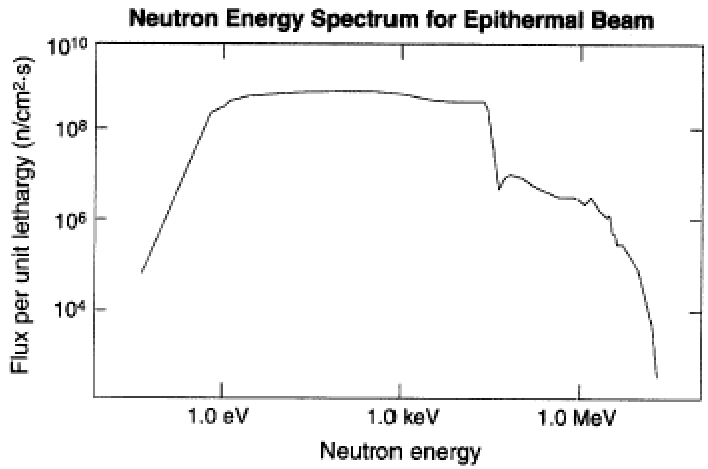
NEUTRON INTERACTIONS WITH SOFT TISSUE

- Neutrons below about 20 MeV:
  - ✓ Thermal neutrons (0<E<0.5 eV);

$$\frac{dN}{dE} \propto E e^{-E/kT}$$

  - ✓ Epithermal neutrons (0.5 eV<E<100 keV);
  - ✓ Fast neutrons (100 keV<E<20 MeV).





NEUTRON INTERACTIONS WITH SOFT TISSUE

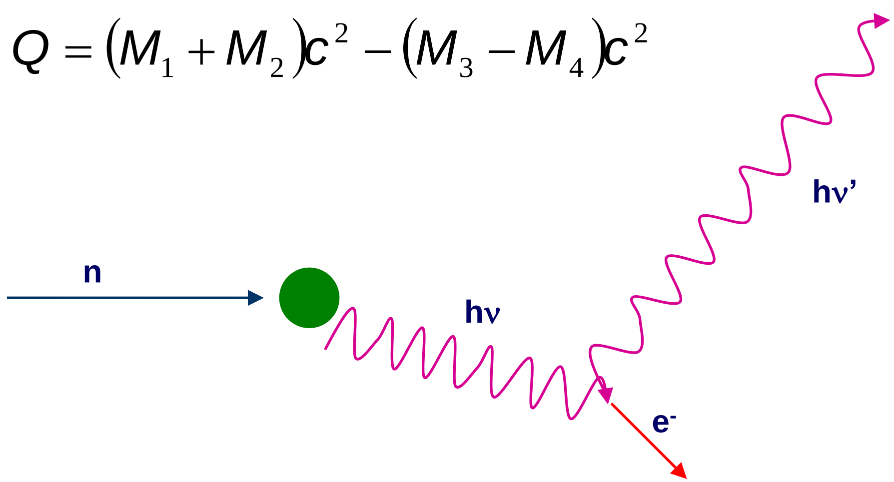
- Soft tissue:

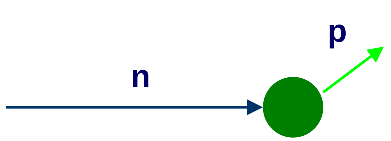
Element	Weight percent
H	10.2
C	12.3
N	3.5
O	72.9
Na	0.08
Mg	0.02
P	0.2
S	0.5
K	0.3
Ca	0.007

THERMAL NEUTRONS

Element	Reaction	Q (MeV)	Cross section
H	$^{1}H(n,\gamma)^{2}H$	2.223	332 mb
C	$^{12}C(n,\gamma)^{13}C$	4.946	3.4 mb
N	$^{14}N(n,\gamma)^{15}N$	10.833	75 mb
N	$^{14}N(n,p)^{14}C$	0.626	1.81 b
O	$^{16}O(n,\gamma)^{17}O$	4.143	0.178 mb

$$Q = (M_1 + M_2)c^2 - (M_3 - M_4)c^2$$





EPITHERMAL NEUTRONS

- Neutron absorption cross sections depend on 1/v;
- Elastic scattering occurs and recoil nuclei can contribute to the absorbed dose.

FAST NEUTRONS

- Elastic scattering occurs and recoil nuclei contribute to the absorbed dose.

$$E_R = \frac{4A}{(A+1)^2}\left(\cos^2 \vartheta\right)E_n$$

Target Nucleus	ER,max/En
H	1
C	0.284
N	0.249
O	0.221

- Inelastic reactions:

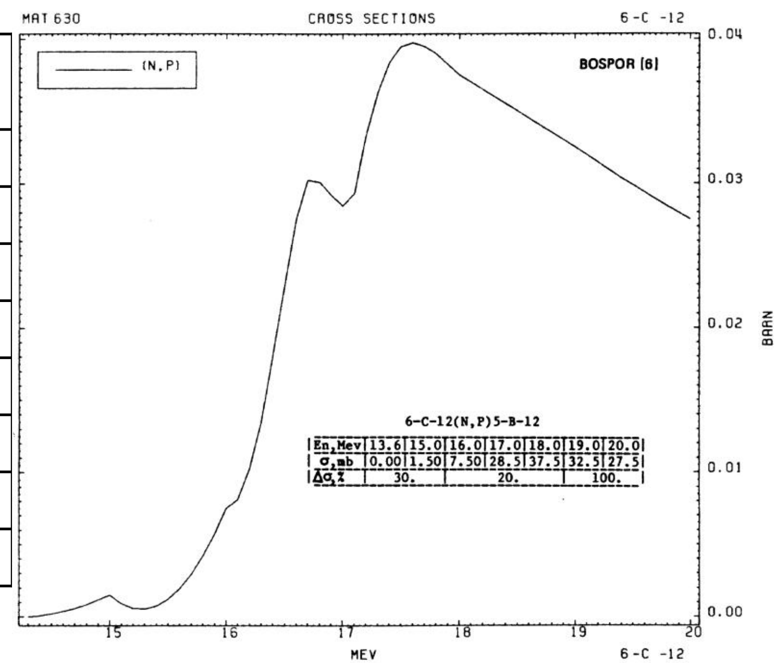
$$E_{th} = -Q\left(\frac{M_3 + M_4}{M_3 + M_4 - M_1}\right)$$

$$E_{th} = -Q\left(\frac{M_1 + M_2}{M_2}\right) \qquad M_2 >> Q/c^2$$

FAST NEUTRONS

- Some inelastic reactions:

Target Nucleus	Reaction	Q (MeV)	Threshold Energy (MeV)
C	$^{12}C(n,\alpha)^{9}Be$	-5.70122	6.18044
C	$^{12}C(n,p)^{12}B$	-12.58665	13.64462
C	$^{12}C(n,2n)^{11}C$	-18.72201	20.29569
N	$^{14}N(n,\alpha)^{11}B$	-0.15816	0.16955
N	$^{14}N(n,2n)^{13}N$	-10.55345	11.31363
O	$^{16}O(n,\alpha)^{13}C$	-2.21561	2.35534
O	$^{16}O(n,p)^{16}N$	-9.63815	10.24595
O	$^{16}O(n,2n)^{15}O$	-15.66384	16.65162



$^{12}C(n,p)^{12}B$

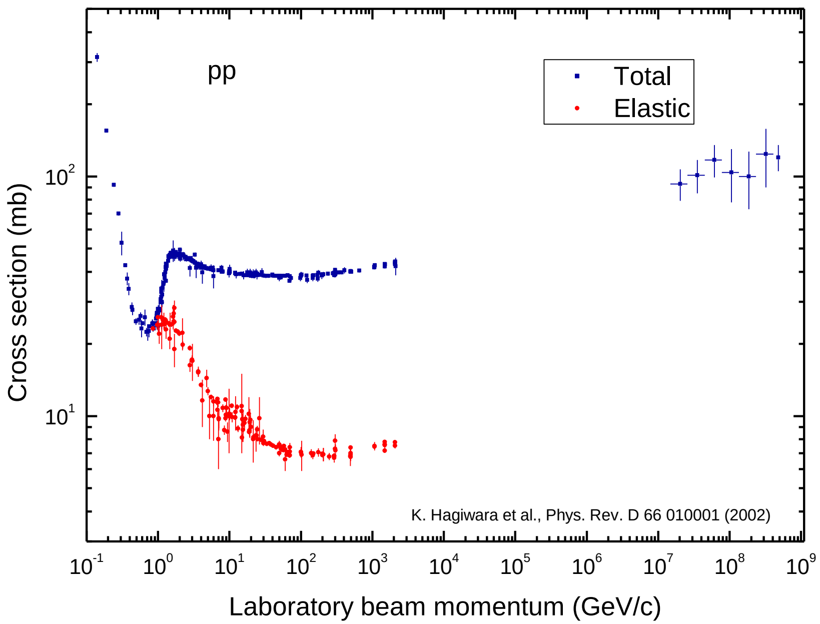
SECONDARY RADIATION AT INTERMEDIATE AND HIGH ENERGIES

- The main mechanisms for secondary hadron production from particles other than ions at intermediate energies (from about 50 MeV up to a few GeV) will be outlined.
- It should be underlined that for particle momenta higher than a few GeV/c, the hadron-nucleus cross section tends to its geometric value:

$$\sigma_{hN} \cong \pi r^2 = \pi (r_0 A^{1/3})^2 = 45 A^{2/3}\ \mathrm{mb}$$

- The interaction length scales with:

$$\lambda_{hN} = \frac{1}{\frac{N}{\rho}\sigma_{hN}} \cong 37 A^{1/3}\ \mathrm{g\,cm^{-2}}$$

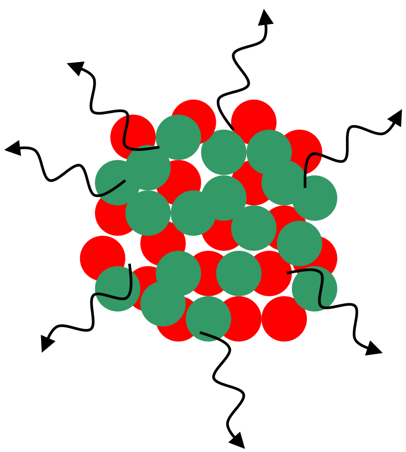


- References:
  - ✓ Ferrari, A. and Sala, P.R. The Physics of High Energy Reactions. Proceedigs of the Workshop on Nuclear Reaction Data and Nuclear Reactors Physics, Design and Safety, International Centre for Theoretical Physics, Miramare-Trieste (Italy) 15 April-17 May 1996, Gandini A. and Reffo G., Eds.World Scientific, 424-532 (1998).
  - ✓ ICRU 28.

INTRANUCLEAR CASCADE

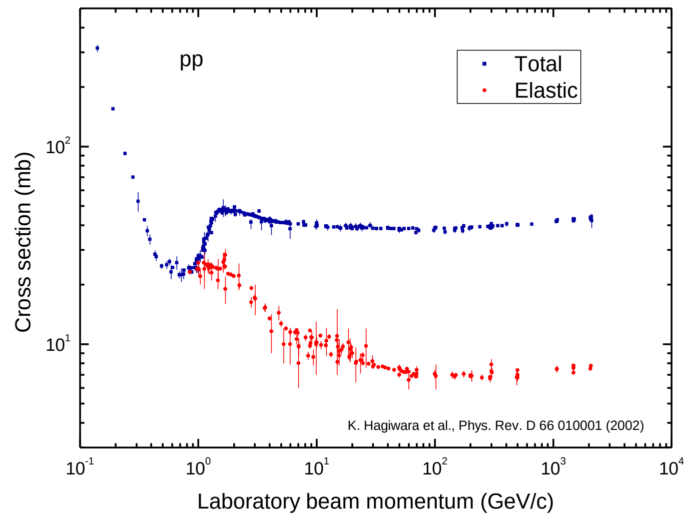
- Intermediate energy reactions can be described through the intranuclear cascade model. Its main steps are:
  - ✓ direct hadron-nucleon interactions (10^{-23} s);
  - ✓ pre-equilibrium stage;
  - ✓ nuclear evaporation (10^{-19} s);
  - ✓ de-excitation of the residual nucleus.

- Secondary particles can interact with other nuclei giving rise to an extra-nuclear cascade.



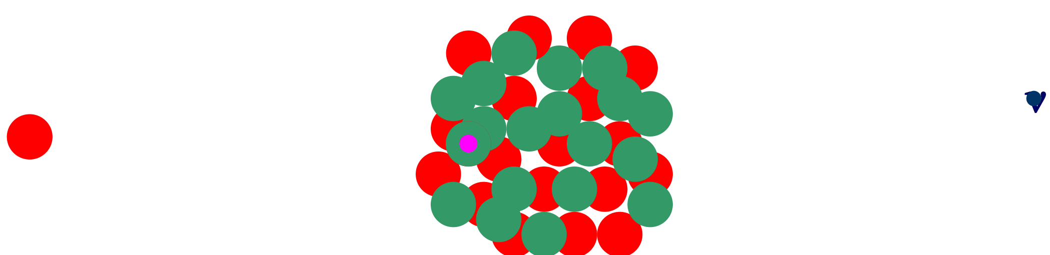
INTRANUCLEAR CASCADE

- Direct h-n interactions can be treated by assuming that hadrons are transported in the target nucleus like free particles interacting with nucleons with a probability ruled by free-space x-sections.
- Of course other more complex phenomena should be accounted for (quantum effects, effects of the nuclear field, Pauli blocking, local nuclear density, etc.).
- The free-particle model can be applied for particle momenta higher than 1 GeV/c (i.e. above about 200 MeV).
- Secondary nucleons from direct interactions are forward peaked and their energy distribution extends up to about the primary beam energy.



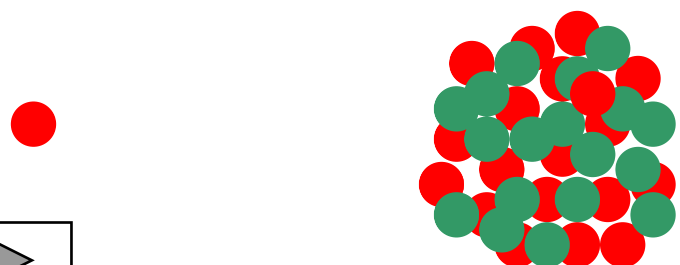
INTRANUCLEAR CASCADE

- At intermediate energies the inelastic reactions are mainly limited to pion production:
  - ✓ threshold energy: about 290 MeV, significant production above about 700 MeV;
  - ✓ $\pi^{\pm}$ half life: 2.6×10^{-8} s ($\pi^{+}\rightarrow \mu^{+} + \nu$)
  - ✓ π^{0} half-life: 8.4×10^{-17} s ($\pi^{0}\rightarrow 2\gamma$).
- At high-energy accelerators (above tens GeV) muons give a significant contribution to the stray radiation field past the shield (high-energy and low LET).
- Neutral pions decay into two high-energy photons and may switch on an electromagnetic cascade.



INTRANUCLEAR CASCADE

- Pre-equilibrium is a transition between the first interaction and the final thermalisation of nucleons:
  - ✓ this stage occurs when the excitation energy is shared among a few nucleons;
  - ✓ secondaries are forward peaked;
  - ✓ the energy distribution extends up to about the maximum beam energy.

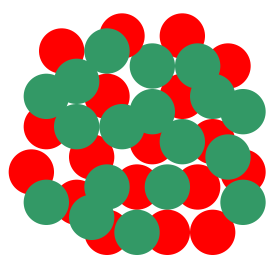


INTRANUCLEAR CASCADE

- The last step of the INC occurs when the nuclear excitation is shared among a large number of nucleons:
  - ✓ the compound nucleus has lost memory about the former steps;
  - ✓ nucleons and light fragments (d, t, α) can be emitted;
  - ✓ for A>16 (nuclear level density approximated to a continuum):
    - → evaporation model;
    - → the energy distribution can be described by a Maxwellian function (peaked at T):

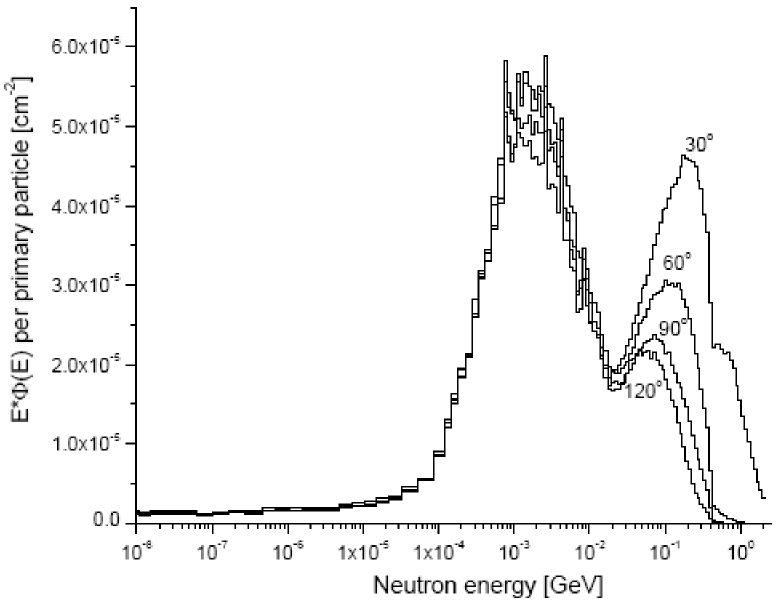
$$T = \sqrt{\frac{U}{a}} \ \ \mathrm{MeV} \qquad a = \frac{A}{8} \ \mathrm{MeV}^{-1}$$

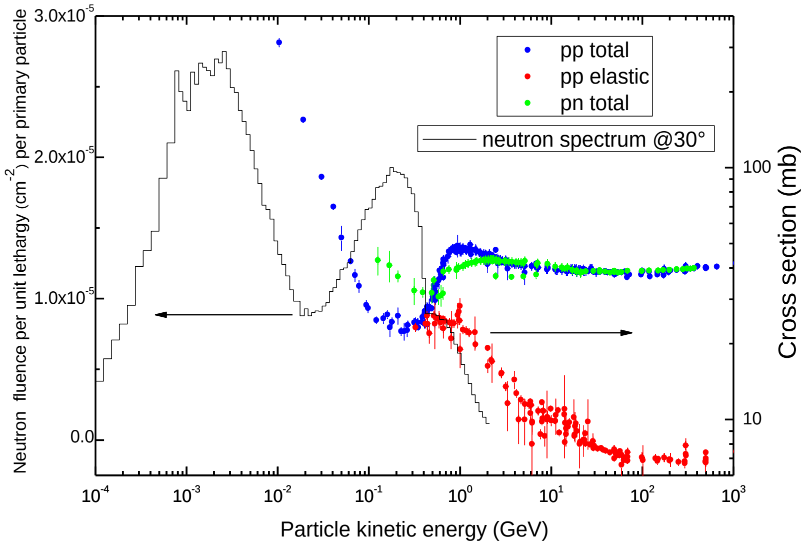
    - → the angular distribution is isotropic.



- For light nuclei (A<16) other models such as the Fermi break-up model can be applied for describing secondary particle and fragment generation.

INTRANUCLEAR CASCADE

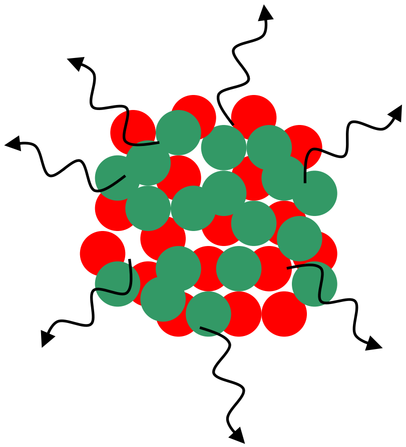




Neutron spectral fluence [EΦ(E)] per primary hadron from 40 GeV/c protons/pions on a 50 mm thick silver target, at emission angles of 30°, 60°, 90° and 120°. Agosteo et al. NIM B 229 (2005) 24-34.

INTRANUCLEAR CASCADE

- After evaporation, the residual nucleus can be still in an excited state:
  - ✓ if the excitation energy is lower than the binding energy of the less bounded nucleon;
    - → the residual energy is lost through the emission of photons.
  - ✓ the angular distribution of the de-excitation photons is isotropic;
  - ✓ for heavy nuclei the energy spectrum can be described by a Maxwellian function;
  - ✓ for light nuclei the excitation levels should be accounted for.



INTRANUCLEAR CASCADE

- Of the various particles generated by a target bombarded by a high-energy beam only neutron, photons and muons can contribute significantly to the dose past a shield:
  - ✓ protons and light fragments from evaporation are of low energy and are completely stopped in the air inside the hall (the range of a 5 MeV proton in air is 34 cm);
  - ✓ pions decay with a very short half-life;
  - ✓ high-energy hadrons interact with the shielding barrier and generate secondary radiation which should be accounted for;
  - ✓ The radiation field generated inside the barrier is composed by neutrons (mainly), protons, photons, electrons, positrons and pions.

REFERENCES

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- E. Segrè, Nuclei and Particles, (1964) W.A. Benjamin inc.
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- ICRU, Basic Aspects of High Energy Particle Interactions and Radiation Dosimetry (1978) ICRU, Bethesda, Maryland.
- S. Agosteo, M. Silari, L. Ulrici, Instrument Response in Complex Radiation Fields, Radiation Protection Dosimetry, 137 (2009) 51-73 doi: 10.1093/rpd/ncp186.

Additional Slides

Lethargy plots

- Conservative in terms of area for semi-logarithmic plots

$$\int_{E1}^{E2} f(E)dE = \int_{E1}^{E2} E\, f(E)dE/E = \int_{E1}^{E2} E\, f(E)d(\ln E) = \ln 10 \int_{E1}^{E2} E\, f(E)\, d(\log E)$$

- Therefore:

$$f(E)dE = Ef(E)d(\ln E) \quad \text{and}: \; Ef(E) = \frac{f(E)dE}{d(\ln E)}$$

- Histogram:

$$E_i f_i(E) = \frac{f_i(E)\times(E_{i+1}-E_i)}{\ln E_{i+1} - \ln E_i} = \frac{f_i(E)\Delta E}{\ln(E_{i+1}/E_i)}$$

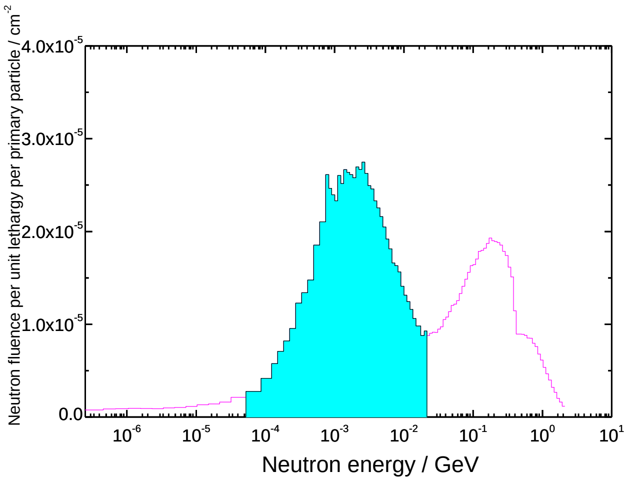
- Lethargy (definition):

$$u = \ln\frac{E_0}{E} = \ln E_0 - \ln E$$

$$du = -\frac{dE}{E}$$

$$F(u)du = -F(E)dE$$

$$F(u) = EF(E)$$



PARTICLE FLUENCE:
COSINE-WEIGHTED BOUNDARY CROSSING

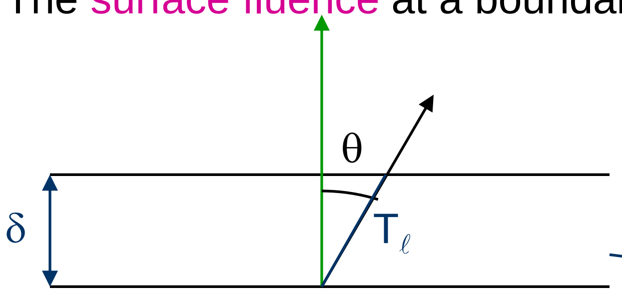
- The spectral distribution of particle radiance is defined as:

$$p_E = \frac{d^4N}{da\,d\Omega\,dE\,dt} = vn(\vec{r},\Omega,E)$$

  - ✓ v=particle velocity;
  - ✓ n=particle density (number of particles N per unit volume).
- The particle fluence averaged over a region of volume V can be estimated as:

$$\Phi = \int_V\int_\Omega\int_E\int_t vn(\vec{r},\Omega,E)\,dt\,dE\,d\Omega\,\frac{dV}{V} = \frac{\int\int nv\,dt\,dV}{V} = \frac{\int nds\,dV}{V} = \frac{T_\ell}{V}$$

  - ✓ nds is a “track-length density”;
  - ✓ T_ℓ sum of track lengths.
- The surface fluence at a boundary crossing is, for one particle of weight w:



$$\Phi_S = \lim_{\delta\to 0} w\frac{T_\ell}{V} = w\frac{\delta/|\cos\theta|}{S\delta} = \frac{w}{S|\cos\theta|}$$

Infinitely thin region of volume Sδ