# Higher order QCD corrections
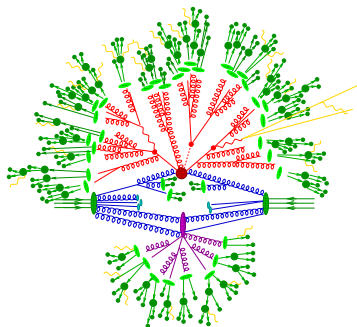
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# QCD at the LHC

Complicated environment, QCD must be understood/modeled as best as feasible

⇒ parton model - beams of partons

⇒ radiation off incoming partons

⇒ primary hard scattering ($\mu \simeq Q \gg \Lambda_{QCD}$)

⇒ radiation off outgoing partons ($Q > \mu > \Lambda_{QCD}$)

⇒ hadronization and heavy hadron decay ($\mu \simeq \Lambda_{QCD}$)
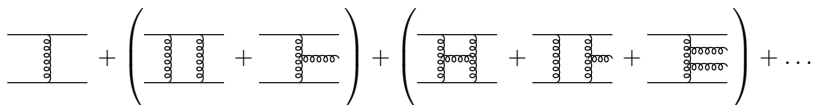
⇒ multiple parton interactions, underlying event

# The hard process in perturbation theory

The scale of the hard scattering is $\mu \gg \Lambda_{QCD}$, so by asymptotic freedom, it can be treated in perturbation theory, i.e. by expansion in powers of the strong coupling, $\alpha_S(\mu)$.

Consider a generic cross section for producing $m$ jets

$$\sigma_m = \alpha_S^p\Big(\sigma_m^{LO} + \alpha_S\sigma_m^{NLO} + \alpha_S^2\sigma_m^{NNLO} + \dots\Big)$$

Representative Feynman-diagrams



How many terms to compute?

# Why NNLO?

LO prediction: order of magnitude estimate, rough shapes of distributions

NLO is mandatory for meaningful normalization and shape predictions

NNLO may be relevant
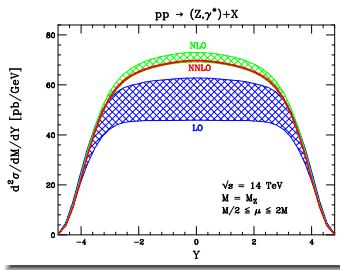
➠ NLO corrections are large:
  ▸ Higgs production from gluon fusion in hadron collisions

➠ for benchmark processes measured with high experimental accuracy:
  ▸ $\alpha_s$ measurements form $e^+e^-$ event shapes
  ▸ $W$, $Z$ production
  ▸ heavy quark hadroproduction

➠ reliable error estimate is needed:
  ▸ processes relevant for PDF determination
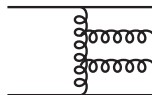  ▸ important background processes



(Anastasiou, Dixon, Melnikov, Petriello,
Phys. Rev. **D69** (2004) 094008.)

# NNLO ingredients

A generic $m$-jet cross section at NNLO involves

➡ Tree-level squared matrix elements
  ‣ with $m + 2$ parton kinematics
  ‣ known from LO calculations
  ‣ 'doubly-real' contribution (RR)

➡ One-loop squared matrix elements
  ‣ with $m + 1$ parton kinematics
  ‣ usually known from NLO calculations
  ‣ 'real-virtual' contribution (RV)

➡ Two-loop squared matrix elements
  ‣ with $m$ parton kinematics
  ‣ known for all massless $2 \to 2$ processes
  ‣ 'doubly-virtual' contribution (VV)

# NNLO ingredients
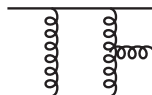
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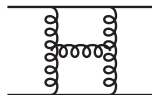  ▶ known from LO calculations
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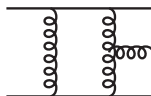
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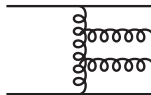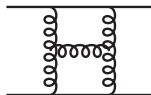Assuming we know the relevant matrix elements, can we use those matrix elements to compute cross sections?

# The problem - IR singularities

Consider the NNLO correction to a generic $m$-jet observable

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m \,.$$

### Doubly-real

- $\mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2}$
- Tree MEs with $m+2$-parton kinematics
- kin. singularities as one or two partons unresolved: up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration
- no explicit $\epsilon$ poles

### Real-virtual

- $\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1}$
- One-loop MEs with $m+1$-parton kinematics
- kin. singularities as one parton unresolved: up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration
- explicit $\epsilon$ poles up to $\mathrm{O}(\epsilon^{-2})$

### Doubly-virtual

- $\mathrm{d}\sigma_m^{\mathrm{VV}} J_m$
- One- and two-loop MEs with $m$-parton kinematics
- kin. singularities screened by jet function: PS integration finite
- explicit $\epsilon$ poles up to $\mathrm{O}(\epsilon^{-4})$

# The problem - IR singularities

Consider the NNLO correction to a generic $m$-jet observable

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_m \mathrm{d}\sigma^{\mathrm{VV}}_m J_m \,.$$

### THE KLN THEOREM

Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (i.e. IR safe) observables.

### HOWEVER

How to make this cancellation explicit, so that the various contributions can be computed numerically? Need a method to deal with implicit poles.

# Basics of subtraction

Strategy: rearrange IR singularities between various contributions by subtracting and adding back suitably defined approximate cross sections.

- ➡ subtraction terms match the singularity structure of real emission point wise (in $d$ dimensions) $\Rightarrow$ phase space integrals over real radiation rendered convergent
- ➡ integrated forms of subtraction terms have the same pole structure as virtual contribution $\Rightarrow$ explicit $\epsilon$-poles cancel point by point

The construction of a general (i.e. process- and observable-independent) subtraction algorithm

- ➡ made possible by the universal structure of IR singularities, embodied in so-called IR factorization formulae
- ➡ is not unique, hence several approaches (FKS, dipole, antenna,. . . )

# Subtraction - a caricature

Want to evaluate (at $\epsilon \to 0$)

$$\sigma = \int_0^1 d\sigma^R(x) + \sigma^V \qquad \text{where} \qquad \begin{aligned} &d\sigma^R(x) = x^{-1-\epsilon} R(x) \\ &R(0) = R_0 < \infty \\ &\sigma^V = R_0/\epsilon + V \end{aligned}$$

➡ define the counterterm

$$d\sigma^{R,A}(x) = x^{-1-\epsilon} R_0$$

➡ use it to reshuffle singularities between R and V contributions

$$\begin{aligned} \sigma &= \int_0^1 \left[ d\sigma^R(x) - d\sigma^{R,A}(x) \right]_{\epsilon=0} + \left[ \sigma^V + \int_0^1 d\sigma^{R,A}(x) \right]_{\epsilon=0} \\ &= \int_0^1 \left[ \frac{R(x) - R_0}{x^{1+\epsilon}} \right]_{\epsilon=0} + \left[ \frac{R_0}{\epsilon} + V - \frac{R_0}{\epsilon} \right]_{\epsilon=0} \\ &= \int_0^1 \frac{R(x) - R_0}{x} + V \end{aligned}$$

The last integral is finite, computable with standard numerical methods.

# Structure of the NNLO correction

Rewrite the NNLO correction as a sum of three terms

$$\sigma^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}} = \sigma_{m+2}^{\mathrm{NNLO}} + \sigma_{m+1}^{\mathrm{NNLO}} + \sigma_m^{\mathrm{NNLO}}$$

each integrable in four dimensions

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} J_m - \left[ \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} J_m \right] \right\}$$

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[ \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right] J_{m+1} - \left[ \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left( \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[ \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[ \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left( \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

1. $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$ regularizes the doubly-unresolved limits of $\mathrm{d}\sigma_{m+2}^{\mathrm{RR}}$
2. $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$ regularizes the singly-unresolved limits of $\mathrm{d}\sigma_{m+2}^{\mathrm{RR}}$
3. $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$ accounts for the overlap of $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$ and $\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$
4. $\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1}$ regularizes the singly-unresolved limits of $\mathrm{d}\sigma_{m+1}^{\mathrm{RV}}$
5. $\left( \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1}$ regularizes the singly-unresolved limit of $\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$

# Defining a subtraction scheme

Two issues must be addressed

1. What to subtract?
2. How to add it back?

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2. How to add it back?

Strategy: IR limits are process independent and known

1. Start by defining subtraction terms based on IR limit formulae $\Rightarrow$ the result is trivially general and explicit ✔

2. Worry about integrating them later, since this is *in principle* a very narrowly defined problem, given 1., but in practice turns out to be very cumbersome ✘

# Defining a subtraction scheme

### The following three problems must be addressed

1. Matching of limits to avoid multiple subtraction in overlapping singular regions of PS. Easy at NLO: collinear limit + soft limit - collinear limit of soft limit.

$$\mathbf{A}_1|\mathcal{M}_{m+1}^{(0)}|^2 = \sum_i \left[ \sum_{i \neq r} \frac{1}{2}\mathbf{C}_{ir} + \mathbf{S}_r - \sum_{i \neq r} \mathbf{C}_{ir}\mathbf{S}_r \right]|\mathcal{M}_{m+1}^{(0)}|^2$$

2. Extension of IR factorization formulae over full PS using momentum mappings that respect factorization and delicate structure of cancellations in all limits.

$$\{p\}_{m+1} \xrightarrow{r} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+1}(\{p\}_{m+1}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{1,m}]$$
$$\{p\}_{m+2} \xrightarrow{r,s} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+2}(\{p\}_{m+2}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{2,m}]$$

3. Integration of the counterterms over the phase space of the unresolved parton(s).

# Defining a subtraction scheme

Specific issues at NNLO

1. Matching is cumbersome if done in a brute force way. However, an efficient solution that works at any order in PT is known.

2. Extension is delicate. E.g. counterterms for singly-unresolved real emission (unintegrated and integrated) must have universal IR limits. This is not guaranteed by QCD factorization.

3. Choosing the counterterms such that integration is (relatively) straightforward generally conflicts with the delicate cancellation of IR singularities.

# NNLO subtraction terms - general features

## Based on universal IR limit formulae

➡ Altarelli-Parisi splitting functions, soft currents (tree and one-loop, triple AP functions)

➡ simple and general procedure for matching of limits using physical gauge

➡ extension based on momentum mappings that can be generalized to any number of unresolved partons

## Fully local in color $\otimes$ spin space

➡ no need to consider the color decomposition of real emission ME's

➡ azimuthal correlations correctly taken into account in gluon splitting

➡ can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit

## Straightforward to constrain subtractions to near singular regions

➡ gain in efficiency

➡ independence of physical results on phase space cut is strong check

## Given completely explicitly for any process with non colored initial state

# Conclusions and outlook

### NNLO is the new precision frontier

### Two bottlenecks

1. can we compute the relevant (2-loop) amplitudes?
2. if yes, can we use those to compute cross sections?

### Subtraction is the traditional solution to 2. We have set up

➡ general, explicit, local subtraction scheme for computing NNLO jet cross sections
➡ construction based on IR limit formulae
➡ worked out in full detail for processes with no colored particles in the initial state

### To Do:

➡ extension to hadron initiated processes