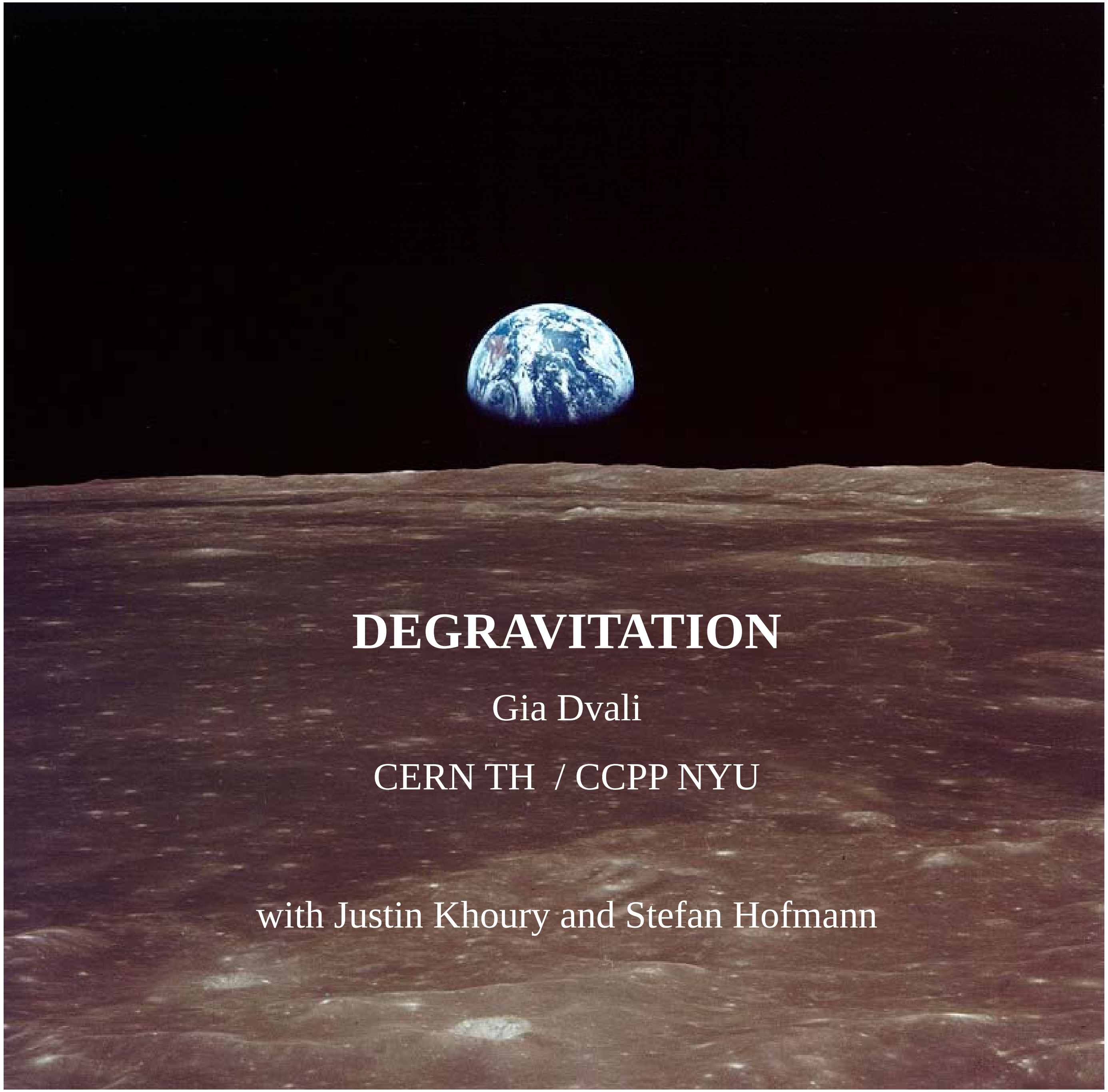


DEGRAVITATION

Gia Dvali

CERN TH / CCPP NYU

with Justin Khoury and Stefan Hofmann

Dvali, Gabadadze, Porrati;
Dvali, Gabadadze, Kolanovic, Nitti

Deffayet, Dvali, Gabadadze;

Dvali, Gruzinov and Zaldarriaga;

Dvali, Gabadadze, Shifman;

Arkani-Hamed, Dimopoulos, Dvali, Gabadadze

OUTLINE

1) **MOTIVATION**

2) **THE IDEA OF DEGRAVITATION**

3) **THE PUZZLE**

4) **DEGRAVITATION = GRAVITON MASS / DECAY WIDTH**

5) **STRONG COUPLING AND r_* -PHENOMENON**

6) **GRAVITON MASS AND THE SCALE OF QUANTUM GRAVITY**

7) **EXPERIMENTAL TESTS:**

   - Collider Probes of Quantum Gravity
   - Lunar Ranging
   - Cosmology

The standard question:

Why is the vacuum energy so small?

However, since nobody has ever meusered the energy of the vacuum by any means other than gravity,

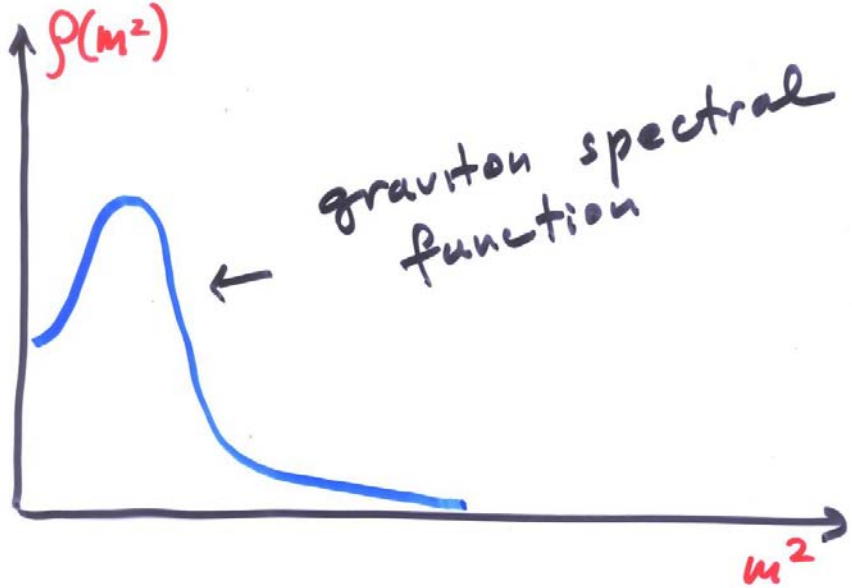
the right question to ask is:

Why does the vacuum energy gravitate so little?

In General Relativity, graviton, $h_{\mu\nu}$, is a massless spin-2 particle, and the two questions are equivalent.

However, the story is different if graviton is effectively massive or is a resonance!

If graviton has a mass or is a resonance, then gravity is modified at large distances, and vice versa!



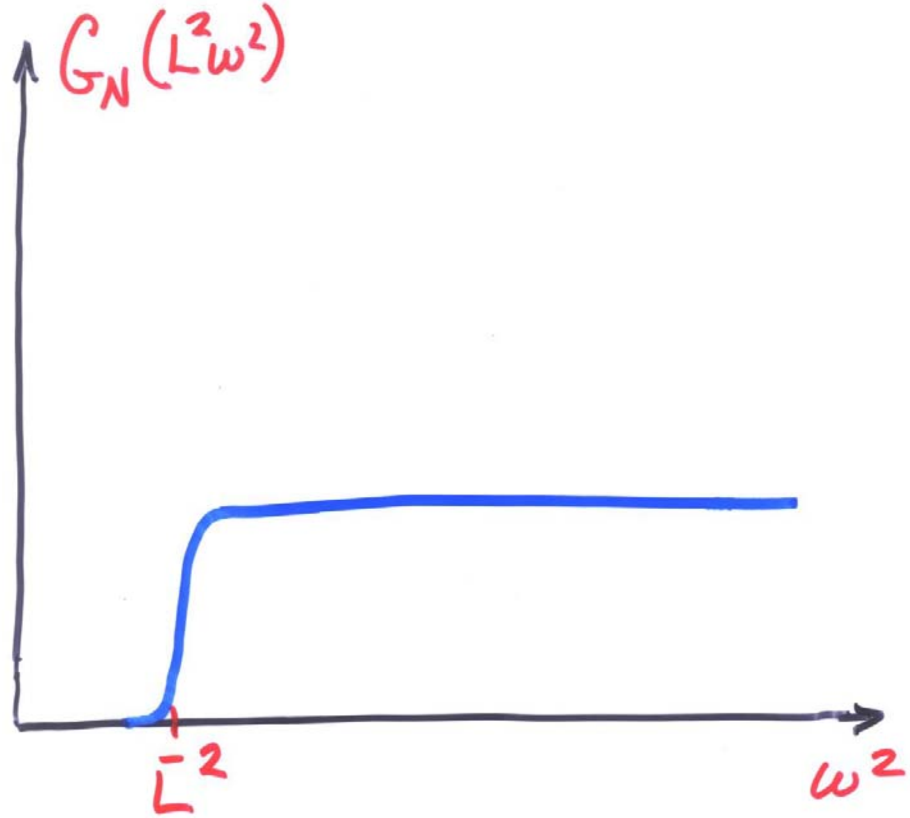
Thus, in large-distance-modified gravity theories one can ask, whether the vacuum energy appears to be small because it is effectively degravitated?

An effective phenomenological equation:

$$G_N^{-1}(L^2\Box)\, G_{\mu\nu} = 8\pi\, T_{\mu\nu}$$

Newton's constant is promoted into a high-pass filter.

G_N as a high-pass filter:



puzzle:

Take the linearized theory:

$$\left(1 - \frac{m^2(\Box)}{\Box}\right) \mathcal{E}\tilde{h}_{\mu\nu} = -T_{\mu\nu}$$

where

$\mathcal{E}\tilde{h}_{\mu\nu} \equiv$ Linearized Einstein tensor

and

$$16\pi G_N(\Box) \equiv \left[1 - \frac{m^2(\Box)}{\Box}\right]^{-1}$$

The gauge-invariant action:

$$\mathcal{L} = \frac{1}{2}\tilde{h}^{\mu\nu}\left[1 - \frac{m^2(\Box)}{\Box}\right]\mathcal{E}\,\tilde{h}_{\mu\nu} + \tilde{h}^{\mu\nu}T_{\mu\nu}$$

The puzzle:

Take $m^2 \equiv$ constant. Then $\tilde{h}_{\mu\nu}$ describes a massive spin-2 particle.

Nevertheless, it propagates $\underline{\underline{2}}$ degrees of freedom.

This is impossible!

The only ghost-free theory of linearized massive gravity

Pauli-Fierz mass

$$m_g^2 \left(h_{\mu\nu} h^{\mu\nu} - (h_\mu^\mu)^2 \right)$$

$h_{\mu\nu}$ contains 5 polarizations.

$$5 = 2 + 2 + 1$$

massless graviton contains 2.

The only consistent linear theory of massive spin-2 (Pauli-Fierz theory) propagates 5 states:

$$5 = 2 + 2 + 1$$

$$\text{helicity:} \quad 2 + 1 + 0$$

leading to van Dam-Veltman-Zakharov discontinuity

$$A_{2+0} = \frac{T_{\mu\nu}\tilde{T}^{\mu\nu} - \frac{1}{3} T_\mu^{\ \mu} T_\nu^{\ \nu}}{\Box - m^2}$$

vDVZ - discontinuity

Metric produced by a static gravitating source

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Massive $\rightarrow$ $h_{00} = 2G_N M \frac{1}{r} \frac{4}{3}$

Massless $\rightarrow$ $h_{00} = 2G_N M \frac{1}{r}$

One graviton exchange amplitude:

$$A_2 = \frac{T_{\mu\nu}\tilde{T}^{\mu\nu} - \frac{1}{2}T_\mu^\mu \tilde{T}_\nu^\nu}{\Box - m^2}$$

has a missing helicity-0 piece!

$$A_0 = \frac{1}{6}\,\frac{T_\mu^\mu \tilde{T}_\nu^\nu}{\Box - m^2}$$

So our starting action cannot possibly describe a consistent theory of a spin-2 field!

Promoting $m^2 \longrightarrow m^2(\Box)$ <u>does not</u> help.

The amplitude can be spectrally expanded:

$$\frac{1}{\Box - m^2(\Box)} = \int_0^\infty ds \, \frac{\rho(s)}{\Box - s}$$

and to each s the same objection applies!

Goldstone - Stückelberg Story.

The most general ghost-free theory of a massive/resonance spin-2

$$\mathcal{E} h_{\mu\nu} - m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu} h) = -T_{\mu\nu}$$

Stückelberg vector A_μ

$$h_{\mu\nu} \equiv \hat{h}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

gauge symmetry:

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$A_\mu \to A_\mu - \xi_\mu$$

Integrating out A_μ

this is what we get

for helicity-2 part

$$\left[1 - \frac{m^2(\Box)}{\Box}\right] \mathcal{E} \bar{h}_{\mu\nu} = -T_{\mu\nu}!$$

$$\left[1-\frac{m^2(\Box)}{\Box}\right]\mathcal{E}\,\tilde{h}_{\mu\nu} = -\,T_{\mu\nu}$$

Thus, $\tilde{h}_{\mu\nu}$ describes the Einsteinian (helicity-2) part of a massive/resonance graviton!

Degravitation $\equiv$ Graviton mass/width!

Gravitational analog of the background charge-screening in superconductors.

Cosmological constant problem in theory of linear massless spin-2.

$$\mathcal{E}\tilde{h}_{\mu\nu} = \Lambda\eta_{\mu\nu}$$

de Sitter space-times:

1 Closed FRW slicing

$$\tilde{h}_{00} = 0, \quad \tilde{h}_{0j} = 0 \quad \tilde{h}_{ij} = \frac{\Lambda}{6}\left(t^2\delta_{ij} + x_i x_j\right)$$

2 Static patch of de Sitter

$$\tilde{h}_{00} = \frac{\Lambda}{6}x_i x^i, \quad \tilde{h}_{0i} = 0, \quad \tilde{h}_{ij} = \frac{\Lambda}{6}x_i x_j$$

De-gravitation by graviton mass:

$$-\mathcal{E}h_{\mu\nu} + m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = -\Lambda\eta_{\mu\nu}$$

$$\Downarrow$$

Flat space

$$h_{\mu\nu} = \frac{\Lambda}{3m^2}\eta_{\mu\nu} \;!$$

Understanding degravitation in the language of different helicites

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} - \eta_{\mu\nu}\frac{1}{3}\Pi_{\alpha\beta}\tilde{h}^{\alpha\beta} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

In terms of helicity-2 ($\tilde{h}_{\mu\nu}$) and helicity-0 (χ), $h_{\mu\nu}$ can be written as:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{1}{6}\eta_{\mu\nu}\chi + \frac{1}{3}\frac{\partial_\mu\partial_\nu}{m^2}\chi$$

We wish to understand degravitation in terms of $\tilde{h}_{\mu\nu}$ ~~m~~ and χ.

First, as said before, integrating out extra helicities equation for $\tilde{h}_{\mu\nu}$ is:

$$\left(1-\frac{m^2}{\Box}\right)\mathcal{E}\tilde{h}_{\mu\nu} = \Lambda\eta_{\mu\nu}$$

$$\Downarrow$$

$$\tilde{h}_{00} = -\frac{\Lambda}{m^2}(1-\cos mt)$$

$$\tilde{h}_{0j} = x_j\frac{\Lambda}{3m}\sin mt$$

$$\tilde{h}_{ij} = \frac{\Lambda}{m^2}(1-\cos mt)\delta_{ij} + \underset{i\neq j}{\frac{\Lambda}{6}\cos(mt)\,x_i x_j}$$

For $mt \ll 1$ this is de Sitter:

$$\tilde{h}_{00} \approx -\frac{\Lambda}{2}t^2, \quad \tilde{h}_{0j} \approx \frac{\Lambda}{3}t x_j,$$

$$\tilde{h}_{ij} \approx \frac{\Lambda}{2}t^2\delta_{ij} + \underset{i\neq j}{\frac{\Lambda}{6}x_i x_j}$$

Can be recast:

$$ds^2 = -dt^2 + \left(1 + \frac{\Lambda t^2}{3}\right)\delta_{ij}\, dx^i dx^j$$

$$+ \frac{\Lambda}{3} x_i x_j\, dx^i dx^j \approx$$

$$\approx -dt^2 + \cosh\sqrt{\frac{\Lambda}{3}}\, t \left(\frac{dr^2}{1-\frac{\Lambda r^2}{3}} + r^2 d\Omega^2\right)$$

de Sitter in closed FRW slicing.

In the same time,
the helicity-0 behaves as

$$\chi = \frac{4\Lambda}{m^2}(\cos mt - 1)$$

and ~~compensated~~ compensates the effect of $\tilde{h}_{\mu\nu}$!

In fact degravitation
persists also in the limit

$$m^2 \to 0$$

In the limit $m^2 \to 0$

the equations for $\tilde{h}_{\mu\nu}$ and χ

split

$$\mathcal{E}\tilde{h}_{\mu\nu} = -T_{\mu\nu}$$

$$\Box \chi = -T$$

solutions:

$$\tilde{h}_{\mu\nu} = -\frac{\Lambda}{12}\eta_{\mu\nu} x_\alpha x^\alpha$$

$$\chi = \frac{\Lambda}{2} x_\alpha x^\alpha$$

Thus,

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\frac{\chi}{6} = \text{flat metric!}$$

Constraints on $m^2(\Box)$

$$\mathcal{E} h_{\mu\nu} + m^2(\Box)\left(h_{\mu\nu} - \eta_{\mu\nu} h\right) = -T_{\mu\nu}$$

For $\Box \to 0$, we must have

$$m^2(\Box) \approx L^{2(\alpha-1)} \Box^{\alpha}$$

with $\alpha < 1$.

From absence of ghosts we have a constraint

$$\alpha > 0$$

Thus,

$$0 < \alpha < 1$$

vDVZ - discontinuity

Metric produced by a static gravitating source

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

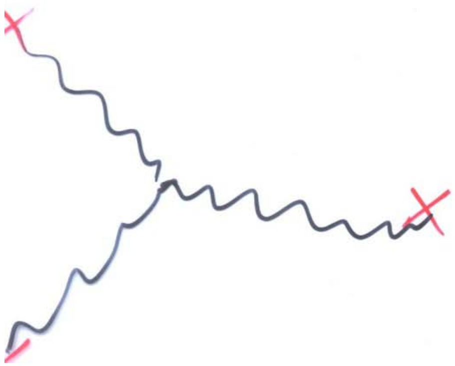
Massive $\rightarrow$ $h_{00} = 2 G_N M \dfrac{1}{r} \, \dfrac{4}{3}$

Massless $\rightarrow$ $h_{00} = 2 G_N M \dfrac{1}{r}$

In GR metric produced by a source can be found in $\frac{r_g}{r}$ expansion.

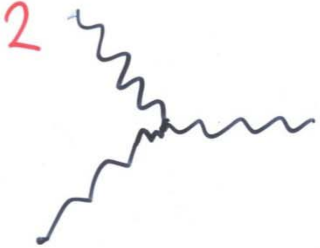
$$r_g \equiv 2 G_N M$$

$$\longrightarrow \quad \frac{r_g}{r}$$



$$\longrightarrow \quad \left(\frac{r_g}{r}\right)^2$$

For massive graviton, say for $r \sim$ solar system

 $= 10^{-32}$  !

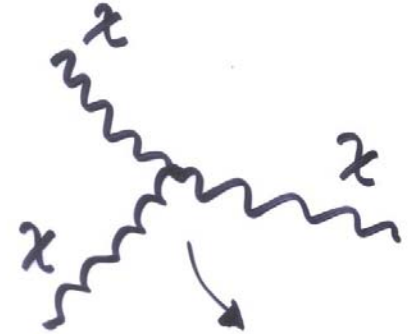
Deffayet, G.D., Gabadadze, Vainshtein

This happens because the helicity – 0 graviton is strongly coupled.

Ignoring helicity-1, we then get

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\frac{\chi}{6} + \frac{L^{2(\alpha-1)}}{3}\frac{\partial_\mu \partial_\nu}{\Box^\alpha}\chi$$

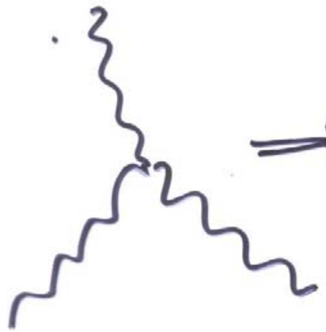
At non-linear level χ is strongly coupled



$$\frac{\chi\,\Box\left(\Box^{1-\alpha}\chi\right)^2}{\Lambda_s^{5-4\alpha}}$$

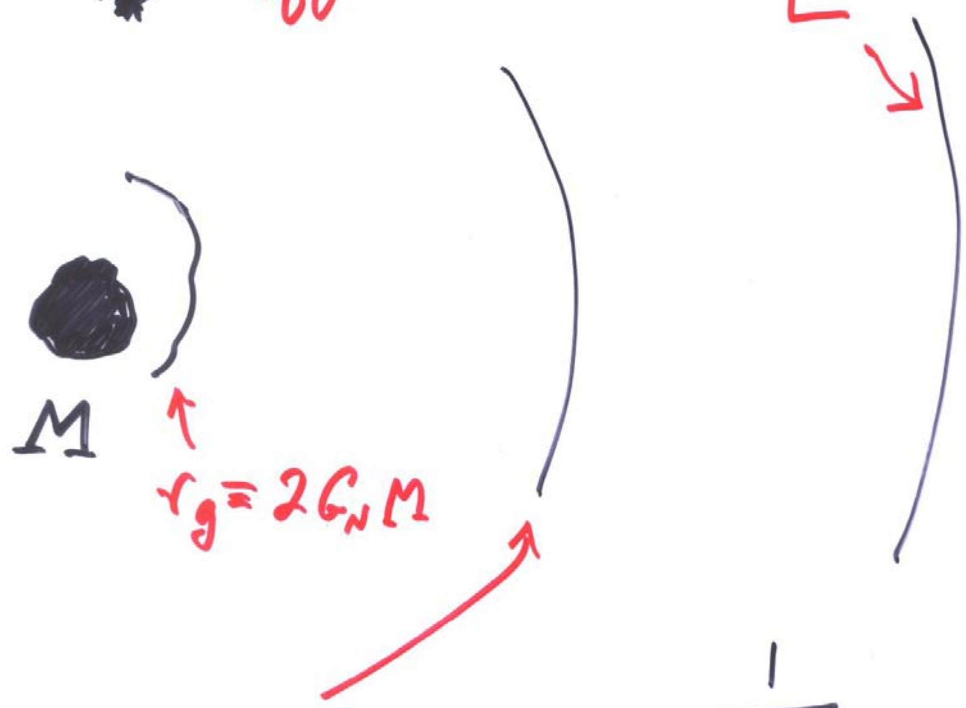
Strong coupling scale of helicity-0 graviton

$$\Lambda_s \equiv \left(L^{4(\alpha-1)} M_{pl}\right)^{\frac{1}{5-4\alpha}}$$



$$\Longrightarrow \frac{1}{\Lambda_s^{5-4\alpha}}\, \chi \,\Box \left(\Box^{1-\alpha}\chi\right)^2$$

r_* - effect



$$r_* \equiv \left(L^{4(1-\alpha)} r_g\right)^{\frac{1}{5-4\alpha}}$$

$$r \ll r_* \longrightarrow \chi \sim \frac{r_g}{r_*}\left(\frac{r}{r_*}\right)^{\frac{3}{2}-2\alpha}$$

$r \gg r_* \longrightarrow \chi \sim$ linear regime

Deviation from Einsteinian potential near gravitating objects ($r \ll r_*$)

$$\delta \approx \left(\frac{r}{10^{28}\,\mathrm{cm}}\right)^{2-2\alpha} \sqrt{\frac{r}{r_g}}$$

where $0 \leq \alpha \leq 1$

$$r_g \equiv 2 G_N M$$

Degravitation in the decoupling limit

$L \to \infty$, $M_{pl} \to \infty$, $\Lambda_s =$ fixed

$$\Lambda_s \equiv \left(L^{4(\alpha-1)} M_{pl}\right)^{\frac{1}{5-4\alpha}}$$

$$\mathcal{E}\tilde{h}_{\mu\nu} = -\frac{T_{\mu\nu}}{M_{pl}}$$

$$\Box\chi - \frac{1}{\Lambda_s^{5-4\alpha}}\left[3\Box\left(\Box^{1-\alpha}\chi\right)^2 - \Box\left(\frac{\partial_\mu\partial_\nu}{\Box^\alpha}\chi\right)^2 - 2\partial_\mu\partial_\nu\left(\frac{\partial^\mu\partial^\nu}{\Box^\alpha}\chi\,\Box^{1-\alpha}\chi\right)\right] = -\frac{T_\mu^\mu}{M_{pl}}$$

$$T_{\mu\nu} = \Lambda\,\eta_{\mu\nu}$$

Degravitated solution:

$$\tilde{h}_{\mu\nu} = -\frac{\Lambda}{12 M_{pl}} \eta_{\mu\nu}\, x_\beta x^\beta$$

$$\chi = \frac{\Lambda}{2 M_{pl}} x_\beta x^\beta$$

This will be the case if

$$\frac{\partial_\mu \partial_\nu \partial_\beta}{\Box^\alpha} x^2 = 0$$

$$\Downarrow$$

$$\boxed{\alpha < \frac{1}{2}}$$

The relation between the graviton mass and quantum gravity scale $\equiv M_*$

$$L = \left(\frac{M_*^{2-\alpha}}{M_p}\right)^{\frac{1}{\alpha-1}}$$

For $\alpha = 0$ (massive graviton) we have:

$$L^{-1} \equiv m_g = \frac{M_*^2}{M_p} \quad !$$

The two immediate predictions are:

$$m_g = H_0 \sim 10^{-33}\,eV$$

and

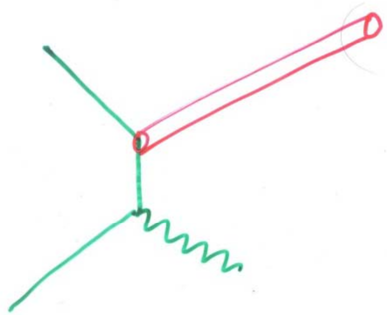
$$M_* \sim 10^{-3}\,eV$$

Modeling QG with a closed string spectrum, with $M_* \sim 10^{-3}\,eV$, coupled to the Standard Model as they should.
(G.D, Gabadadze, Kolanovic, Nitti)

This sounds crazy, because we would see production of $\sim 10^{1000}$ string states already in a scattering of the visible light!

But, this intuition is false.

In fact, most of the string states are extremely weakly coupled.



The rate of production at energies E is:

$$\Gamma = \sum_n E \left(\frac{E^2}{M_p M_*} \right)^{2n-2}$$

The critical scale is

$\boxed{M_p M_*} \sim \text{TeV}$ not $M_* \sim 10^{-3}\,\text{eV}$

So if graviton has a mass, then LHC should hit a "Hagedorn wall"?

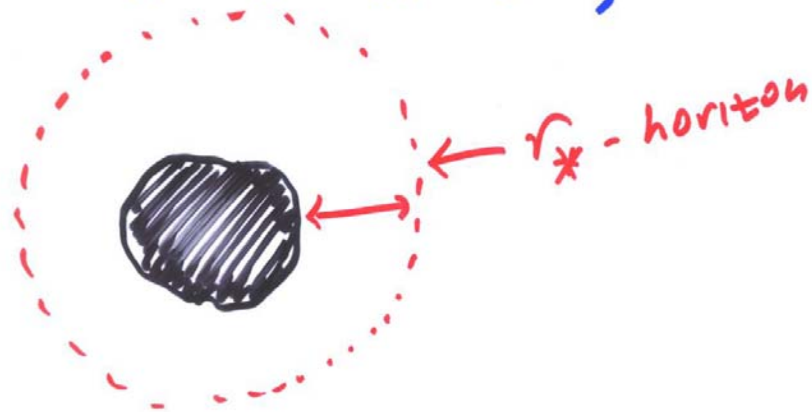
$$\frac{d\sigma}{dt} \sim \sum_n \frac{1}{s^2} \left(\frac{s}{M_p M_*} \right)^{2n-2}$$

The model building issue for embedding in string theory in SM not gravity sector.

See however constructions

The rule:

For a source that is localized within its own r_*, gravity is almost Einsteinian,

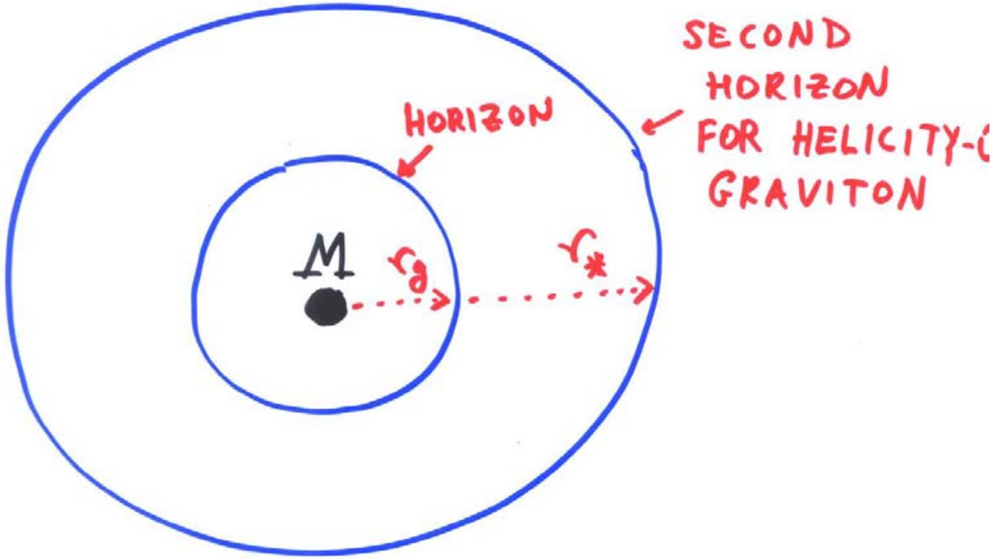


with small corrections given by:

$$\delta \simeq \left(\frac{r}{10^{28}\,\mathrm{cm}}\right)^{2-2\alpha} \sqrt{\frac{r}{r_g}}$$

THE CONCEPT OF r_*

FOR A SOURCE OF GRAVITATIONAL RADIUS $r_g \equiv 2 G_N M$



$$r_* \equiv \left(10^{56} cm^2 \, r_g\right)^{\frac{1}{3}}$$

Currently the only known theory is $\alpha = \frac{1}{2}$ (DGP)

The predicted correction

$$\delta \simeq \frac{r}{10^{28}\,\mathrm{cm}} \sqrt{\frac{r}{r_g}}$$

will (probably) be tested by APOLLO

Adelberger, Stubbs, Murphy . . .

see, Murphy, Adelberger, Strasburg, Stubbs

LUNAR RANGING TEST

G.D., Gruzinov, Zaldarriaga;
Lue, Starkman.

PREDICTED ANOMALOUS PERIHELION PRECESSION:

$$\delta\phi = \frac{3\pi}{4} \frac{r}{10^{28}\,\mathrm{cm}} \sqrt{\frac{r}{r_g}}$$

FOR THE MOON:

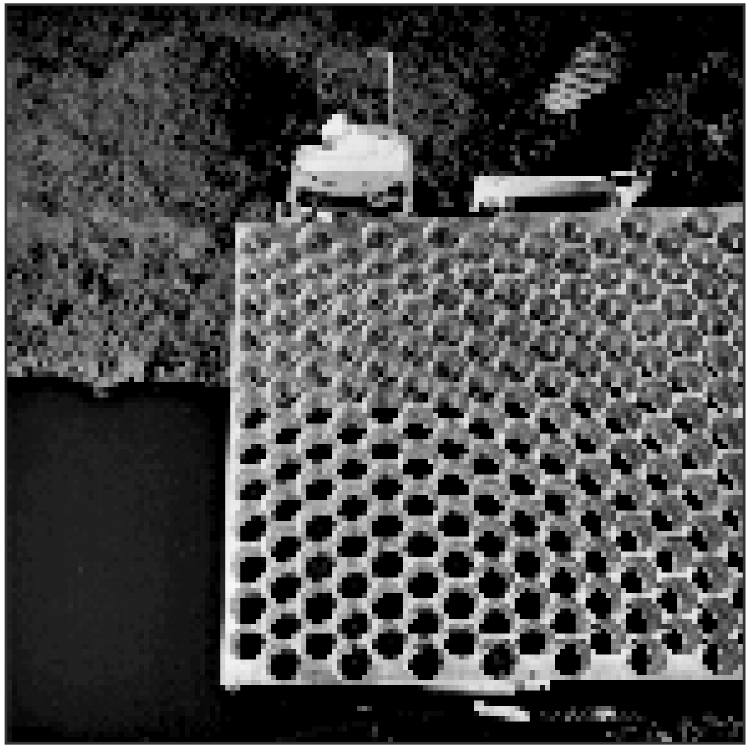
$$\delta\phi = 1.4 \times 10^{-12}$$

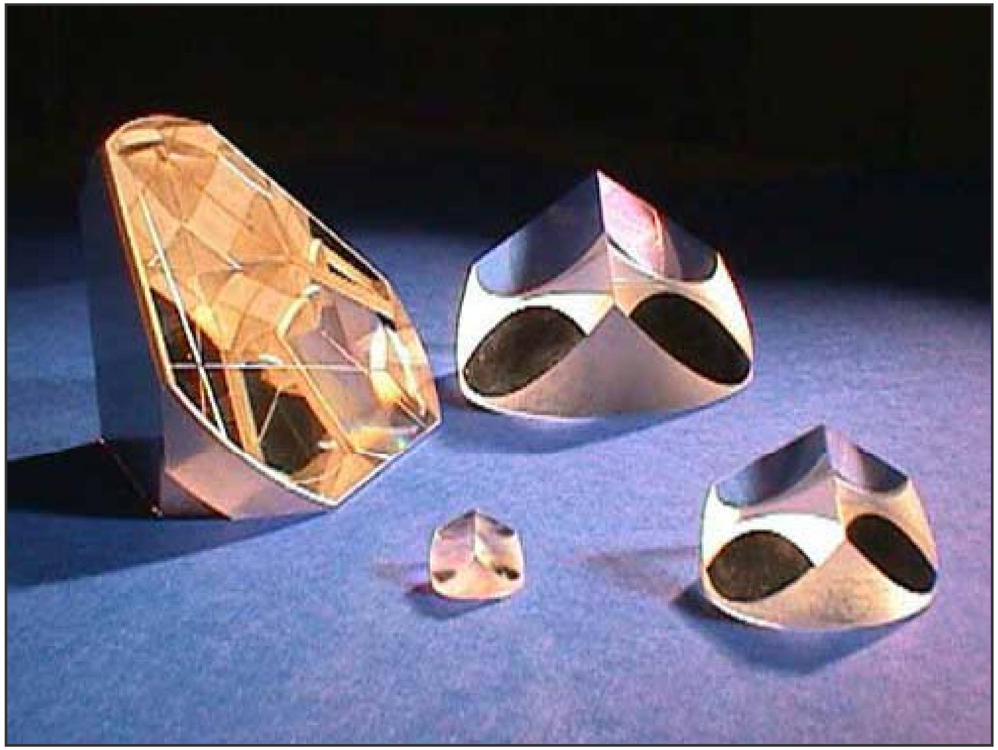
TODAY'S ACCURACY:

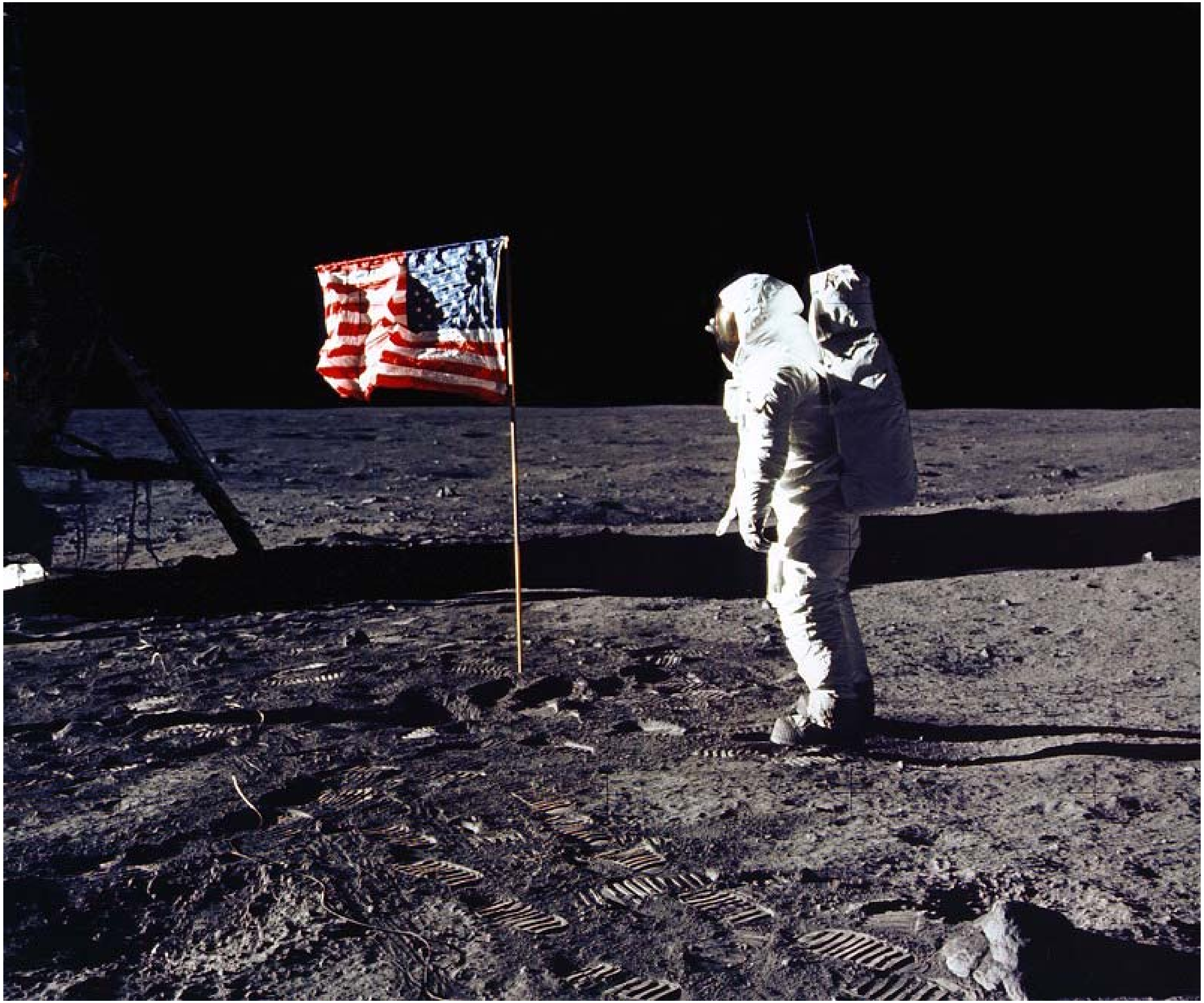
$$\sigma_\phi = 2.4 \times 10^{-11}$$

10-FOLD IMPROVEMENT IS EXPECTED:

Adelberger, Stubbs, Murphy....








UNITED
STATES

Cosmological tests:

For $L \simeq H_0^{-1}$ Universe is at its r_* !

$$a^3(t) \propto t^2 \left[1 \pm (t H_0)^{-2\alpha} \right]$$