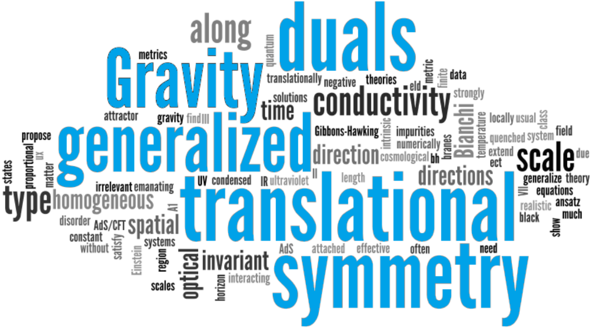
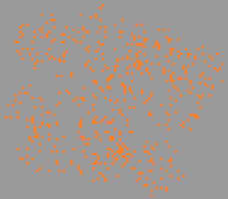


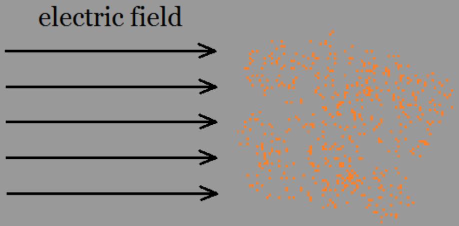
David Vegh

CERN Theory Group Retreat – November 9, 2012

Naive picture of electrons in metals



Naive picture of electrons in metals



electrons accelerate forever $\rightarrow$ DC conductivity is ∞

two ways to resolve this issue:

- dilute charge carriers
- break translational invariance

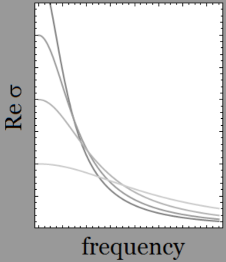
Conductivity

Ohm's law: $\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$

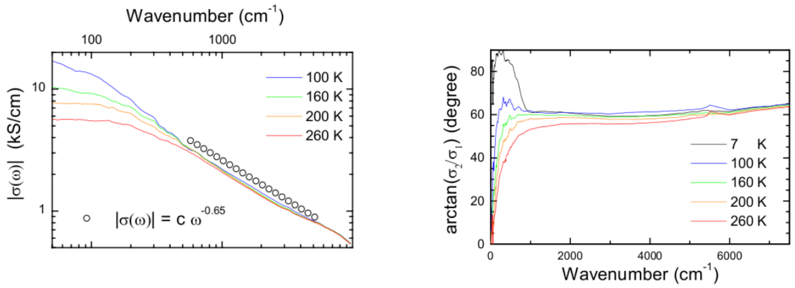
Drude model: $m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = q\vec{E}$

$\vec{j} = nq\vec{v} \quad \rightarrow \quad \sigma(\omega) = \left(\frac{nq^2\tau}{m}\right)\frac{1}{1-i\omega\tau}$

Paul Drude



Cuprate high temperature superconductors



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

[van der Marel, et. al., Nature 425 (2003) 271]

Gauge / gavity duality

[Maldacena, 1997] [Gubser-Klebanov-Polyakov] [Witten]

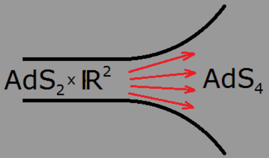
d-dimensional *conformal field theory*	$\Longleftrightarrow$	string theory in (d+1)-dim. *anti-de Sitter* spacetime
global symmetries	$\Longleftrightarrow$	gauge symmetries
finite T	$\Longleftrightarrow$	black hole in AdS
finite μ	$\Longleftrightarrow$	electrically charged black hole

Charged black hole in AdS_4

Relativistic CFT_3 with gravity dual and conserved $U(1)$ global symmetry

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{R^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots \right)$$

Charged black hole solution



$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + R^2 \frac{dr^2}{f(r)r^2}$$

$$f(r) = 1 + \frac{Q^2}{r^4} - \frac{M}{r^3} \qquad A = \mu\left(1 - \frac{r_0}{r}\right) dt$$

μ: chemical potential

horizon at $r = r_0$.

Conductivity

- compute conductivity using Kubo formula

$$\sigma(\omega) = \frac{1}{i\omega} \langle J_x(\omega) J_x(\omega) \rangle_{ret.}$$

- J_x dual to a_x (which mixes with g_{tx} in the bulk)
- the result:

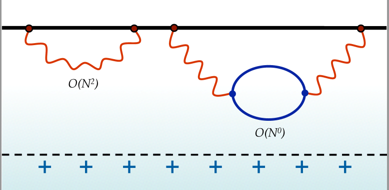
$$\sigma(\omega) = \frac{\rho^2}{\varepsilon + P} \cdot \frac{i}{\omega} + \mathcal{K}\omega^2$$

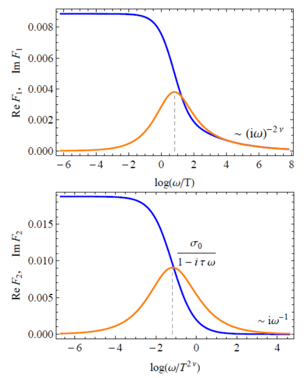
- $i\varepsilon$ prescription gives $\delta(\omega)$ in the real part $\rightarrow \sigma_{DC} = \infty$
- two ways to resolve this issue:
  - dilute charge carriers
  - break translational invariance

Conductivity (dilute)

contribution of holographic Fermi surfaces to conductivity

[Faulkner-Iqbal-Liu-McGreevy-DV]





$\nu < \frac{1}{2}$ $\qquad \sigma(\omega) \sim T^{-2\nu} F_1(\omega/T)$

for $\omega \gg T$: $\sigma(\omega) \sim (i\omega)^{-2\nu}$

$\nu > \frac{1}{2}$ $\qquad \sigma(\omega) \sim T^{-2\nu} F_2(\omega/T^{2\nu})$

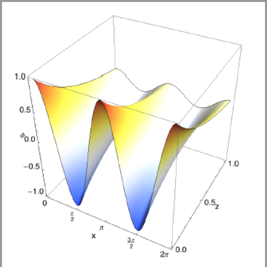
for $\omega \sim T^{2\nu}$: $\sigma(\omega) \sim \frac{\sigma_0}{1+i\omega\tau}$ (Drude)

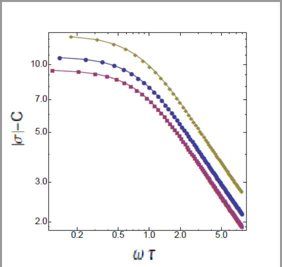
for $\omega \gg T$: $\sigma(\omega) \sim \frac{ia}{\omega} + b(i\omega)^{2\nu-2}$

Conductivity (lattice)

[Horowitz-Santos-Tong]

- inhomogeneous boundary conditions on fields: 1d lattice





- optical conductivity: $|\sigma(\omega)| = A + \frac{B}{\omega^{2/3}}$
- DC resistivity explained by [Hartnoll-Hofman] : $\rho \sim T^{2\nu-1}$

$$\nu = \frac{1}{2}\sqrt{5 + 2(k_L/\mu)^2 - 4\sqrt{1+(k_L/\mu)^2}}$$

Bianchi classification

Killing vectors satisfy 3d algebra $[\xi_i, \xi_j] = C_{ij}^k \xi_k$

invariant one-forms $d\omega^i = \frac{1}{2} C_{ij}^k \omega^j \wedge \omega^k$

- type I $(C_{ij}^k = 0)$

  $\xi_i = \partial_i$ $\qquad \omega^i = dx^i$

- type II $(C_{23}^1 = -C_{32}^1 = 1)$

  $\xi_{1,2} = \partial_{1,2}$ $\qquad \omega^1 = dx^1 - x^3 dx^2$

  $\xi_3 = \partial_3 + x^2 \partial_1$ $\qquad \omega^{2,3} = dx^{2,3}$

- type III $(C_{13}^1 = -C_{31}^1 = 1)$

  $\xi_{1,2} = \partial_{1,2}$ $\qquad \omega^1 = e^{-x^3} dx^1$

  $\xi_3 = \partial_3 + x^1 \partial_1$ $\qquad \omega^{2,3} = dx^{2,3}$

- etc...

Summary

- in translationally invariant systems $\sigma_{DC} = \infty$ (if $\rho \neq 0$)
- this ∞ can be resolved:
  - dilute charge carriers
  - break translational invariance
    - homogeneous: Bianchi spaces (can include time)
    - inhomogeneous: lattice (PDEs, not easy)