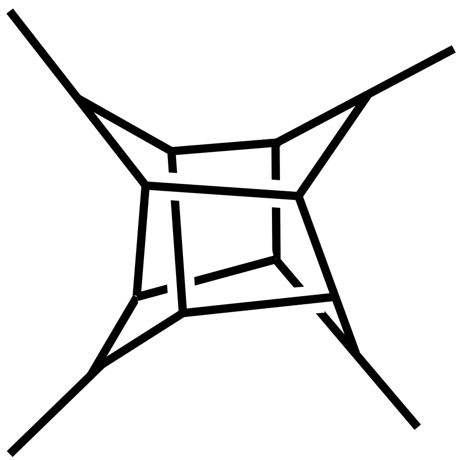
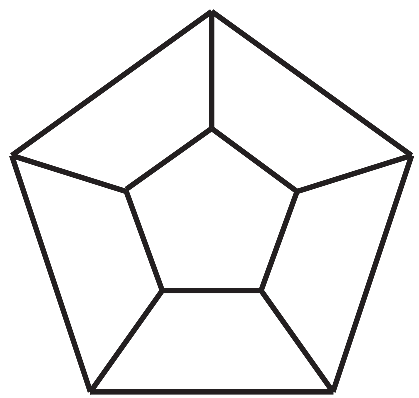
UV divergences of SUGRA and SYM and Color-Kinematics Duality

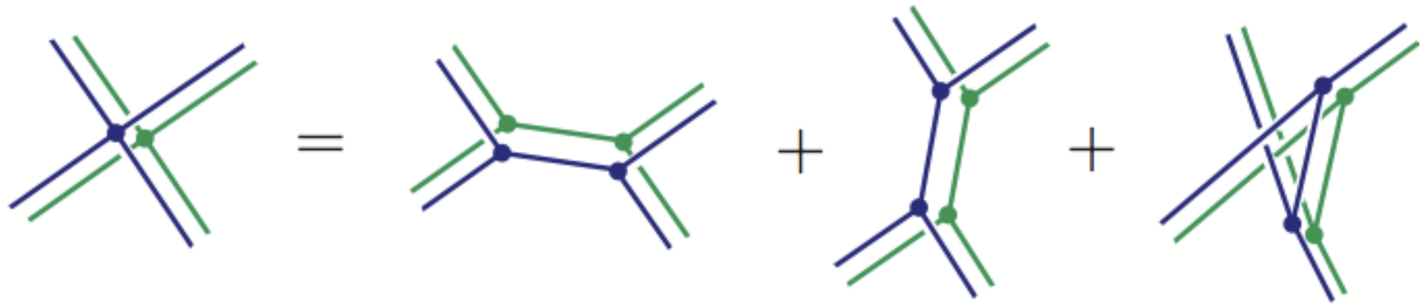


Henrik Johansson



TH Retreat
Thoiry - St Genis

Nov 9, 2012



Interests...

- Scattering amplitudes in *D*=4
  - Planar and non-planar *N*=4 SYM
  - *N*=8 supergravity
  - General multi-loop methods…towards QCD
  - *D*=3 BLG/ABJM theory

This talk

- UV divergences in *D*=4 and higher dimensions
  - Question of SUGRA UV finiteness/divergences
  - *D*=5 SYM and the relation to (2,0) theory
- Hidden structures in gauge theory and gravity
  - Duality between color and kinematics
  - Gravity as a double copy of gauge theory

Why Amplitudes ?

- Simplicity
  - The most basic gauge-invariant structures of a theory
  - High degree of universality – rules are constrained
  - Controlled by Lorentz sym, factorization and unitarity
  - on-shell simplicity – natural for massless gauge theory
  - Strikingly simpler than Lagrangian would suggest

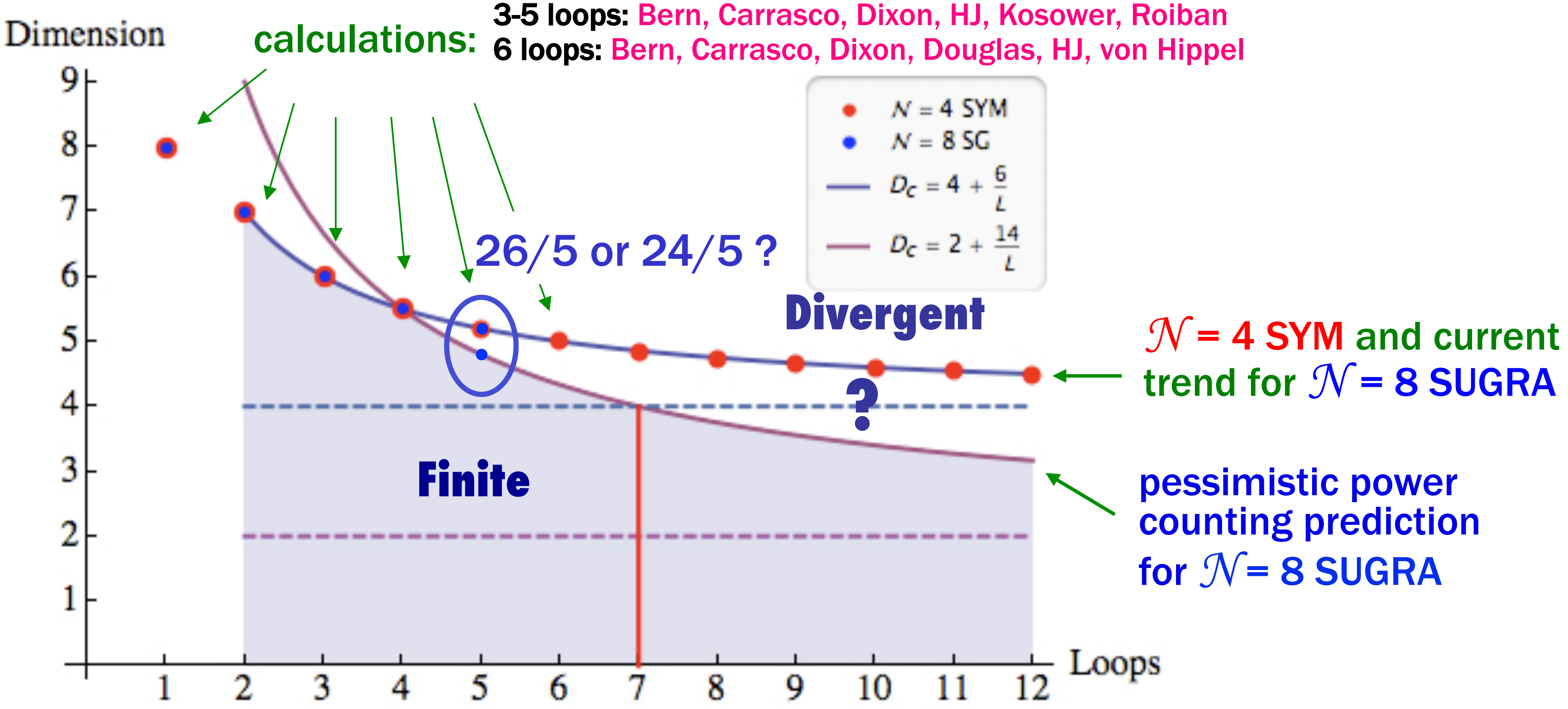
- Richness
  - Infinite amount of data to study
  - Expose hidden symmetries in established theories
  - Can deeply probe a theory – multi-loop calculations
  - Allows precision phenomenology !

UV divergences

Question of supergravity finiteness...

Parameter space for UV divergences in $\mathcal{N} = 8$ SUGRA and $\mathcal{N} = 4$ SYM

1-2 loops: Green, Schwarz, Brink; Marcus and Sagnotti
3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban
6 loops: Bern, Carrasco, Dixon, Douglas, HJ, von Hippel

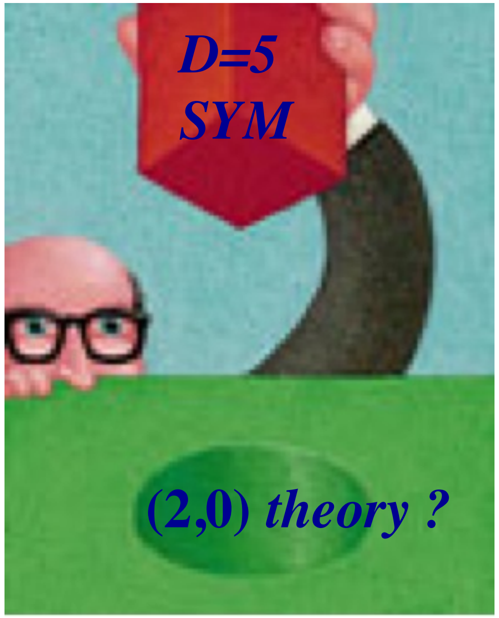


5-loop UV calc. will give strong indication of $\mathcal{N} = 8$ finiteness/divergence

Gauge Theory Analogy

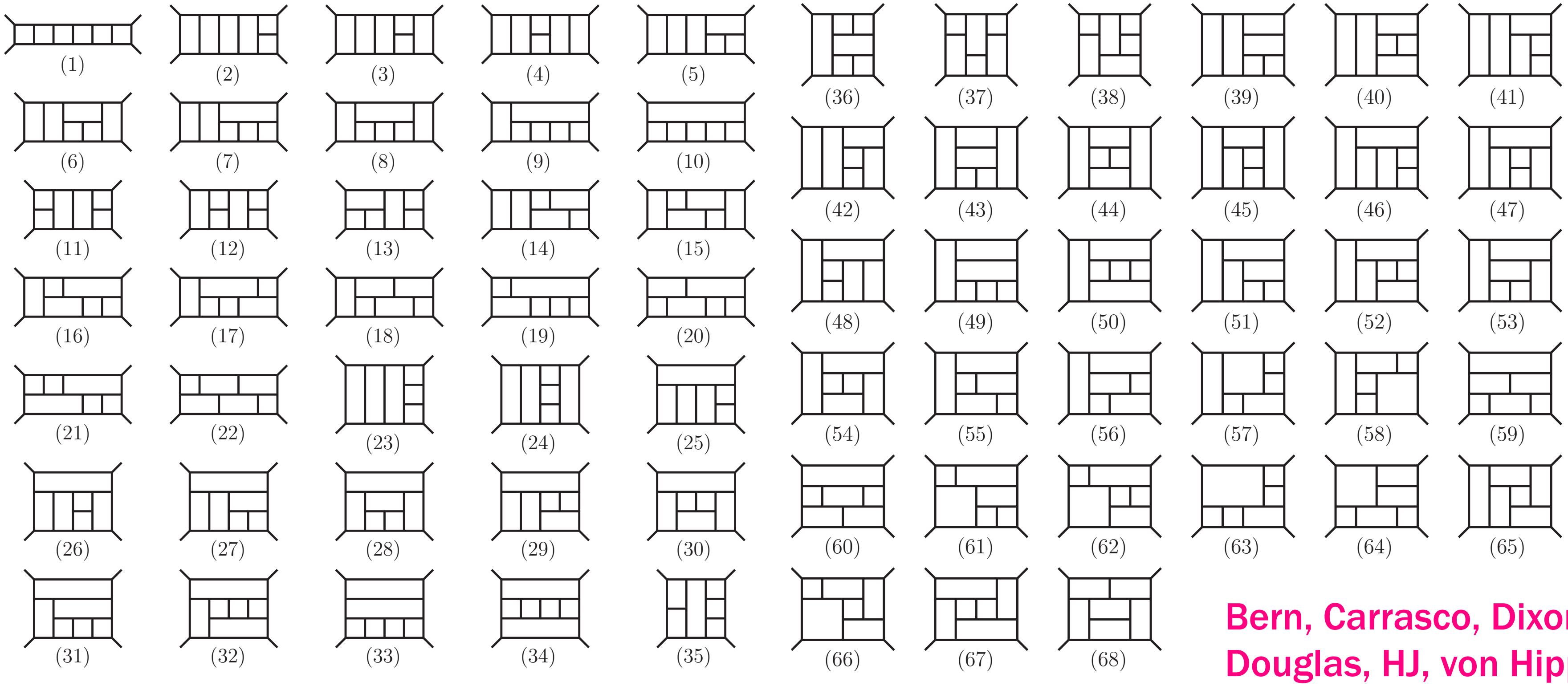
- Gauge theory in *D*>4 have same problem as *D*=4 gravity
- Non-renormalizable due to dimensionful coupling
- However, *D*=5 SYM has a UV completion: (2,0) theory in *D*=6
  - Is *D*=5 SYM perturbatively UV finite ? Douglas; Lambert et al.
  - If yes, how does it work ?
  - If no, what do we need to add ?
  - Solitons, KK modes ? Douglas; Lambert et al.

    Hohenegger et al.



- Understanding *D*=5 SYM might (or might not) give clues to how to understand *D*=4 gravity.

6-Loop Planar *D*=5 SYM

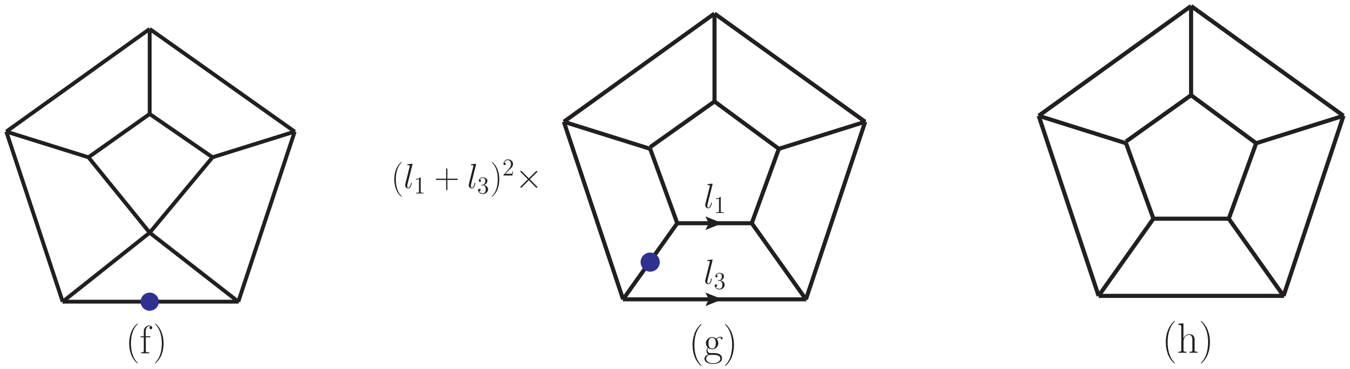


Bern, Carrasco, Dixon, Douglas, HJ, von Hippel

- 68 planar diagrams
- Given by dual conformal invariance (up to integer 0,1,-1,2,... prefactors)
- Independently constructed by: Eden, Heslop, Korchemsky, Sokatchev; Bourjaily, DiRe, Shaikh, Spradlin, Volovich

6-Loop D=5 SYM divergence

Bern, Carrasco, Dixon, Douglas, HJ, von Hippel



$$A_4^{(6)}\Big|_{D=5,\ \mathrm{div.}} = 6stuA_4^{\mathrm{tree}}(1,2,3,4)(10V^{(\mathrm{f})}+5V^{(\mathrm{g})}-V^{(\mathrm{h})})$$

- Using integration by parts identities, div. simplifies to 3 integrals
- Div. cannot be written on positive definite form ?
- Numerical integration required – modified version of FIESTA & 1000 node cluster at Stony Brook
- Result: divergence is nonzero.

$$A_4^{(6)}\Big|_{D=5,\ \mathrm{div.}} = \frac{1}{\epsilon}\frac{1}{(4\pi)^{15}}stuA_4^{\mathrm{tree}}(1,2,3,4)(68.68\pm 0.17)$$

- What cancels this divergence ? Solitons/KK modes ? Douglas; Lambert et al.

Color-Kinematics Duality

Henrik Johansson

Color-Kinematics Duality

Yang-Mills theories are controlled by a kinematic Lie algebra

- Amplitude represented by cubic graphs:

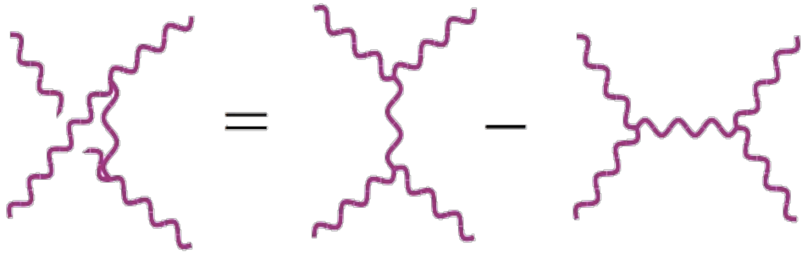
numerators

color factors

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

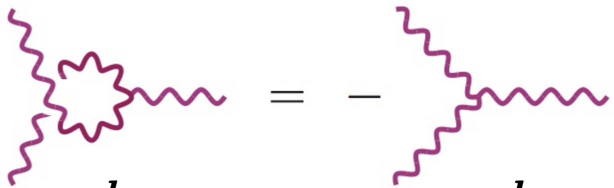
propagators

Color & kinematic numerators satisfy same relations:



Jacobi identity

$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$



antisymmetry

$$f^{bac} = - f^{abc}$$

Duality: color ↔ kinematics

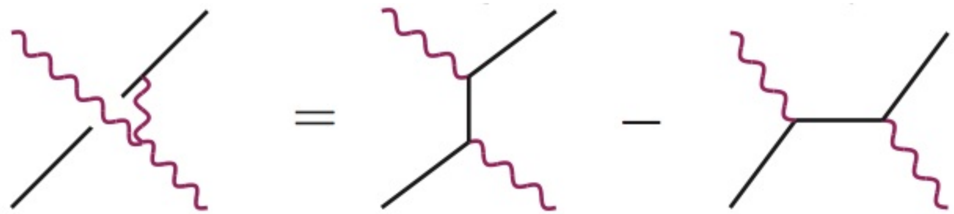
Bern, Carrasco, HJ

Some details of color-kinematics duality

Bern, Carrasco, HJ

can be checked for 4pt on-shell ampl. using Feynman rules

Example with two quarks:



$$\varepsilon_2 \cdot (\overline{u}_1 \not{V} u_3) \cdot \varepsilon_4 = \overline{u}_1 \not{\varepsilon}_4 \not{p}_t \not{\varepsilon}_2 u_3 - \overline{u}_1 \not{\varepsilon}_2 \not{p}_s \not{\varepsilon}_4 u_3$$

$$f^{cba} T^c_{ik} = T^b_{ij} T^a_{jk} - T^a_{ij} T^b_{jk}$$

1. $(A^\mu)^4$ contact interactions absorbed into cubic graphs
   - by hand 1=*s*/*s*
   - or by auxiliary field $B \sim (A^\mu)^2$
2. Beyond 4-pts duality not automatic ➔ Lagrangian reorganization
3. Known to work at tree level: all-*n* example Kiermaier; Bjerrum-Bohr et al.
4. Enforces (BCJ) relations on partial amplitudes ➔ (*n*-3)! basis
5. Same/similar relations control string theory S-matrix

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger

Gravity is a double copy

- Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

BCJ

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can belong to different theories:

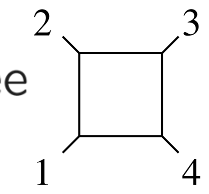
n_i		$\tilde{n}_i$		
$(\mathcal{N}=4)$	$\times$	$(\mathcal{N}=4)$	$\rightarrow$	$\mathcal{N}=8$ sugra
$(\mathcal{N}=4)$	$\times$	$(\mathcal{N}=2)$	$\rightarrow$	$\mathcal{N}=6$ sugra
$(\mathcal{N}=4)$	$\times$	$(\mathcal{N}=0)$	$\rightarrow$	$\mathcal{N}=4$ sugra
$(\mathcal{N}=0)$	$\times$	$(\mathcal{N}=0)$	$\rightarrow$	Einstein gravity + axion+ dilaton

similar to Kawai-Lewellen-Tye but works at loop level

C-K amplitudes, 1-loop examples

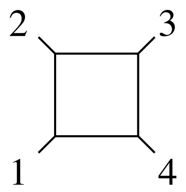
Known duality-satisfying loop amplitudes:

N=4 SYM: $st A_4^{\text{tree}}$



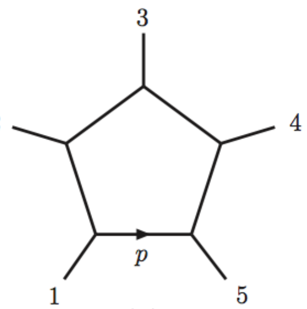
Green, Schwarz, Brink (1982)

All-plus QCD: $\dfrac{\langle 1\,2\rangle\,\langle 3\,4\rangle}{[1\,2]\,[3\,4]}\,\mu^4$



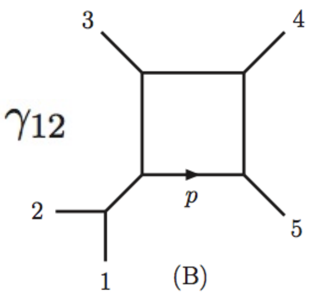
N=4 SYM and All-plus QCD:

β_{12345}



(P)

γ_{12}



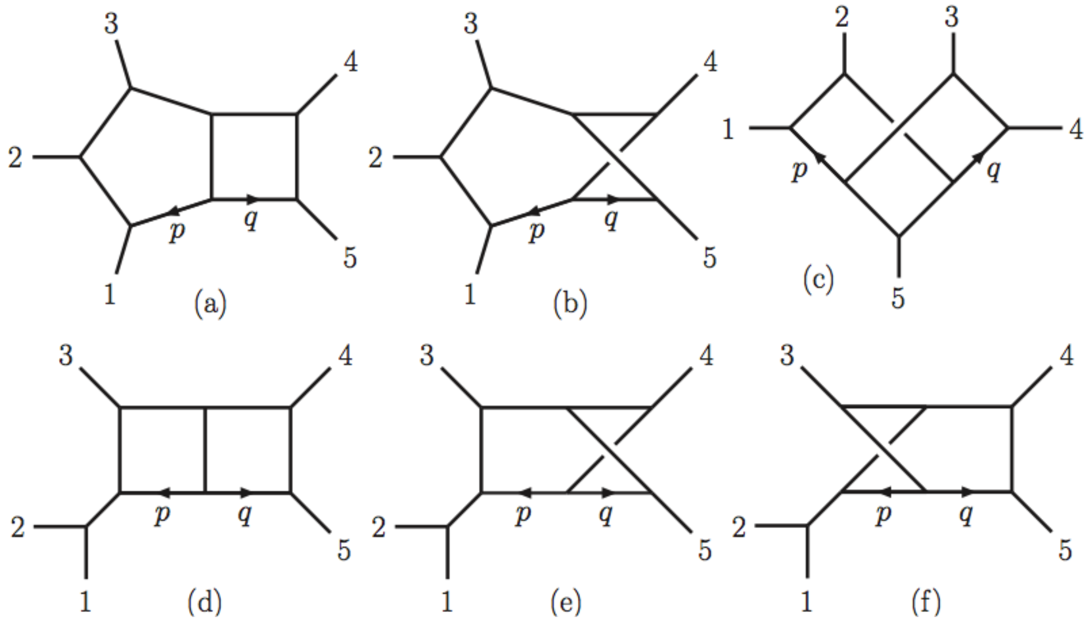
(B)

1106.4711 [hep-th] Carrasco, HJ

$$\beta_{12345}^{\mathcal{N}=4} = \delta^{(8)}(Q)\,\frac{[12][23][34][45][51]}{4\epsilon(1,2,3,4)} \qquad \gamma_{12}^{\mathcal{N}=4} = \delta^{(8)}(Q)\,\frac{[12]^2[34][45][35]}{4\epsilon(1,2,3,4)}$$

$$\beta_{12345}^{+++++} = \mu^4\,\frac{[12][23][34][45][51]}{4\epsilon(1,2,3,4)} \qquad \gamma_{12}^{+++++} = \mu^4\,\frac{[12]^2[34][45][35]}{4\epsilon(1,2,3,4)}$$

2-loop 5-pts $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG



Carrasco, HJ
1106.4711 [hep-th]

The 2-loop 5-point amplitude with duality exposed

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p})$ $+2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q})$ $+\gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$

$\mathcal{N}$= 8 SG obtained from numerator double copies

$$\tau_{ip} = 2k_i \cdot p$$

Summary

- Explicit calculations in $\mathcal{N}=8$ SUGRA up to four loops show that the power counting exactly follows that of $\mathcal{N}=4$ SYM – a finite theory
- 5 loop calculation in $D=24/5$ probes the potential 7-loop $D=4$ counterterm – will provide critical input to the $\mathcal{N}=8$ question !
- D=5 SYM have a 6-loop UV divergence, showing that the standard perturbative expansion misses some of the (2,0) theory contributions.
- Color-Kinematics Duality shows that Yang-Mills contains a hidden kinematic Lie Algebra – gravity being the double copy of this
- Allows gravity calculations simply by reorganizing the Yang-Mills amplitude – greatly facilitating UV analysis in gravity
- Stay tuned for the 5-loop SUGRA result…