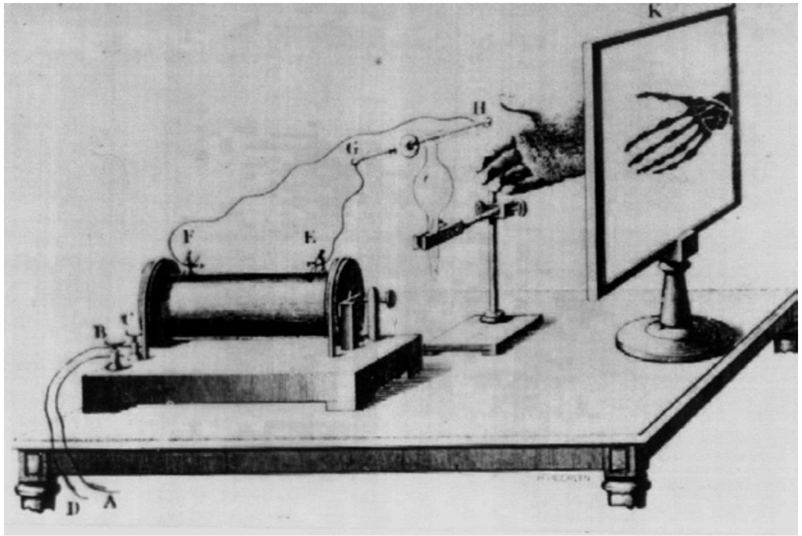
TRAINING COURSE ON RADIATION DOSIMETRY

# Quantities and units in radiation dosimetry

Marco SILARI, *CERN*

*1th Annual ARDENT Workshop, Vienna, 20-23 November 2012*



1895

Discovery of X rays

**Wilhelm C. Röntgen**

1897

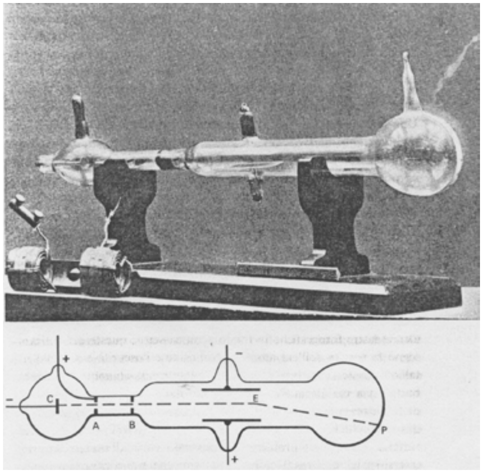
First treatment of tissue with X rays

**Leopold Freund**

**J.J. Thompson**

1897

"Discovery" of the electron



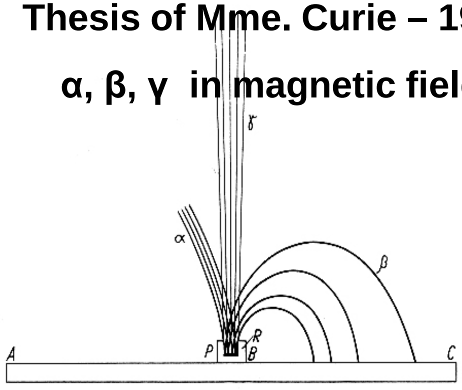
**Henri Becquerel (1852-1908)**

1896

Discovery of natural radioactivity

**Thesis of Mme. Curie – 1904**

**α, β, γ in magnetic field**




Hundred years ago

1898

Discovery of polonium and radium

**Marie Curie (1867 – 1934)** **Pierre Curie (1859 – 1906)**

- 1911: **first practical application of a radioisotope** (as *radiotracer*) by G. de Hevesy, a young Hungarian student working with naturally radioactive materials in Manchester

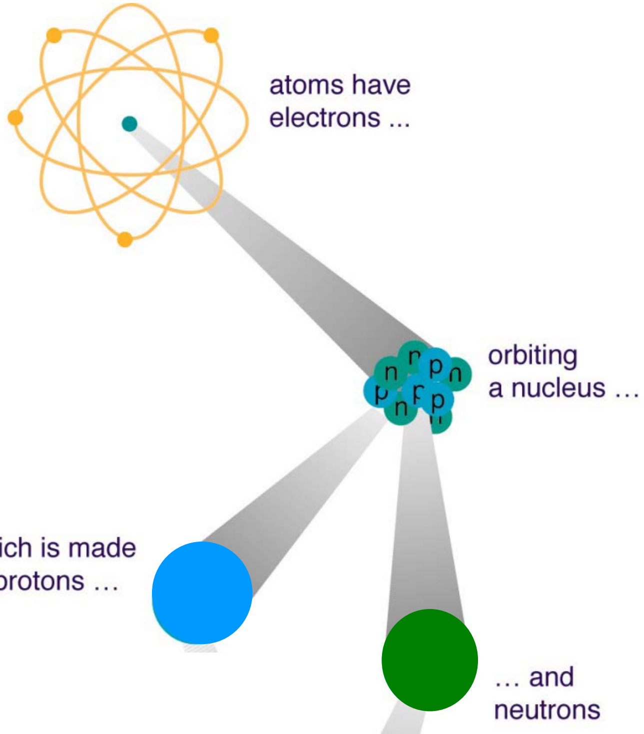
- 1924: de Hevesy, who had become a physician, used radioactive isotopes of lead as tracers in bone studies

**James Chadwick**

**(1891 – 1974)**

1932

Discovery of the neutron




**Cyclotron + neutrons = first attempt of radiation therapy with fast neutrons at LBL (R. Stone and J. Lawrence, 1938)**

**Directly ionizing radiation:**

- fast charged particles (e.g., electrons, protons, alpha particles), which deliver their energy to matter directly, through many small Coulomb-force interactions along the particle's track

**Indirectly ionizing radiation:**

- X- or $\gamma$-rays photons or neutrons (i.e., uncharged particles), which first transfer their energy to charged particles in the matter through which they pass in a relatively few large interactions, or cause nuclear reactions
- The resulting fast charged particles then in turn deliver the energy in matter

The deposition of energy in matter by indirectly ionizing radiation is a two-step process

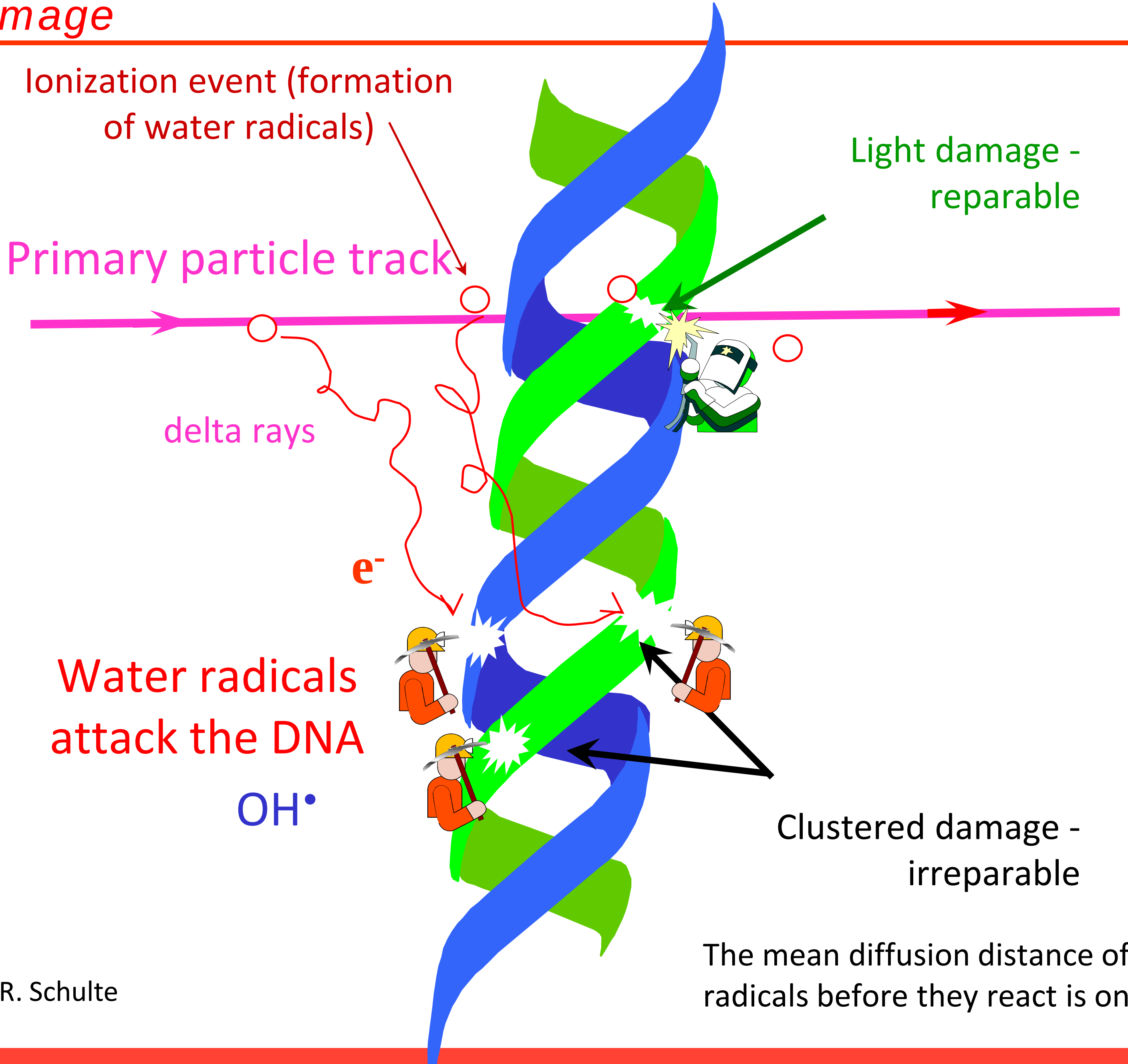
photon → electron
neutron → proton or recoiling nuclei

## Unique effects of interaction of ionizing radiation with matter

- Biological systems (humans in particular) are particularly susceptible to damage by ionizing radiation
- The expenditure of a trivial amount of energy (~4 J/kg or Gy) to the whole body is likely to cause death
- Even if this amount of energy can only raise the gross temperature by about 0.001 °C
- This is because of the ability of ionizing radiation to impart their energy to individual atoms and molecules
- The resulting high local concentration of absorbed energy can kill a cell either ***directly*** or through the formation of highly reactive chemical species such as ***free radicals*** (atom or compound in which there is an unpaired electron, such as H or $CH_3$) in the water medium that constitutes the bulk of the biological material

Main aim of **dosimetry** = measurement of the absorbed dose (energy/mass)

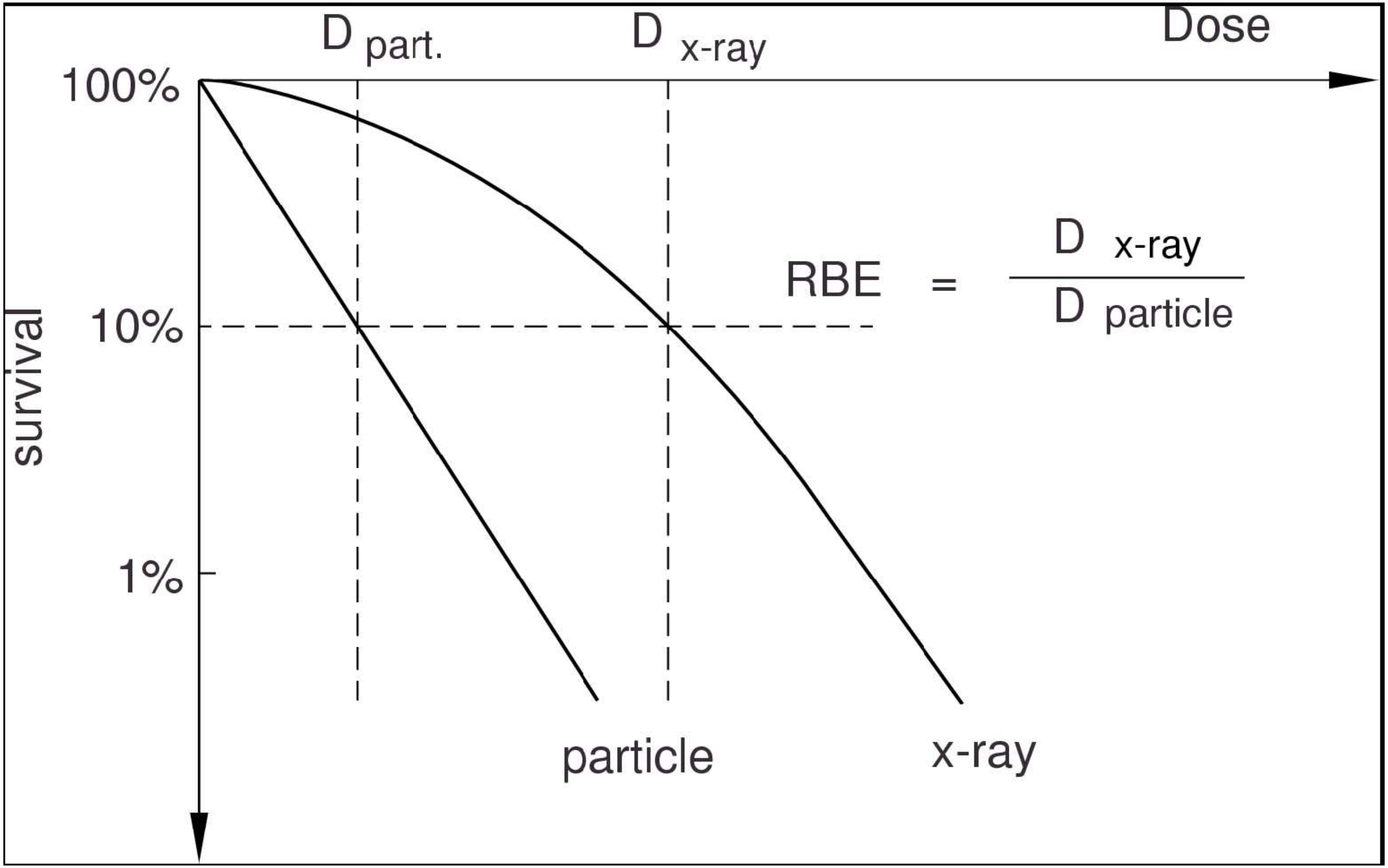
# DNA damage




The mean diffusion distance of OH radicals before they react is only 2-3 nm

Courtesy R. Schulte

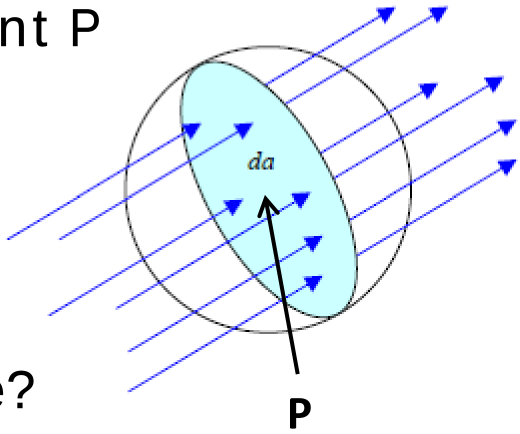
# Radiobiological effectiveness (RBE)



- How many *rays* (i.e. photons or particles) will strike a point P in radiation field?
- The answer is ***zero***, since a point has no cross-sectional area with which the rays can collide
- So: how can we describe the radiation field at P?

➢ Associate some nonzero volume to the point P

➢ Simplest volume is a sphere centered at P (it presents the same cross-sectional area to rays incident from all directions)




➢ How large should this imaginary sphere be?

  ➢ It depends on whether the physical quantities we wish to define w.r.t. the radiation field are:

    ➢ Stochastic

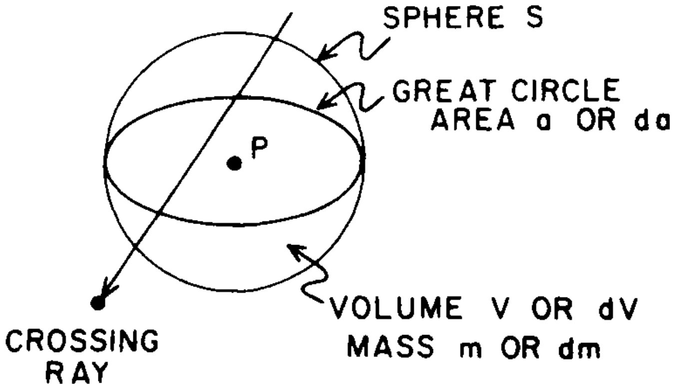
    ➢ Non-stochastic

- Its values occur randomly and cannot be predicted. However, the probability of any particular value is determined by a probability distribution
- It is defined for finite (i.e. non-infinitesimal) domains only. Its values vary discontinuously in space and time, and it is meaningless to speak of a gradient or rate of change
- In principle, its values can each be measured with an arbitrarily small error
- The ***expectation value*** $N_e$ of a stochastic quantity is the mean $\bar{N}$ of its measured values $N$ as the number $n$ of observations approaches ∞:

$$\bar{N} \rightarrow N_e \text{ as } n \rightarrow \infty$$

- For given conditions its value can, in principle, be predicted by calculations
- It is, in general, a “point function” defined for infinitesimal volumes; hence it is a continuous and differentiable function of space and time, and one may speak of its spatial gradient and time rate of change
- ***Its value is equal to, or based upon***, the ***expectation value*** of a related stochastic quantity, if one exists. Although non-stochastic quantities in general need not be related to stochastic quantities, they are so related in the context of ionizing radiation

- The volume of the imaginary sphere surrounding P may be small but must be ***finite*** if dealing with stochastic quantities
- This volume may be ***infinitesimal***, *dV*, in reference to non-stochastic quantities
- Likewise the great-circle area ***da*** and contained mass ***dm***, as well as the irradiation time ***dt*** may be expressed as infinitesimal with non-stochastic quantities




- Most common and useful quantities for describing radiation fields and their interaction with matter are non-stochastic
- Stochastic quantities are mostly involved with ***microdosimetry*** (the determination of energy spent in a small but finite volume) → *next year workshop at the Politecnico of Milano, October 2013*
- Microdosimetry is of particular interest in relation to **biological cell damage** → *T. Waker's lecture*

# Counting statistics

- In general a "*constant*" radiation field is ***random*** w.r.t. how many rays or particles arrive at a given point per unit area and time interval
- The number of rays or particles observed (counted by a detector) in repetitions of the measurement follows a ***Poisson distribution***, which can be approximated by a Gaussian for large number of events
- Standard deviation σ of a single random measurement *N* relative to $N_e$:

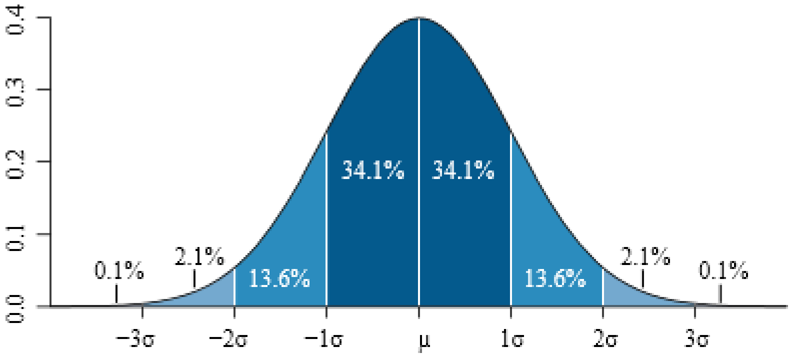
$$\sigma = \sqrt{N_e} \cong \sqrt{\bar{N}}$$

(Remember that $\bar{N} \to N_e$ as $n \to \infty$)

$N_e$ = expectation value of the number of rays detected per measurement

Percent standard deviation *S*:

$$S = \frac{100\sigma}{N_e} = \frac{100}{\sqrt{N_e}} \cong \frac{100}{\sqrt{\bar{N}}}$$




A single measurement *N* has 68.3% chance of lying within ±σ of $N_e$, 95.5% chance of lying within ±2σ of $N_e$ and 99.7% chance within ±3σ

In radiation dosimetry we have:

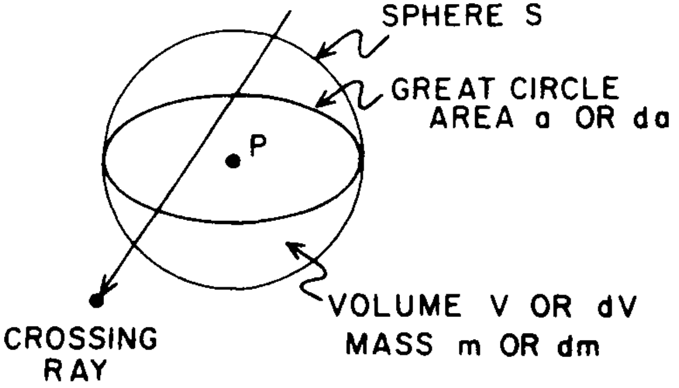
- Quantities describing the **radiation field** (*e.g., fluence*)
- Quantities describing the **medium** with which the radiation field interacts (*e.g., stopping power*)
- **Dosimetric quantity** =

  = quantity describing the field *x* constant of the medium

***Radiation fields can be described by a set of non-stochastic quantities***

## FLUENCE at a point P

$$\Phi = \frac{dN_e}{da} \qquad (\text{ m}^{-2} \text{ or } \text{cm}^{-2})$$




$N_e$ = expectation value of the number of rays or particles striking a finite sphere surrounding point *P* during a time interval from a starting time $t_0$ to a later time $t$

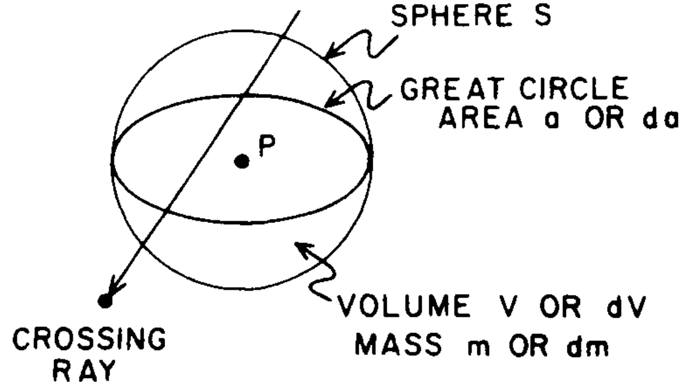
The sphere around *P* is reduced to an infinitesimal with great-circle area *da*

FLUX DENSITY OR FLUENCE RATE at a point P

$\Phi$ may be defined for all values of $t$ through the interval from $t_0$ (for which $\Phi$ = 0) to $t = t_{max}$ (for which $\Phi = \Phi_{max}$). Then at some time $t$ within the interval $t_0 \rightarrow t$:

$$\varphi = \frac{d\Phi}{dt} = \frac{d^2 N_e}{dtda}$$

( $m^{-2}\ s^{-1}$ or $cm^{-2}\ s^{-1}$)




$d\Phi$ is the increment of fluence during the infinitesimal time interval $dt$ at time $t$

The fluence at *P* for the time interval $t_0 \rightarrow t_1$

$$\Phi(t_0, t_1) = \int_{t_0}^{t_1} \varphi(t)dt$$

For a time-independent radiation field, $\varphi(t)$ = *constant* and

$$\Phi(t_0, t_1) = \varphi \cdot (t_1 - t_0) = \varphi\, \Delta t$$

It is important to note that:

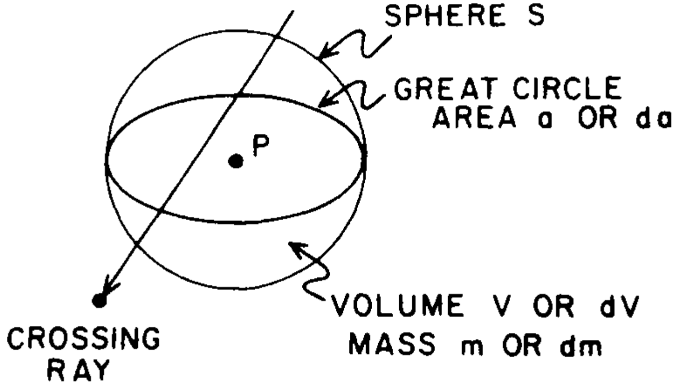
- $\varphi$ and $\Phi$ express the sum of rays or particles incident from ***all directions***, and irrespective of their quantum or kinetic energies → **basic information**
- a radiation field is often composed of various components (e.g., photons, neutrons, charged particles), which are – as far as possible – measured separately, as their interaction with matter are fundamentally different

The energy fluence $\Psi$ sums the energy of all individual rays or particles

$$\Psi = \frac{dR}{da}$$

(J $m^{-2}$ or erg $cm^{-2}$)

$$\boxed{1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}}$$




*R* = expectation value of the total energy (exclusive of rest-mass energy) carried by all the $N_e$ rays striking a finite sphere surrounding point *P* during a time interval from $t_0$ to $t$

The sphere around *P* is reduced to an infinitesimal with great-circle area *da*

For the special case where only a single energy *E* of rays is present:

$$R = E\, N_e$$

$$\Psi = \frac{d(EN_e)}{da} = E\Phi$$

ENERGY FLUX DENSITY OR ENERGY FLUENCE RATE at a point *P*

$\Psi$ may be defined for all values of *t* through the interval from $t_0$ (for which $\Psi = 0$) to $t = t_{max}$ (for which $\Psi = \Psi_{max}$). Then at some time *t* within the interval $t_0 \rightarrow t$:

$$\psi = \frac{d\Psi}{dt} = \frac{d}{dt}\left(\frac{dR}{da}\right)$$

(J m$^{-2}$ s$^{-1}$ or erg cm$^{-2}$ s$^{-1}$)

$d\Psi$ = increment of energy fluence during the infinitesimal time interval *dt* at time *t*

Similarly to what written above for the fluence rate:

$$\Psi(t_0, t_1) = \int_{t_0}^{t_1} \psi(t) dt$$

and for constant $\psi(t)$

$$\Psi(t_0, t_1) = \psi \cdot (t_1 - t_0) = \psi \, \Delta t$$

For monoenergetic rays of energy *E* (for which $\Psi = E\Phi$) the energy flux density $\psi$ may be related to the flux density $\varphi$ by:

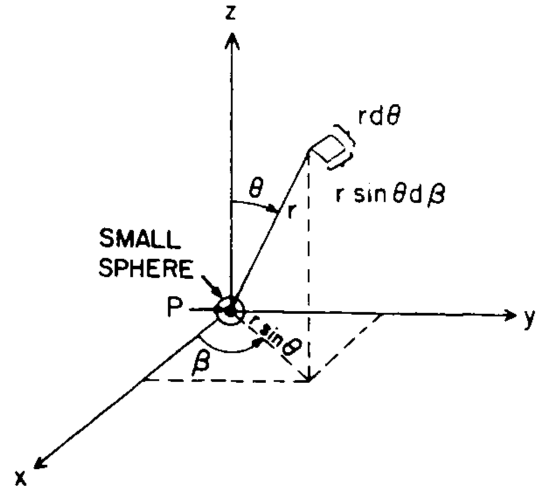
$$\psi = \frac{d\Psi}{dt} = E \frac{d\Phi}{dt} = E\varphi$$

Most radiation interactions are dependent upon the energy of the ray as well as its type, and the sensitivity of radiation detectors typically depends on the direction of incidence of the rays striking it

The radiation field must usually be described in terms of its energy and angular distributions

In principle one could measure the fluence rate at any time *t* and point *P* as a function of kinetic energy or quantum energy *E* and of the polar angles of incidence *θ* and *β*, to obtain the differential fluence rate:

$\varphi'(\theta, \beta, E)$ $(m^{-2}\ s^{-1}\ sr^{-1}\ eV^{-1})$

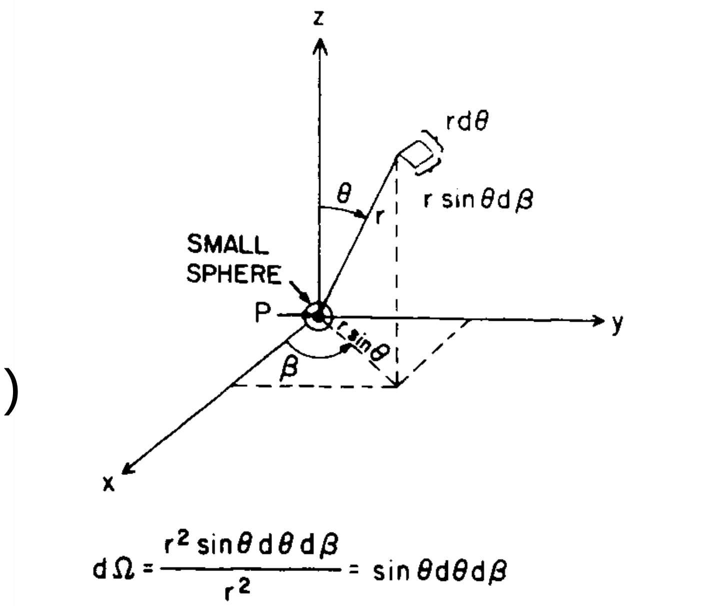



$$d\Omega = \frac{r^2 \sin\theta\, d\theta\, d\beta}{r^2} = \sin\theta\, d\theta\, d\beta$$

(According to the energy range, one uses $keV^{-1}$ or $MeV^{-1}$ instead of $eV^{-1}$)

Number of rays per unit time having energies between *E* and *E* + *dE* which pass through the element of solid angle $d\Omega$ at the given angles $\theta$ and $\beta$ before striking the small sphere centered at *P*, per unit great-circle area of the sphere:

$\varphi'(\theta, \beta, E)\, d\Omega\, dE$ $\quad$ ($m^{-2}\ s^{-1}$ or $cm^{-2}\ s^{-1}$)




Integrating over all angles and energies, one obtains the flux density $\varphi$:

$$\varphi = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \int_{E=0}^{Emax} \varphi'(\theta, \beta, E) \sin\theta\, d\theta\, d\beta\, dE \qquad (m^{-2}\ s^{-1} \text{ or } cm^{-2}\ s^{-1})$$

Similar expressions are valid for the energy fluence rate, fluence and energy fluence

# *Energy spectra*

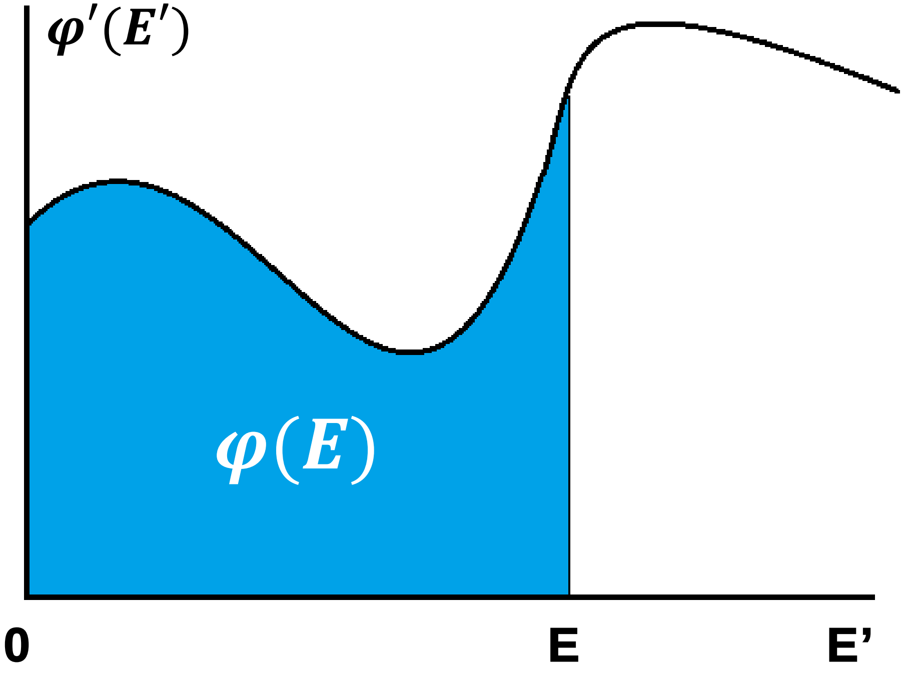
- Simpler and more useful differential distributions of fluence, fluence rate, energy fluence and energy fluence rate are those which are functions of only one of the variables $\theta$, $\beta$ or $E$
- When $E$ is the chosen variable, the resulting differential distribution is called the ***energy spectrum*** of the quantity
- For example:

Energy spectrum of the fluence rate summed over all directions, $\varphi'(E)$:

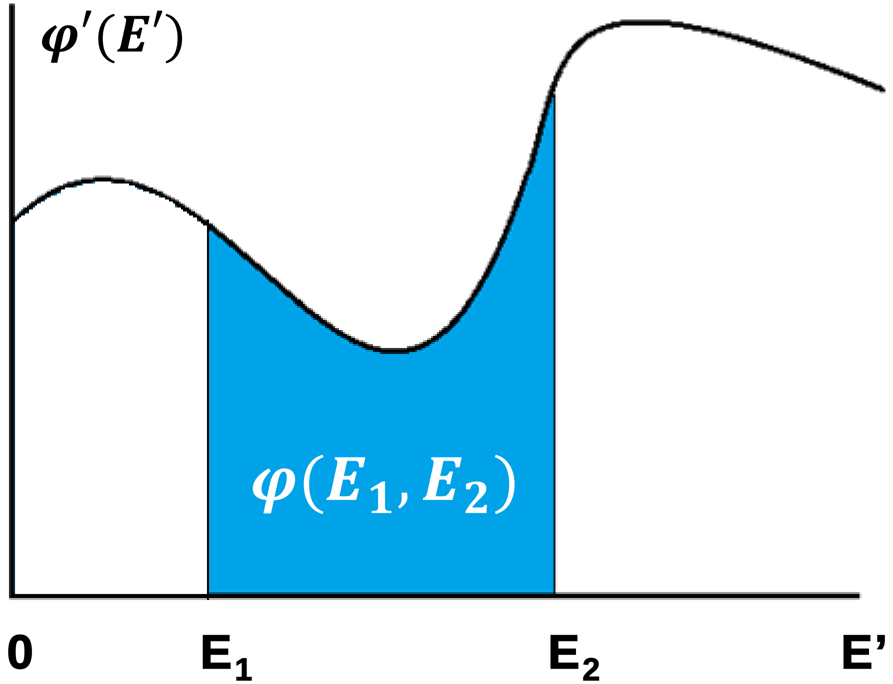
$$\varphi'(E)=\int_{\theta=0}^{\pi}\int_{\beta=0}^{2\pi}\varphi'(\theta, \beta, E)\sin\theta\, d\theta\, d\beta \qquad (m^{-2}\ s^{-1}\ keV^{-1}\ \text{ or }\ cm^{-2}\ s^{-1}\ keV^{-1})$$

and integrating over all energies of the rays gives of course

$$\varphi = \int_{0}^{E_{max}} \varphi'(E)dE$$




$$\varphi = \int_0^{E_{max}} \varphi'(E)dE$$

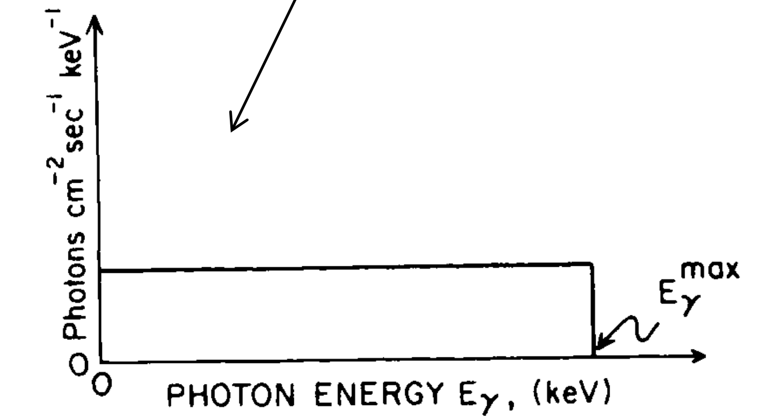


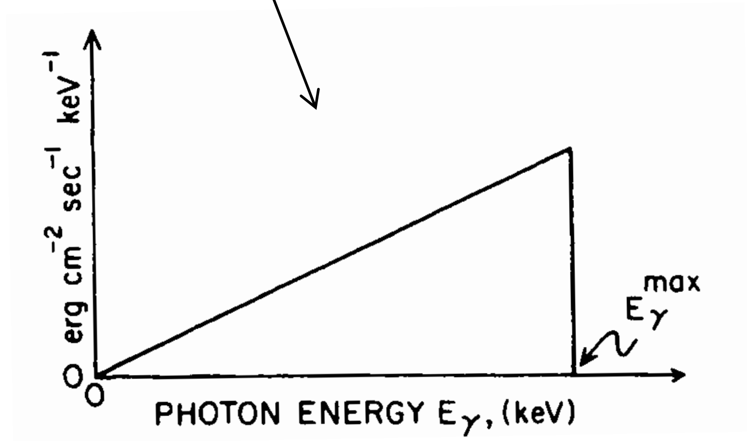

$$\varphi(E_1, E_2) = \int_{E_1}^{E_2} \varphi'(E)dE$$

**Example**

Flat spectrum of photon fluence rate $\varphi'(E)$ versus photon energy *E*

Corresponding spectrum of energy fluence rate $\psi'(E)$

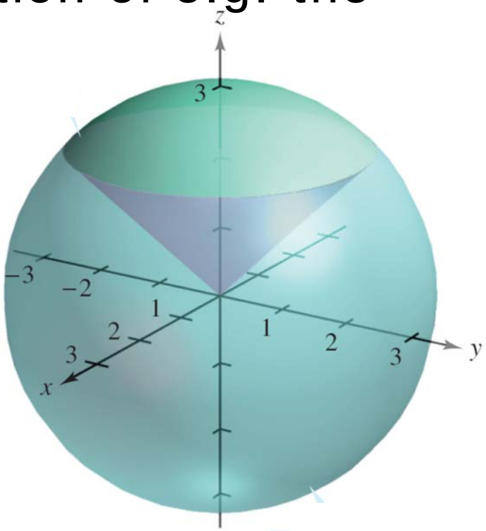



$$\psi'(E) = E\,\varphi'(E)$$

$$\psi = \int_{E=0}^{E_{max}} \psi'(E)dE = \int_{E=0}^{E_{max}} E\varphi'(E)dE$$

If the radiation field is symmetrical w.r.t. the vertical axis *z*, it can be described in terms of the differential distribution of e.g. the fluence rate *as a function of polar angle θ*



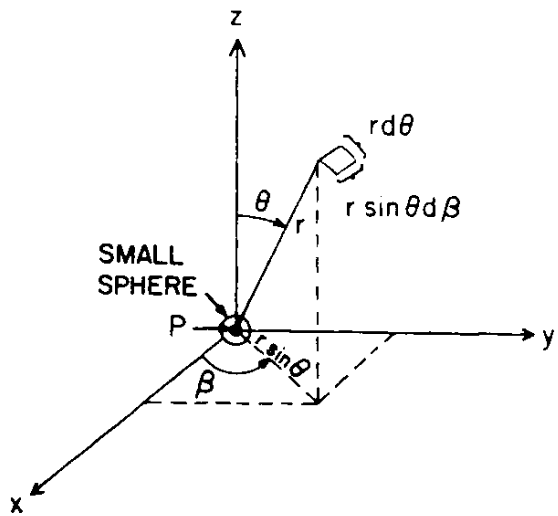
$$\varphi'(\theta)=\int_{\beta=0}^{2\pi}\int_{E=0}^{Emax}\varphi'(\theta,\beta,E)\sin\theta\, d\beta\, dE$$

The component of the fluence rate consisting of the particles of all energies arriving at *P* through the annulus lying between the two polar angles $\theta_1$ and $\theta_2$ is:

$$\varphi(\theta_1,\theta_2)=\int_{\theta_1}^{\theta_2}\varphi'(\theta)d\theta$$

$\varphi'(\theta)$ is expressed e.g. in m$^{-2}$ s$^{-1}$ radian$^{-1}$

If $\theta_1=0$ $and$ $\theta_2=\pi$ → $\varphi(\theta_1,\theta_2)=\varphi$



Differential distribution of fluence rate *per unit solid angle*, for particles of all energies:

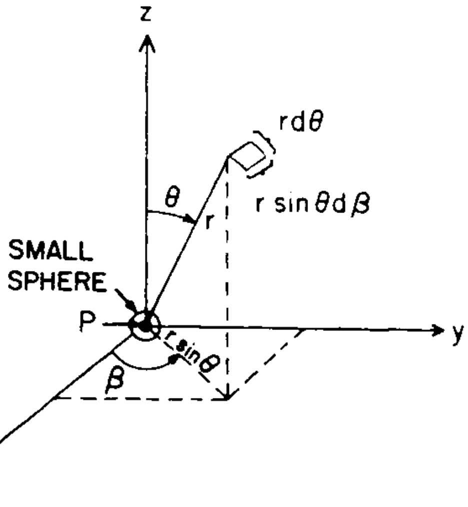
$$\varphi'(\theta,\beta) = \int_{E=0}^{E_{max}} \varphi'(\theta,\beta,E)\,dE \qquad (\mathrm{m^{-2}\ s^{-1}\ sr^{-1}})$$

and integrating over all directions:

$$\varphi = \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \varphi'(\theta, \beta) \sin\theta \, d\theta \, d\beta$$

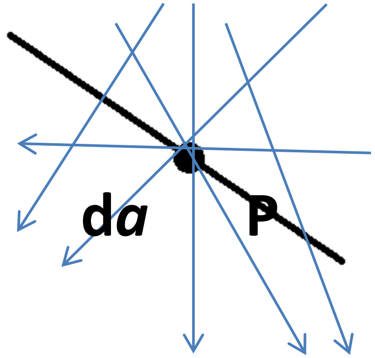
For a field that is symmetrical about the *z*-axis, $\varphi'(\theta,\beta)$ is independent of $\beta$, and integrating the previous expression over all $\beta$–values:

$$\varphi = 2\pi \int_{\theta=0}^{\pi} \varphi'(\theta, \beta) \sin\theta \, d\theta$$



**PLANAR FLUENCE**: the number of particles crossing a fixed plane in either direction (=summed by scalar addition) per unit area of the plane

$$\Phi_p = \frac{dN_e}{da}\overline{\cos\theta}$$

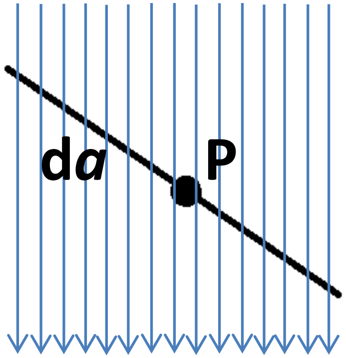



$\theta$ = angle of incidence of rays or particles on surface d*a*

If the angular distribution of the rays or particles is isotropic:

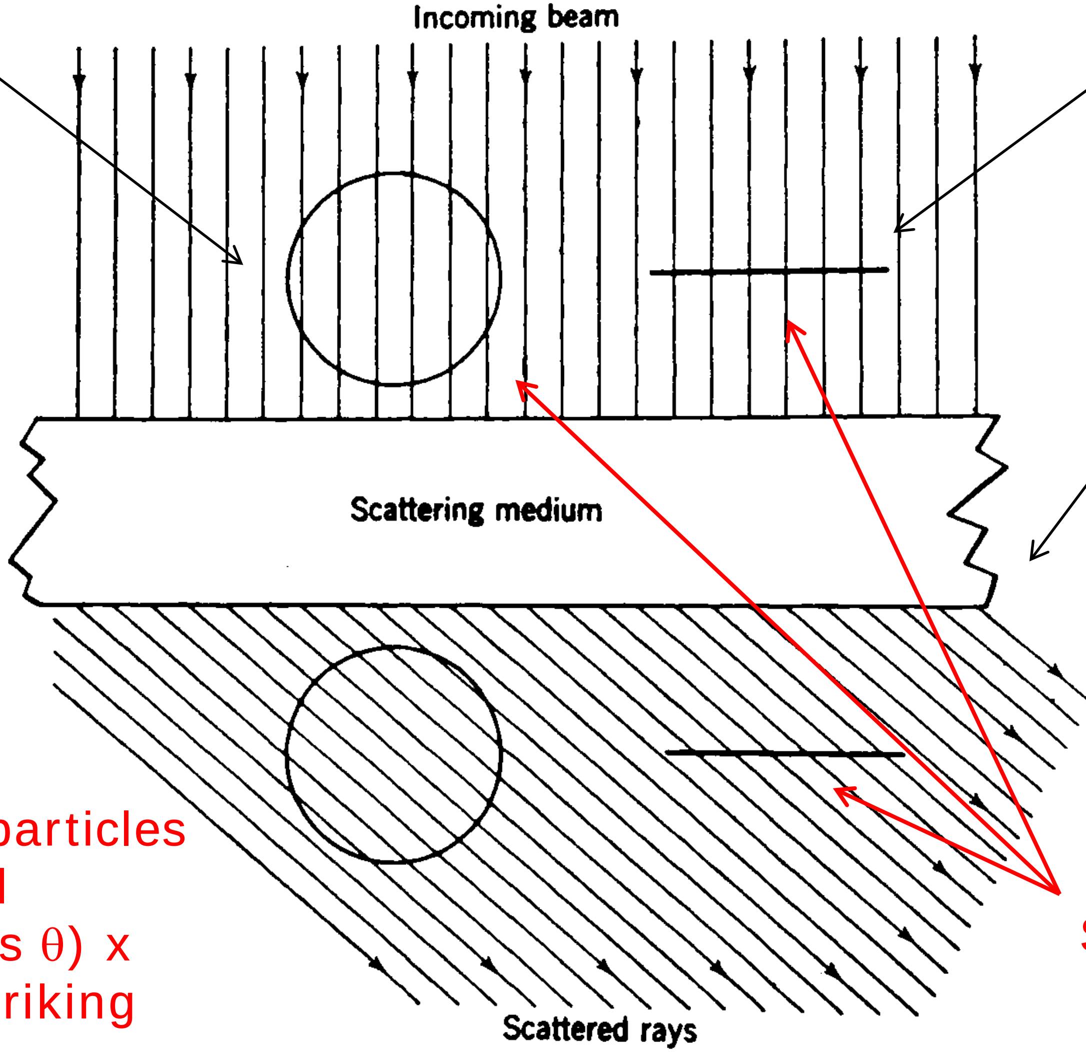
$\overline{\cos\theta}$= 1/2 $$\Phi_p = \frac{1}{2}\frac{dN_e}{da} = \frac{1}{2}\Phi \qquad \psi_p = \frac{1}{2}\psi$$

If the radiation field is unidirectional:




$$\Phi_p = \frac{dN_e}{da}\cos\theta$$

Spherical and flat detectors of equal cross-sectional areas




Particles scattered through the same angle $\theta$

No of scattered particles striking spherical detector = $(1/\cos\theta)$ x no of particles striking flat detector

Same fluence

In radiation dosimetry three **non-stochastic quantities** describe the interaction of a radiation field with matter:

- the ***kerma K***, describing the first step in energy dissipation by indirectly ionizing radiation = energy transfer to charged particles
- the ***absorbed dose D***, describing the energy imparted to matter by all kinds of ionizing radiations, but delivered by the charged particles
- the ***exposure X***, which describes x- and $\gamma$-fields in terms of their ability to ionize air
  - A related quantity is the mean energy expended per ion par production in a gas, $\overline{W}$

- Uncharged ionizing radiation lose their energy in relatively few large interactions, whereas charged particles typically undergo many small collisions, losing their kinetic energy gradually
- An uncharged particle ***has no limiting range*** in matter, beyond which it cannot go
- Charge particles encounter such a ***range limit*** as they run out of kinetic energy
- For comparable energies, uncharged particles penetrate much farther through matter, on the average, than charged particles, although this difference gradually decreases at energies above 1 MeV

$$\frac{\mu}{\rho} = \frac{1}{\rho N}\frac{dN}{dl}$$

$\boxed{\text{mass attenuation coefficient}}$

$$\frac{\mu}{\rho} = \sigma\frac{N_A}{A}$$

σ = cross section
$N_A$ = Avogadro's constant ($6.022 \cdot 10^{23}$ mol$^{-1}$)
A = atomic weight

Total mass attenuation coefficient for γ-ray interactions (neglecting photonuclear reactions):

$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{k}{\rho} + \frac{\sigma_R}{\rho}$$

(cm$^2$ g$^{-1}$ or m$^2$ kg$^{-1}$)

Rayleigh scattering

pair production

Compton effect

photoelectric effect

Mass energy-transfer coefficient

$$\frac{\mu_{tr}}{\rho} = \frac{1}{\rho EN}\frac{d\varepsilon_{tr}}{dl}$$

$\frac{d\varepsilon_{tr}}{EN}$ = fraction of energy of incident particles transferred to kinetic energy of secondary particles

For photons:

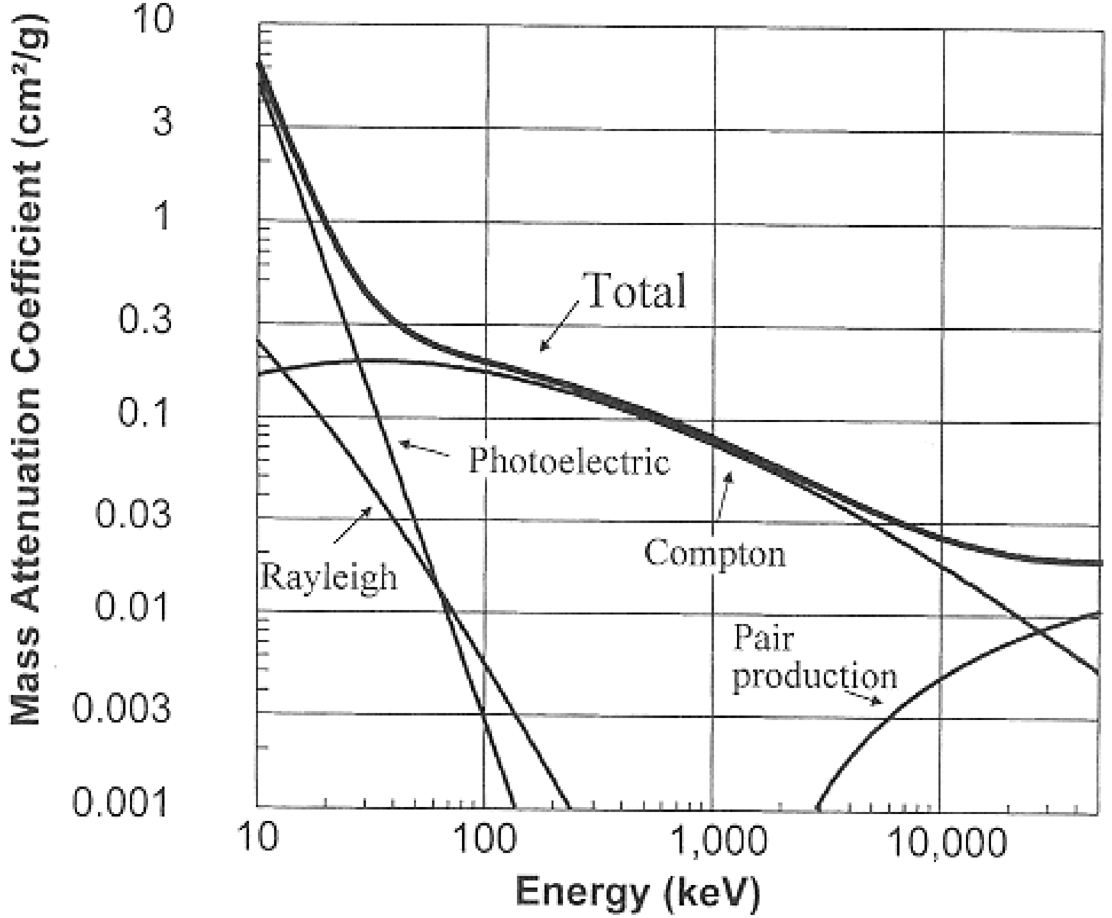
$$\frac{\mu_{tr}}{\rho} = \frac{\tau_{tr}}{\rho} + \frac{\sigma_{tr}}{\rho} + \frac{k_{tr}}{\rho}$$

Mass energy-absorption coefficient

$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho}(1-g)$$

average fraction of secondary electron energy lost in radiative interactions (bremsstrahlung and $\beta^+$ annihilation)
For low Z and low hv, g → 0

Example: mass attenuation coefficient for soft tissue (Z = 7)



# Exponential attenuation of uncharged ionizing radiation ($\gamma$ and neutrons)

$$dN = -\mu N dl$$

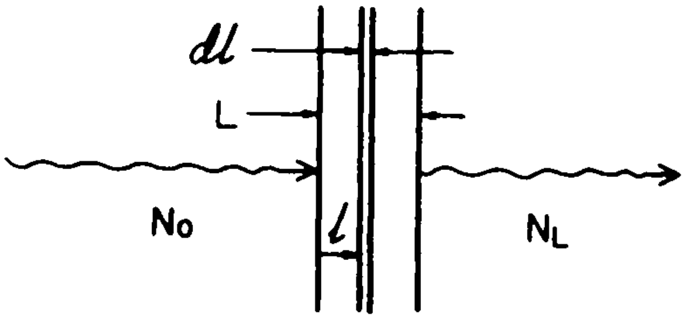
$$\frac{dN}{N} = -\mu dl$$

$$\int_{N=N_0}^{N_L} \frac{dN}{N} = -\int_{l=0}^{L} \mu dl$$

$$lnN_L - lnN_0 = ln\frac{N_L}{N_0} = -\mu L$$

$$\frac{N_L}{N_0} = e^{-\mu L}$$

$$\frac{N_L}{N_0} = e^{-(\mu_1+\mu_2+\mu_3+\cdots)L}$$




$\mu dl$ = probability of interaction in an infinitesimal thickness $dl$

$\mu$ = linear attenuation coefficient (narrow beam) ($cm^{-1}$ or $m^{-1}$)

$1/\mu$ = mean free path = average distance a single particle travels in a medium before interacting

A distance of $3/\mu$ reduces the beam intensity to 5%, $5/\mu$ to <1%

$$S = \frac{dE}{dl} \qquad \frac{S}{\varrho} = \frac{dE}{\varrho dl}$$

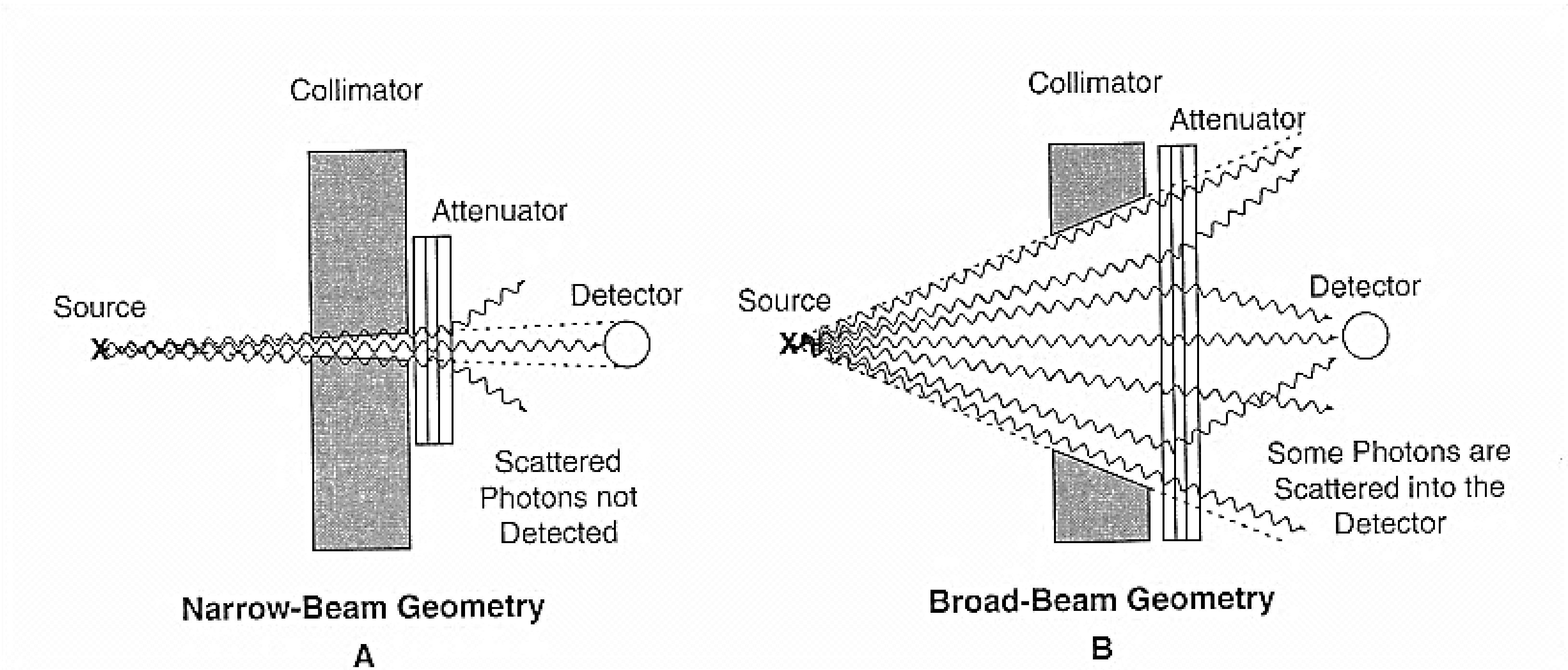
$$\frac{S}{\varrho} = \left(\frac{dE}{\varrho dl}\right)_{collision} + \left(\frac{dE}{\varrho dl}\right)_{radiative}$$

*dE* = energy lost by the particle in path-length $dl$

$$L_{\Delta} = \left(\frac{dE}{dl}\right)_{\Delta}$$

*dE* = energy *locally* imparted to the medium in collision events
$\Delta$ = cut-off on energy of $\delta$-rays
The expression "***locally***" can be more or less restrictive

Strictly speaking, exponential attenuation is only observed for a monoenergetic beam of identical uncharged particles that are absorbed without producing scattered secondary radiation

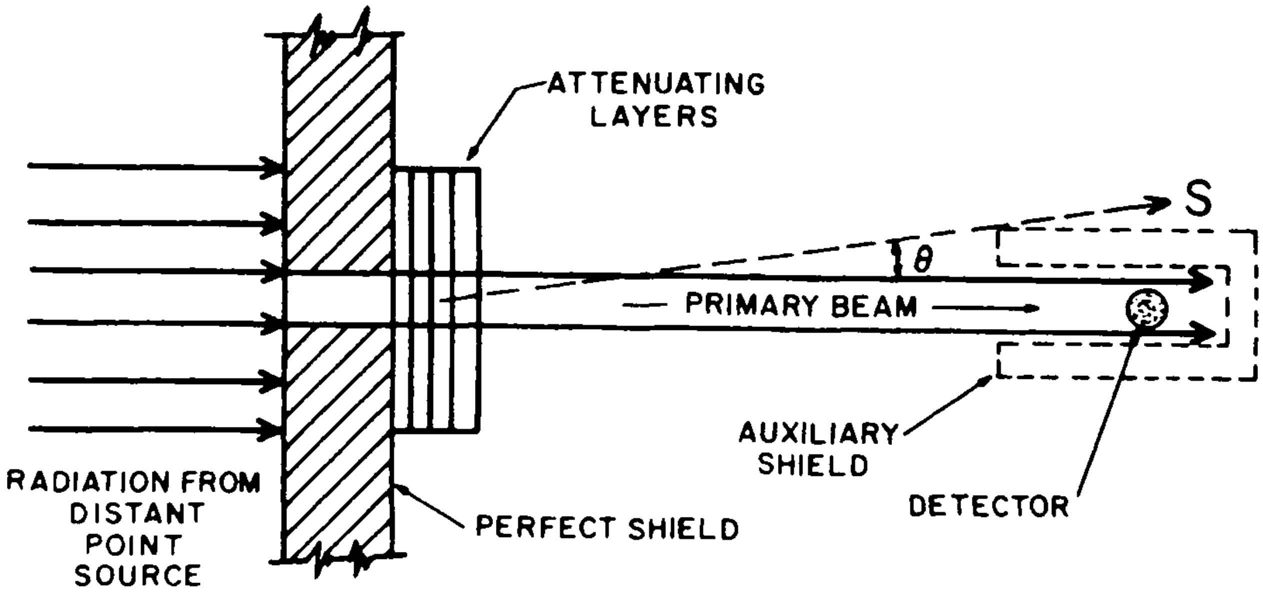



In broad-beam geometry, the *effective attenuation coefficient* $\mu' > \mu$

Two methods to achieve narrow-beam attenuation:

- Discrimination against all scattered and secondary particles that reach the detector

  (on the basis of particle energy, penetrating ability, direction, etc)
- Narrow-beam geometry, which prevents any scattered or secondary particle from reaching the detector



$$B = \frac{Quantity\ due\ to\ primary + scattered\ and\ secondary\ radiation}{Quantity\ due\ to\ primary\ alone}$$

For narrow-beam geometry B = 1
For broad-beam geometry B > 1

B is a function of radiation type and energy, attenuation medium and depth, geometry and measured quantity (e.g. energy fluence, kerma, dose)

For example, for energy fluence $\Psi$:

$$\frac{\Psi_L}{\Psi_0} = Be^{-\mu L}$$

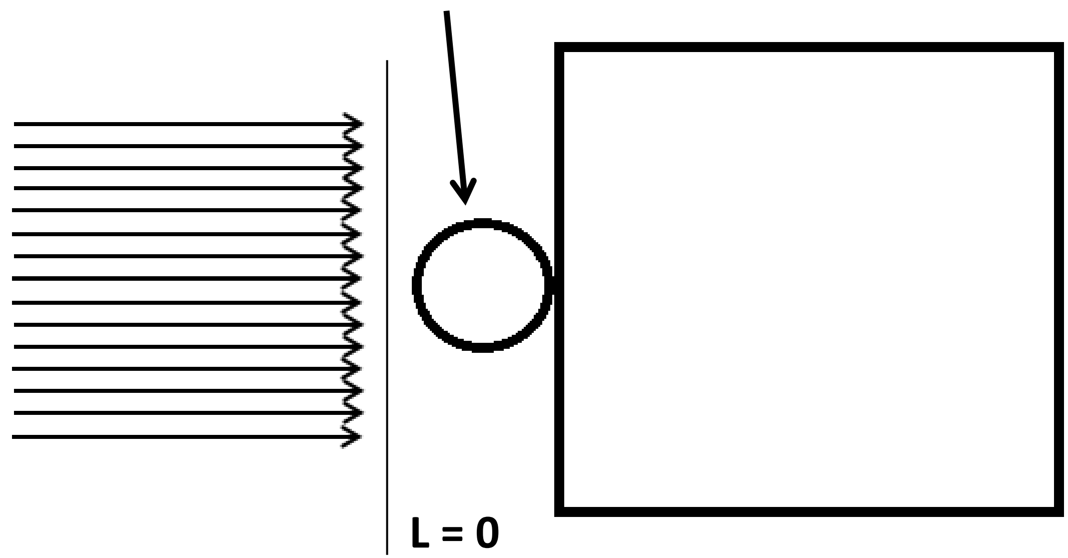
$\Psi_0$ = unattenuated primary energy fluence

$\Psi_L$ = total energy fluence at the detector behind a medium thickness L

$\mu$ = narrow-beam attenuation coefficient

$If\ L = 0 \rightarrow B = B_0 \equiv \frac{\Psi_L}{\Psi_0} = 1$ for most broad-beam geometries, *except* when detector on phantom surface

No attenuation between source and **detector**




Here $\Psi_L > \Psi_0$ and $B_0 > 1$

$B_0$ is called *backscatter factor*

For $^{60}Co$ photons on a water phantom → $B_0 = 1.06$ for tissue dose

The **energy transferred** is a stochastic quantity defined as:

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

where:

$(R_{in})_u$ = radiant energy of uncharged particles entering the volume *V*

$(R_{out})_u^{nonr}$ = radiant energy of uncharged particles leaving *V*, *except* that which originated from radioactive losses of kinetic energy by charged particles while in *V*

$\sum Q$ = net energy derived from rest mass in *V* (m →E positive, E →m negative)

The ***radiant energy*** is the energy of particles (except rest energy) emitted, transferred or received

The ***energy transferred*** is just the kinetic energy received by charged particles in *V* (regardless of where or how they in turn spend that energy)

Relevant only for ***indirectly ionizing*** radiation (photons and neutrons)

$$K = \frac{d(\varepsilon_{tr})_e}{dm} \equiv \frac{d\varepsilon_{tr}}{dm}$$ (Gy) (1 Gy = 1 J/kg = 100 rad)

$(\varepsilon_{tr})_e$ = expectation value of the energy transferred in the finite volume V (the sum of the initial kinetic energies of all charged particles produced by the indirectly ionizing particles in V)

$d(\varepsilon_{tr})_e$ = expectation value for the infinitesimal volume *dv* at point *P*

*dm* = mass of *dv*

The average value of *K* in a volume *V* of mass *m* is $(\varepsilon_{tr})_e$ /m

# Relation of kerma to energy fluence for photons

For monoenergetic photons, the kerma is related to the energy fluence by:

$$\mathrm{K} = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \qquad \text{(Gy)}$$

$\mu_{tr}$ is the linear energy-transfer coefficient ($m^{-1}$ or $cm^{-1}$) and $\mu_{tr}/\rho$ is the mass energy-transfer coefficient (function of the photon energy *E* and atomic number *Z* of the medium)

For a spectrum of photons:

$$K = \int_{E=0}^{E_{max}} \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE$$

$\Psi'(E) =$ differential distribution of photon energy fluence
$\mu_{tr}/\rho$ are numerical values <u>tabulated</u> for selected photon energies and materials

Average value of $\mu_{tr}$ / $\rho$ for the spectrum $\Psi'(E)$:

$$\left(\frac{\bar{\mu}_{tr}}{\rho}\right)_{\Psi'(E),Z} = \frac{K}{\Psi} = \frac{\int_E \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE}{\int_E \ \Psi'(E)\, dE}$$

# Relation of kerma to fluence for neutrons

Neutron fields are usually described in terms of flux density (or fluence) instead of energy flux density (or energy fluence)

Kerma factor $(F_n)_{E,Z}$:

$$(F_n)_{E,Z} = \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \cdot E \qquad \text{(Gy cm}^2\text{)}$$

and:

$$\mathrm{K} = \Phi \cdot (F_n)_{E,Z} \qquad \text{(Gy)}$$

For neutrons with energy spectrum $\Phi'(E)$ [cm$^{-2}$ MeV$^{-1}$] of particle fluence:

$$K = \int_{E=0}^{E_{max}} \Phi'(E) \cdot (F_n)_{E,Z} dE \qquad \text{(Gy)}$$

$(F_n)_{E,Z}$ are numerical values <u>tabulated</u> for selected neutron energies and materials

Average value of $F_n$ for a neutron spectrum $\Phi'(E)$:

$$\overline{(F_n)}_{\Phi'(E),Z} = \frac{K}{\Phi} = \frac{\int_E \Phi'(E) \cdot (F_n)_{E,Z}\, dE}{\int_E \Phi'(E)\, dE}$$

Kerma factors are used to convert dose or kerma measured in a tissue-equivalent material to absorbed dose or kerma in tissue

$\Rightarrow$ the correction to be applied is the ratio of the kerma factors

For photons, the kerma consists of energy transferred to ***electrons and positrons*** *per unit mass of medium*

$K = K_c + K_r$

$K_c$ = collision kerma = energy spent by the electrons in collisions (ionization and excitation in or near the electron track)

$K_r$ = radiative kerma = energy spent by the electrons in radiative-type interactions or by positrons through in-flight annihilation

For neutrons, the resulting charged particles are ***protons and heavier recoiling nuclei***:

$K_r << K_c \rightarrow K = K_c$

The ***net energy transferred*** is a stochastic quantity defined for a volume *V* as:

$$\varepsilon_{tr}{}^{n} = (R_{in})_u - (R_{out})_u{}^{nonr} - R'_u + \sum Q = \varepsilon_{tr} - R'_u$$

$R'_u$ = radiant energy emitted as radiative losses by the charged particles which originated in the volume V, regardless of where the radiative loss event occur

$\varepsilon_{tr}$ and *K* include energy that goes to radiative losses

$\varepsilon_{tr}{}^{n}$ and *Kc* <u>do not</u> include such losses

$$Kc = \frac{d{\varepsilon_{tr}}^{n}}{dm}$$

= expectation value of the net energy transferred to charged particles per unit mass at the point of interest, excluding both the radiative-loss energy and the energy passed from one charged particle to another.

Average value of collision kerma throughout a volume of mass *m*:

$$Kc = \frac{({\epsilon_{tr}}^{n})_{e}}{m}$$

For monoenergetic photons, *Kc* is related to the energy fluence $\Psi$ via the *mass energy-absorption coefficient* $(\mu_{en}\,/\rho)_{E,Z}$ :

$$Kc = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,Z}$$ (similarly to $\mathrm{K} = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$ )

And similar as seen above for an energy spectrum $\Psi'(E)$

$$Kc = \int_{E=0}^{E_{max}} \Psi'(E) \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,Z} dE$$

For a low-Z medium and small photon energy *E* (small radiative losses):

$$(\mu_{en} / \rho)_{E,Z} \approx (\mu_{tr} / \rho)_{E,Z}$$

$$Kc \approx K$$

Percentage by which $(\mu_{en} / \rho)_{E,Z}$ is less than $(\mu_{tr} / \rho)_{E,Z}$

| γ-ray Energy (MeV) | $100\,(\mu_{tr} - \mu_{en})/\mu_{tr}$ | | |
|---|---|---|---|
| | $Z = 6$ | 29 | 82 |
| 0.1 | 0 | 0 | 0 |
| 1.0 | 0 | 1.1 | 4.8 |
| 10 | 3.5 | 13.3 | 26 |

Kerma rate at point *P* and time *t*

$$\dot{K} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{d\varepsilon_{tr}}{dm}\right)$$

(Gy s$^{-1}$ or Gy h$^{-1}$)

Integrated kerma between times $t_0$ and $t_1$:

$$K(t_0, t_1) = \int_{t_0}^{t_1} \dot{K}(t)\, dt$$

and for constant kerma rate:

$$K(t_0, t_1) = \dot{K}(t_1 - t_2)$$

The **energy imparted** is a stochastic quantity defined as:

$$\varepsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q$$

where:

$(R_{in})_u$ = radiant energy of uncharged particles entering the volume *V*

$(R_{out})_u$ = radiant energy of all uncharged radiation leaving *V*

$(R_{in})_c$ = radiant energy of the charged particles entering *V*

$(R_{out})_c$ = radiant energy of the charged particles leaving *V*

$\sum Q$ = net energy derived from rest mass in V
(m → E positive, E → m negative)


The absorbed dose is relevant to ***all types*** of ionizing radiation fields and to any ionizing radiation source distributed within an absorbing medium

$$D = \frac{d\epsilon}{dm} \qquad \text{(Gy)}$$

$$\dot{D} = \frac{dD}{dt} = \frac{d}{dt}\left(\frac{d\epsilon}{dm}\right)$$

$\epsilon$ = expectation value of the energy imparted in the finite volume V during a given time interval

d$\epsilon$ = expectation value of the energy imparted in an infinitesimal volume *dV* at point P

*dm* = mass of *dV*

***D is the expectation value of the energy imparted to matter per unit mass at point P***

Average absorbed dose in a volume of mass m: $\overline{D} = (\varepsilon)_e / m$

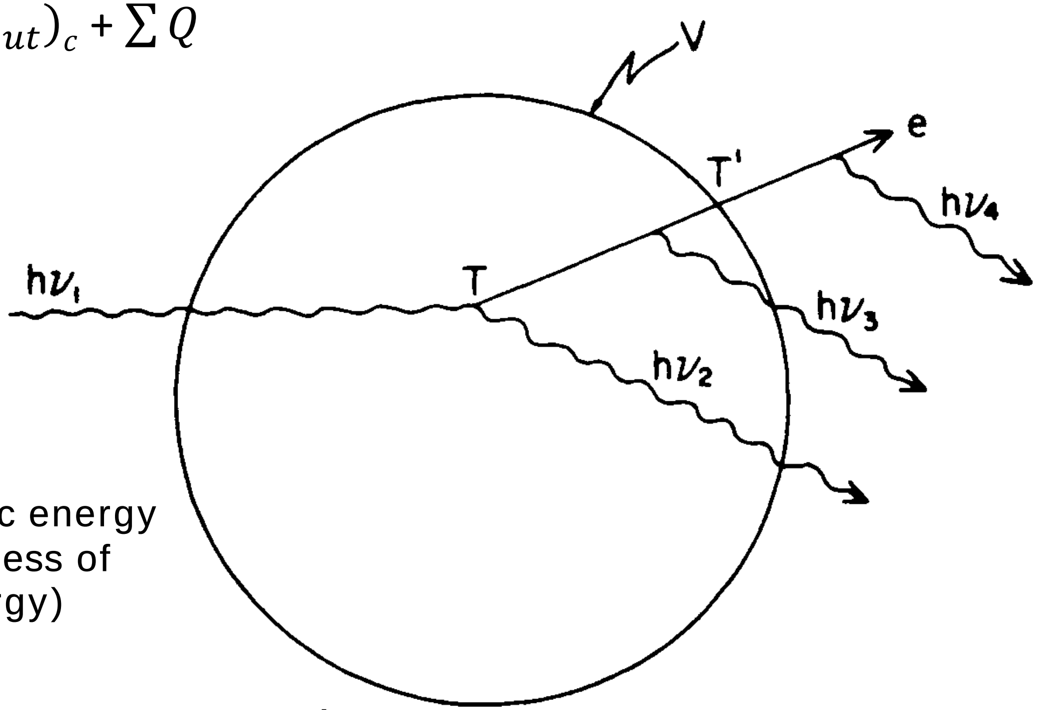
$$\varepsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q$$

$$\varepsilon = h\upsilon_1 - (h\upsilon_2 + h\upsilon_3 + T') + 0$$

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

$$\epsilon_{tr} = h\upsilon_1 - h\upsilon_2 + 0 = T$$

The ***energy transferred*** is just the kinetic energy received by charged particles in *V* (regardless of where or how they in turn spend that energy)




$$\varepsilon_{tr}{}^{n} = (R_{in})_u - (R_{out})_u^{nonr} - R'_u + \sum Q = \varepsilon_{tr} - R'_u$$

$$\epsilon_{tr}{}^{n} = h\upsilon_1 - h\upsilon_2 - (h\upsilon_3 + h\upsilon_4) + 0$$
$$= T - (h\upsilon_3 + h\upsilon_4)$$

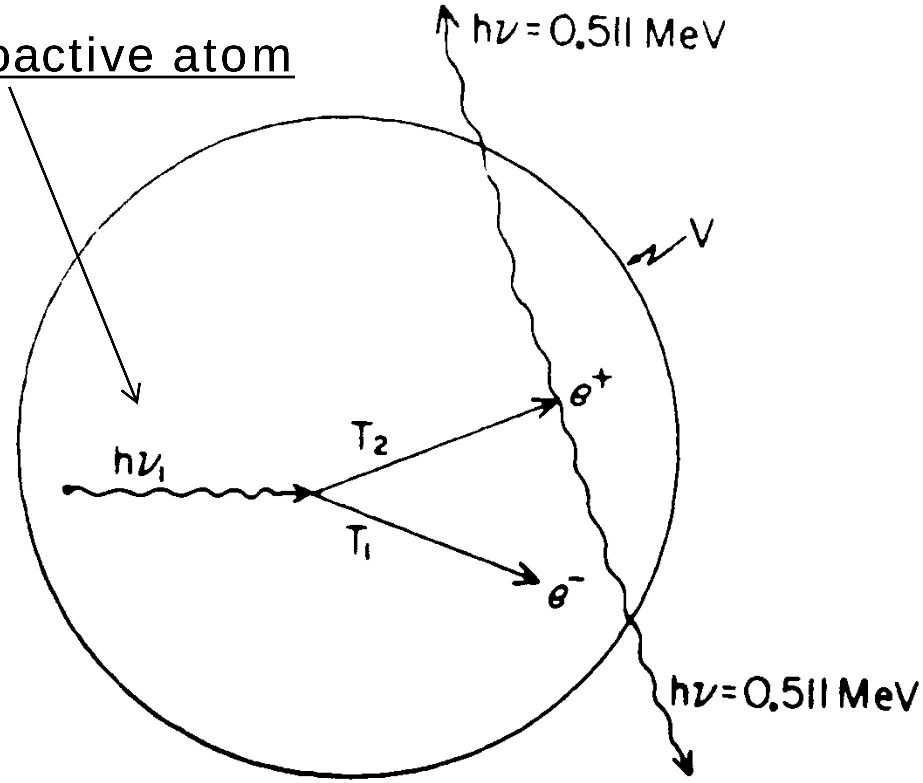
Compton interaction followed by bremsstrahlung emission

<u>γ-ray emitted by a radioactive atom</u>

$$\varepsilon = \varepsilon_{tr} = \epsilon_{tr}{}^{n} = 0 - 1.022\ MeV + \sum Q$$

$$\sum Q = h\upsilon_1 - 2m_0c^2 + 2m_0c^2 = h\upsilon_1$$

$$\varepsilon = \varepsilon_{tr} = \epsilon_{tr}{}^{n} = h\upsilon_1 - 1.022\ MeV = \\ = T_1 + T_2$$




$\sum Q$ = net energy derived from rest mass in *V* (m →E positive, E →m negative)

Example involving γ-ray emission, pair production and β⁺annihilation

# The exposure X

Only defined for x-ray and $\gamma$-ray photons

$$X = \frac{dQ}{dm} \qquad \text{(C/kg)}$$

$dQ$ = absolute value of the total charge of the ions of one sign produced in air when all the electrons and positrons liberated by photons in air of mass $dm$ are completely stopped in air

the ionization arising from the absorption of bremsstrahlung emitted by the electrons is not to be included in $dQ$ (*only relevant at high energies*)

$$\bar{W} = \text{mean energy expended in a gas per ion pair formed}$$

$$\frac{\bar{W}_{air}}{e} \simeq 34\ eV \text{ per ion pair} = 34\ J/C$$

The exposure *X* is the ionization equivalent of the collision kerma in air, for x- and $\gamma$-rays

Exposure *X* at a point *P* due to an energy fluence $\Psi$ of monoenergetic photons of energy *E*:

$$X = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,air} \left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air} \left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air}/34 \qquad (\mathrm{C\ kg^{-1}})$$

1 R (**roentgen**) is the exposure that produces in air one esu of charge of either sign per 0.001293 g of air (the mass contained in 1 $cm^3$ at 760 Torr and 0°C)

1 R = 2.58 $x$ $10^{-4}$ C $kg^{-1}$

For a photon spectrum with energy fluence $\Psi'(E)$:

$$X = \int_{E=0}^{E_{max}} (\mu_{en}/\rho)_{E,air} (e/\overline{W})_{air} \Psi'(E) dE$$

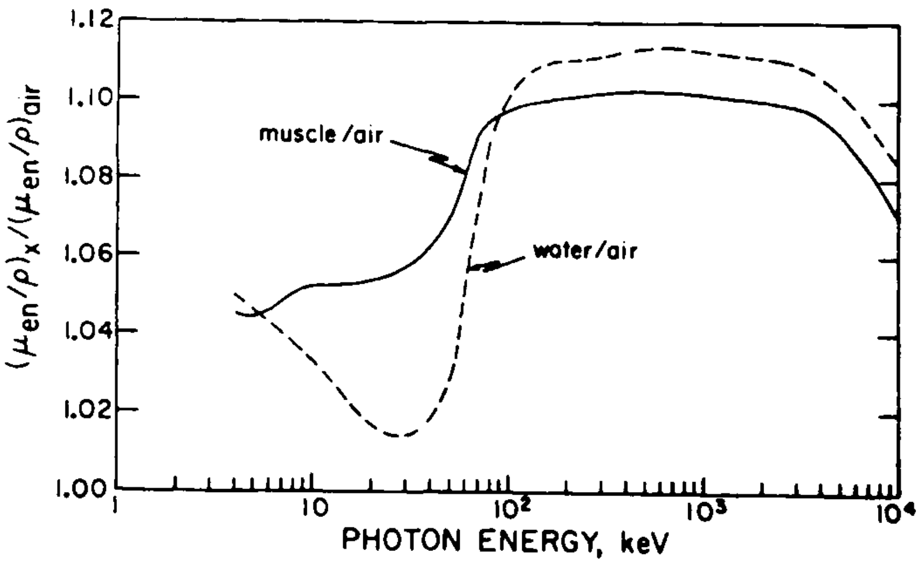
$$\dot{X} = \frac{dX}{dt}$$ (C kg$^{-1}$ s$^{-1}$ or R s$^{-1}$)

Exposure occurring between times $t_0$ and $t_1$:

$$X = \int_{t_0}^{t_1} \dot{X}(t)\, dt$$

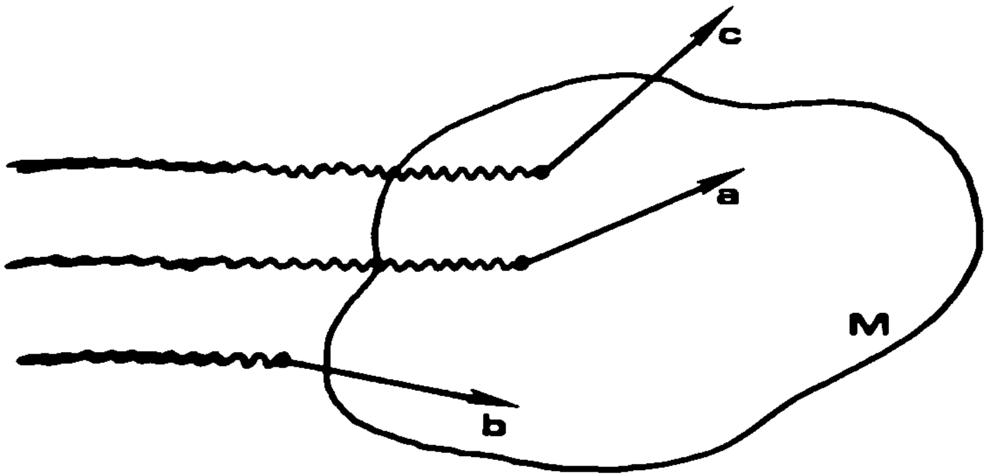
- The energy fluence $\Psi$ is proportional to $X$ for any given photon energy or spectrum
- Air is similar to soft tissue (muscle) in effective atomic number, so that it is "tissue-equivalent" w.r.t. x- or $\gamma$-ray energy absorption
- If one is interested in the effect of x- or $\gamma$-radiation in tissue, air may be substituted as a reference medium in a measuring instrument

- X $\alpha$ $(\mu_{en}/\rho)_{E,air}$
- Kc in muscle $\alpha$ $(\mu_{en}/\rho)_{E,muscle}$
- $(\mu_{en}/\rho)_{E,muscle}$ / $(\mu_{en}/\rho)_{E,air}$
  $\approx 1.07 \pm 3\%$ for E = 4 keV – 10 MeV



Generally, the transfer of energy (kerma) from a photon beam to charged particles at a particular location ***does not lead*** to the absorption of energy by the medium (absorbed dose) at the same location

***(e.g., a 10 MeV electron has a range in water of about 5 cm)***

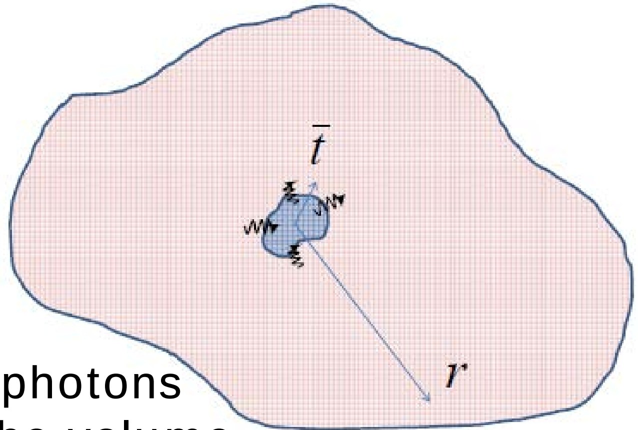



a) All energy transferred by photons to electrons is deposited in M ⇒ D = K
b) Electrons originates outside M but deposit part of their energy in M ⇒ D > K
c) Electrons originates in M but deposit part of their energy outside M ⇒ D < K

If (b) and (c) compensate each other ⇒ CPE ⇒ $D = K$

The dimensions of the volume are much larger than ***the mean free path of the radiation***

- Medium of homogeneous atomic composition
- Medium of homogeneous density
- Radioactive source uniformly distributed
- No electric and magnetic fields present to perturb the charged-particle paths



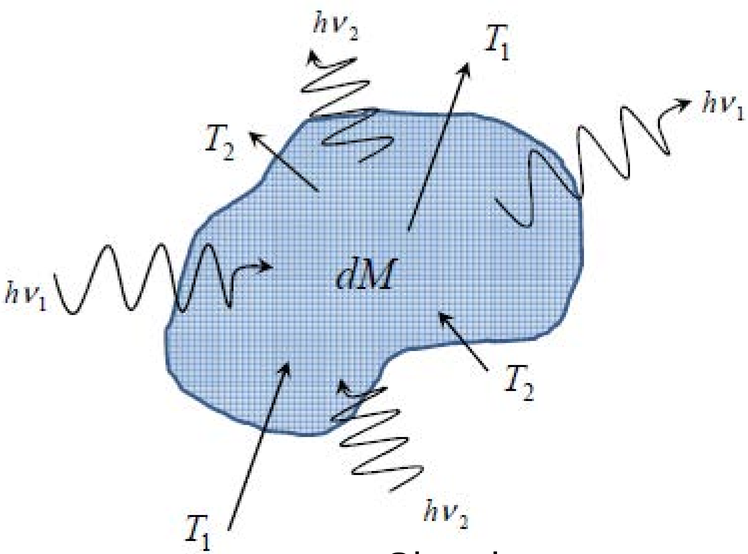

$\bar{t}$ = mean free path of the photons
r = radius to the edge of the volume

Radiation equilibrium

$$(R_{in})_u = (R_{out})_u \quad \text{and} \quad (R_{in})_c = (R_{out})_c$$

$$\varepsilon = \sum Q$$

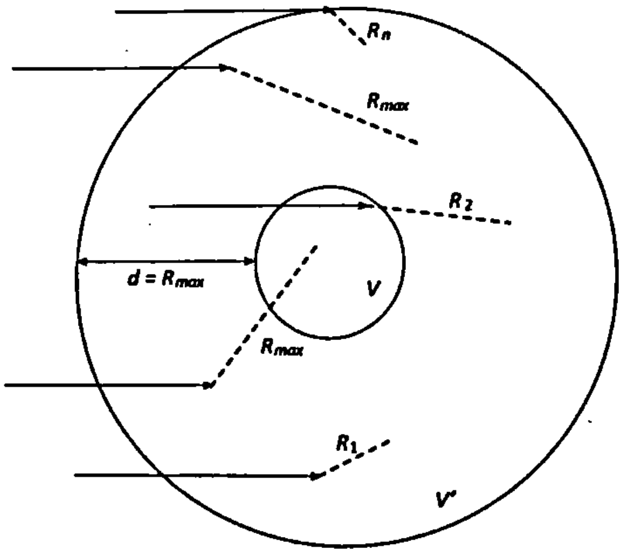
$$D = \frac{d(\sum Q)}{dm}$$




Sketch courtesy M. Kissick

The dimensions of the volume are much larger than the ***mean free path of the secondary charged-particles***

- Medium of homogeneous atomic composition
- Medium of homogeneous density
- Radioactive source uniformly distributed
- No inhomogeneous electric and magnetic fields present



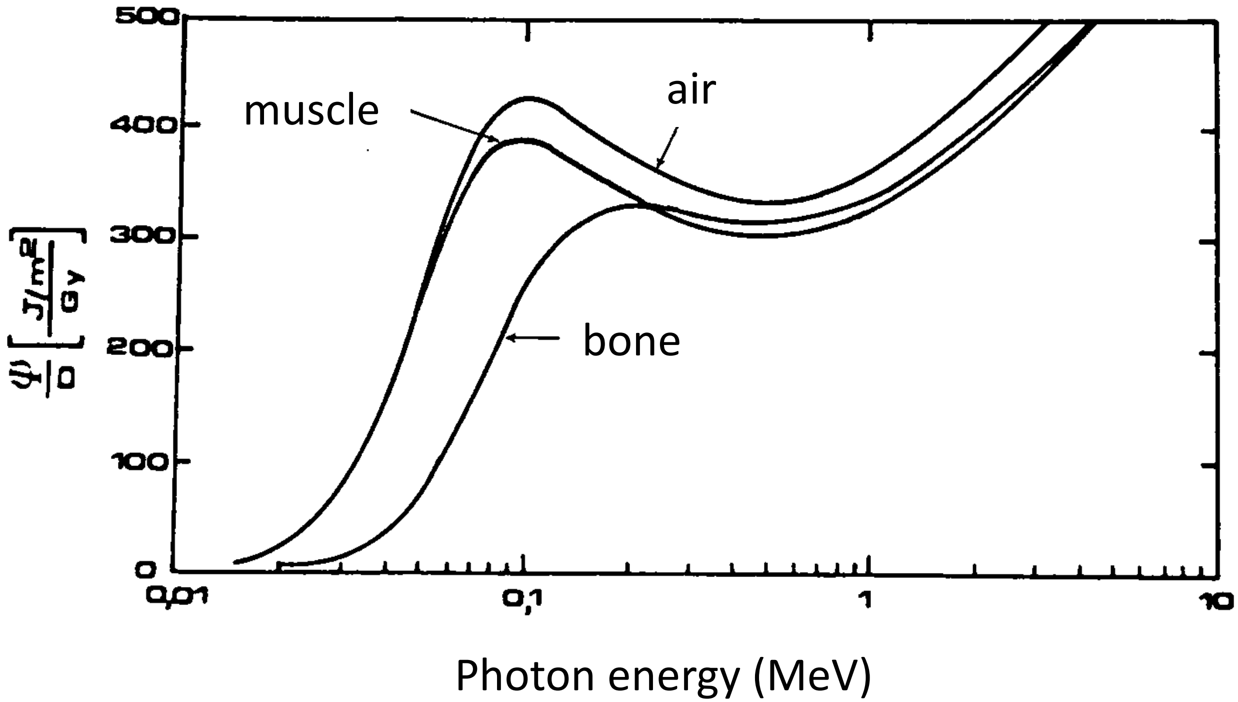

$$\boxed{\text{Charged-particle equilibrium}}$$

$$(R_{in})_c = (R_{out})_c$$

$$\varepsilon = (R_{in})_u - (R_{out})_u + \sum Q = \epsilon_{tr}$$

$$D = \frac{d\varepsilon}{dm} = \frac{d\epsilon_{tr}}{dm} = K \quad and \quad \Psi = \frac{D}{(\mu_{tr}/\rho)}$$

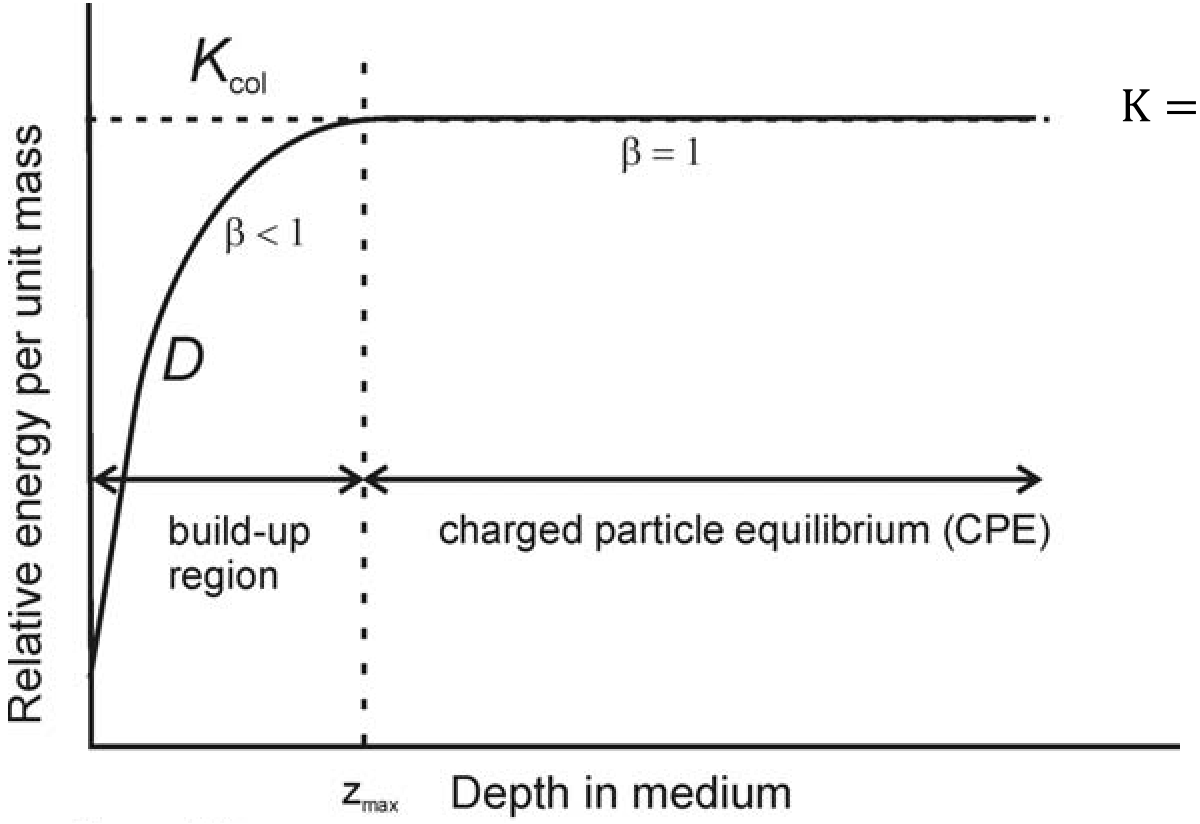
$$\mathrm{K} = \Psi \cdot (\mu_{tr}/\rho)$$




Conversion coefficients from energy fluence to absorbed dose
(for CPE and negligible radiative losses)

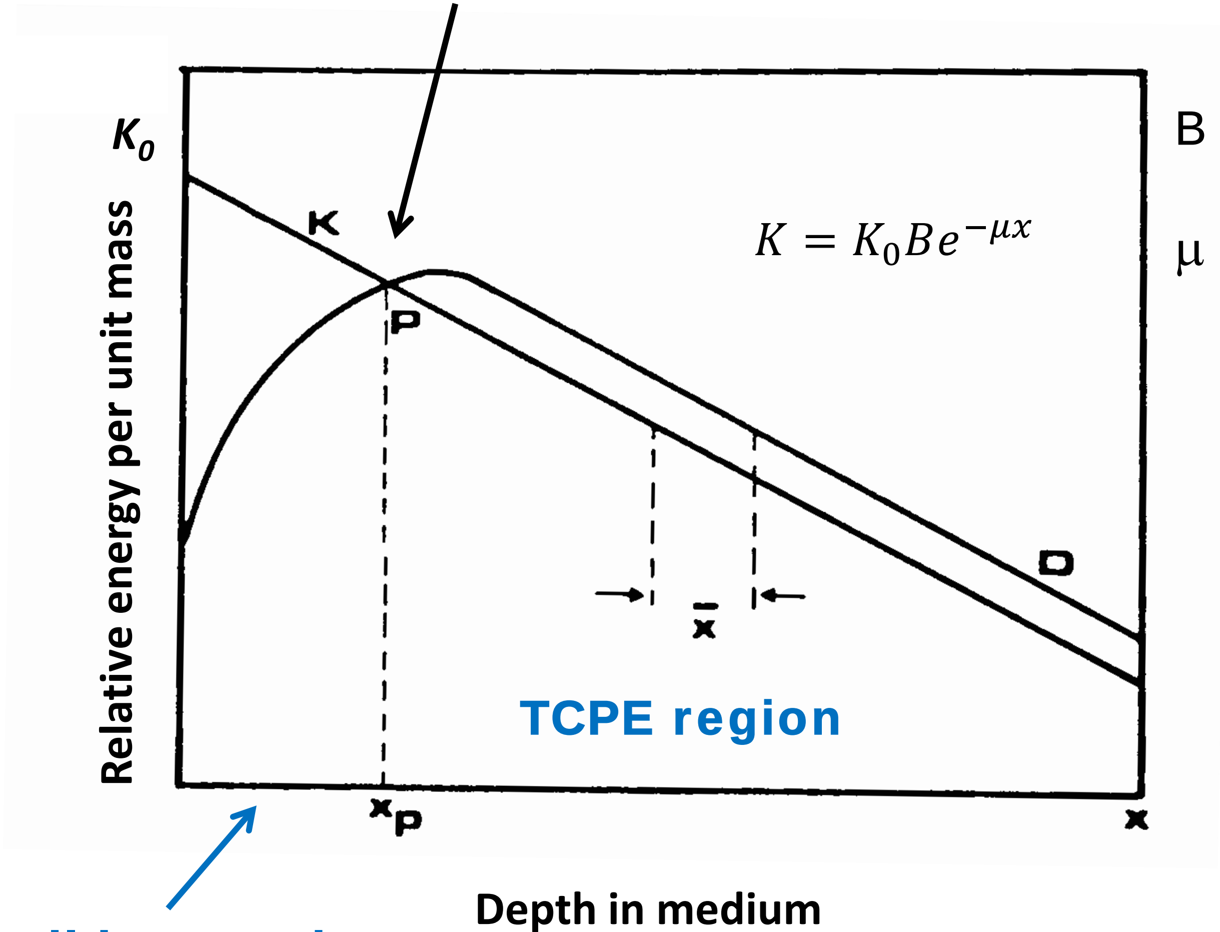
β = D / Kc

$K = Kc$ if radiative losses are negligible




$$K = \Psi \cdot (\mu_{tr}/\rho)$$

Point at which CPE exists



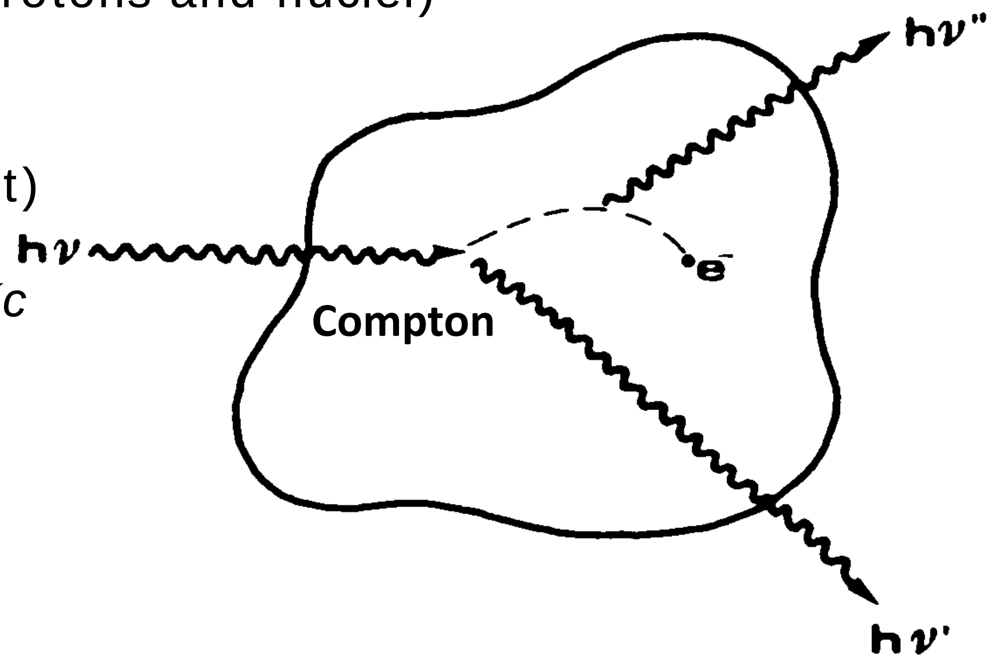

**Build-up region**

B = build-up factor

μ = attenuation coefficient

$$D = K\,(1 + \mu' \bar{x})$$

Radiative losses are negligible (and $K = Kc$) for low energy photons and neutrons (secondaries are protons and nuclei)




- CPE exists (no CP enter or exit)
- hν'' is included in $K$
- hν'' in not included in $D$ and $Kc$

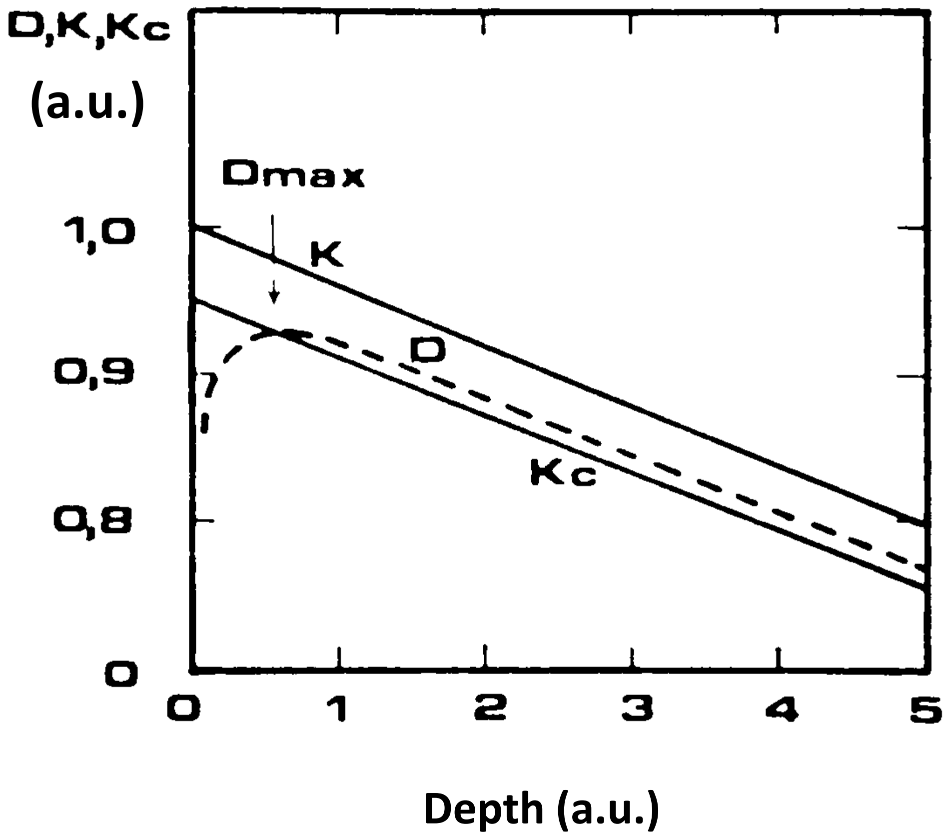
$\Rightarrow D = Kc < K$

$K - D = Kr$

If the electron would not radiate part of its energy $\Rightarrow Kr = 0$

In carbon, water, air and other low-Z media, $Kr = K - Kc < 1\%$ for photons up to 3 MeV

Uncollimated beam of high-energy photons impinging perpendicularly on a semi-infinite slab of absorbing material




At the surface:

$$K_0 = \Psi_0 \left(\frac{\mu_{tr}}{\rho}\right)$$

$$K = K_0 B e^{-\mu x} = \Psi_0 \left(\frac{\mu_{tr}}{\rho}\right) B e^{-\mu x}$$

$$Kc = \Psi \left(\frac{\mu_{en}}{\rho}\right) = \frac{\mu_{en}}{\mu_{tr}} K$$

For e.g. 6 MeV photons on Al:

$$\left(\frac{\mu_{en}}{\mu_{tr}}\right)_{6\ MeV} = 0.95$$

## For photons

CPE

$$D = Kc = \Psi \left( \frac{\mu_{en}}{\rho} \right)$$

TCPE

$$D = Kc\ (1 + \mu' \bar{x}) = \Psi \left( \frac{\mu_{en}}{\rho} \right) (1 + \mu' \bar{x})$$

CPE

$$\frac{D_A}{D_B} = \frac{(Kc)_A}{(Kc)_B} = \frac{\overline{(\mu_{en}/\rho)_A}}{\overline{(\mu_{en}/\rho)_B}}$$

- μ' is the common slope of the *K, D and Kc* curves
- $\bar{x}$ is the mean distance the secondary charged particles carry they kinetic energy in the direction of the primary rays while depositing it as dose

## For neutrons

CPE

$$D = K = \Phi F_n$$

TCPE

$$D = K\ (1 + \mu' \bar{x}) = \Phi F_n (1 + \mu' \bar{x})$$

CPE

$$\frac{D_A}{D_B} = \frac{(K)_A}{(K)_B} = \frac{\overline{(F_n)_A}}{\overline{(F_n)_B}}$$

$$\underbrace{D_{air}}_{\text{J/kg}} \overset{\text{CPE}}{=} \underbrace{(Kc)_{air}}_{\text{J/kg}} = \underbrace{X}_{\text{C/kg}} \cdot \underbrace{\left(\frac{\overline{W}}{e}\right)_{air}}_{\text{34 J/C}}$$

If D is in Gy and X is in Roentgen, remembering that:

$$X = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,air} \left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air} \left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air}/34$$

and 1 R = 2.58 *x* $10^{-4}$ C $kg^{-1}$

$$D_{air} = (Kc)_{air} = 2.58x10^{-4} x\ 34\ X\ (Gy) = 8.76x10^{-3} X\ (Gy) = 0.876\ X\ (rad)$$

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