

Particle Detectors

Part 1

Lectures at the CHIPP winter School 2013
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CERN)

INTRODUCTION

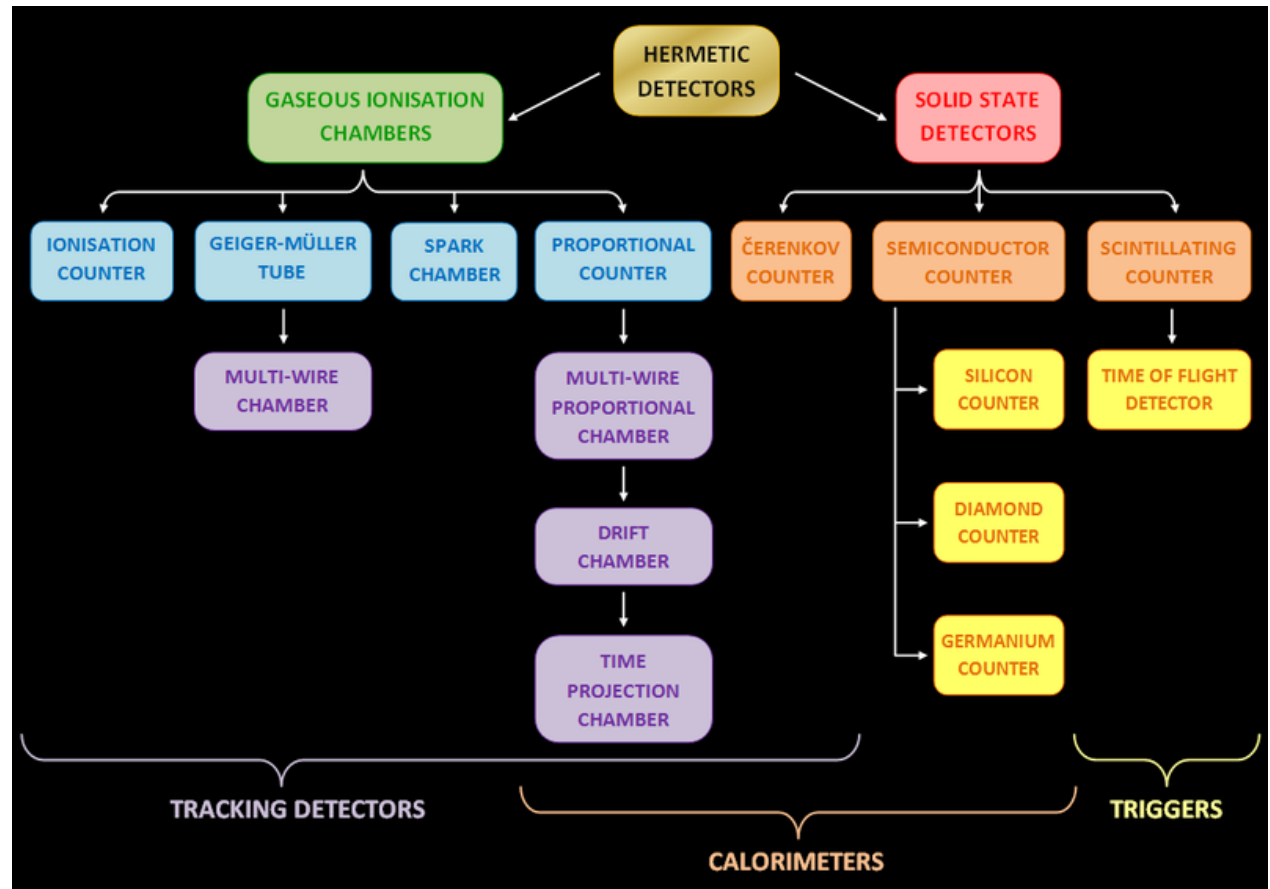
Usual disclaimer

- These lectures cannot cover all of the complex subjects of particle detectors
 - Cannot describe the full variety
 - And even less go in full depth
- Tried to give a balanced overview of the techniques and the reason of their choice
- Tried not to be too CMS-biased
- Working on detector id fun!

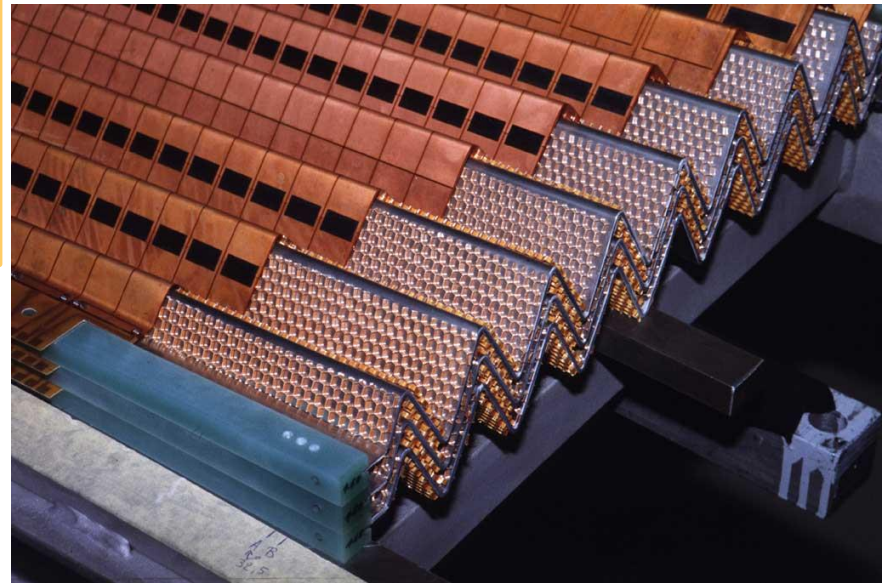
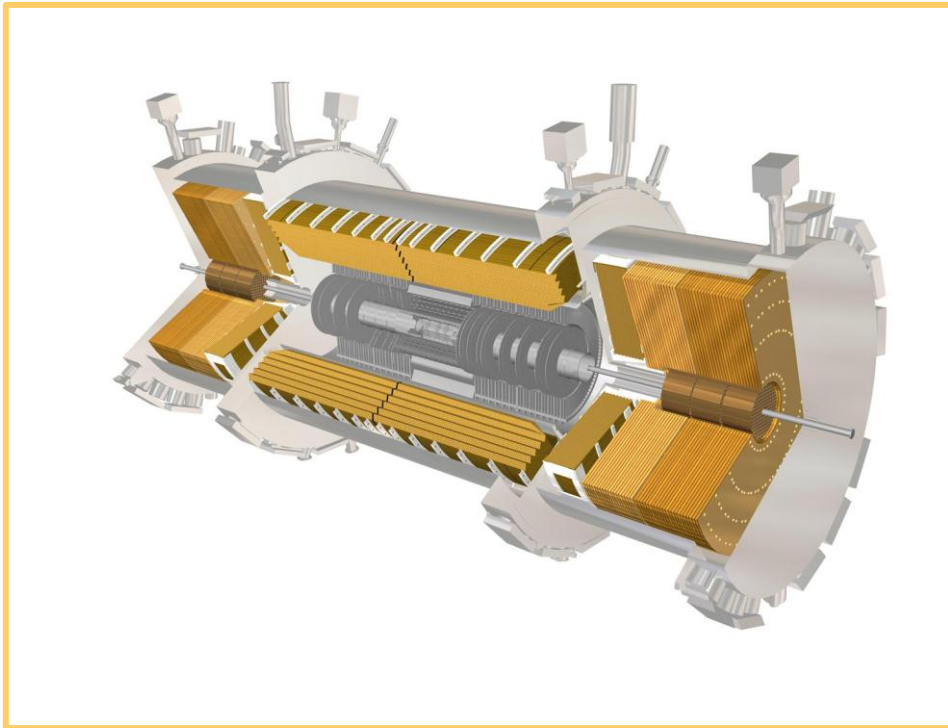


Classification of particle detectors

- Go to Wikipedia and get all information you need



- Well not really
 - The Atlas Liquid Argon “accordion” calorimeter is neither a “gaseous” nor a “solid state” detector

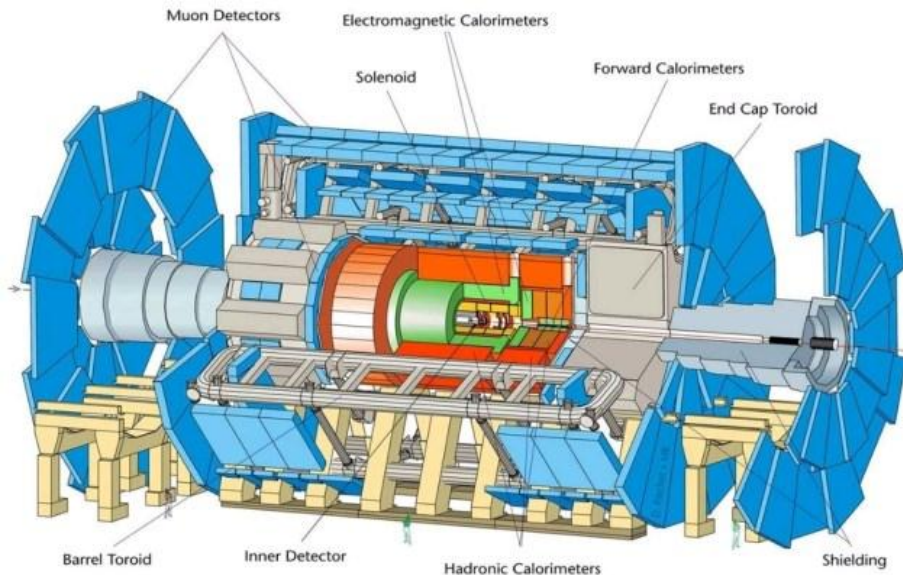
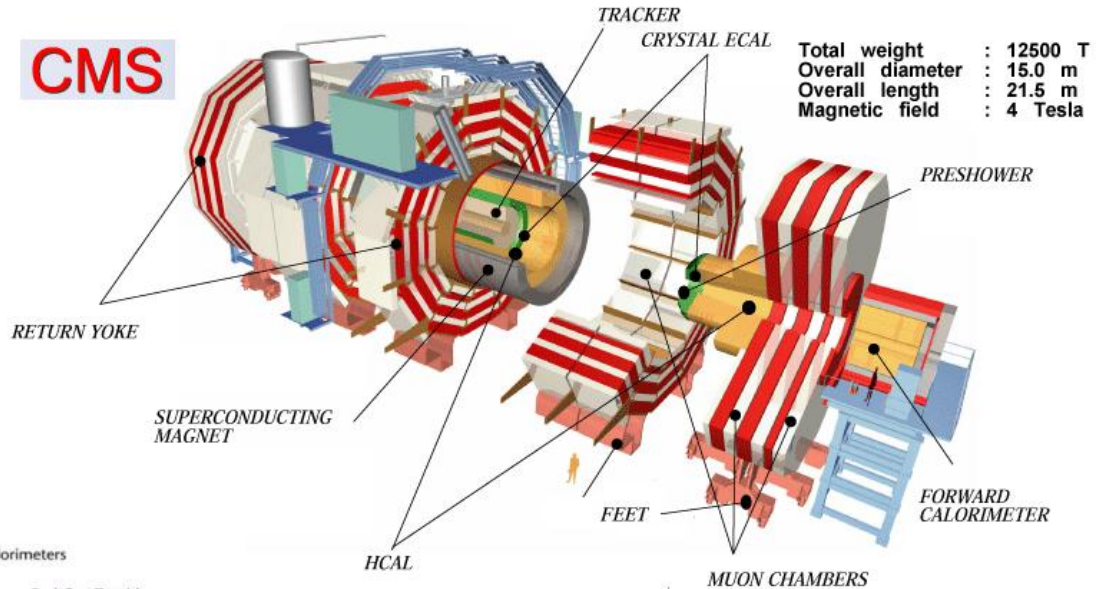


Many possible classifications

- Signal generation
 - Ionization
 - Scintillation light
 - Cherenkov light
 - Transition Radiation
- Use
 - Tracking detectors
 - Vertex, Central, Muons
 - Calorimeters
 - Electromagnetic, Hadron
 - Particle Identification
 - Trigger
- Technologies used
 - Gaseous detectors
 - Multi-wire, Drift chambers, Limited Streamer Tubes, RPCs, GEMs
 - Scintillators
 - Crystals, Plastic, Liquid
 - Semiconductors
 - Pixels, Strip

... and then detectors get combined

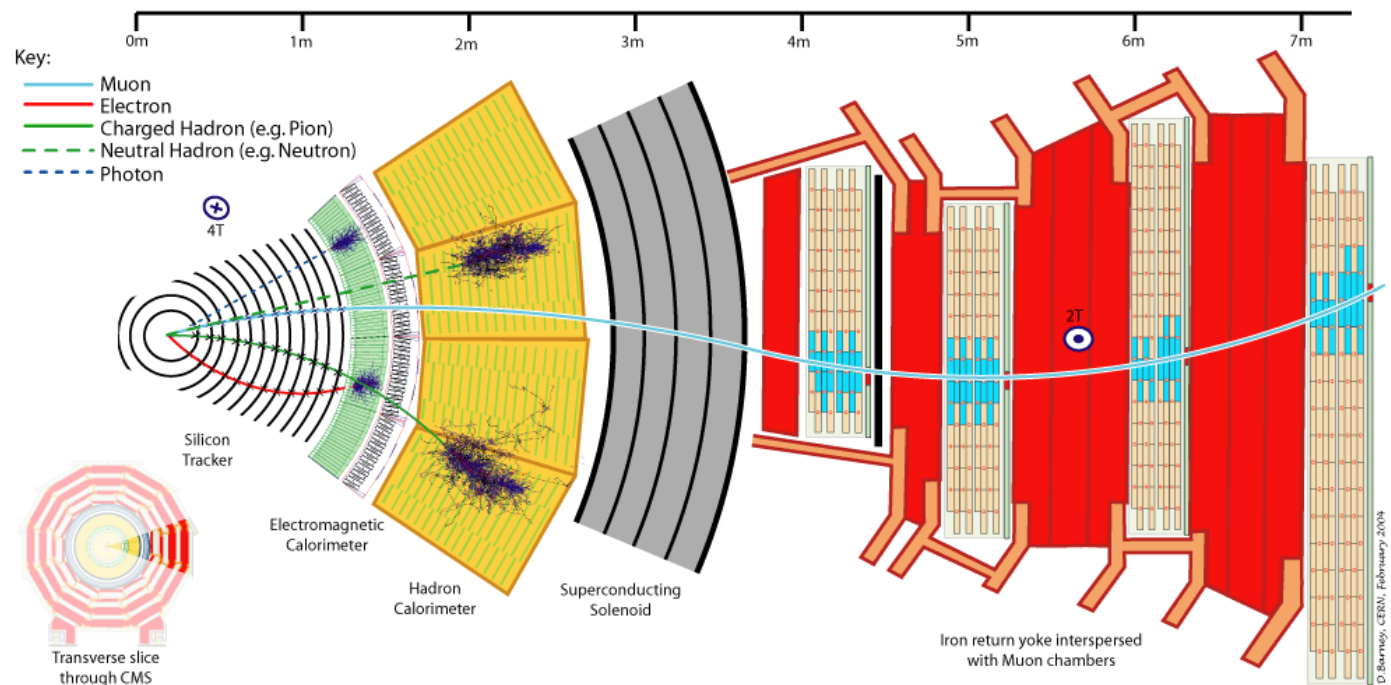
- Modern large experiments are complex combination of detectors
 - Often with combined tasks, e.g. calorimeters and muon detectors are used for fast trigger



- Similar requirements, the aspect is similar
 - But the specific choice of the technologies are quite different
 - **There is space for imagination**

Plan of the lectures

- First some recap on interaction of radiation with matter
- Then a description of the main classes of detectors
 - with the different technologies used to build them
- Finally a real life example (...CMS)

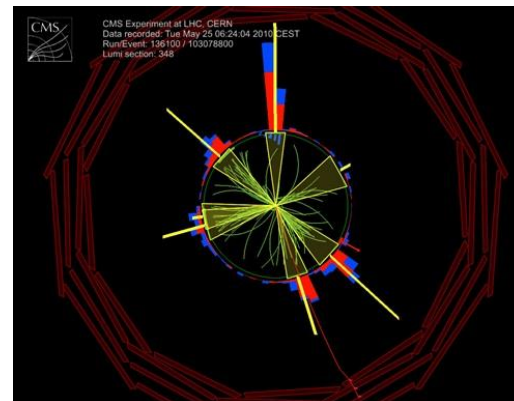
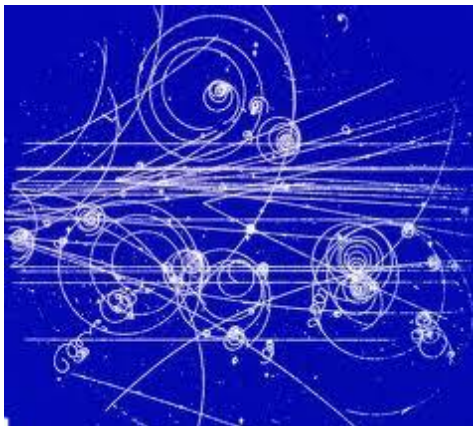


What are the particles we detect?

- Stable particles, or unstable particles with long enough lifetimes to transverse the detectors
 - Other particles are identified, when needed, by their decay produces



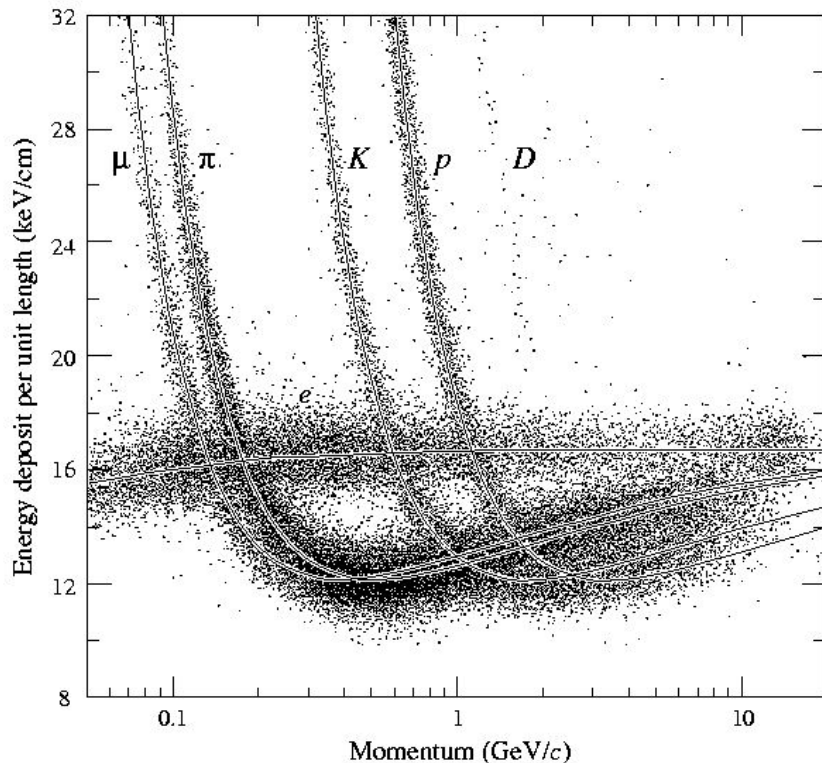
- Electrons, muons
- Photons
- Neutrinos
- Charged and neutral nucleons, pions, kaons
 - Most of the times hadrons are inside jets of particles coming from hadronization of the partons



INTERACTIONS WITH MATTER

Bethe-Bloch

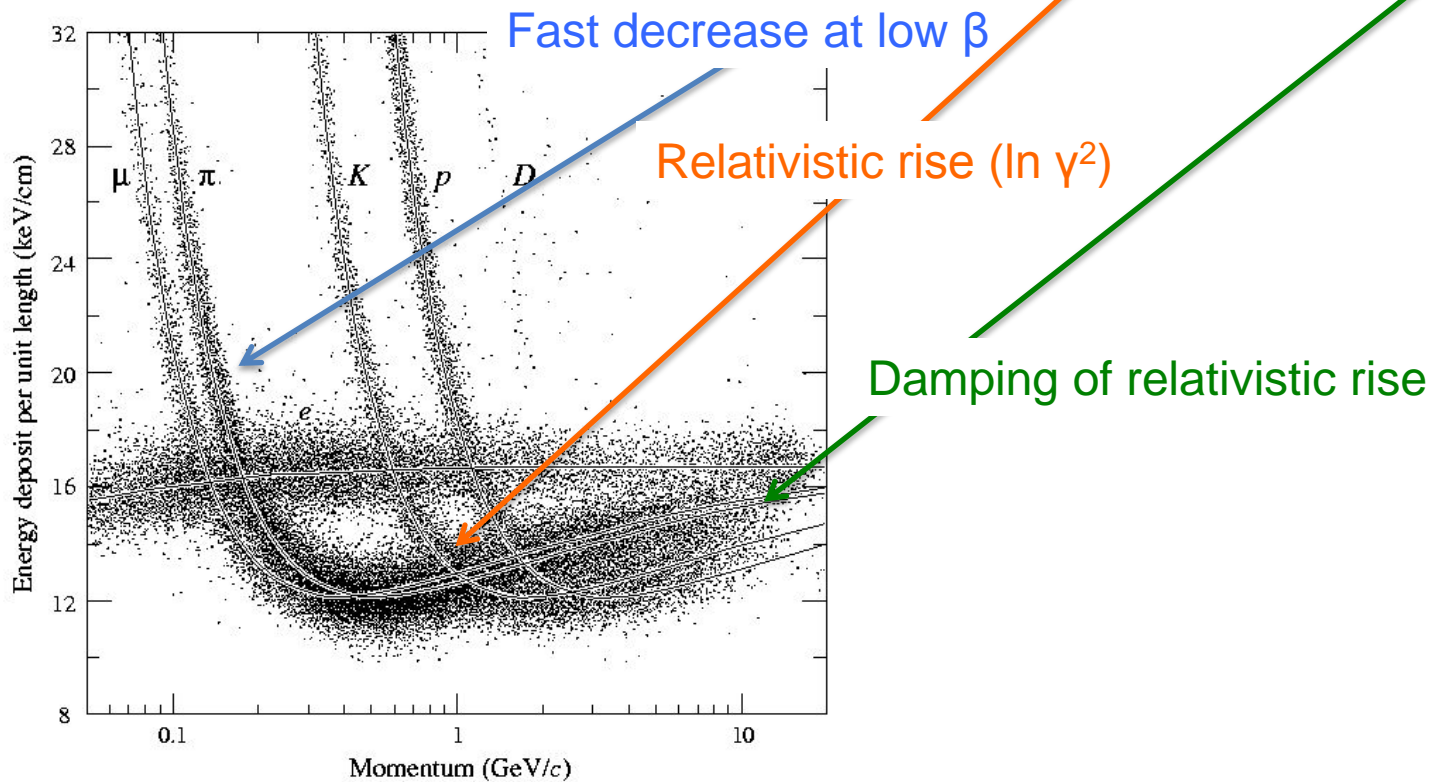
$$-\frac{dE}{dx} \left[\frac{\text{MeV}}{\text{cm}} \right] = 4\rho N_A r_e^2 m_e c^2 z^2 r \frac{Z}{A} \frac{1}{b^2} \left(\ln \frac{2m_e c^2 g^2 b^2}{I} - b^2 - \frac{d}{2} \right)$$



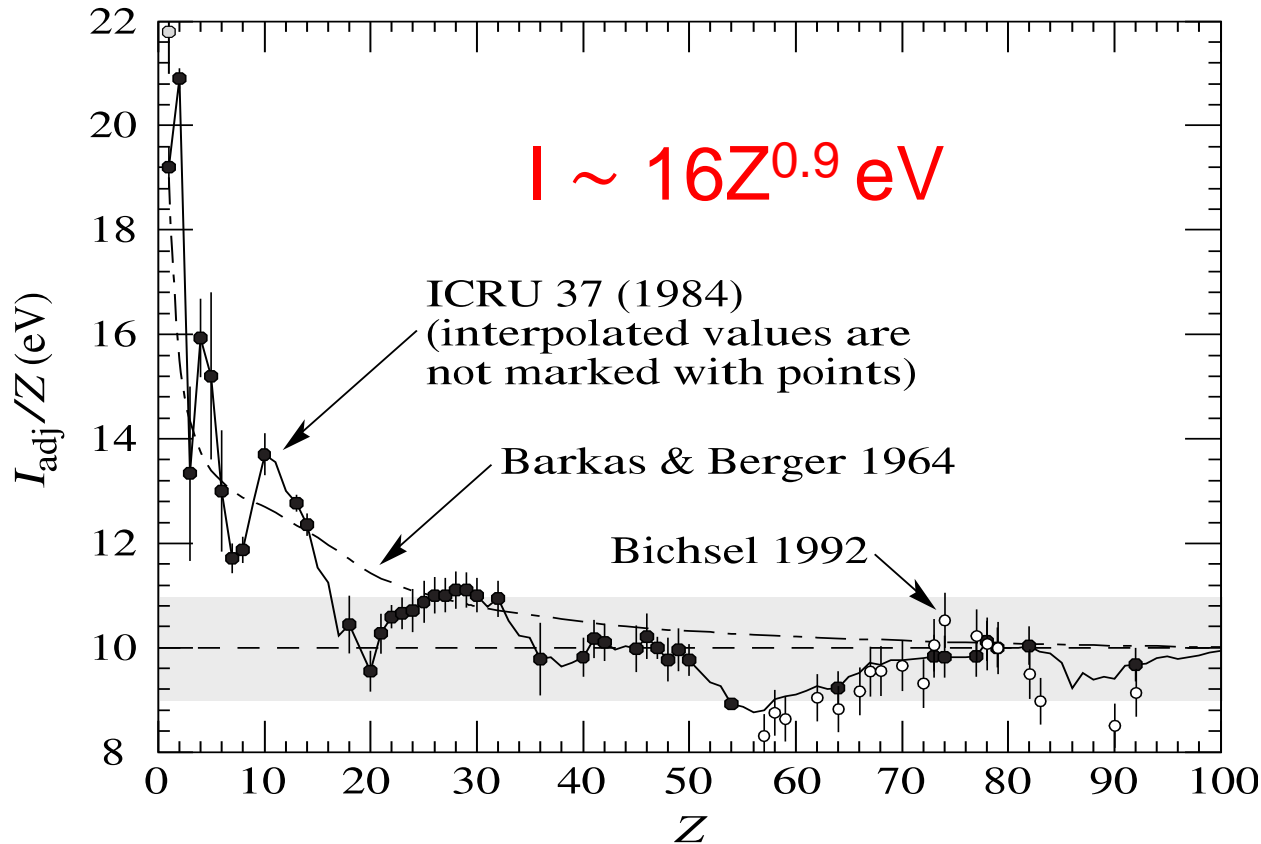
- Energy loss of charged particles per unit length
 - z = charge of the particle
 - Z, A of the absorber
 - I = mean excitation energy of the absorber
 - $4\pi N_A r_e^2 m_e c^2 = D = 0.3071 \text{ MeV}/(\text{g}/\text{cm}^2)$
 - δ describes the E.M. screening effect of the absorber

Bethe-Bloch

$$-\frac{dE}{dx} \left[\frac{\text{MeV}}{\text{cm}} \right] = 4\rho N_A r_e^2 m_e c^2 z^2 r \frac{Z}{A} \frac{1}{b^2} \left(\ln \frac{2m_e c^2 g^2 b^2}{I} - b^2 - \frac{d}{2} \right)$$

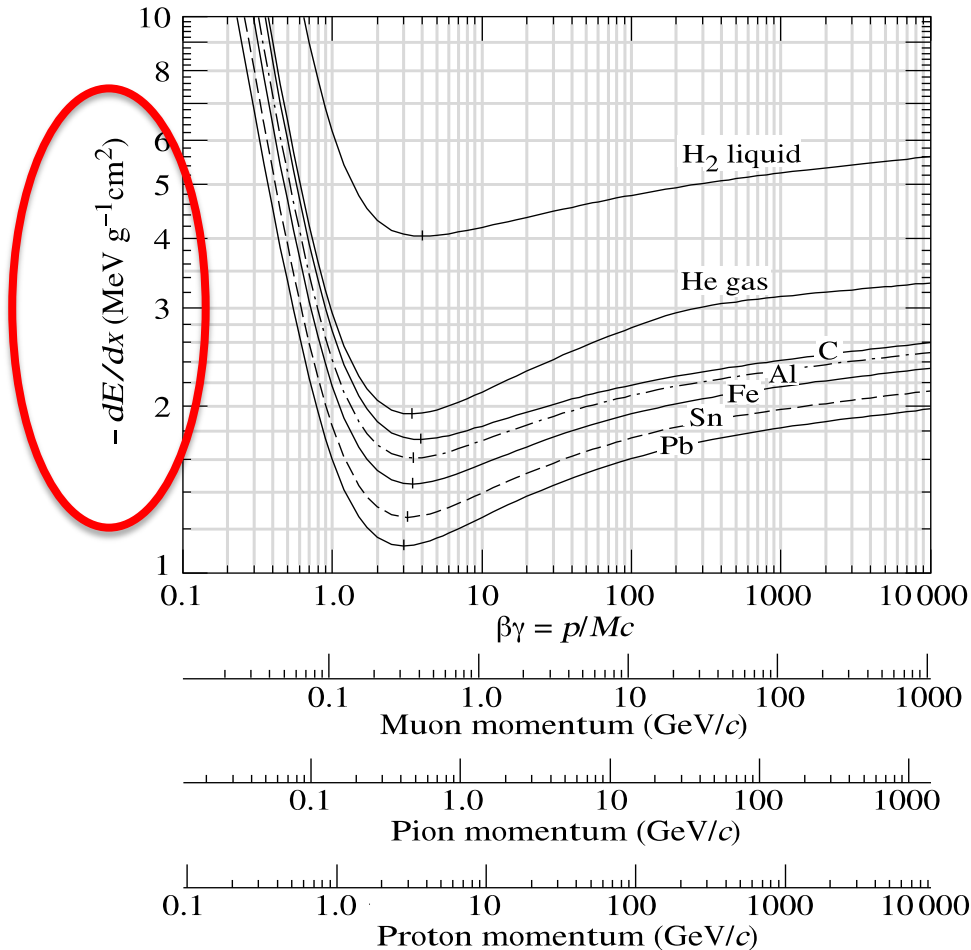


Mean excitation energy: I



- NB for light elements I depends on the phase
 - Atomic hydrogen $I=15\text{eV}$
 - Molecular hydrogen $I=19.2\text{eV}$
 - Liquid hydrogen $I=21.8\text{eV}$

Mass thickness

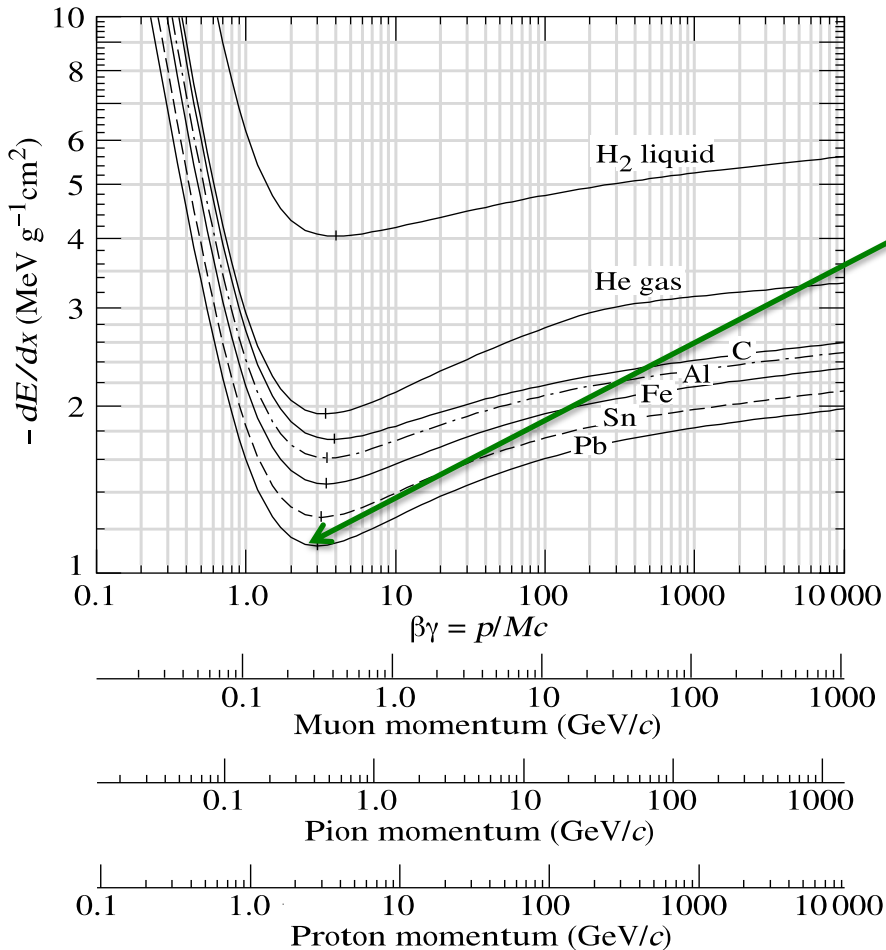


- Useful to express the loss of energy in term of “mass thickness” t

$$-\frac{dE}{dt} \left[\frac{\text{MeV}}{\text{g/cm}^2} \right]$$

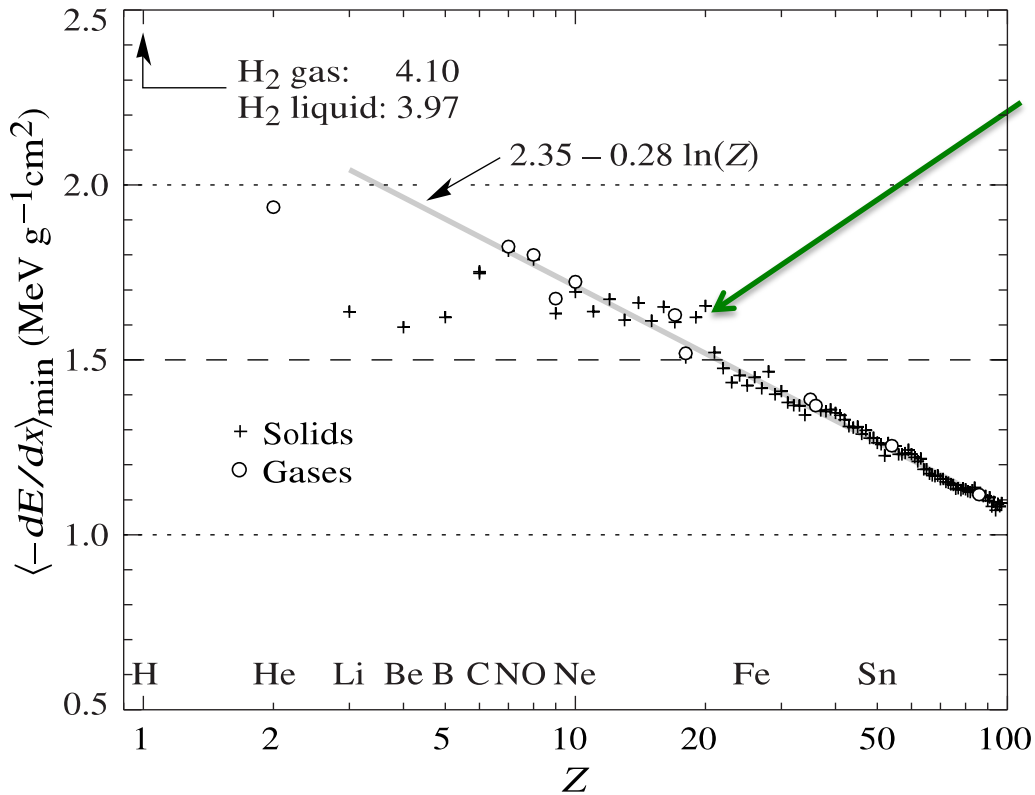
- $dt = p dx$ [g/cm²]
- Look at the units to understand what is used

MIP



- Ionization minimum at $\beta\gamma \sim 4$
 - MIP = minimum ionizing particle (at and above the minimum)
- Dependency on the material is small, apart in the lightest materials
 - in unit of mass thickness, then you have to multiply for the density

dE/dx vs Z



- Notice that, normalized to density, the energy loss at minimum decreases with Z
 - Z/A decreases at high Z

$$-\frac{dE}{dt} = z^2 \frac{Z}{A} f(I)$$

B.B examples

- Argon STP

- 0 °C, 100 kPa: $\rho=1.78 \times 10^{-3} \text{ g/cm}^3$
- $Z=18, A=40, I=16Z^{0.9}=215.7 \text{ eV}$

$$-\frac{dE}{dx} = \frac{0.246 \cdot 10^{-3}}{b^2} \left(\ln \left(8.463 \cdot \frac{b^2}{1-b^2} \right) - b^2 \right) \text{ MeV/cm}$$

- Minimum at $\beta=0.952, \beta\gamma=3.12$
- $dE/dx = 2.66 \text{ keV/cm}$ (1.49 MeV/(g/cm²))
- at $\beta\gamma=100$ increases by 1.54

- Liquid Argon

- $\rho=1.4 \text{ g/cm}^3$
- dE/dx at minimum = 2.09 MeV/cm

- Aluminium

- $\rho=2.7 \text{ g/cm}^3, Z=13, A=27, I=16Z^{0.9}=160.9 \text{ eV}$
- Minimum at $\beta=0.954, \beta\gamma=3.175$
- $dE/dx = 4.47 \text{ MeV/cm}$ (1.65 MeV/(g/cm²)) at minimum

- Liquid hydrogen

- $\rho=0.07 \text{ g/cm}^3, Z=1, A=1, I=21.8 \text{ eV}$
- Minimum at $\beta=0.962, \beta\gamma=3.504$
- $dE/dx = 0.287 \text{ MeV/cm}$ (4.1 MeV/(g/cm²)) at minimum

dE/dx and detectable energy

- NB the Bethe Bloch equation describes the energy lost by the particle in an absorber, not the signal useful to detect it
 - In a slab of lead, lot of energy is lost but none is detectable
 - Losses by ionization, or atomic excitations, can be detectable in some material
- Let's look at some phenomena useful for direct detection
 - Cherenkov radiation
 - Transition radiation

Cherenkov

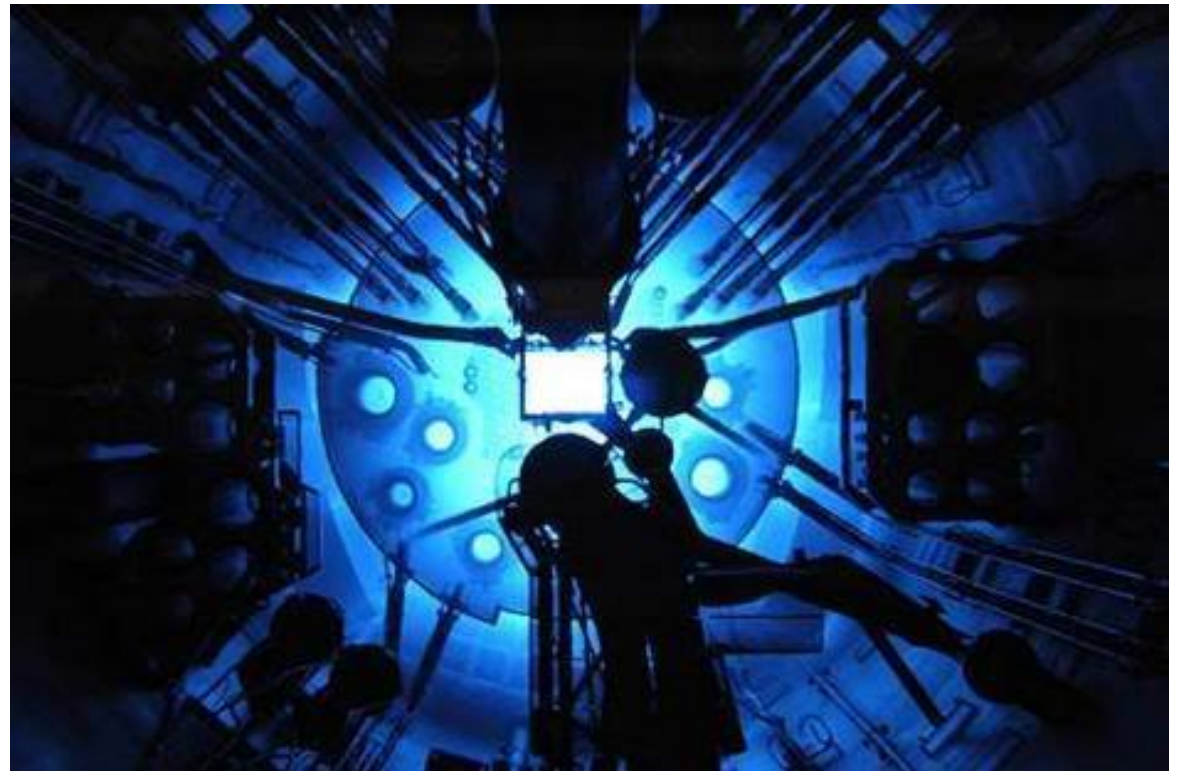


- Electromagnetic shock wave
 - Generated when the speed of the particle in the material is higher than the speed of light

- $\beta c \geq c/n$

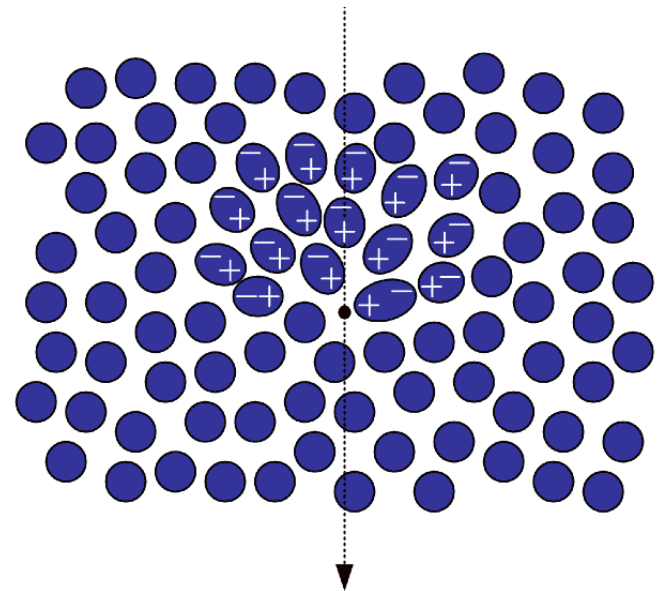
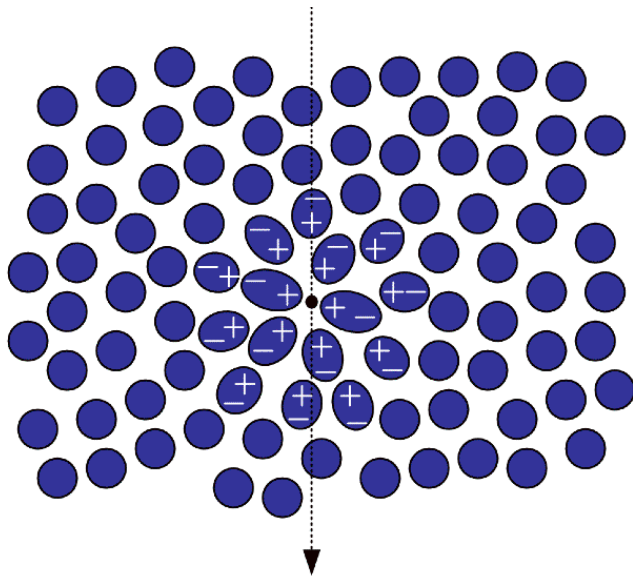
- $\beta \geq 1/n$

- Has a threshold in β , useful to measure it



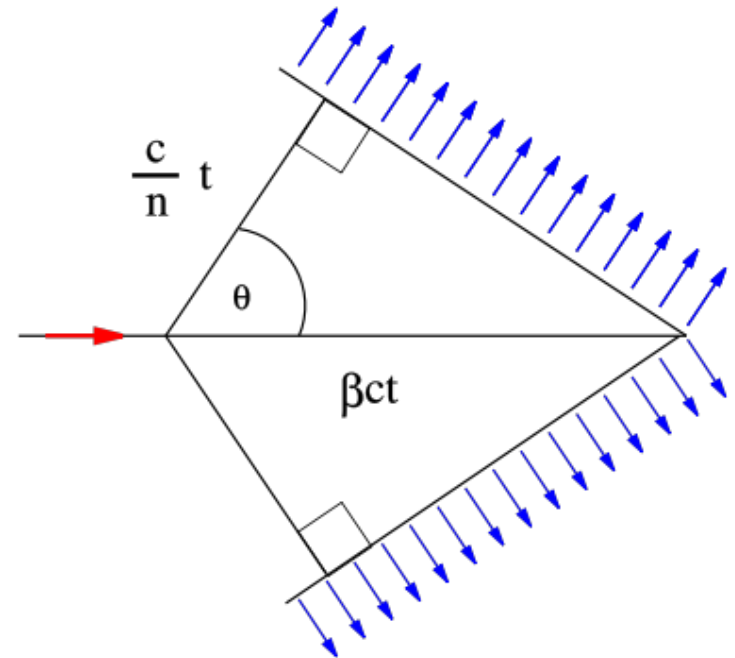
Cherenkov

- $n \approx \sqrt{\epsilon}$ is the refraction index
 - If $v < c/n$ the induced polarization is symmetrical, no radiation emitted
 - If $v > c/n$ the induced polarization is asymmetrical, dipoles emit radiation



Cherenkov

- From the geometry:
 - $\cos\vartheta_c = 1/n(\omega)\beta$
 - The refractive index depend on the frequency of the light (dispersion)
 - The angle itself depend on the frequency
 - The angle increases with β and n
 - glass, $n \approx 1.5$, $\beta \approx 1$ $\vartheta_c \approx 48^\circ$



Cherenkov

- Number of photons emitted:

$$\frac{d^2 N}{dx dl} = \frac{2pz^2 a}{l^2} \left(1 - \frac{1}{n(l)^2 b^2} \right)$$

- Spectrum diverges as $1/\lambda^2$
 - But for small λ , $n \rightarrow 1$, no Cherenkov emission
 - Light tend to be on the blue side
- Visible photon emitted (400÷700nm) in glass

$$\frac{dN}{dx} = 2pz^2 a \left(\frac{1}{l_{\min}} - \frac{1}{l_{\max}} \right) \sin^2 \theta_C$$

- 273 photons/cm. Very small number



Transition radiation

- Transition radiation is produced by relativistic charged particles when they cross the interface of two media of different dielectric constant
 - Very interesting characteristics: the emitted energy is proportional to the Lorentz γ

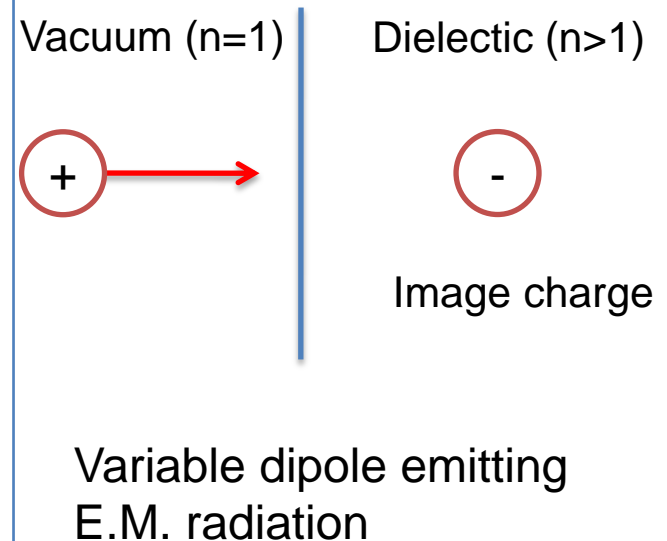
$$E = z^2 \frac{a}{3} \hbar W_P \times g$$

$$W_P = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}, \quad n_e = r N_A \frac{Z}{A}$$

- Very small number of photon emitted per transition

$$N_{TR} \sim \alpha$$

- Photon energy is in the X-ray region (keV)



Bremsstrahlung

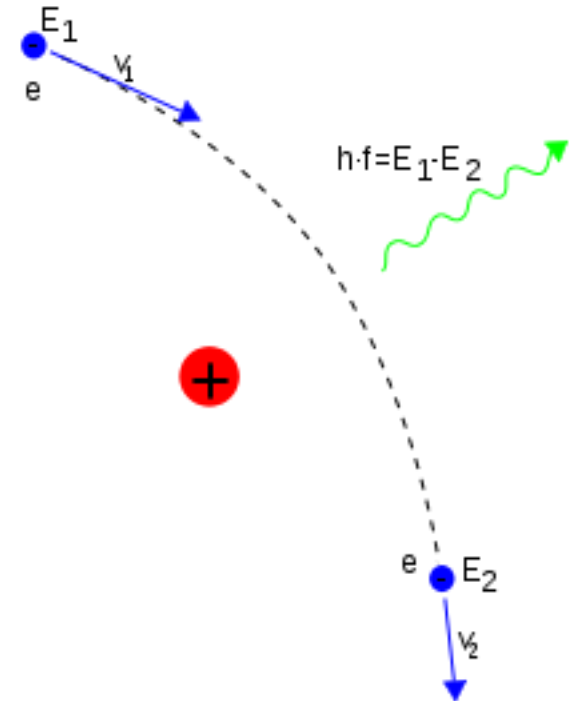
- Radiation emitted by charged particle when decelerated in the field of a nucleus
 - Emission probability is proportional to $1/m^2$ so the effect is typical of electrons

$$S \propto \left(\frac{e^2}{mc^2} \right)^2$$

- For muons, the radiation probability at the same energy is $1/200^2 = 1/40000$
- The energy radiated per unit length is proportional to the energy, and function of the material

$$-\frac{dE}{dx} \gg \frac{E}{X_0} \quad E = E_0 e^{-\frac{x}{X_0}}$$

- So the energy decreases exponentially with x , X_0 is called the “radiation length”



Bremsstrahlung

- An approximated formula for X_0 :

$$\frac{1}{X_0} = 4ar_0^2 N_A \frac{Z^2}{A} r \ln \frac{183}{Z^{1/3}}$$

$$a = \frac{1}{137}$$

$$r_0 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2}$$

- Or, taking into account electrons in the material and Coulomb corrections

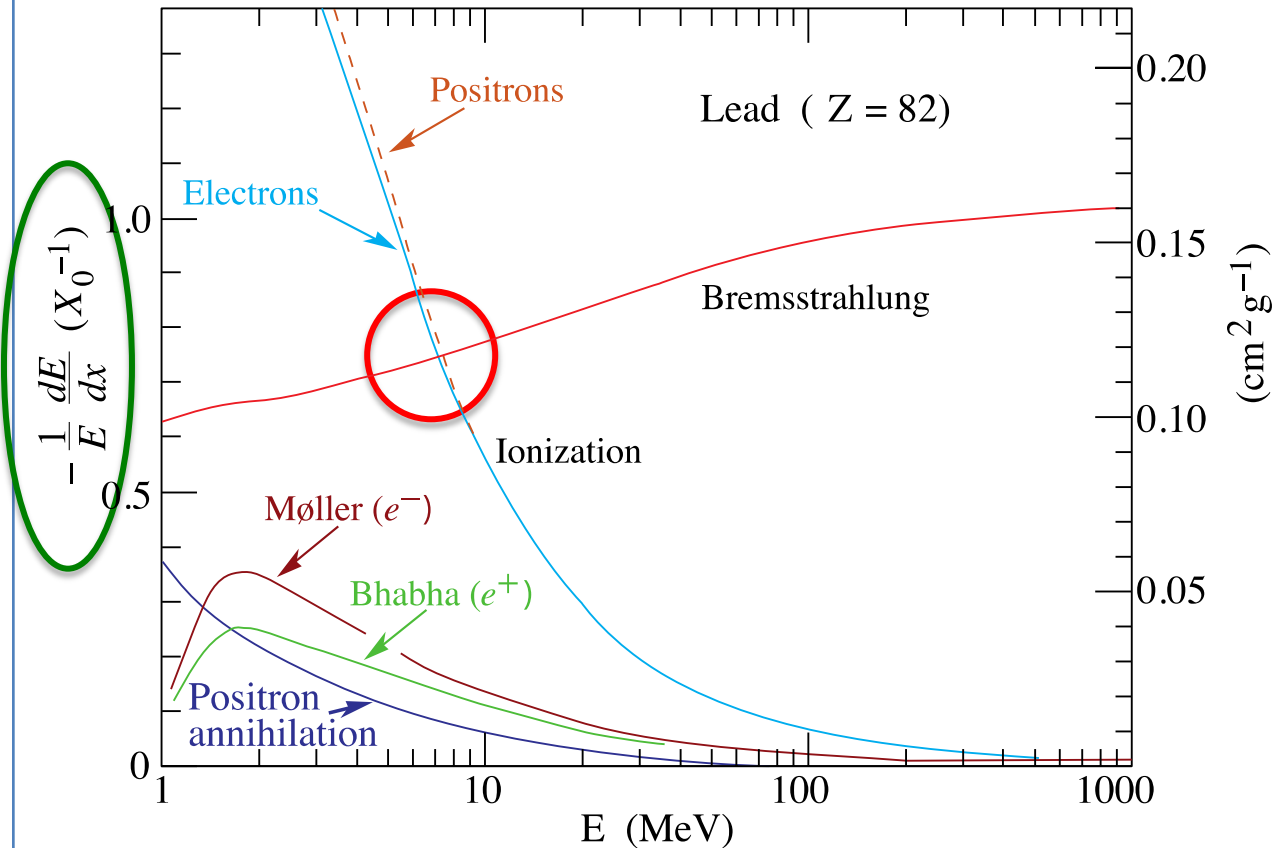
$$\frac{1}{X_0} = 4 \left(Z(Z+1) N_A \frac{r}{A} \right) ar_0^2 \left(\ln \frac{183}{Z^{1/3}} - f(Z) \right)$$

$$F(Z) = a^2 \left(\frac{1}{1+a^2} + 0.202 - 0.036a^2 + 0.008a^4 - 0.002a^6 \right)$$

$$a = Z/137$$

Critical Energy

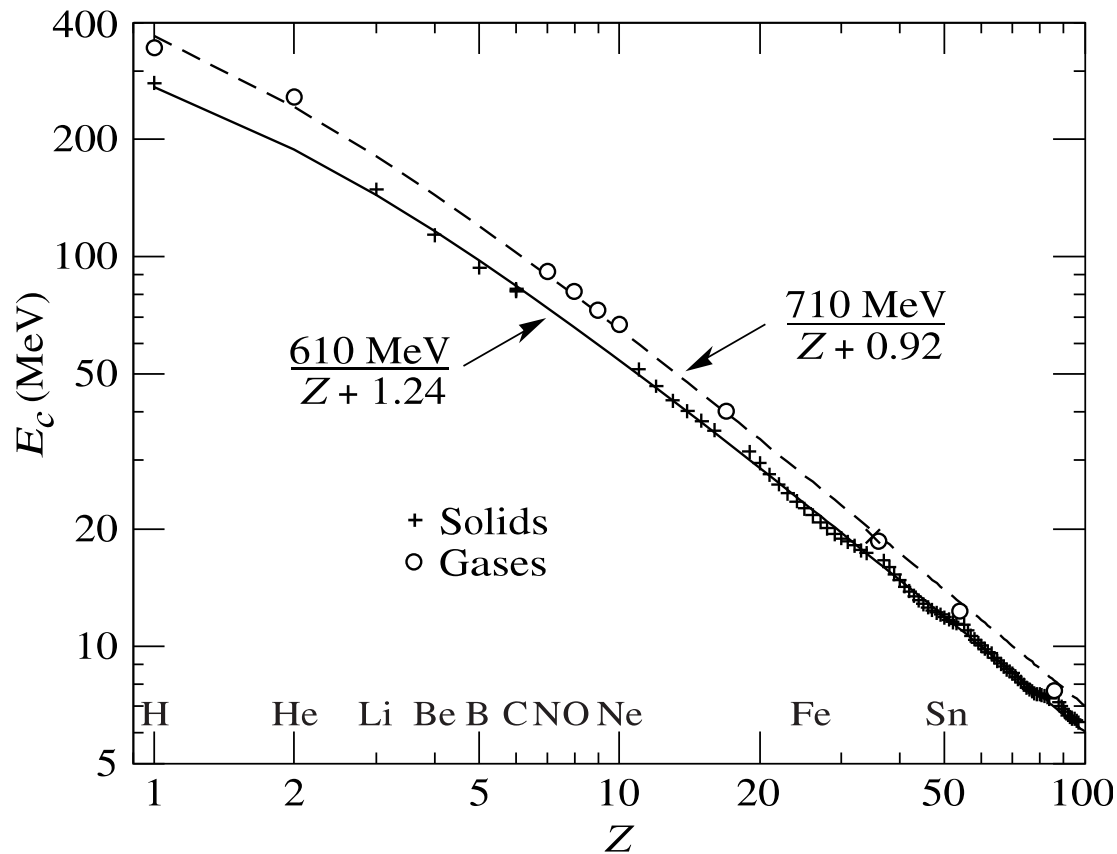
- E_C
 - Energy where the energy lost for bremsstrahlung is equal to that for collisions



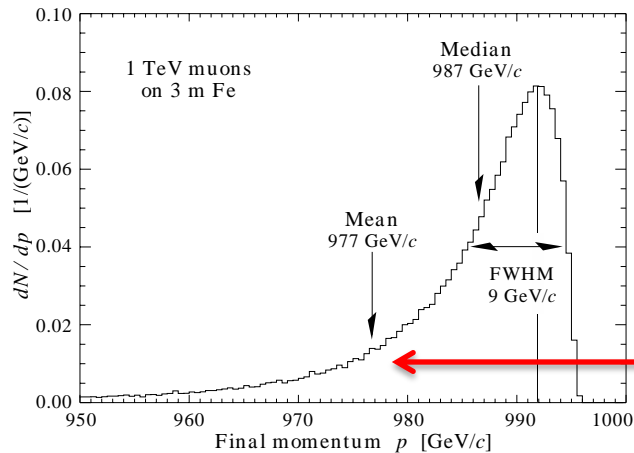
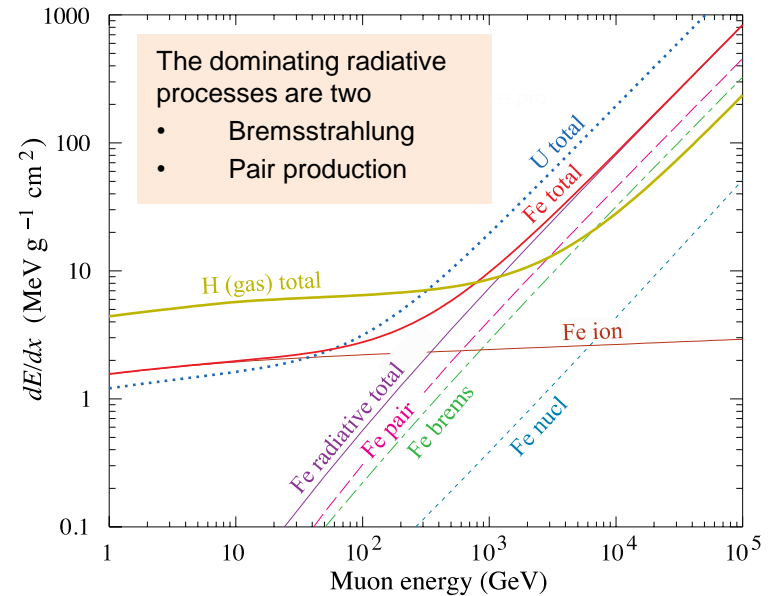
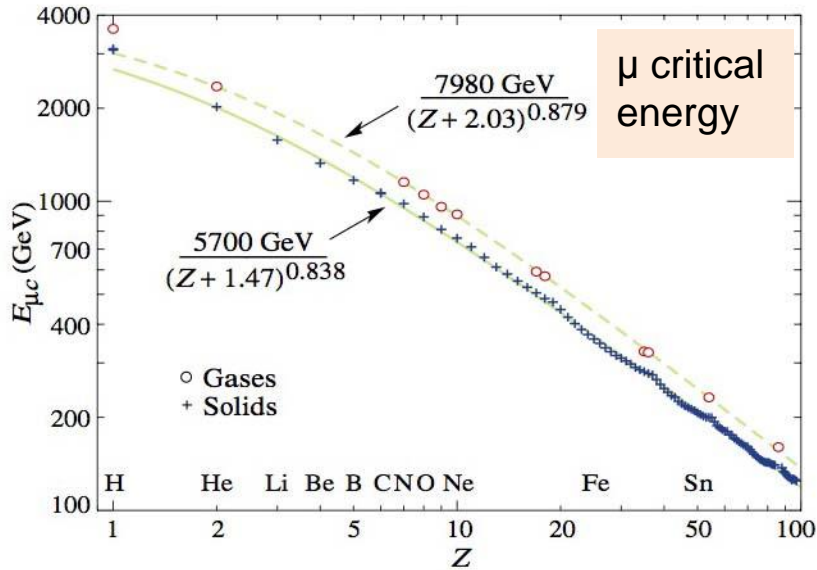
$$\left(\frac{dE}{dx} \right)_{rad} = \left(\frac{dE}{dx} \right)_{coll}$$

Critical Energy

NB the E_C gets smaller at high Z , you lose more energy from bremsstrahlung for longer in heavy materials



Muon radiation losses

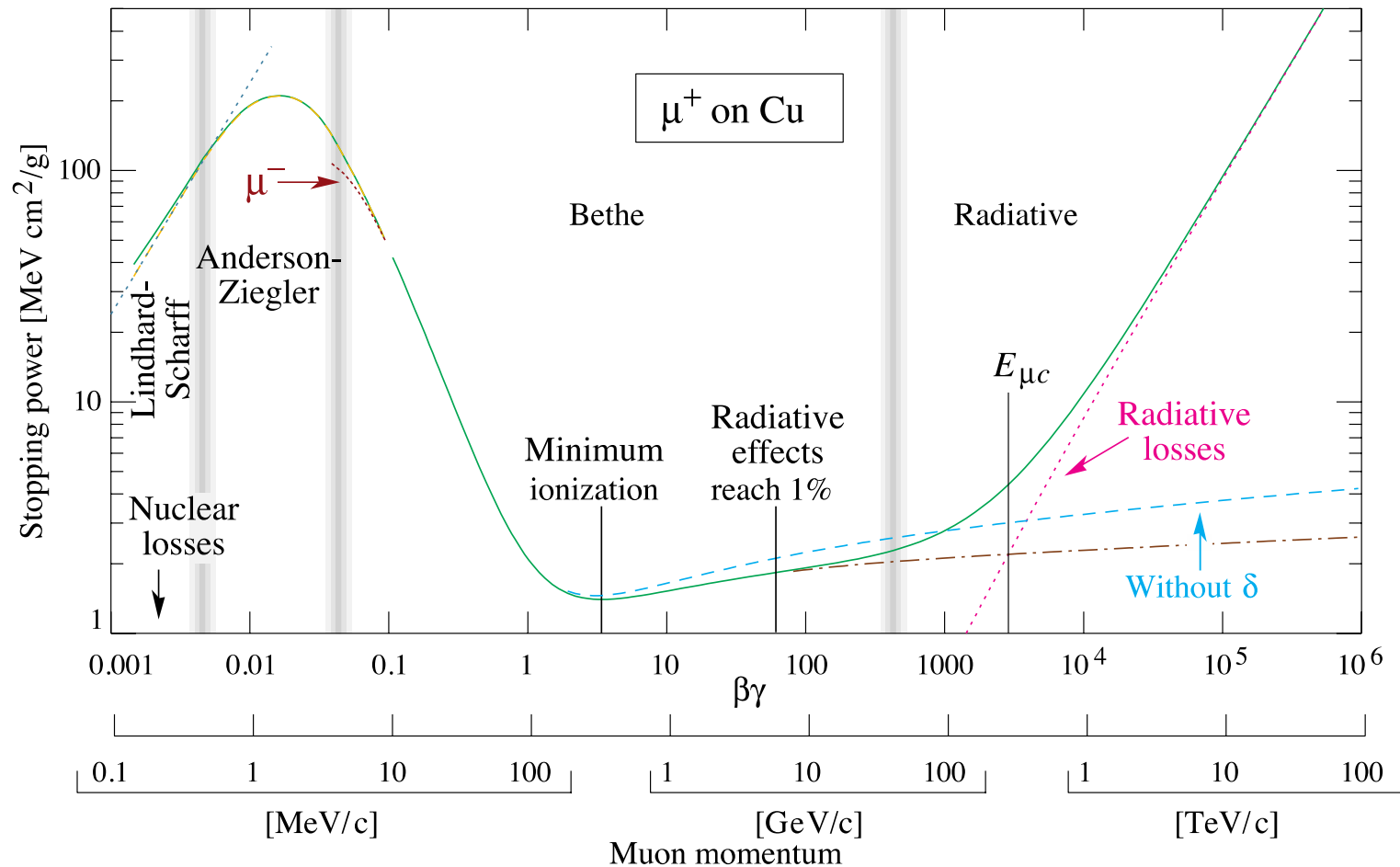


Above few hundreds GeV also muons radiate in heavy absorbers

- Relevant for LHC and cosmic rays
- Expected energy loss from ionization for a 1TeV muon in 3m of Fe less than 5GeV
- Large tails from radiative processes

Summary of energy losses

For charged particle (μ^- through copper in this plot)



Interactions of photon

We cannot talk of energy loss

- Either photons scatter at large angle, or interact losing all its energy

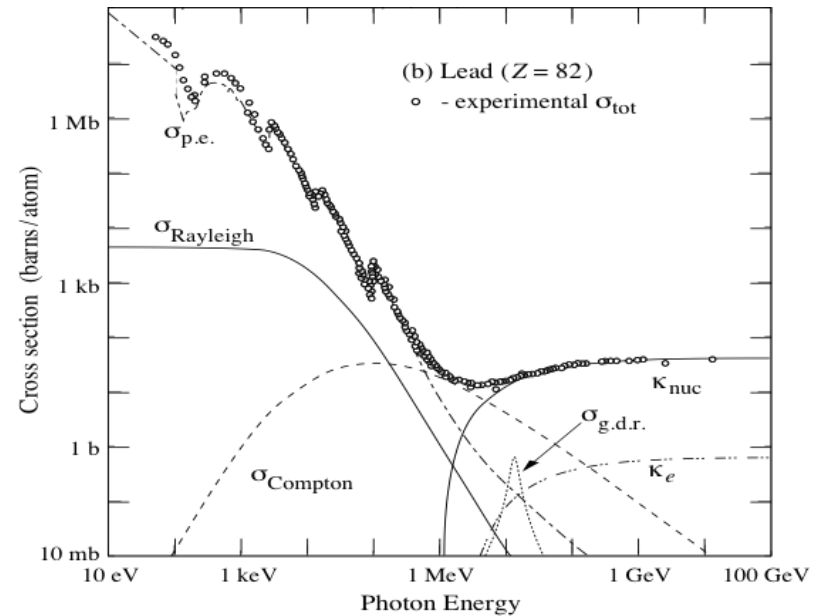
$$-\frac{dI}{dx} = Im$$

$$I(x) = I_0 e^{-mx}$$

$$N_f = \frac{I}{h\nu}$$

- μ = absorption coefficient, measuring the fraction of photon flux lost per unit length
- Three main phenomena for energies >1keV

- | | |
|----------------------------|------------------------|
| • Photoelectric effect | $h\nu \ll m_e c^2$ |
| • Compton scattering | $h\nu \approx m_e c^2$ |
| • e^+e^- pair production | $h\nu \approx m_e c^2$ |



- μ (in cm^{-1} or in cm^2/g) is given by the sum of the different processes:

$$\frac{m}{r} = \frac{N_A}{A} S_{Photo} + Z \frac{N_A}{A} S_{Compton} + \frac{N_A}{A} S_{Pair}$$

- And it depends strongly on the energy of the photon

Photoelectric effect

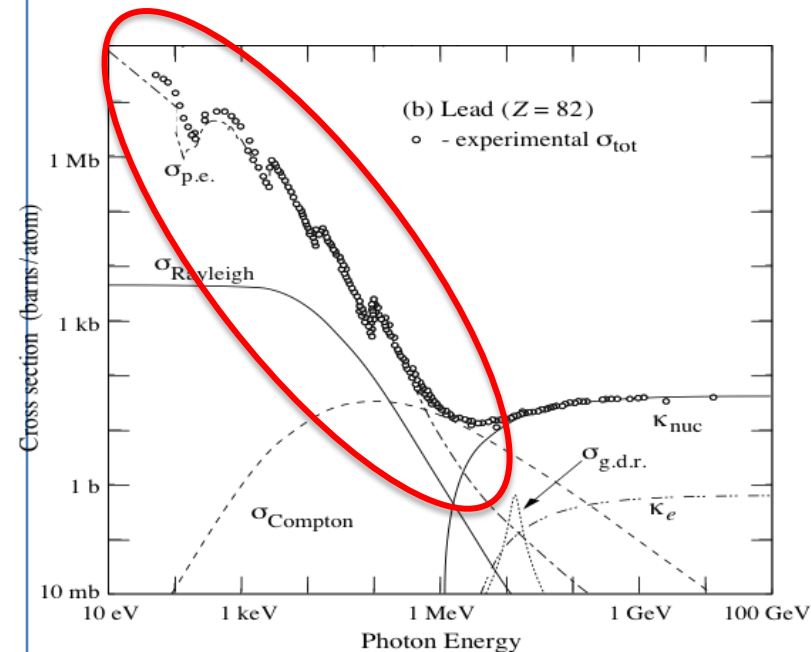
- The energy is absorbed by an atom, which emits an electron
 - For energetic photons the inner levels are interested 1S = K (≈80% of the cross section)
 - Then the rearrangement may generate emissions of X photons or even an (Auger)
- Very strong dependence on energy and on Z

$$S_{photo}^K \approx \sqrt{\left(\frac{32}{e^7}\right)} a^4 Z^5 S_{Th}$$

$$e = \frac{hn}{m_e c^2}$$

$$S_{Th} = \frac{8}{3} \pi r_e^2$$

- Sharp variation close to the atomic levels



Pair production

- $\gamma \rightarrow e^+e^-$
 - There is a threshold energy
 - $h\nu \geq 2m_e c^2 = 1.022 \text{ MeV}$
 - To conserve momentum and energy, it happens with a spectator nucleus

- Approximated cross sections

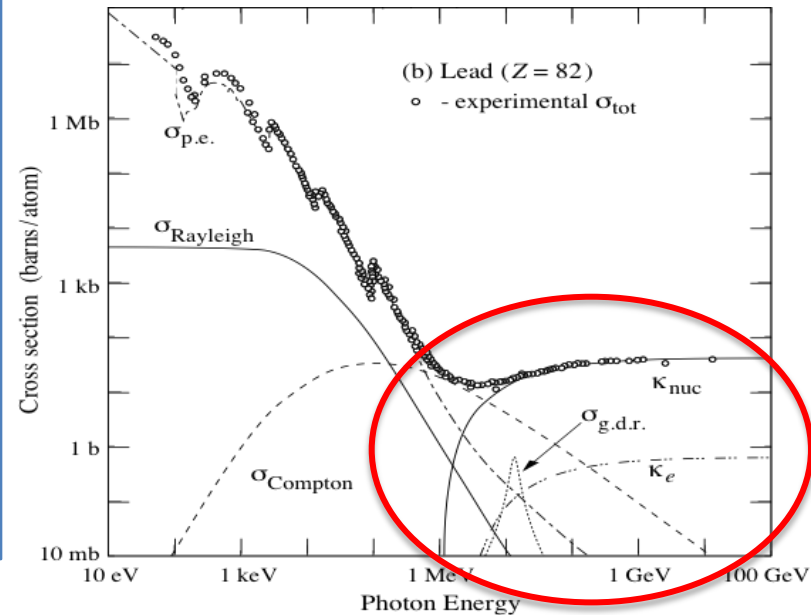
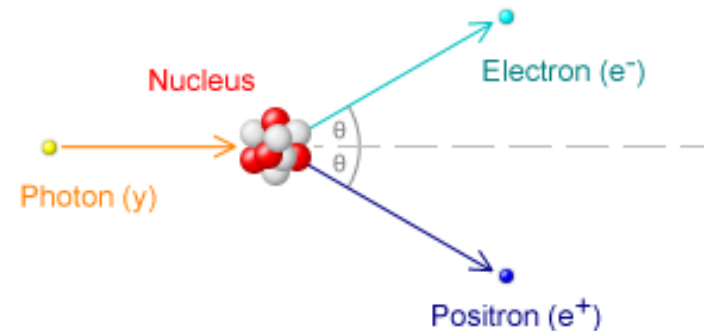
$$\text{for } 2m_e c^2 \ll h\nu \ll \frac{m_e c^2}{a} Z^{-1/3}$$

$$S_{\text{Pair}} = 4Z^2 a r_e^2 \left[\frac{7}{9} \ln \left(\frac{2h\nu}{m_e c^2} - f(z) \right) - \frac{109}{54} \right]$$

$$\text{for } h\nu \gg \frac{m_e c^2}{a} Z^{-1/3}$$

$$S_{\text{Pair}} = 4Z^2 a r_e^2 \left[\frac{7}{9} \ln \left(183Z^{-1/3} - f(z) \right) - \frac{1}{54} \right]$$

- At high energy does not depend on $h\nu$



Pair production

- So for high energy photons

$$m_{Pair} = r \frac{N_A}{A} S_{Pair} = \frac{1}{I_{Pair}}$$

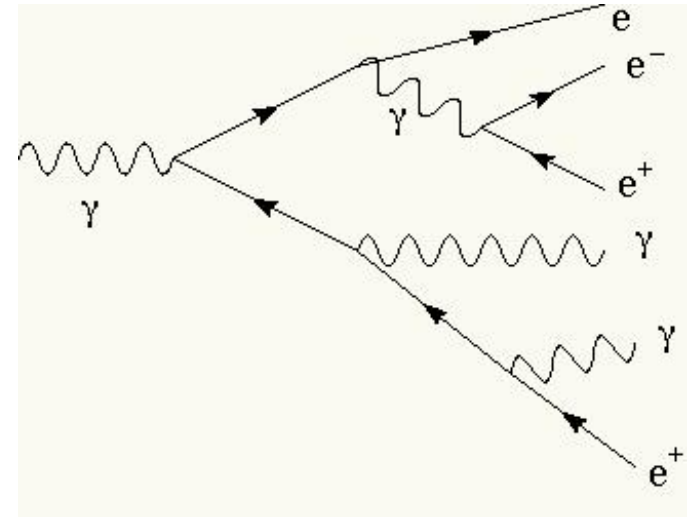
$$\frac{1}{I_{Pair}} \approx \frac{7}{9} 4Z(Z+1) ar_e^2 \left[\ln(183Z^{-1/3}) \right] \approx \frac{7}{9} \frac{1}{X_0}$$

$$I_{Pair} \approx \frac{9}{7} X_0 \approx 1.3 X_0$$

$$I = I_0 e^{-\frac{x}{1.3 X_0}}$$

- Very similar to the energy loss of electrons for bremsstrahlung

$$E = E_0 e^{-\frac{x}{X_0}}$$

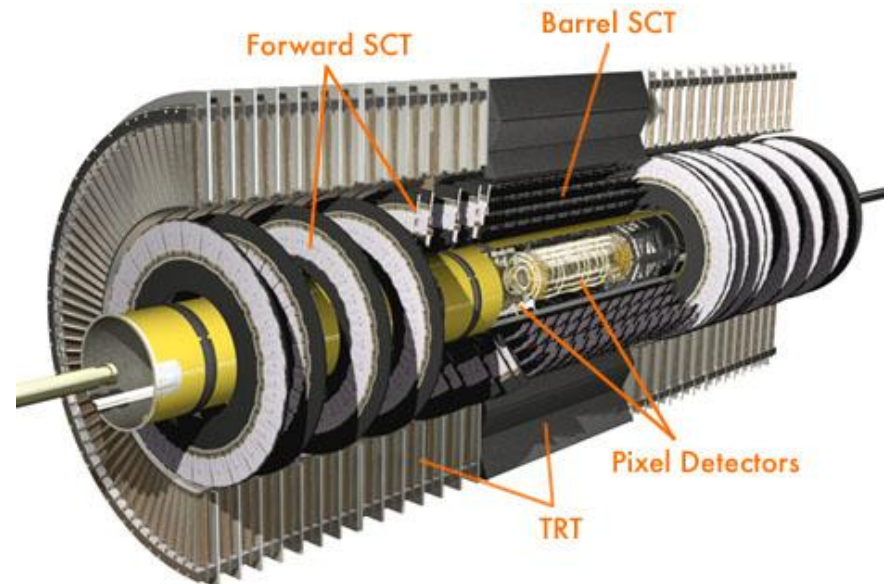


- **Electromagnetic shower**
 - Combined process of bremsstrahlung and pair production
 - Will come back to that when discussing the calorimeters

TRACKING DETECTORS

Essential multi-purpose detectors

- Measurements of:
 - tracks momentum
 - From deflection in magnetic field
 - Event topology
 - Primary and secondary vertexes
 - dE/dx
 - trigger



Momentum measurement

- Bending in magnetic field
 - constant, orthogonal to the velocity

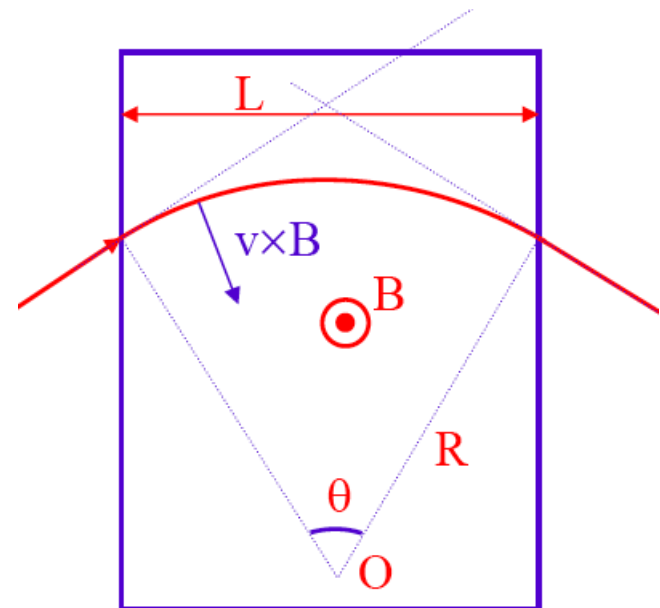
$$R = \frac{p}{qB}$$

if p is in GeV/c q is unit charge, B in Tesla and R in meters

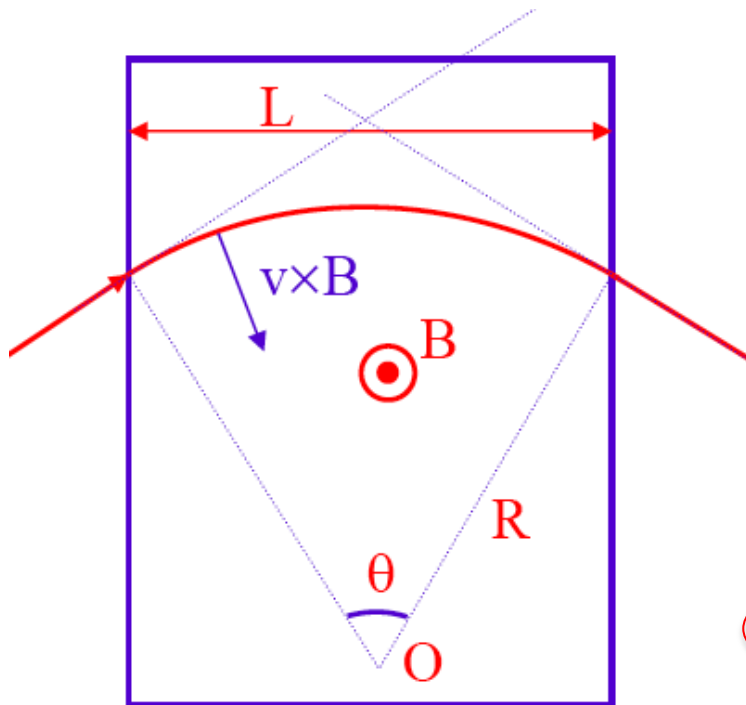
$$p = p [GeV/c] \cdot \frac{10^9 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$$R = \frac{p \times \frac{10^9 \times 1.6 \times 10^{-19}}{3 \times 10^8}}{1.6 \times 10^{-19} \times B} = \frac{10}{3} \frac{p}{B}$$

$$p \gg 0.3RB$$



Momentum measurement



$L \gg Rq$ for small angles

$$q = \frac{L}{R} = 0.3 \frac{BL}{p}$$

$$q = \frac{0.3}{p} \int B dl \text{ (if B is not uniform)}$$

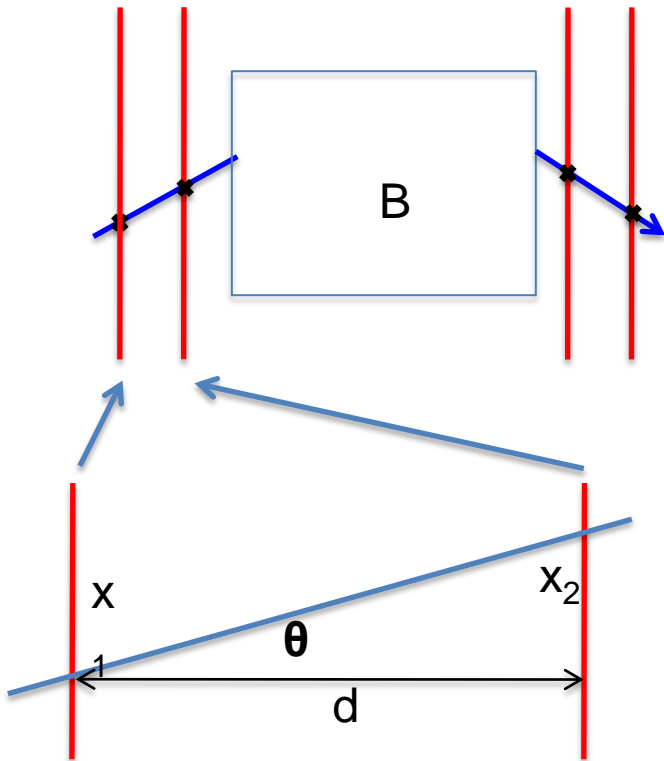
$$p = \frac{0.3}{q} \int B dl$$

$$\frac{s(p)}{p} = \frac{s(q)}{q} = \frac{p}{0.3 \int B dl} s(q)$$

- p can be derived from the bending angle
- Given the error on the angle, $\sigma(p)/p$ increases linearly with p

Measuring the deflection

- To measure the bending we need two directions
 - At least two precise points before and after the magnet



$$q \gg \frac{x_2 - x_1}{d}$$

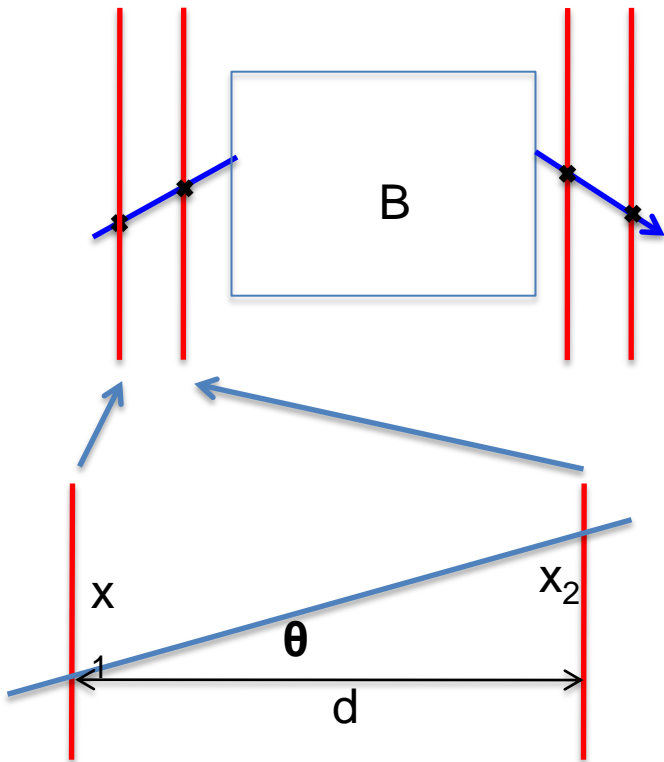
$$s(q) = \frac{1}{d} \sqrt{s^2(x_1) + s^2(x_2)} = \frac{\sqrt{2}}{d} s(x)$$

$$q_{\text{bending}} = q_1 - q_2$$

$$s(q_{\text{bending}}) = \sqrt{2} s(q) = \frac{2}{d} s(x)$$

$$\frac{s(p)}{p} = \frac{p}{0.3 \dot{\theta} B d l} s(q) = \frac{2p}{0.3 d \dot{\theta} B d l} s(x)$$

Example



$$\frac{S(p)}{p} = \frac{p}{0.3 \int B dl} S(q) = \frac{2p}{0.3 d \int B dl} S(x)$$

$$\int B dl = 1 Tm \quad d = 1 m \quad S(x) = 200 mm$$

$$\frac{S(p)}{p} = 1.3 \cdot 10^{-3} p \quad \text{with } p \text{ in } GeV/c$$

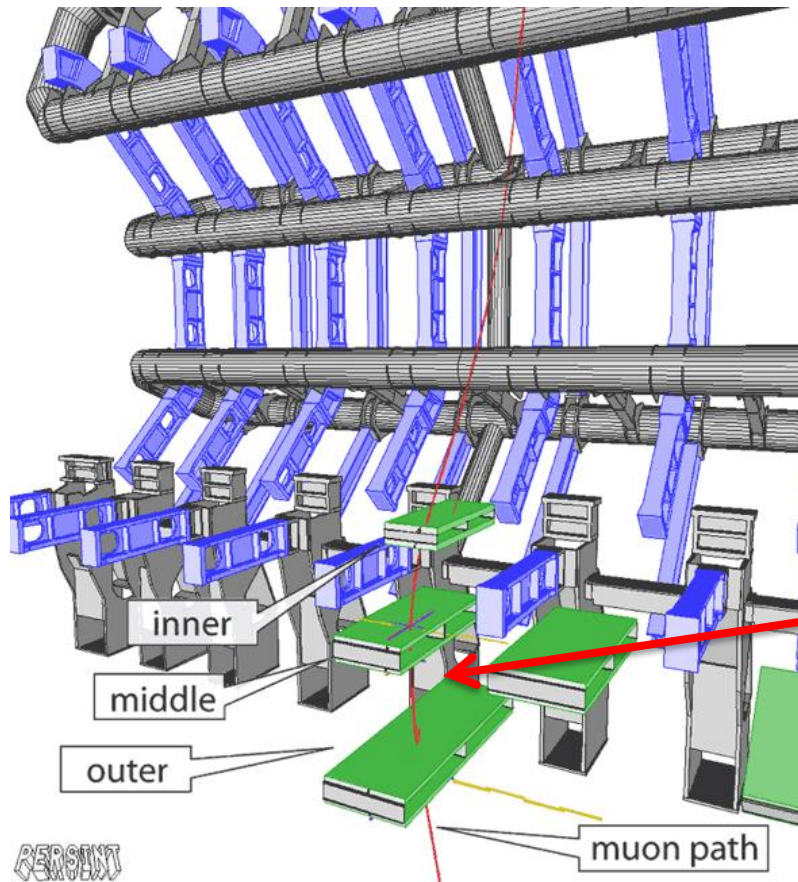
$$p = 1 GeV/c \rightarrow S(p)/p = 1.3 \cdot 10^{-3} \approx 0.1\%$$

$$p = 10 GeV/c \rightarrow S(p)/p = 1.3 \cdot 10^{-2} \approx 1\%$$

$$p = 100 GeV/c \rightarrow S(p)/p = 1.3 \cdot 10^{-1} \approx 10\%$$

Use of bending measurement

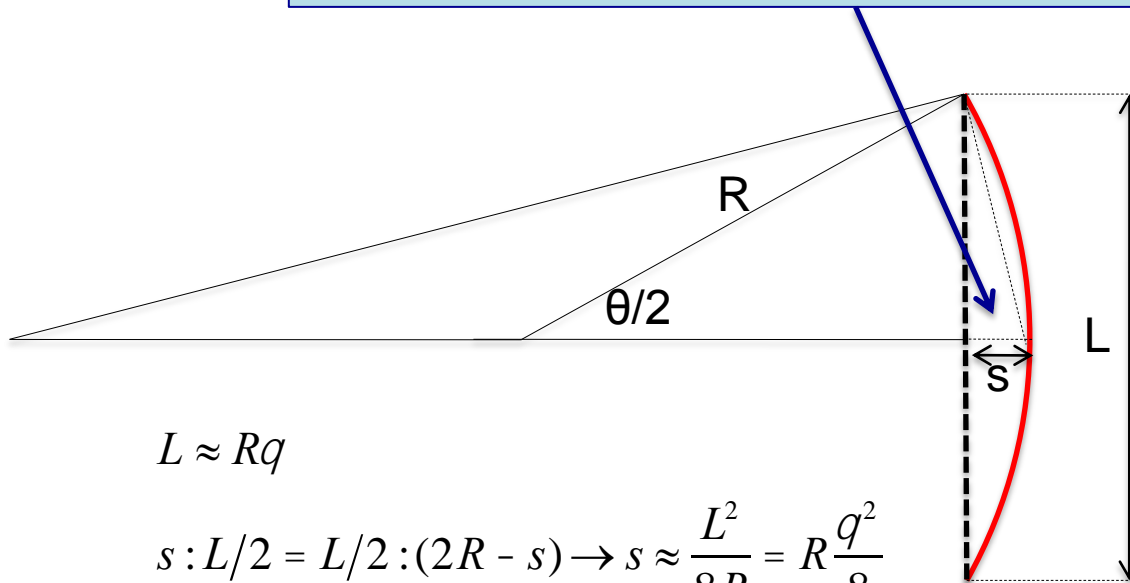
- Bending typically used to measure p for



- Beams
- Fixed target experiments
- Muons
 - bending in magnetized iron
 - or even in air like in Atlas

Sagitta

The track is measured inside the magnetic field



$$L \approx Rq$$

$$s : L/2 = L/2 : (2R - s) \rightarrow s \approx \frac{L^2}{8R} = R \frac{q^2}{8}$$

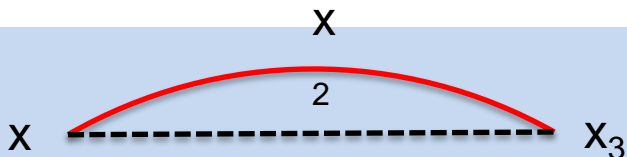
$$s = 0.3 \frac{BL^2}{8p} \quad (\text{if } B \text{ uniform, } p \text{ in GeV}/c)$$

$$\frac{S(p)}{p} = \frac{S(s)}{s} = \frac{8S(s)}{0.3BL^2} p$$

- Again $\sigma(p)/p$ grows with p
- The precision can be improved by
 - Improving the precision of the position
 - Also measuring in more points
 - Increasing B
 - Increasing the lever arm

Sagitta

- Need at least 3 points to make the measurement



$$s = x_2 - \frac{x_1 + x_3}{2}$$

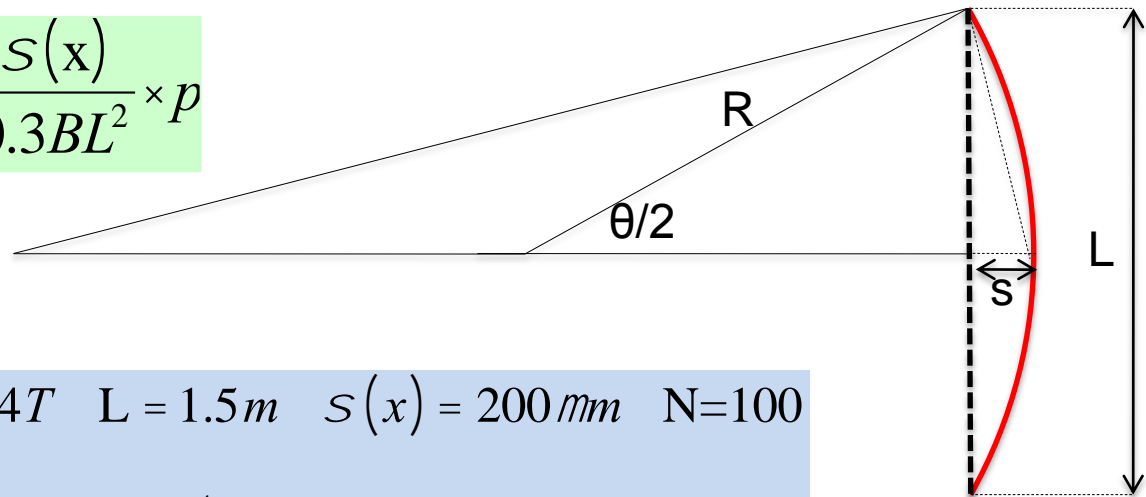
$$S(s) = \sqrt{\frac{3}{2}} S(x) \rightarrow \frac{S(p)}{p} = \frac{S(s)}{s} = \frac{8\sqrt{\frac{3}{2}} S(x)}{0.3BL^2} p$$

- With many points one gets to the following

$$\frac{S(p)}{p} = \sqrt{720/(N+4)} \frac{S(x)}{0.3BL^2} \times p$$

Example

$$\frac{S(p)}{p} = \sqrt{720/(N+4)} \frac{S(x)}{0.3BL^2} \times p$$



$$B = 1.4T \quad L = 1.5m \quad S(x) = 200mm \quad N=100$$

$$\frac{S(p)}{p} = 5.6 \cdot 10^{-4} p \quad \text{whith } p \text{ in } GeV/c$$

$$p = 2GeV/c \rightarrow S(p)/p = 1.1 \cdot 10^{-3} \approx 0.1\%$$

$$p = 20GeV/c \rightarrow S(p)/p = 1.1 \cdot 10^{-2} \approx 1\%$$

$$p = 200GeV/c \rightarrow S(p)/p = 1.1 \cdot 10^{-1} \approx 10\%$$

$$S(p)/p = 100\% \rightarrow p \approx 2TeV$$

Detectors for tracking

- The requirements are clear
 - Be able to measure with high precisions the charged track positions
 - Most times in magnetic field
 - Other possible requirements
 - Minimize dead material (see later the multiple scattering)
 - Linearity if used to measure also dE/dx
- Two main classes of detectors
 - Gas detectors
 - Multi-wire chambers, drift chambers, limited streamer tubes, resistive plate chambers, GEMs ...
 - Semiconductor detectors
 - SI strips, Pixels

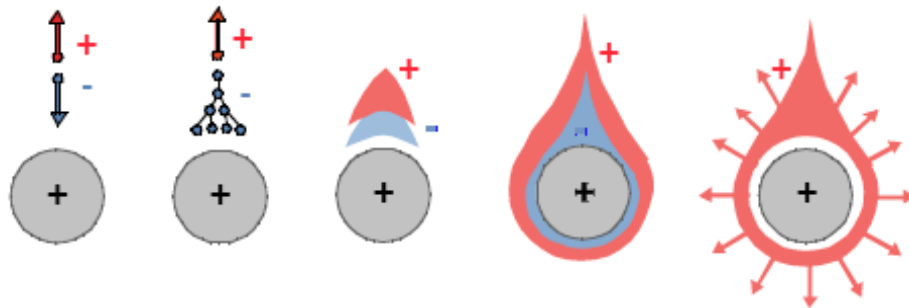
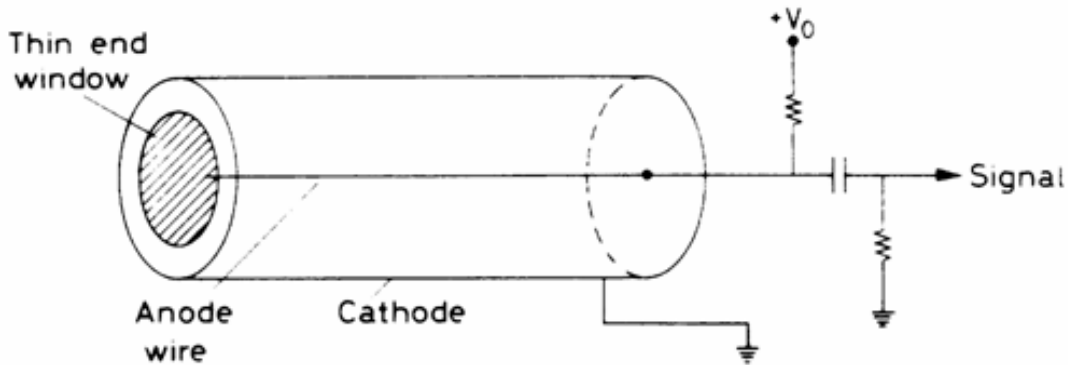
Gas detectors

- Some of the energy lost by a charged particle ionizes the gas
 - Primary ionization
 - The charged particle extracts an electron from an atom
 - Secondary ionization
 - The extracted electron is energetic enough to further ionize the gas
 - W measures the ratio between the energy lost by the particle and the number of ions produced
 - For instances, a MIP produces about 100 ion per cm of Ar at STP

- With an electric field, the electrons and ions can be made drift, to be collected by the electrodes
- The signal is very small
 - $100 e = 1.6 \times 10^{-2} \text{ fC}$
 - Too small even for modern amplifier
- Need a mechanism to amplify the signal in the gas
 - High electric field, avalanche ionization

	H ₂	He	Ar	CH ₄
pot. ion. (eV)	15.4	24.6	15.8	13.1
W (eV)	36.6	41.3	26.4	27.3
dE/dx (keV/cm)	0.34	0.32	2.44	1.48

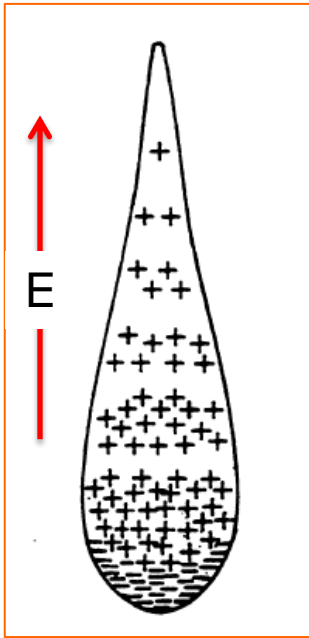
Wire chambers



- **Basic mechanism**

- The anode (+) is a thin wire
- The field between cathode and anode make the electron drift to the wire
- Close to the wire, the field grows as $1/r$ and it becomes high enough to generate an avalanche
- Most of the charge is generated in the last steps around the wire

Avalanche



- Basic mechanism

- An electron from an ionization gets accelerated in the E field and quickly reaches an energy enough to further ionize the gas
 - Max probability to ionize is around 100 eV
- Every mean free path for ionization λ_i the number of electrons doubles
 - $1/\lambda_i$ is called “first Townsend coefficient” α
 - The drift velocity of ions is very small w.r.t. that of the electrons, the ion cloud is left behind
- Gain factor M

$$dn = n \alpha dx$$

$$n = n_0 e^{\alpha x} \rightarrow M = \frac{n}{n_0} = e^{\alpha x}$$

$$M = e^{\int_{x_1}^{x_2} \alpha(x) dx} \quad \text{where E changes}$$

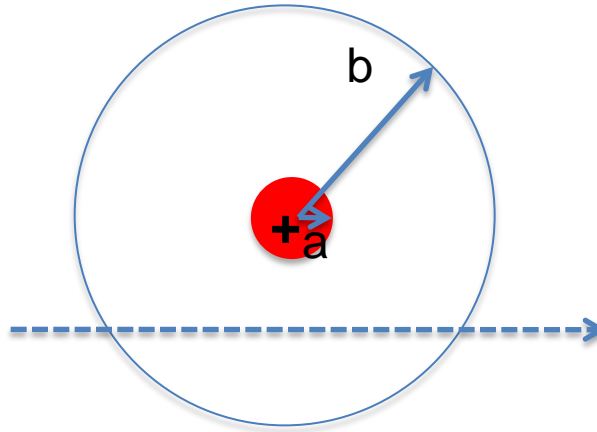
- At too high gains, there is a total discharge in the gas
 - Caused by photon emitted by the excited atoms that ionize elsewhere the gas
 - Gain limit depend mostly on the gas mixture

Wire chambers

$$C' = \frac{2pe}{\ln b/a} \quad \text{capacity per meter}$$

$$E(r) = \frac{C'V_0}{2pe_0} \frac{1}{r} = \frac{V_0}{\ln b/a} \frac{1}{r}$$

$$U(r) = -\frac{C'V_0}{2pe_0} \ln \frac{r}{a} = -\frac{V_0}{\ln b/a} \ln \frac{r}{a}$$



example

$$a = 20 \mu\text{m}, \quad b = 5 \text{mm}, \quad V_0 = 2 \text{kV}$$

$$\rightarrow C' = 10 \text{ pF/m}$$

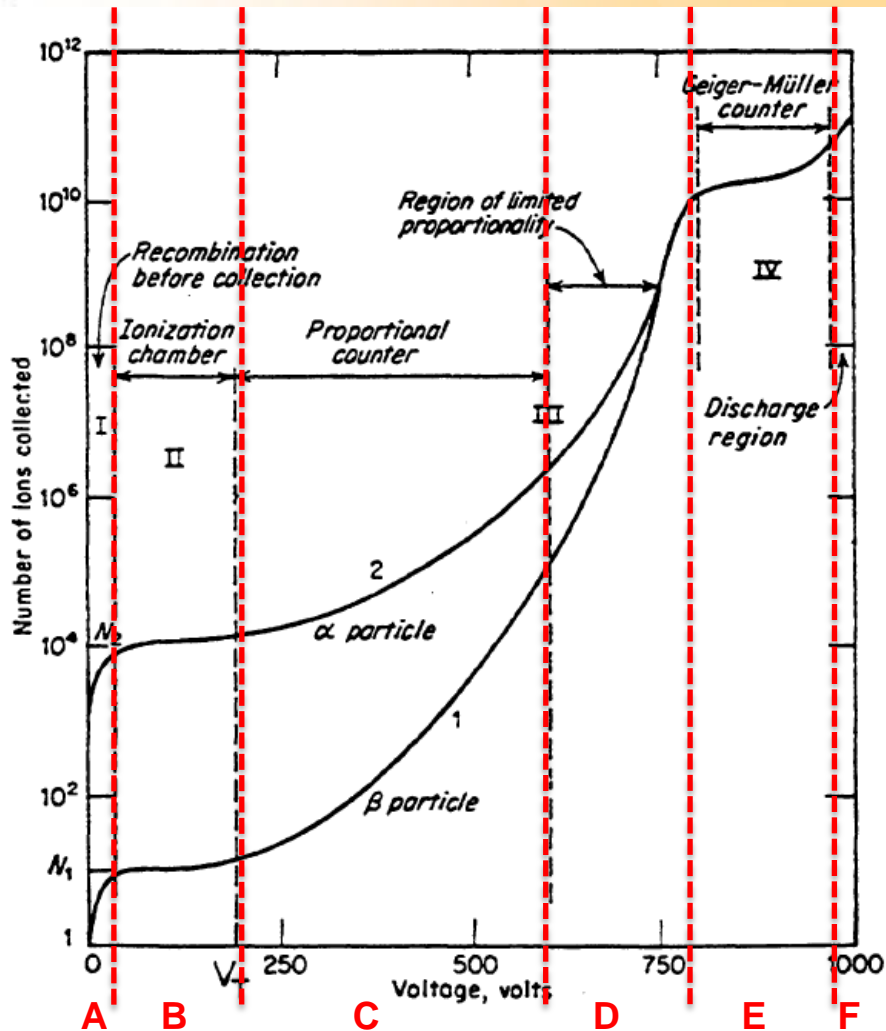
$$\rightarrow E(a) \sim 2 \cdot 10^7 \text{ V/m}$$

- The multiplication gain from the avalanche can be approximated to

$$M = \text{const} \times e^{CV_0}$$

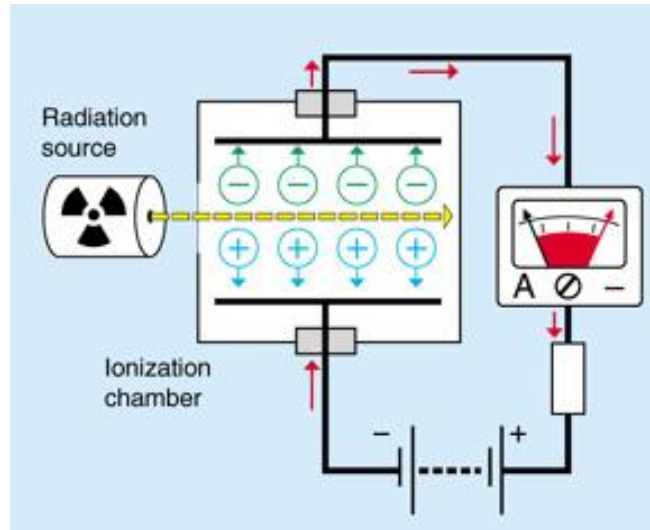
- Grows exponentially with V_0
- The constant depends on the gas

Wire chambers

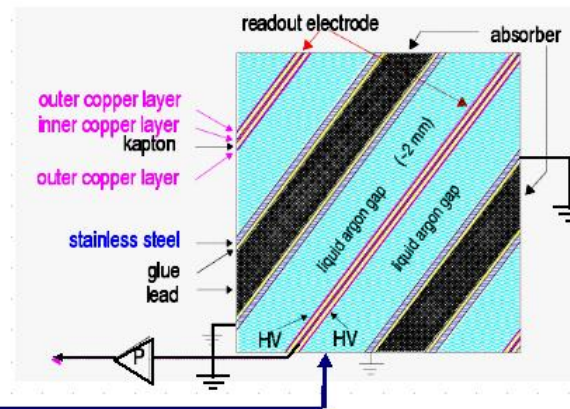
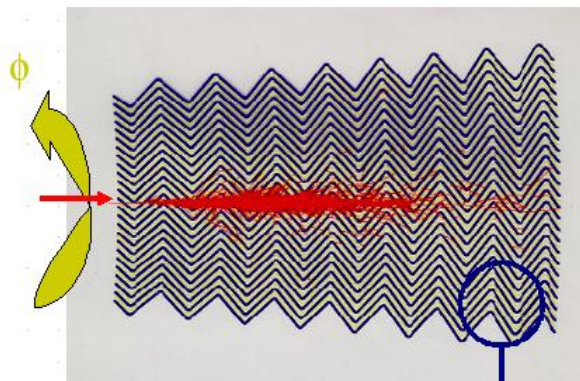


- Amplification regimes
 - Electric field is not enough to collect all the charge, e-ions will recombine
 - The charge is collected without gain (ionization chamber)
 - Gain is modest ($M \leq 10^5$) and the collected charge is proportional to the initial signal (proportional chamber)

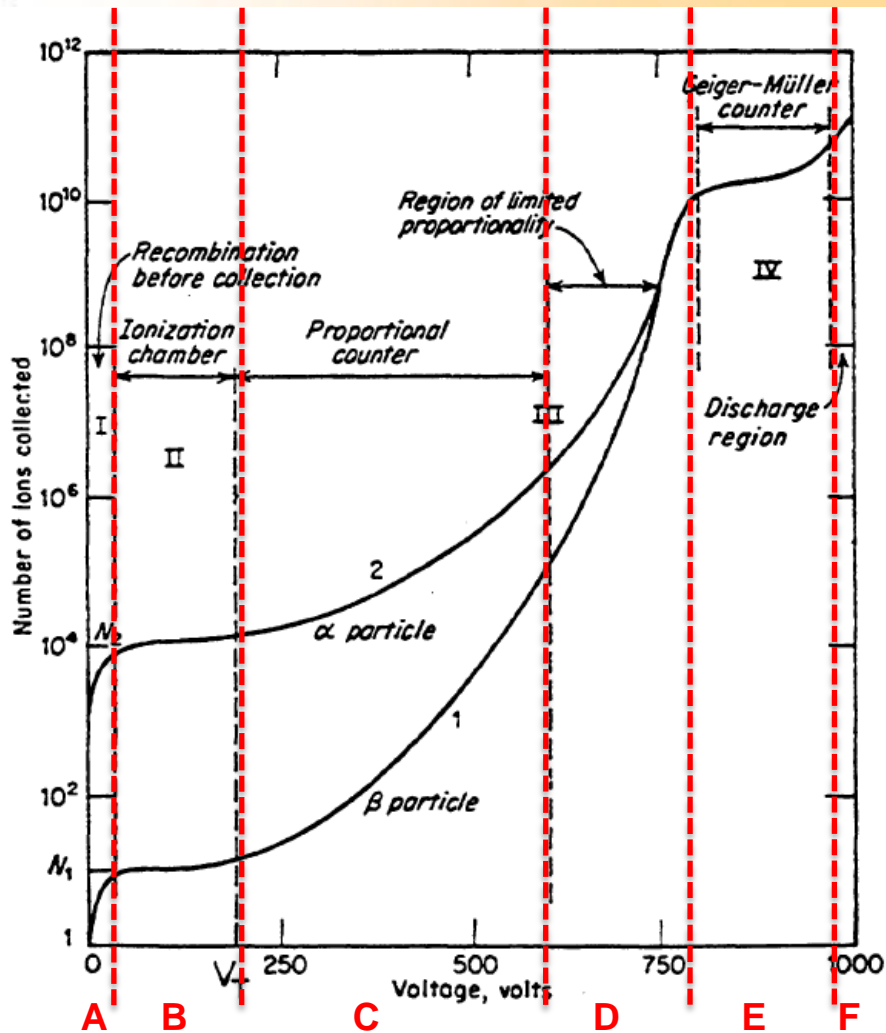
Ionization chambers



- NB, if we use liquid instead of gas, the density is about 10^3 higher and the ionization yield is enough to give enough signal without gain
 - E.g. $\approx 10^5$ ions/cm in liquid argon (LAr) calorimeters



Wire chambers



- Amplification regimes
 - D. Gain is high, space charge effect generate saturation (limited proportionality)
 - E. The avalanche propagates all along the wire because of the emitted photons (Geiger)
 - F. Complete breakdown (discharge even without particle crossing)

Wire chambers

Choice of regime

- Proportional
 - Allows to measure dE/dx
 - Small signal
- Limited proportionality
 - Larger signal, easier electronics readout
- Geiger-Müller
 - Very large signal
 - Slow, large recovery times

Choice of gas

- Principal component is a noble gas (Ar)
 - Easy to generate avalanches, not many degree of freedom to absorb energy
 - Photons from recombination can extract electron from the electrodes and generate discharges
- A polyatomic gas is added to absorb the photons (quencher)
 - Typically hydrocarbons CH_4 , C_3H_8 , C_4H_{10} but also CO_2
 - $\approx 20\%$ of quencher is enough to provide $M \approx 10^5$
- Electrons can be extracted on the cathode by the impacting ions
 - A small fraction of electronegative gasses can be added to reduce the mean free path of electron capture (0.4% CF_3Br , freon)
 - risk to lose efficiency for large drift paths
 - Needed to go into limited-proportionality regime
- Magic mixture : 70%Ar, 29.6% Isobutane, 0.4% Freon



Drift of charges in E field

- $v_D = \mu E$ (μ = mobility)
 - Typical situation of motion with viscous friction

- For ions

$$v_D = m_+ E = \text{const} \cdot \frac{E}{p} \left(\Rightarrow m_+ \propto \frac{1}{p} \right)$$

- The drift velocity scales like E/p (reduced electric field)

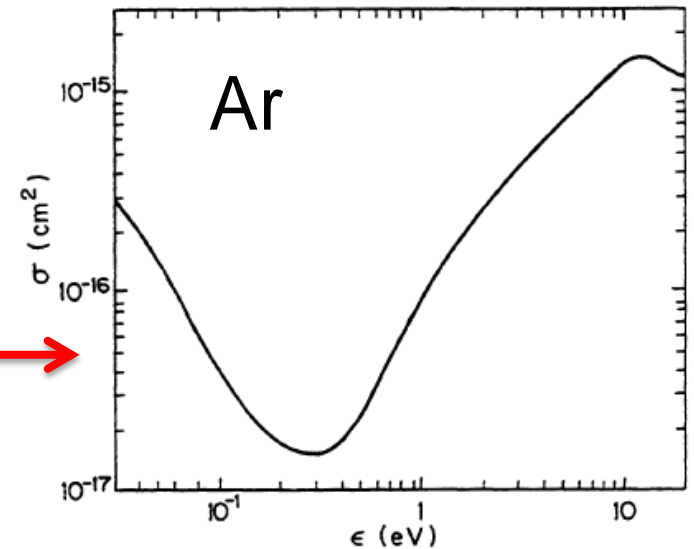
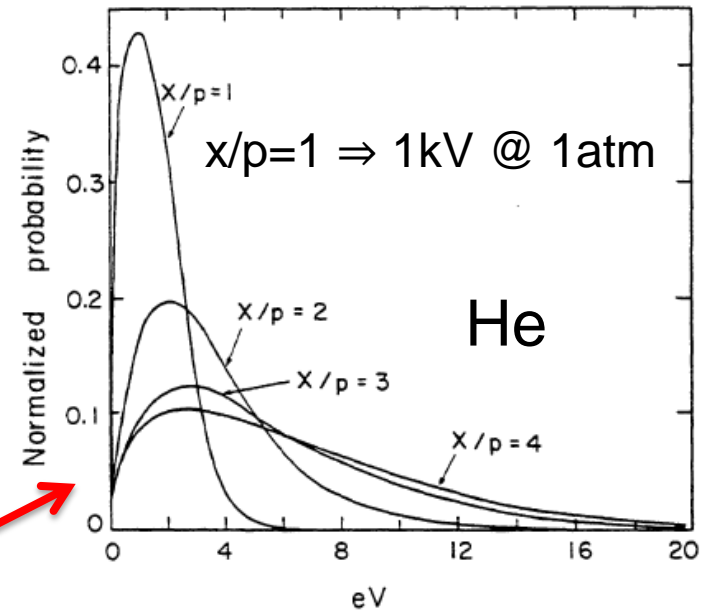
in Ar at STP, with $E=1\text{kV/cm}$

- $v_D = 1.7 \times 1000 = 1.7 \text{ cm/ms}$
- (λ = mean free path between scatterings)

gas (STP)	λ [cm]	μ [cm/s / V/cm]
H ₂	$1.8 \cdot 10^{-5}$	13.0
He	$2.8 \cdot 10^{-5}$	10.2
Ar	$1 \cdot 10^{-5}$	1.7
O ₂	$1 \cdot 10^{-5}$	2.2

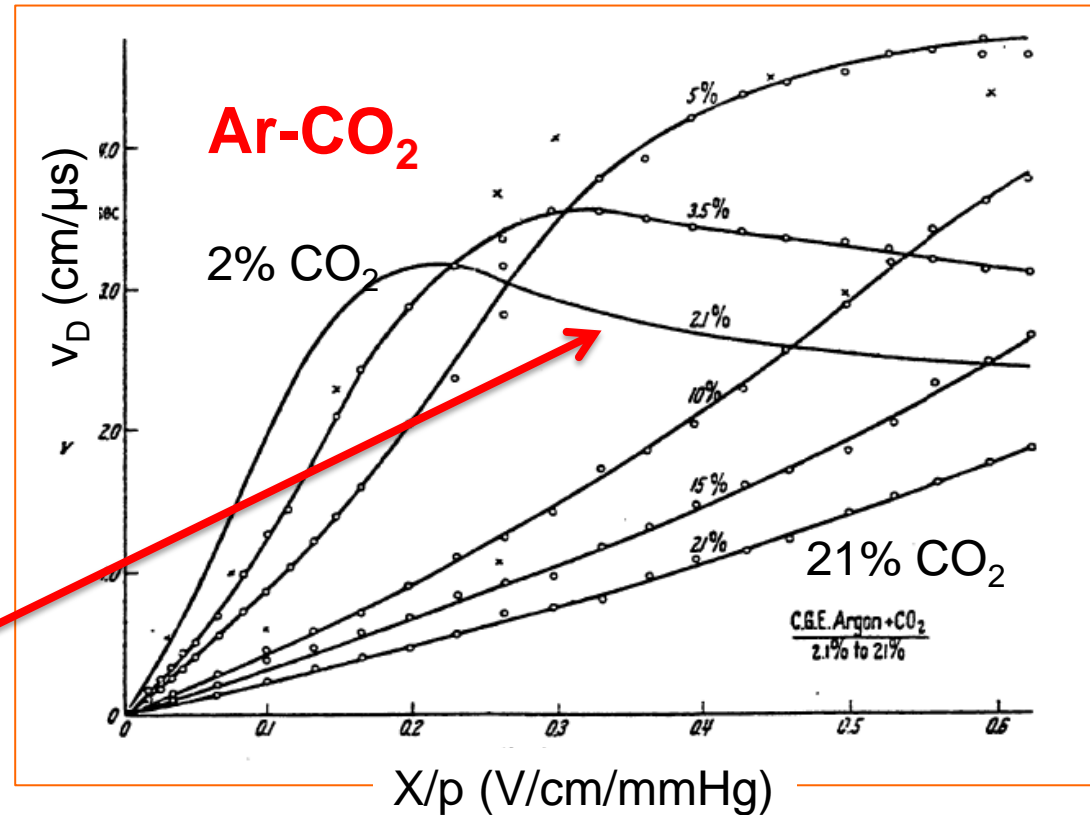
Drift of charges in E field

- Electrons gain much more energy between scatterings
 - Their energy can get similar or larger to the thermal energy ($kT \approx 0.025\text{eV}$)
 - The e-gas scattering cross section varies strongly with energy (Ramsauer effect)



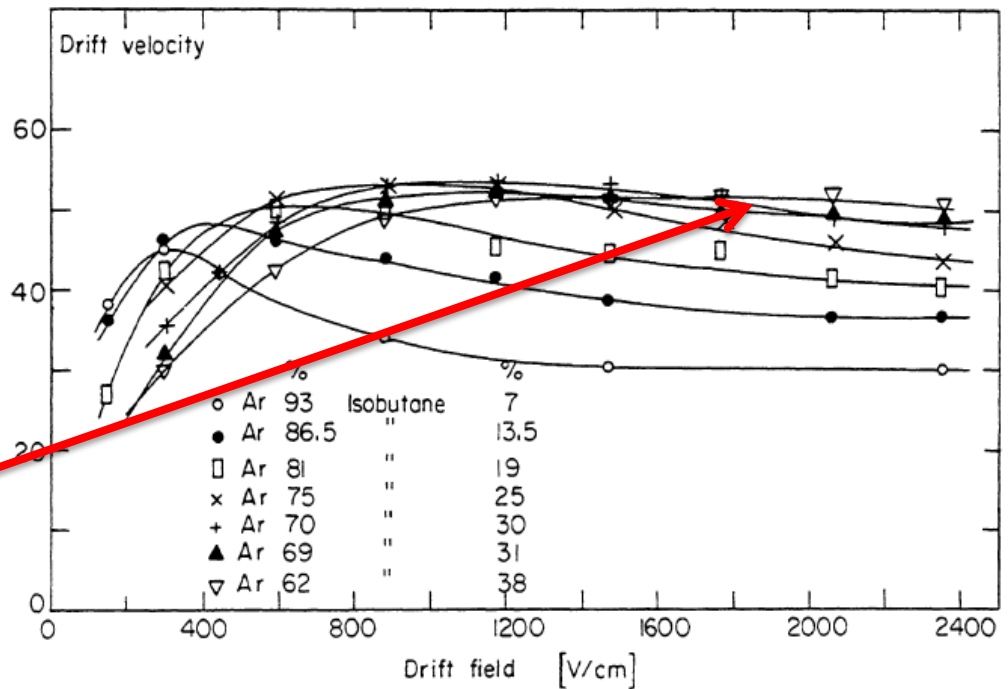
Drift of charges in E field

- We can still write $v_D = \mu E$ but the mobility is not anymore only proportional to $1/p$
 - It is also very sensitive to the gas mixture as the cross section vary a lot
 - Drift velocity can also decrease with increasing E field



Drift of charges in E field

- For some gas mixture the drift velocity saturates
 - Including the “magical” mixture
 - Typical values 5 cm/ μ s (50 μ m/ns, 200ns for cm)



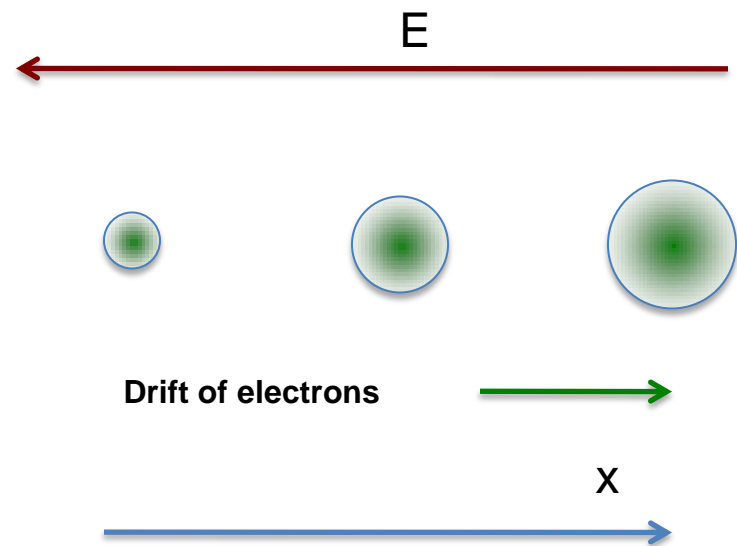
- Can be a very useful feature, v_D does not depend anymore on the details of the E field

Diffusion

- Another important effect is the diffusion
 - Growth in size of the cloud of drifting charges

$$\frac{dN}{dx} = \frac{N_0}{\sqrt{4\rho Dt}} e^{-\frac{x^2}{4Dt}}$$
$$S = \sqrt{2Dt}$$

- The distribution is described by a coefficient D, and grows as \sqrt{Dt}



- There is a correlation between D and mobility

$$D/m \mu k_B T / e$$

Effects of B field on drift

- Described by the “Langevin” equation

$$m \frac{d\vec{v}}{dt} = -\frac{e}{m} \vec{v} - e(\vec{E} + \vec{v} \times \vec{B}) + \vec{\mathcal{H}}(t)$$

- The solution is

$$\vec{v}_D = -\frac{m}{1 + \omega^2 t^2} \left[\vec{E} + \frac{\vec{E} \times \vec{B}}{B} \omega t + \frac{\vec{E} \cdot \vec{B}}{B^2} \vec{B} \omega^2 t^2 \right]$$

- Where τ is the mean free time between collision and $\omega = eB/m$

Effects of B field on drift

- Condition $E \perp B$

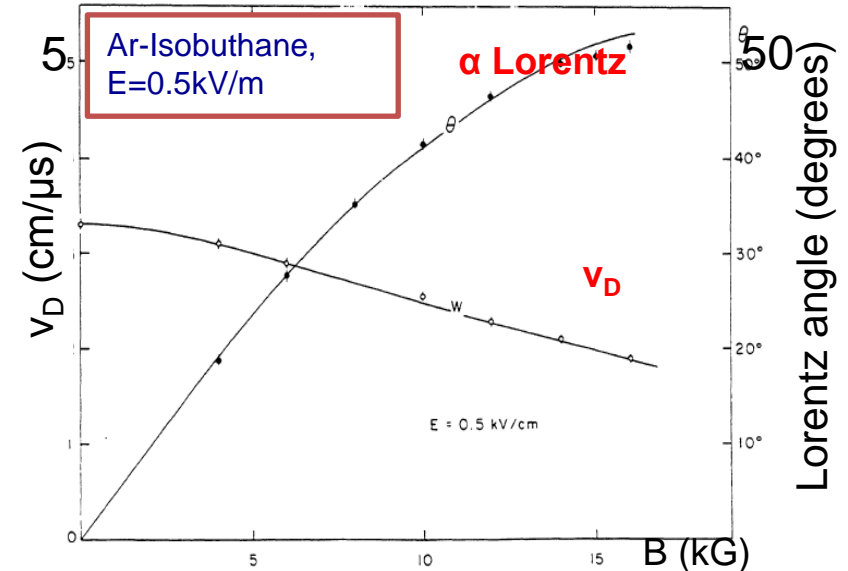
$$\vec{v}_D = -\frac{m}{1+W^2 t^2} \left[\vec{E} + \frac{\vec{E} \times \vec{B}}{B} \omega t + \frac{\vec{E} \cdot \vec{B}}{B^2} \vec{B} \omega^2 t^2 \right]$$

$$v_{D^\wedge} = \frac{\omega t}{1+W^2 t^2} mE$$

$$v_{D\parallel} = \frac{1}{1+W^2 t^2} mE$$

$$v_D = \sqrt{v_{D\parallel}^2 + v_{D^\wedge}^2} = \frac{1}{\sqrt{1+W^2 t^2}} mE$$

$$\text{tga} = \frac{v_{D^\wedge}}{v_{D\parallel}} = \omega t$$



- v_D gets reduced
- The drift has an angle w.r.t E (Lorentz angle)

Effects of B field on drift

- condition $E \parallel B$

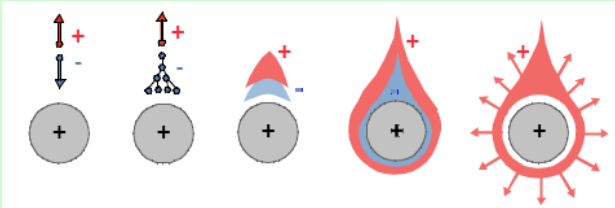
$$\vec{v}_D = -\frac{m}{1 + \omega^2 t^2} \left[\vec{E} + \frac{\vec{E} \times \vec{B}}{B} \omega t + \frac{\vec{E} \cdot \vec{B}}{B^2} \vec{B} \omega^2 t^2 \right]$$

$$v_{D\parallel} = \frac{m}{1 + \omega^2 t^2} (E + 0 + \omega^2 t^2 E) = \frac{mE}{1 + \omega^2 t^2} (1 + \omega^2 t^2) = mE$$

- v_D does not change
- But the transverse diffusions is limited by B

Wire chambers signal

- In the avalanche, most of the charge is generated in the latest λ_1 before the wire



- For wires of $20\mu\text{m}$ mostly within $100\mu\text{m}$
- The signal is generated by the work the E field does to move the charges
 - Cylindrical detector of length l , avalanche of charge Q generated at radius r

$$V^- = + \frac{Q}{2\rho e_0 l} \ln\left(\frac{a}{r}\right)$$

$$V^+ = - \frac{Q}{2\rho e_0 l} \ln\left(\frac{b}{r}\right)$$

$$V^+ + V^- = - \frac{Q}{2\rho e_0 l} \ln\left(\frac{b}{a}\right) = - \frac{Q}{C}$$

- Electrons are already very close to collection, most of the work is done to drift back the ions

- The total time to integrate the signal is typically long
 - Depends on the drift velocity of ions and on the distance anode-cathode
 - Typically $100\mu\text{s} \div 1\text{ms}$
 - $\mu^+ = 1.7 \text{ cm}^2\text{s}^{-1}\text{V}^{-1}\text{atm}^{-1}$ (mobility of ions): $V_0 = 2\text{kV}$, $a = 20\mu\text{m}$, $b = 0.5\text{cm}$, $l = 1\text{m}$, $p = 1\text{Atm}$
 - $T \approx 200\mu\text{s}$
- But the leading edge of the signal is very fast
 - Time to collect $\frac{1}{2}$ charge

$t_{1/2} \gg \frac{a}{b} T$
 - With previous values one gets $t_{1/2} = 800\text{ns}$

MWPC

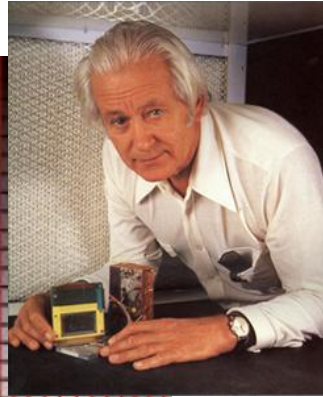
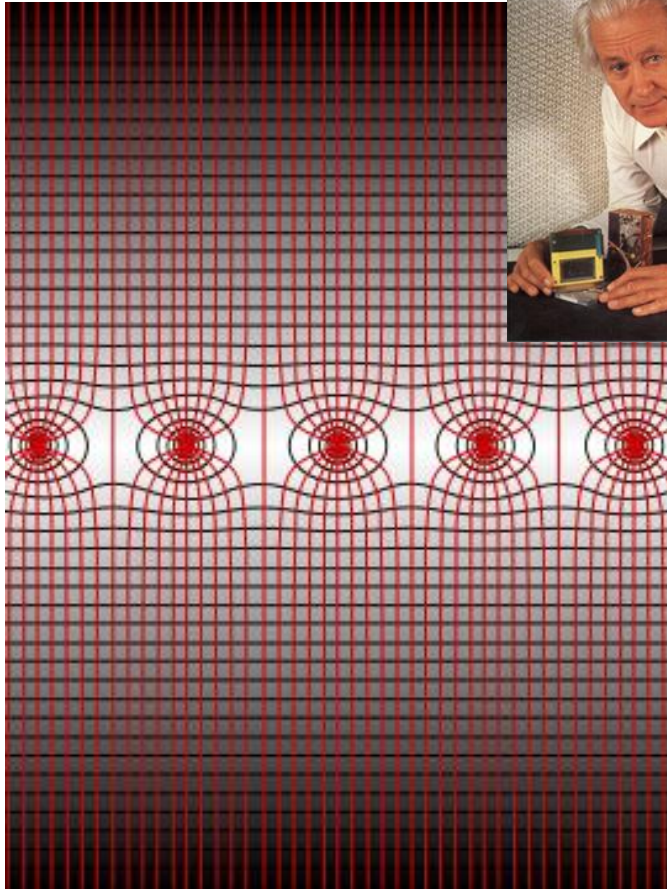
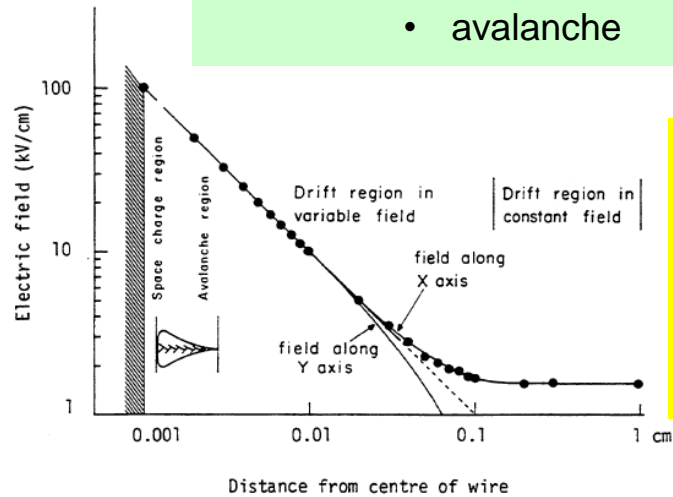


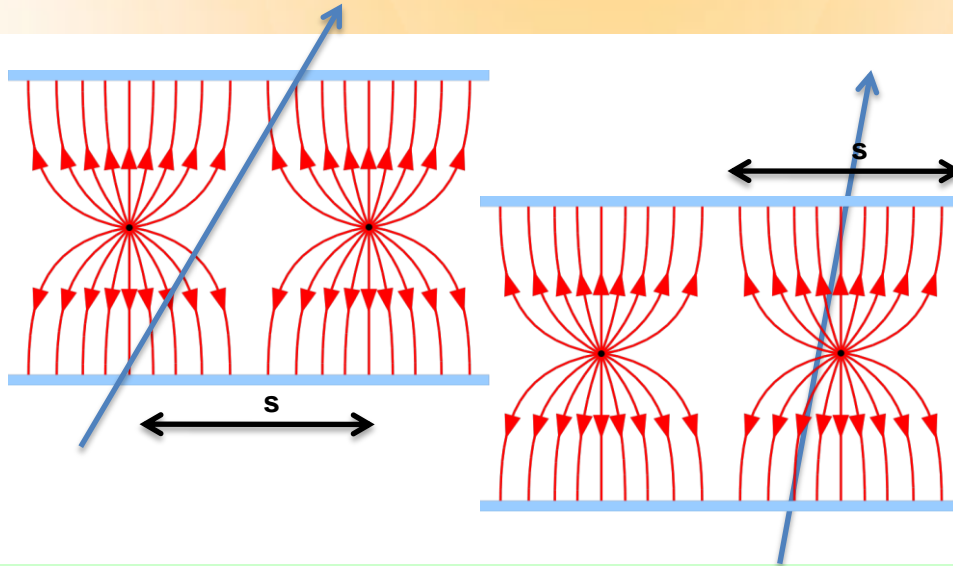
Photo: D. Parker, Science Photo Lab, UK

- Multi Wire Proportional Chambers
 - Charpack 1968 (Nobel Prize 1992)
 - Set of parallel anode wires tightly spaced, between parallel cathodes
 - E field essentially uniform in most of the detector
 - Drift field to collect charges
 - Becomes very intense close to the anode
 - avalanche



- Typical values
 - Wire spacing 2mm
 - Anode-cathode distance 4mm
 - $V \sim 3\text{kV}$
 - Magic mixture

MWPC resolution



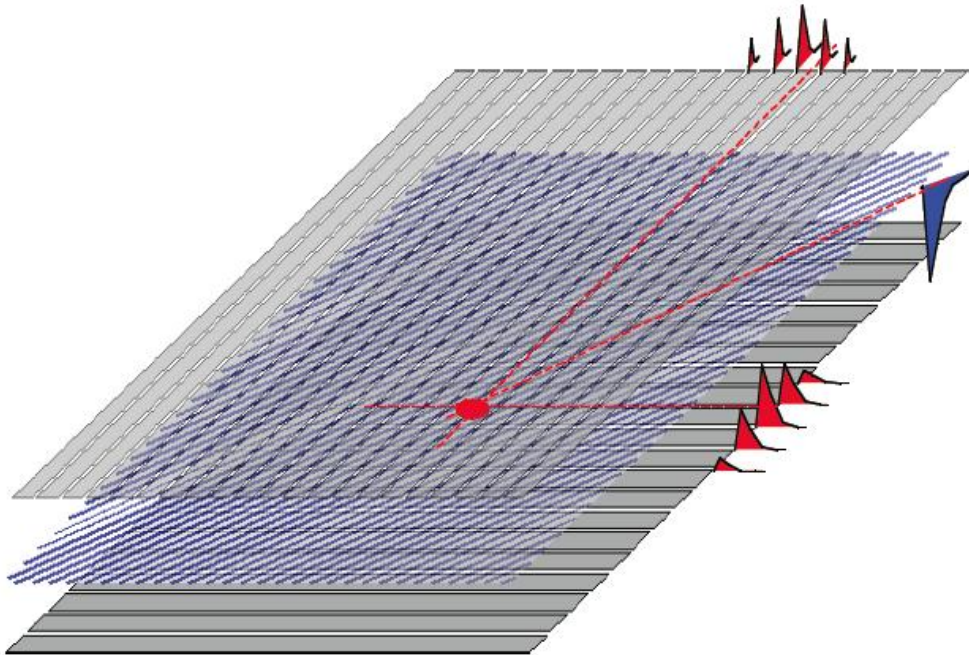
- If wires are readout

$$S_x = \frac{s}{\sqrt{12}}$$

$$S_x = \frac{2}{\sqrt{12}} = 0.6 \text{ mm} \text{ for } 2 \text{ mm wire spacing}$$

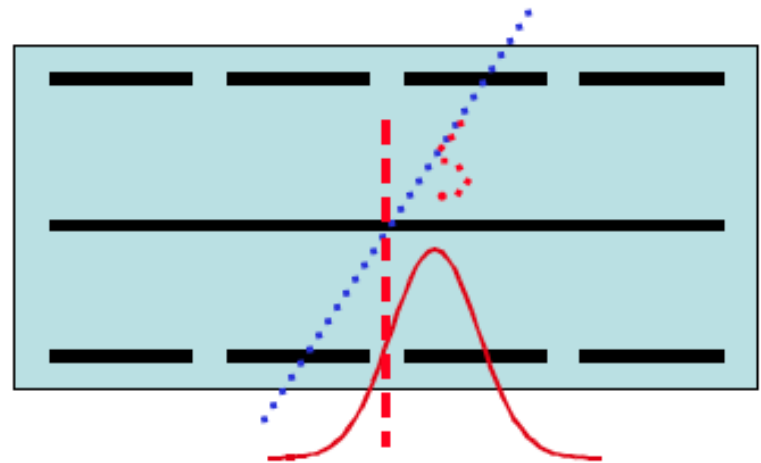
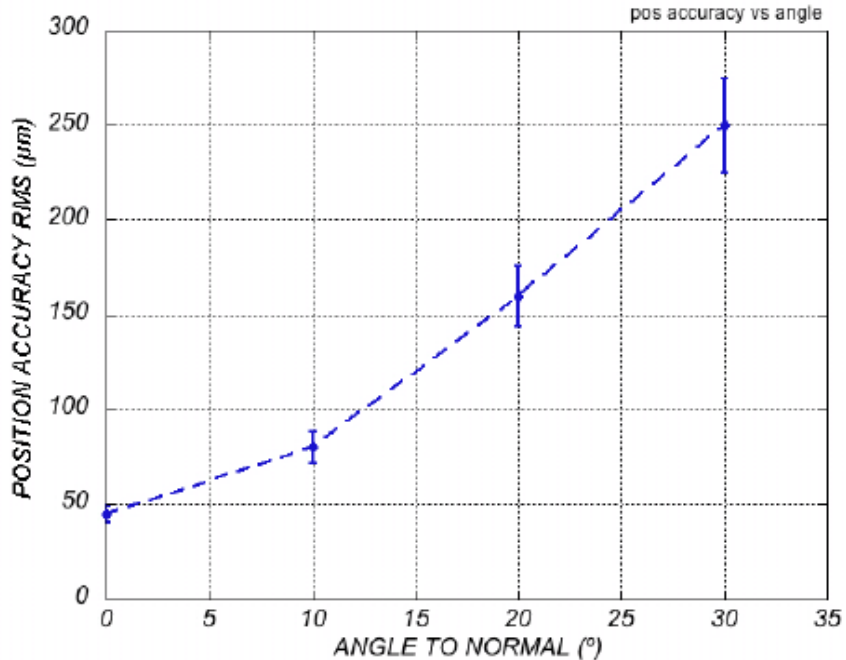
- For non perpendicular tracks more wires can give signal
- The resolution does not change

MWPC resolution



- Cathodes can be readout too
 - Signal induced on more adjacent “strips” (or groups of cathode wires)
 - Position along the anode wire can be reconstructed with a resolution $\approx 100\text{-}200\mu\text{m}$
 - Across the wires nothing changes, the avalanche position is ON the wire
 - Sometimes both cathodes with orthogonal strips are readout
 - Anode at HV, no decoupling capacitors
 - To get high resolution on both coordinates one can use sets of consecutive MWPC with perpendicular anode directions

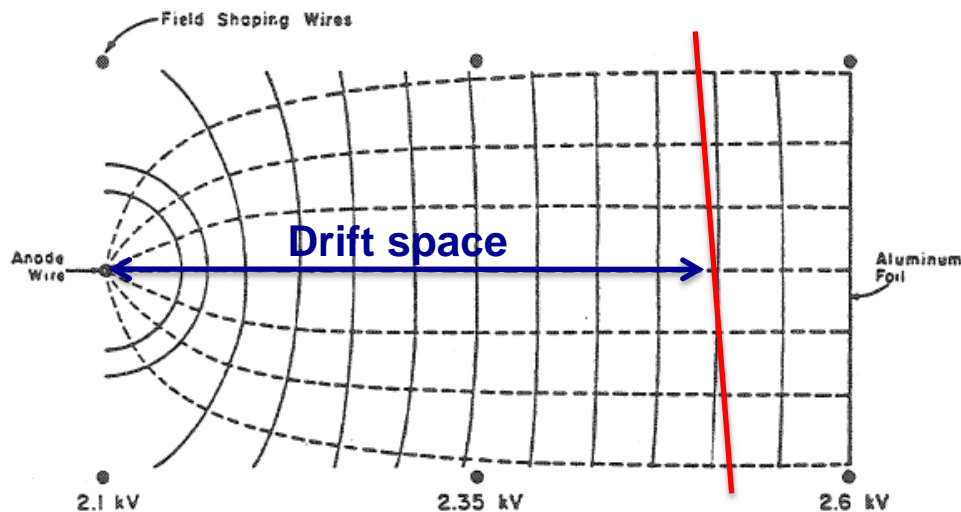
MWPC resolution



- Notice
 - Statistical fluctuations of the primary ionization, and emission of δ rays can influence the resolution, in particular for tracks not orthogonal to the chamber

Drift chambers

- Drift chambers are wire chambers with a long drift path
- The track position is measured by the drift time in a possibly uniform E field
 - Need an external system to give the “start” to the time measurement
 - The “stop” is generated by the signal on the wire

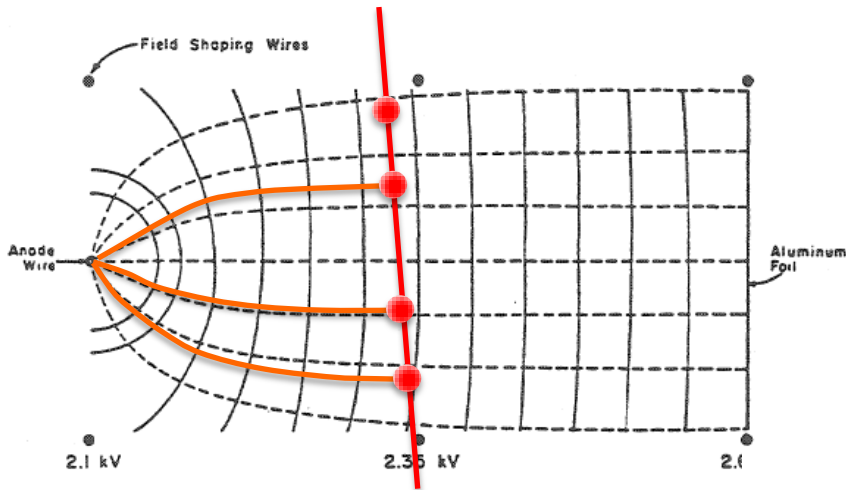


Typical drift velocities are $50\mu\text{m/ns}$ (with magical mixture)

- Need order of ns resolutions to get space resolutions around $100\mu\text{m}$

Trigger system

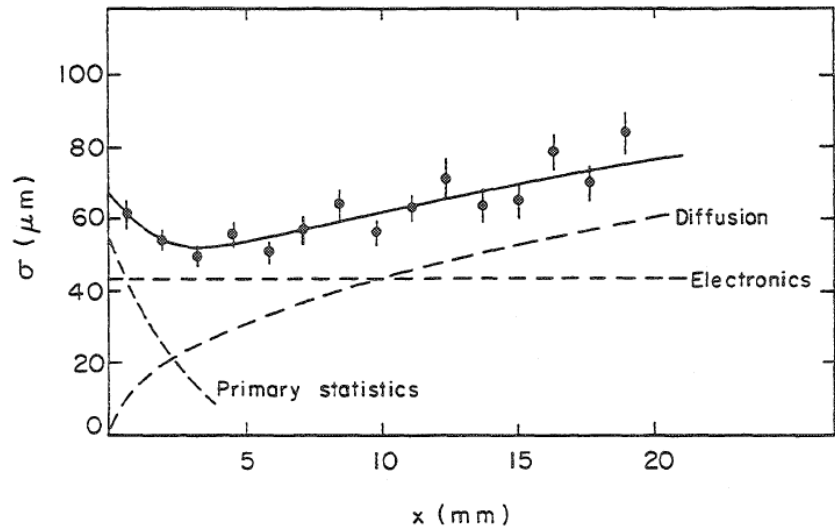
Drift chambers resolution



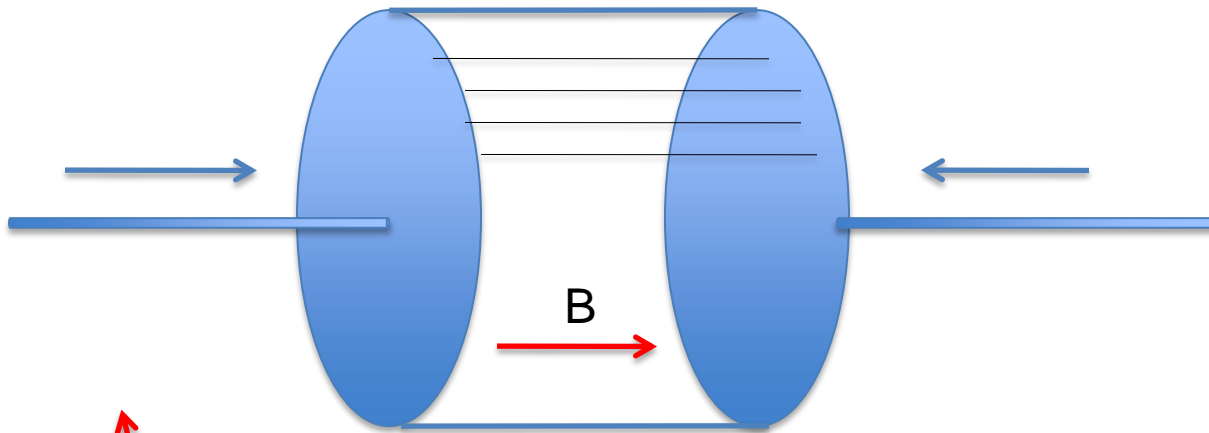
Resolution below $100\mu\text{m}$ can be achieved with drift spaces of several cm

Three important effects

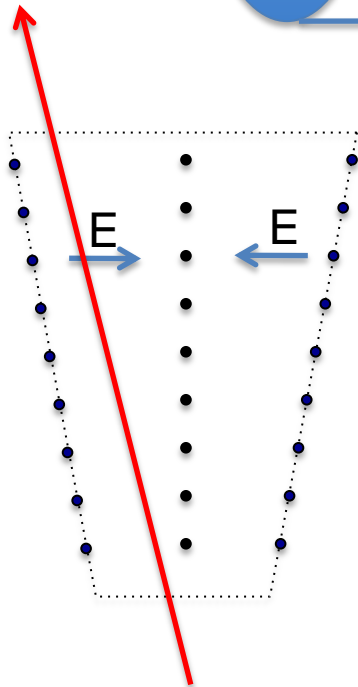
- Electronic noise
- Longitudinal diffusion of the charge
 - Proportional to \sqrt{t} and so to \sqrt{x} for constant drift velocity
- Primary ionization statistics
 - Drift path of primary clusters can be different



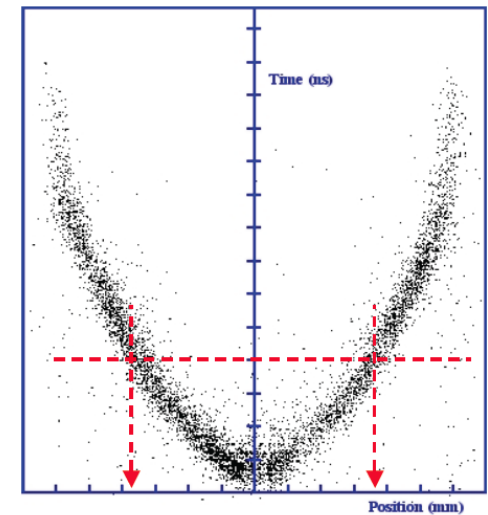
Drift chambers



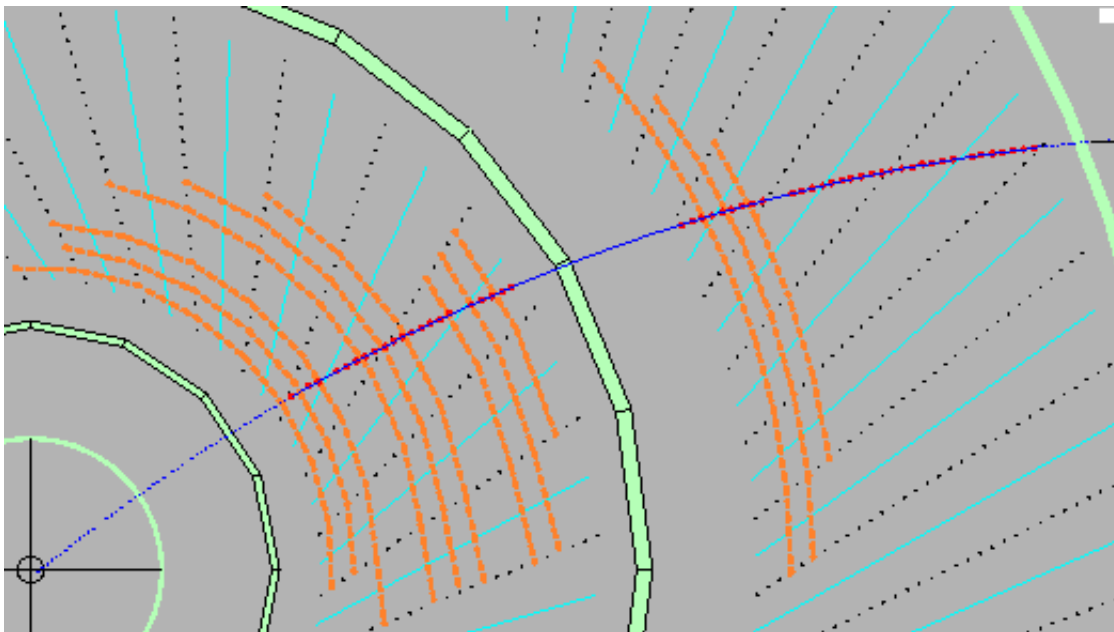
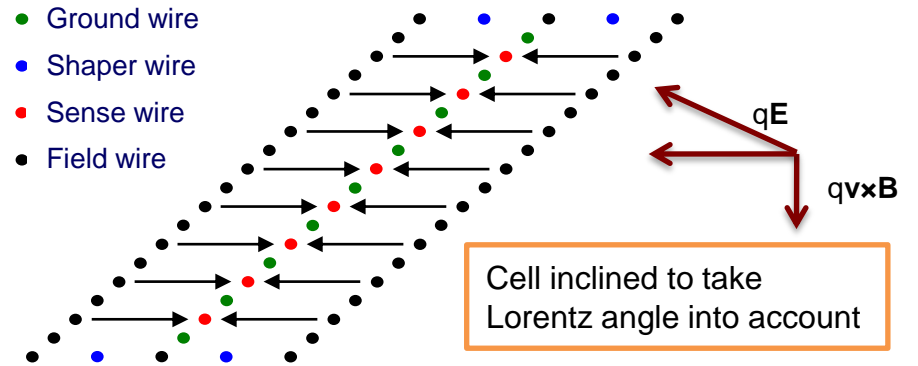
- Often used as central detectors in colliders
 - B parallel to wires so orthogonal to E



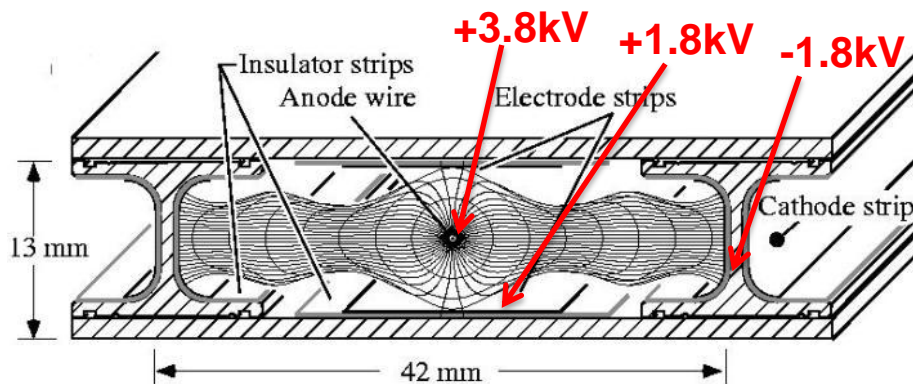
- Typical drift cell
- Time-space relationship
 - Notice the left-right ambiguity



Examples of drift chambers

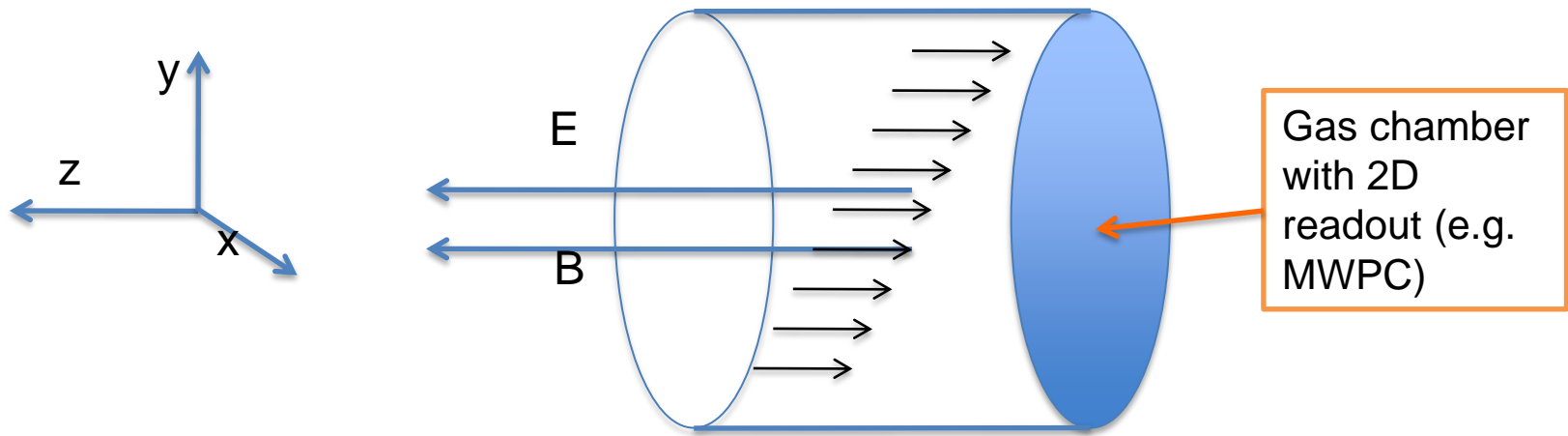


Examples of drift chambers



- Drift chambers of the barrel muon detector of CM
 - Homogeneous drift field
 - Linear space/time relationship using careful field shaping
 - Easier to use in fast trigger
 - Aluminium structure
 - Relatively heavy, not a problem for a muon detector
 - 50 μ m anode wire
 - Gas mixture 85% Ar 15% CO₂
 - Non flammable
 - Maximum drift time \approx 400ns
 - Space resolution \approx 100 μ m

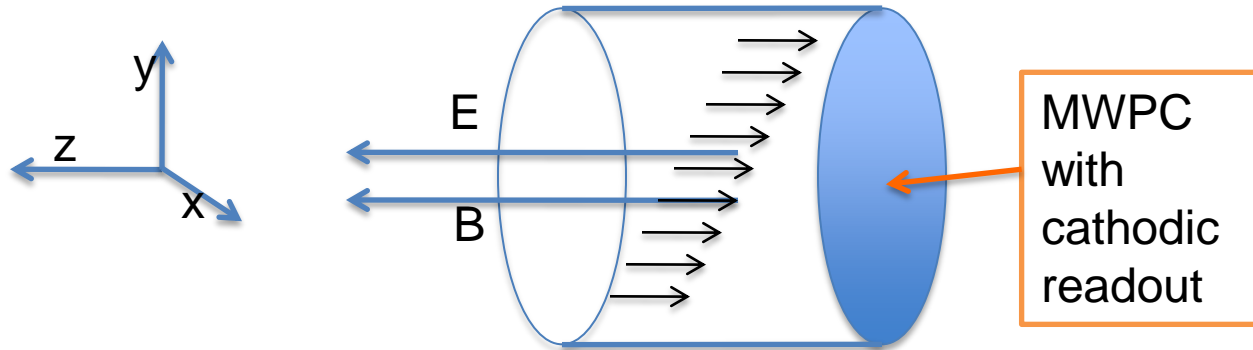
Time Projection Chambers



Time Projection Chamber (TPC)

- Long drift path
 - z readout with drift time
- At the extremity a MWPC or similar (GEM)
 - Reading x,y coordinates

Time Projection Chambers



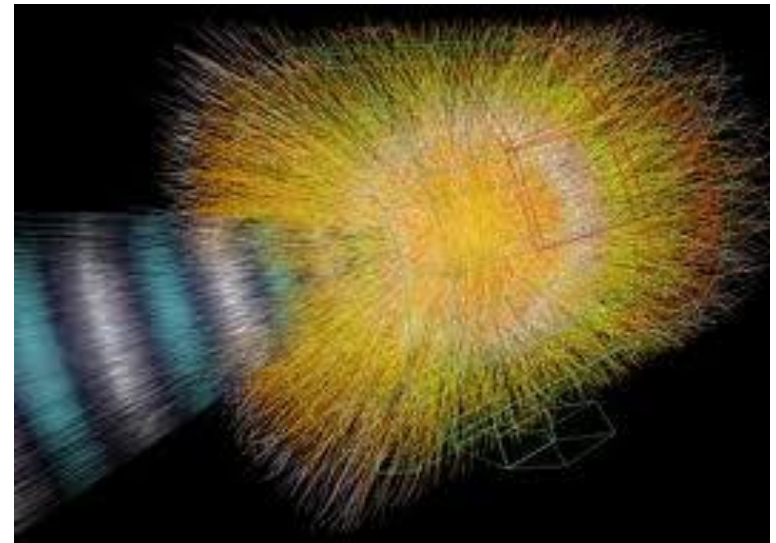
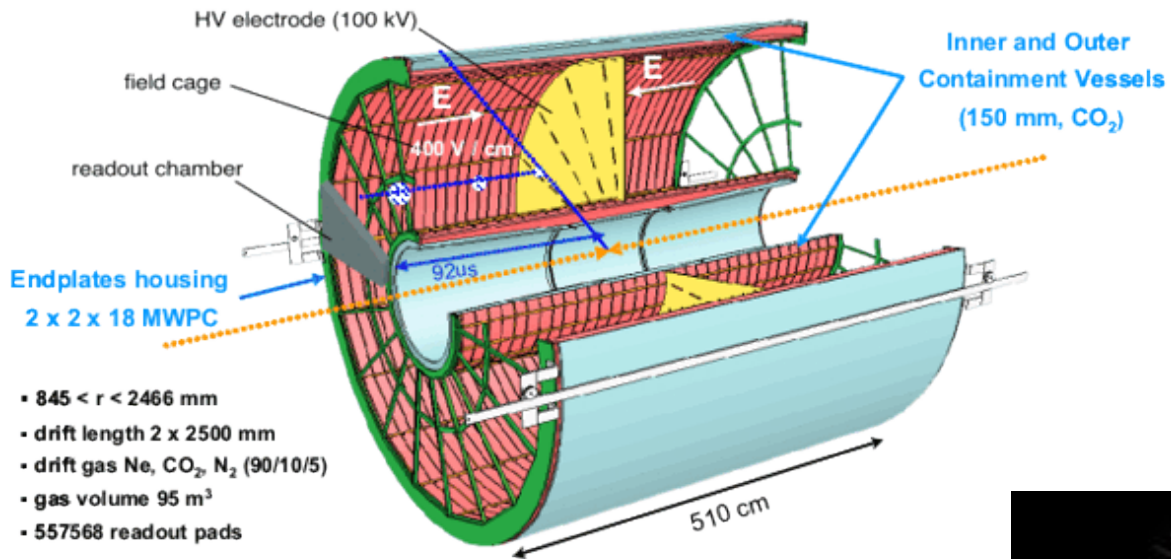
- Advantages

- A true tridimensional readout is possible helps pattern recognition
- Transverse diffusion limited by B, improves x,y, resolution
- Very little material

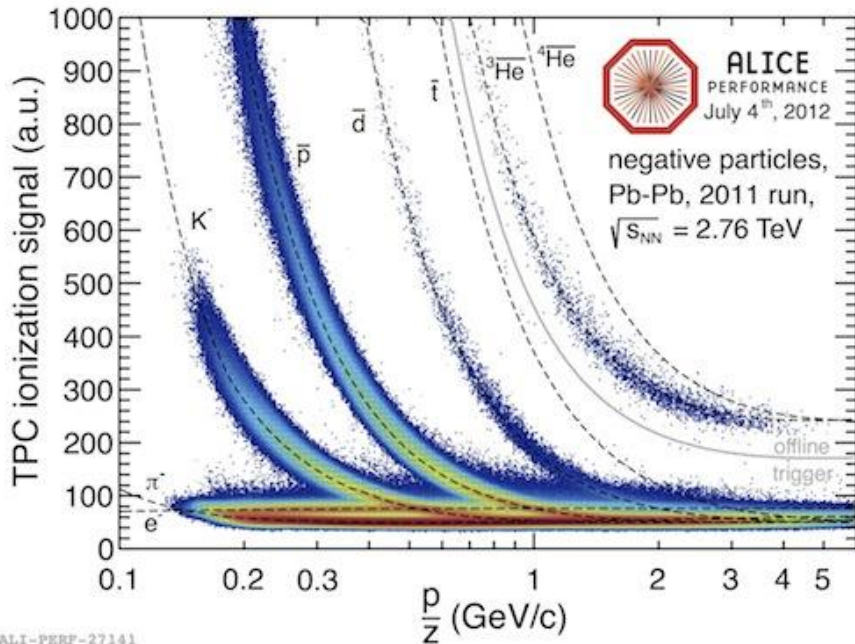
- Disadvantages

- Long drift paths(10÷100 μ s)
 - Sensitive to electronegative impurities
 - Not well suited to very high bunch crossing rates

The TPC of ALICE at LHC

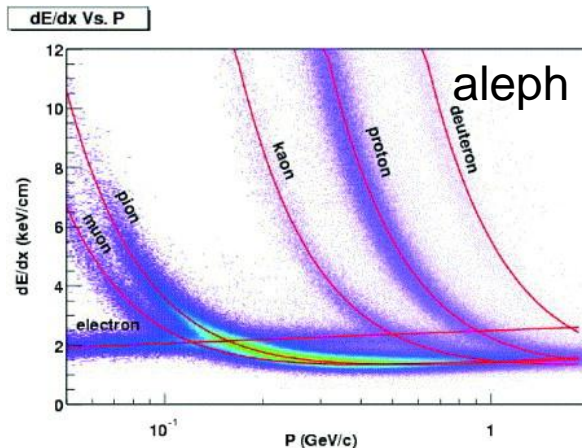


The TPC of ALICE at LHC



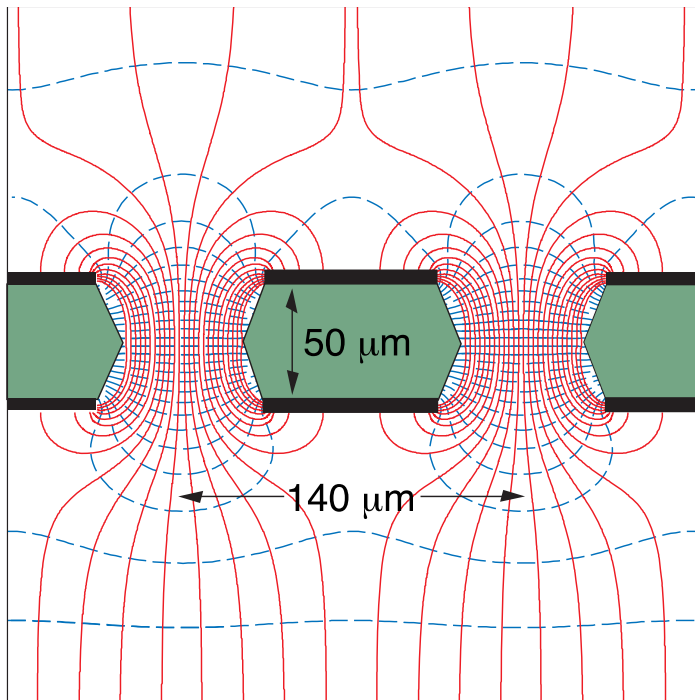
- In the TPC the gain is typically small
 - Long drift times, no electronegative gasses possible
 - Work in proportional mode
 - Large number of samples per track
- Very well suited to measure dE/dx
 - To perform PID

ALI-PERF-27141



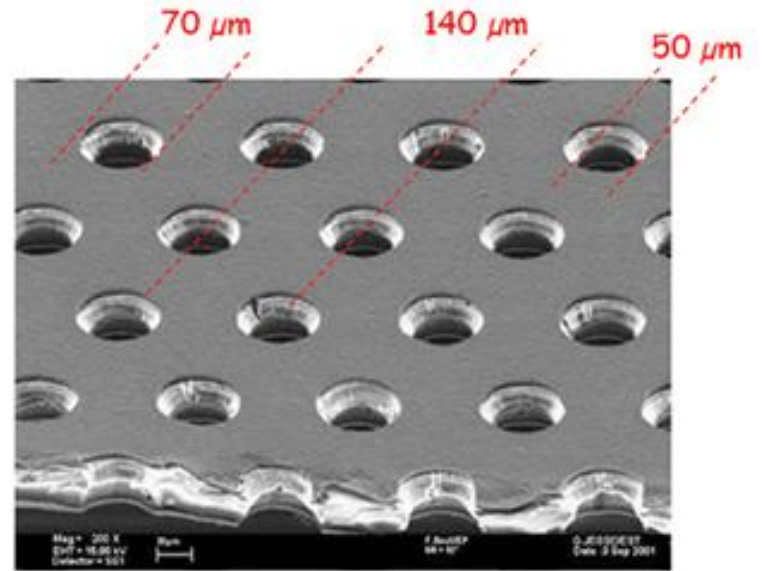
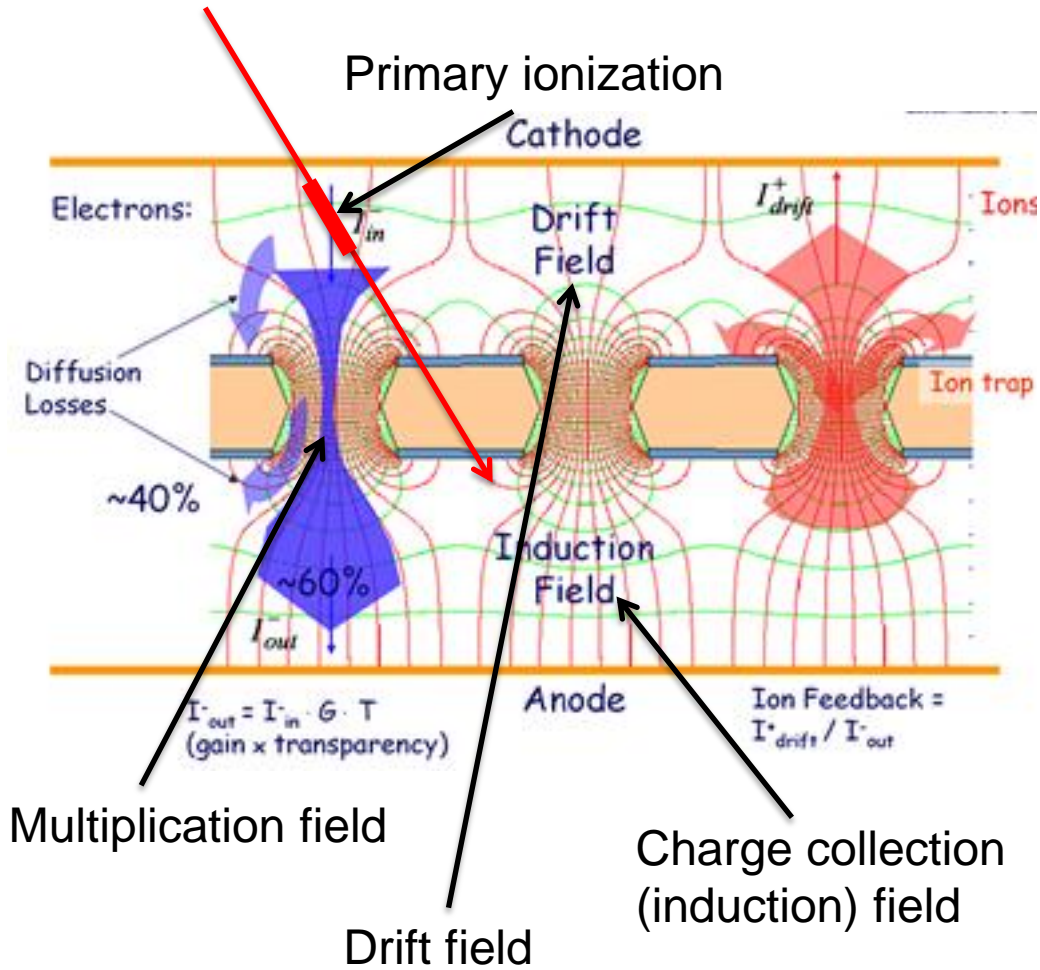
GEM

- **G**as **E**lectron **M**ultiplier



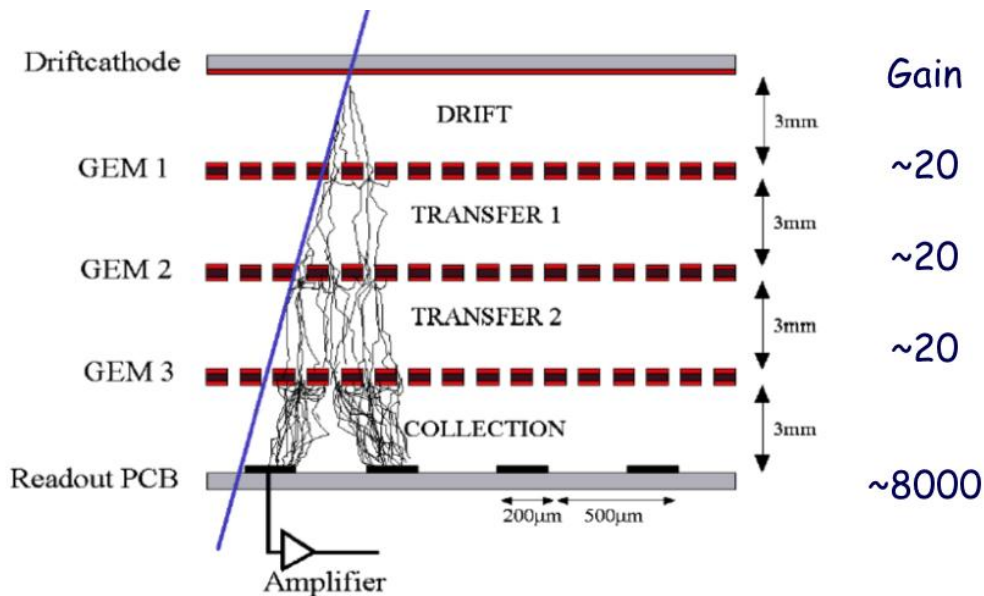
- Kapton foil, metallized on both sides with micro-holes
 - Using lithographic techniques
- HV between the two layers generates an amplification region
 - 400-500V on $50\ \mu\text{m}$
- It is possible to have multiple layers of GEMs with reduced gain/layer
 - Reduced risk of discharge

GEM



GEM

Example with multi-layer configuration

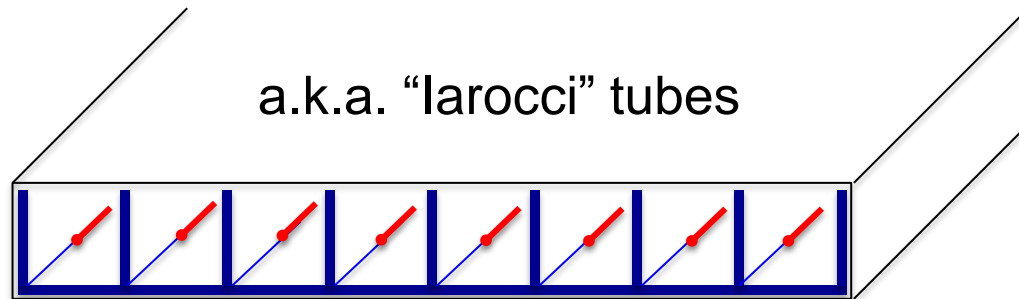


Normally gain is higher (100-1000/ layer)

Advantages

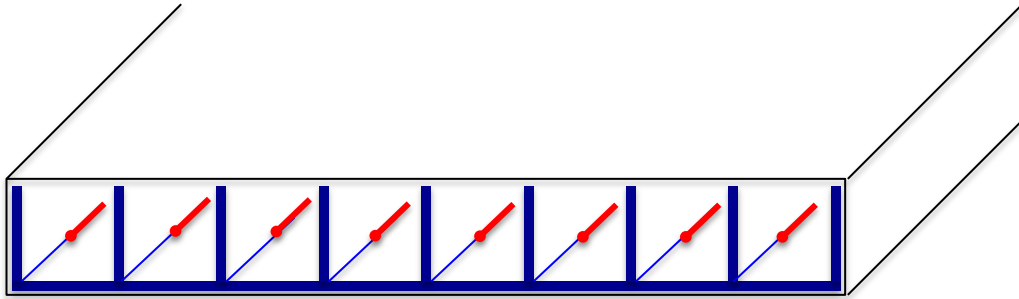
- very good space resolution
 - Down to 30 µm
- Very good separation of adjacent tracks
- Ability to sustain high rates
 - Ion are readily collected by nearby electrodes

Limited Streamer Tubes



- Mechanically a multiwire chamber
 - 100 μm thick anode wire
 - Typically 1 cm spacing
 - Structure made by plastic, painted by a resistive material to provide cathodes
 - HV 4.5 \div 5kV (at STP)
 - Need high field as the wire is thick
 - Very economical construction, suited to cover very large surfaces
 - Muon detectors, cosmic rays large area detectors

Limited Streamer Tubes

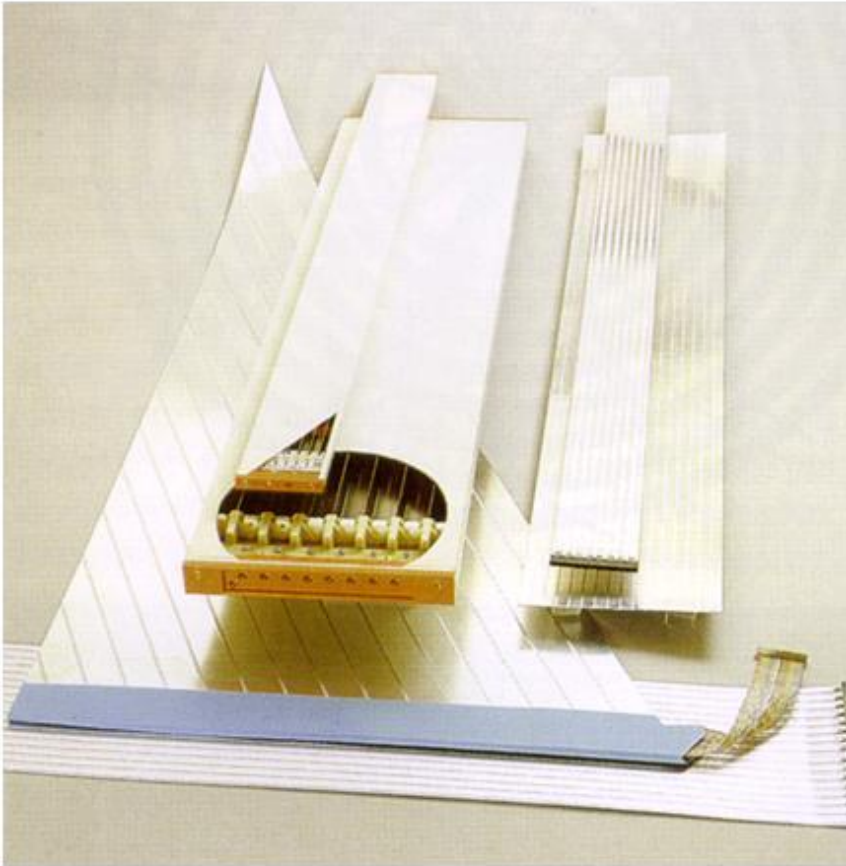


Typical gas mixtures

- Ar - isobutane 30-70%
- CO₂ - Ar – isobutane 89-8-3%

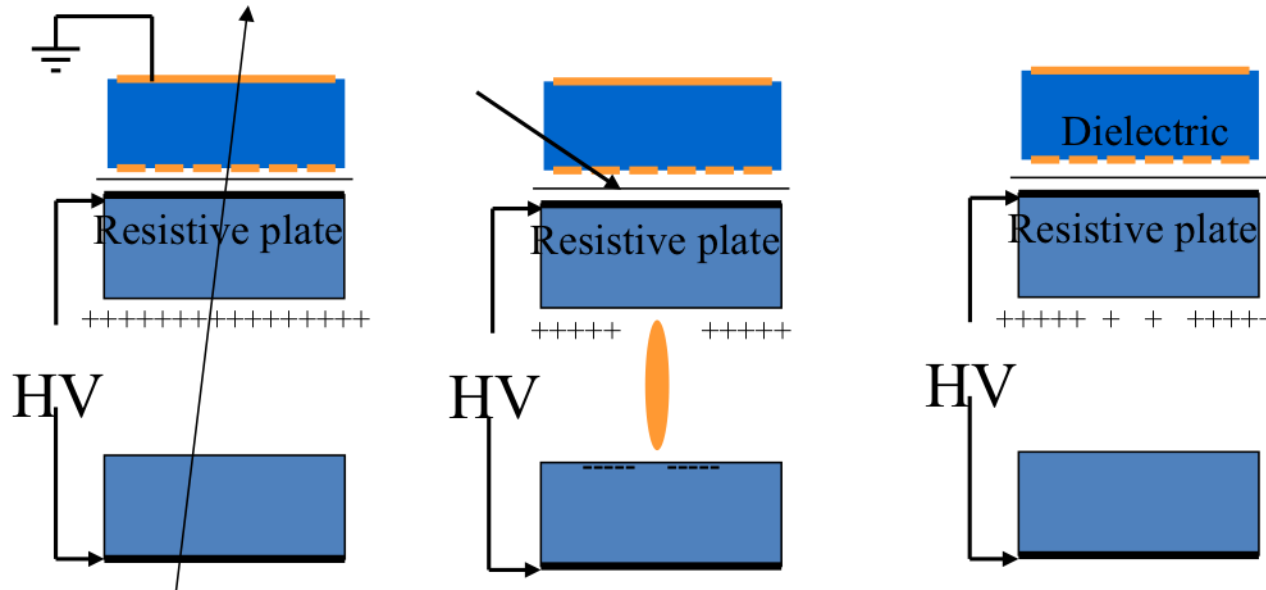
- Work in limited streamer mode:
 - The E field is large in a big region of space, a plasma filament is generated by the avalanche
 - Lots of photons generated, need strongly quenching from the gas
 - Due to the resistive cathodes, the local E field close to the streamer gets reduced, and the streamer ends

Limited Streamer Tubes



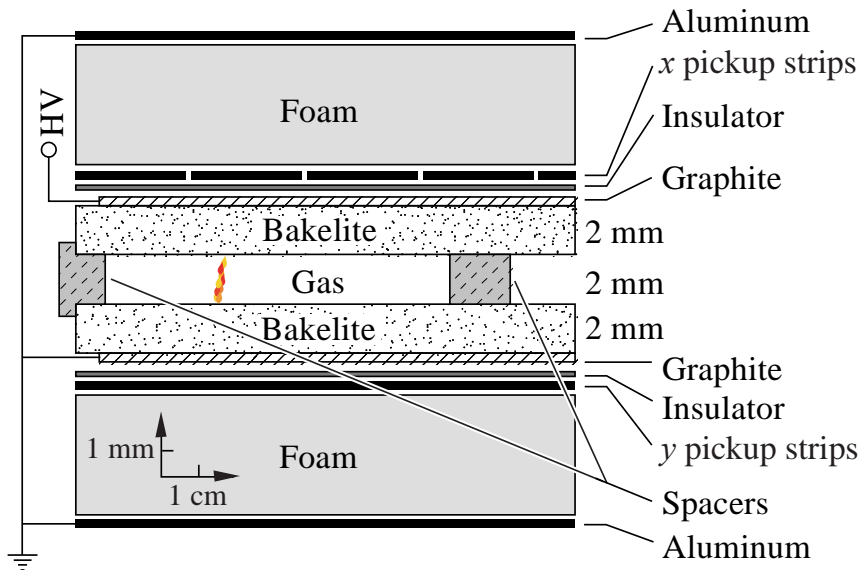
- Large signal
 - $\approx 30\text{pC}$
- Can be readout by external strips
 - Graphite cathodes are transparent to the fast signals
 - Resolution is
 - Wires pitch/ $\sqrt{12}$ across wires
 - Strip pitch/ $\sqrt{12}$ if digital readout, down to $500\mu\text{m}$ if analog (centroid) readout

Resistive Plates Chambers



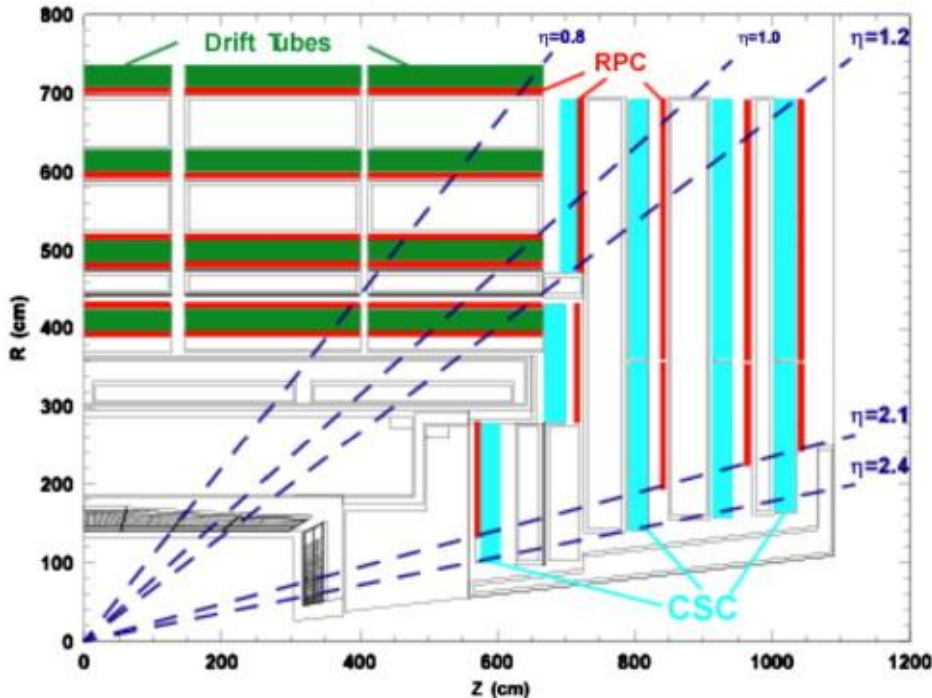
- Flat detectors with large E field between planes
 - Avalanche in the whole space between the planes, quenching concept similar to LST
 - Large signal, readout through external strips/pads
 - No drift, very fast (ns resolution)

Resistive Plates Chambers



- Can be made with bachelite (cheap) or resistive glass
 - HV = 8-10kV
- No wire structure, readout in x-y coordinate with the same resolution
- Very high resistivity, cannot sustain very high particle fluxes

Resistive Plates Chambers



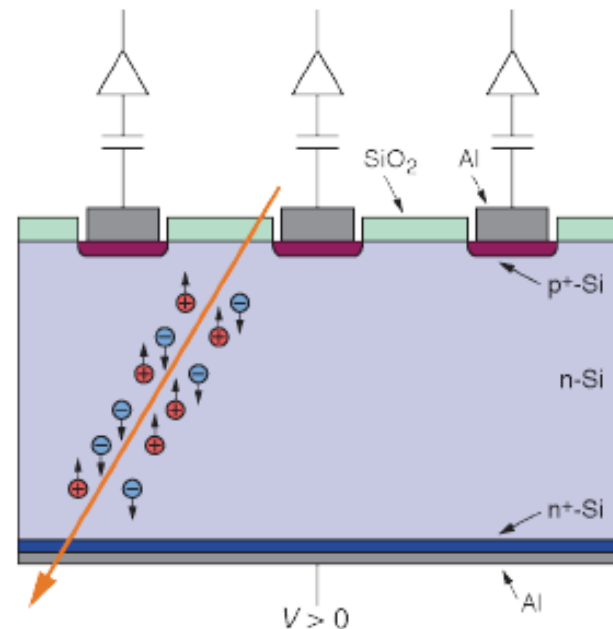
- Fast: used by both Atlas and CMS as detectors for muon trigger
 - Only trigger detector in Atlas, complementing other chambers in CMS

- Notice, in LHC the RPC are used in “avalanche” mode and not in “streamer mode”
 - Reduced gain (10^6 w.r.t. 10^8)
 - Very complex gas mixture to provide high quenching
 - Higher capability to stand particle fluxes ($1\text{kHz}/\text{cm}^2$ w.r.t. $10\text{-}100\text{Hz}/\text{cms}^2$)

SILICON DETECTORS

What are Si detectors?

- Semiconductor (Solid State) detector
 - Essentially, a ionization chamber that collect ionization produced in a solid detector
 - (will discuss later some case where there is also amplification)
 - Need to have a way to collect charge generated inside a solid
- Generally used as position detectors with high resolution



Advantages and disadvantages

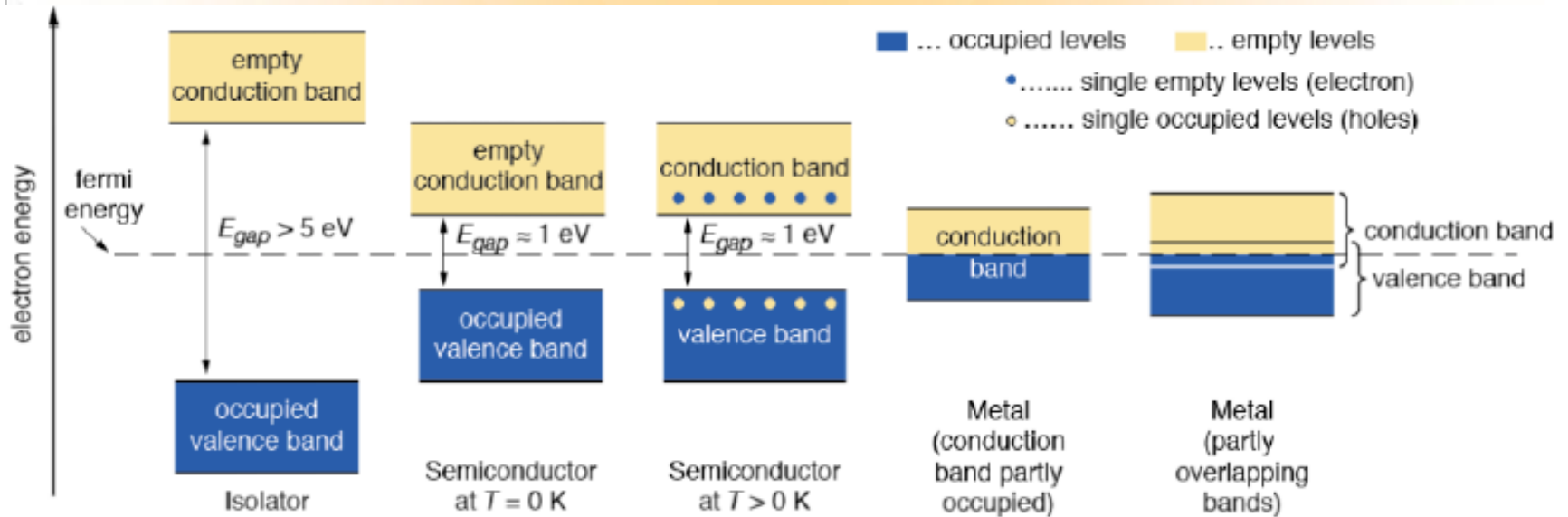
advantages

- High density w.r.t other position detectors (gas chambers)
 - Smaller diffusion which translates in better resolution
- Low ionization energy
 - Few eV to generate a e-h pair, effective in translating energy loss in signal
- Large industrial experience
 - Can use frontier technologies developed for microchips
- Radiation hard

disadvantages

- High density
 - Higher multiple scattering
- No internal gain
 - With exceptions

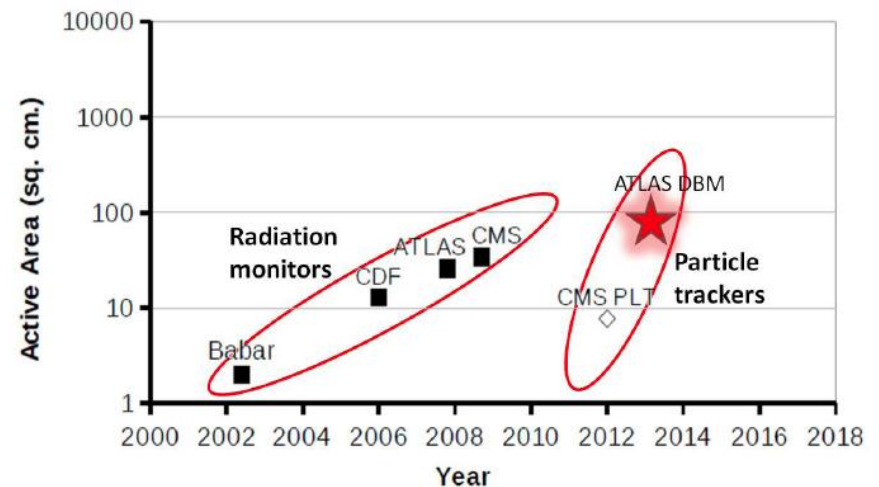
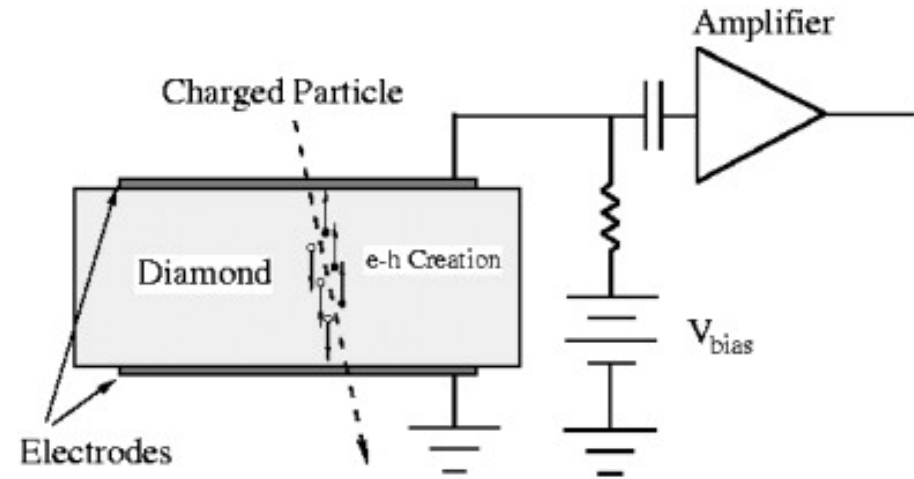
Requirements for solid state detectors



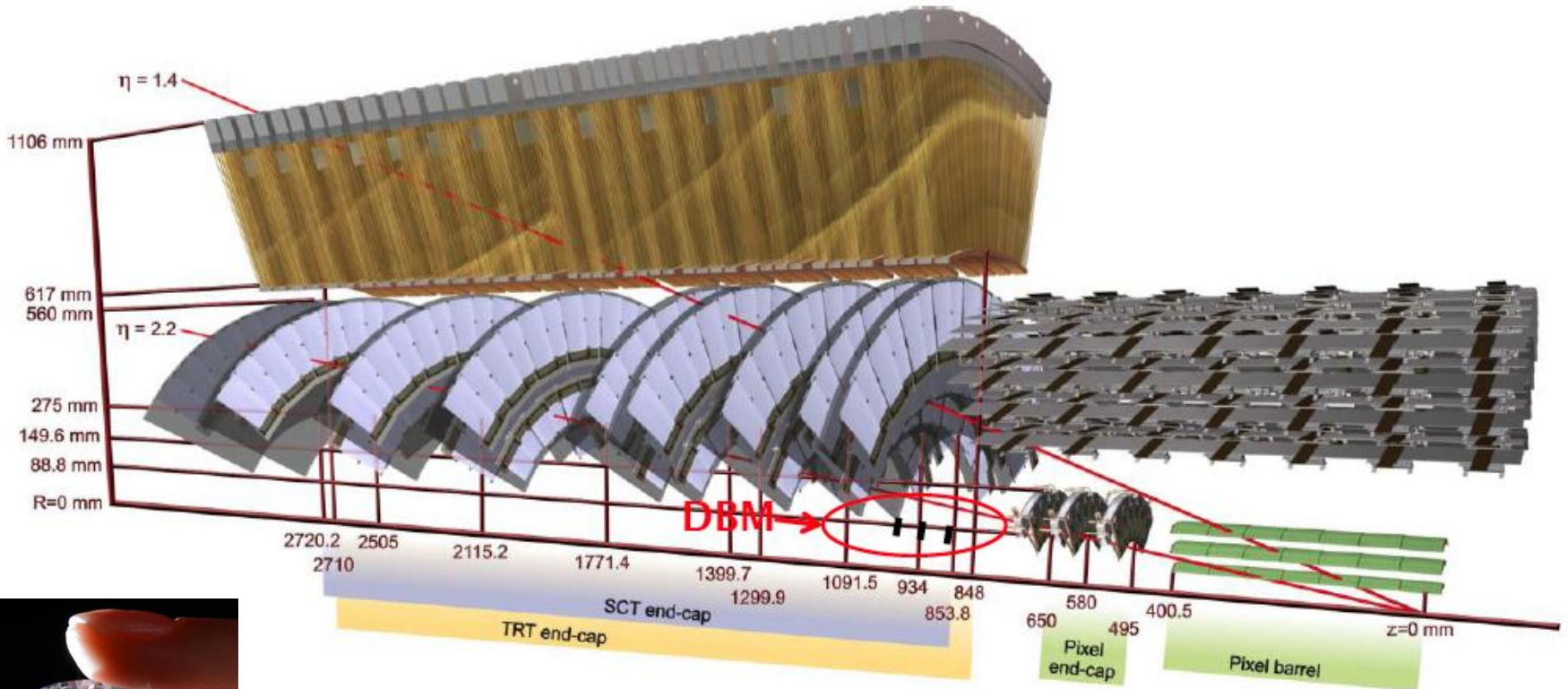
- Signal to noise ratio (SNR) has to be high enough
 - High signal
 - Low ionization energy \rightarrow small band gap
 - Low noise
 - Small number on intrinsic charge carrier \rightarrow large band gap

Requirements for solid state detectors

- **Diamond, ideal material**, band gap $E_g \approx 6\text{eV}$
 - Turn out to be expensive (even artificial diamond)
 - Used where extreme radiation hardness is needed
 - “a diamond is forever”
 - Can stand $\approx 10^{16}\text{p/cm}^2$
 - Beam condition monitors at LHC
- Large detectors being designed, to measure with high precision the luminosity at high intensities of LHC
 - Atlas Diamond Beam Monitor
 - CMS Pixel Luminosity Telescope



Atlas DBM



Weigh equivalent to this diamond (76 carats, 1 carat = 200mg)

Requirements for solid state detectors

- What if we use intrinsic silicon?
 - Ionization energy $I_0=3.62\text{eV}$
 - $dE/dx = 3.87 \text{ MeV/cm}$
 - Density of carriers at $T=300\text{K}$: $n_i=1.45\times 10^{10}/\text{cm}^3$
- Take a detector with
 - Thickness $d=300\mu\text{m}$
 - Surface $A=100\mu\text{m}\times 6\text{cm}=0.06\text{cm}^2$

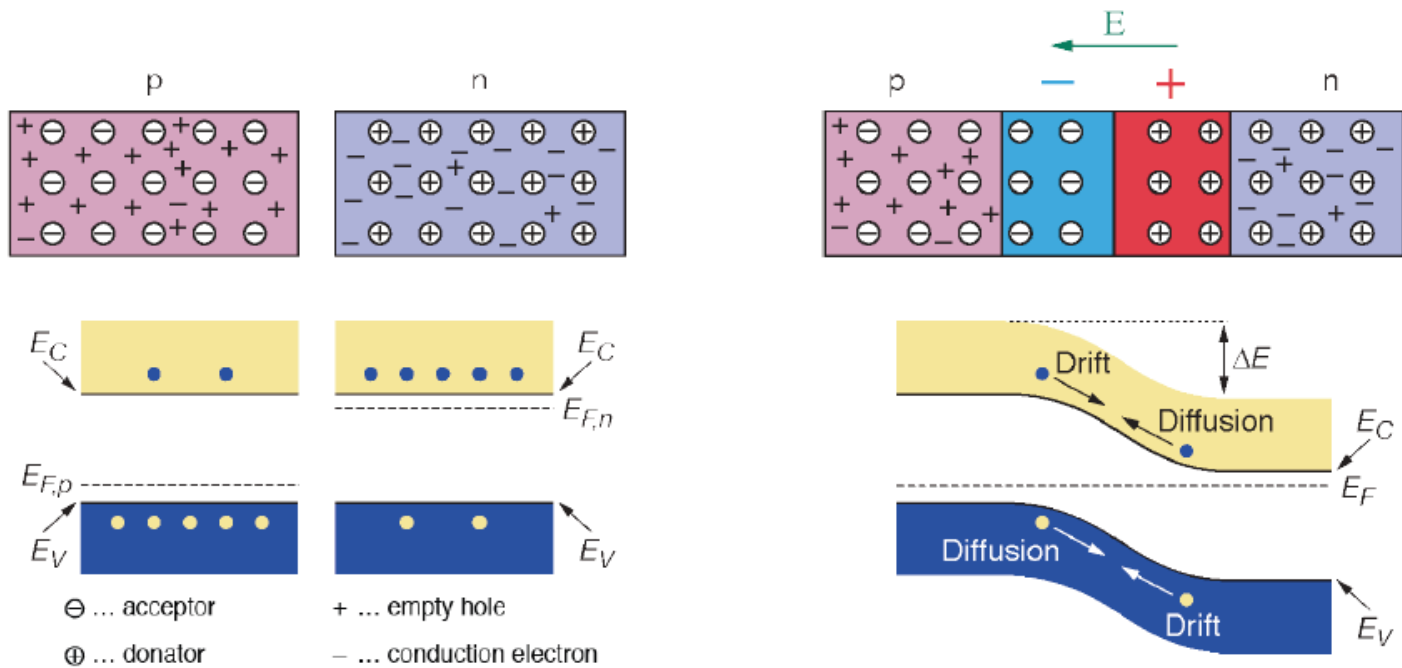
Signal $\frac{dE/dx \cdot d}{I_0} = \frac{3.87 \cdot 10^6 \text{ eV/cm} \cdot 0.03\text{cm}}{3.62\text{eV}} \approx 3.2 \cdot 10^4 e^- h^+ \text{ pairs}$

Noise $n_i \times d \times A = 1.45 \times 10^{10} \text{ cm}^{-3} \times 0.03\text{cm} \times 0.06\text{cm}^2 \gg 2.61 \times 10^7 e^- h^+ \text{ pairs}$

Noise is 3 order of magnitude larger than signal

- Need to remove intrinsic charge carriers
- **p-n junction with large depleted volume**

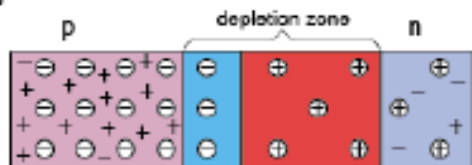
p-n junction



- Two semiconductors, doped p and n are put in contact
- Because of the gradient of the carrier densities, electrons diffuse to P zone, holes to N zone until the electrostatic field that is created stops the process
- Close to the junction there is now a region empty of carriers (depletion layer)

p-n junction

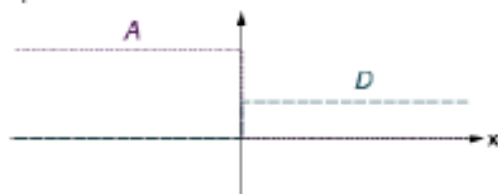
pn junction scheme



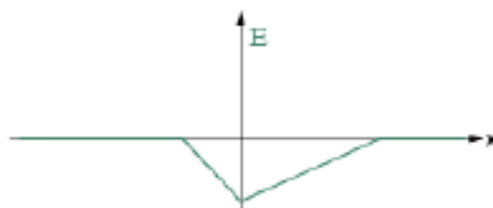
concentration of free charge carriers



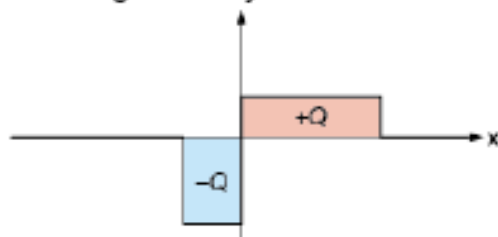
acceptor and donator concentration



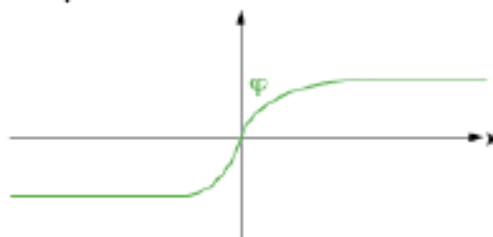
electric field



space charge density



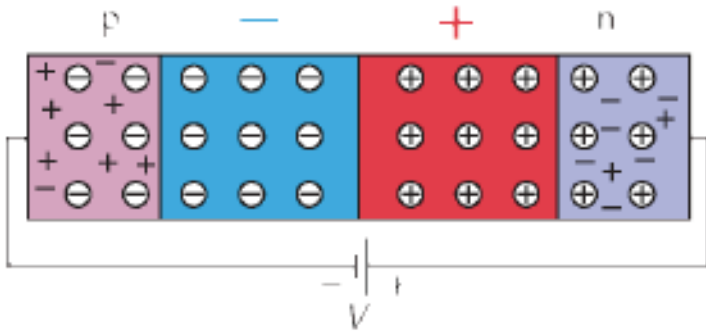
electric potential



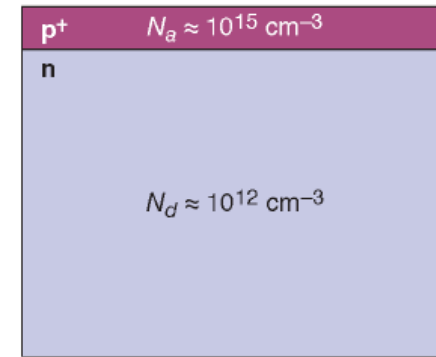
⊖ ... acceptor + ... empty hole
 ⊕ ... donator - ... conduction electron

p-n junction

p-n junction reversely polarized



- By applying an external **bias voltage** $V_N > V_P$ electron and holes move away from the depleted region making it bigger
- The current through the junction is small, the **depletion region can be used as a detector**



p+n junction

- Typical Si detector are largely asymmetric in term of dopant concentration
- The depletion region is asymmetric
- Its width **W** can be shown to be

$$N_A \gg N_D \quad \text{and} \quad x_P \ll x_N$$

$$W \gg x_N \gg \sqrt{\frac{2e|V|}{qN_D}}$$

depletion voltage and leakage current

- **Depletion voltage**

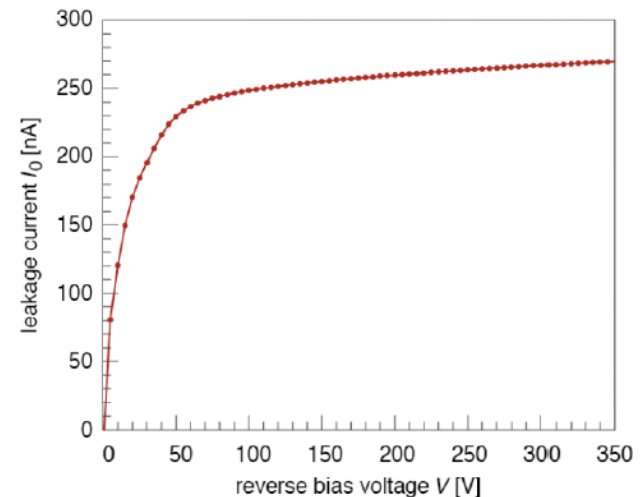
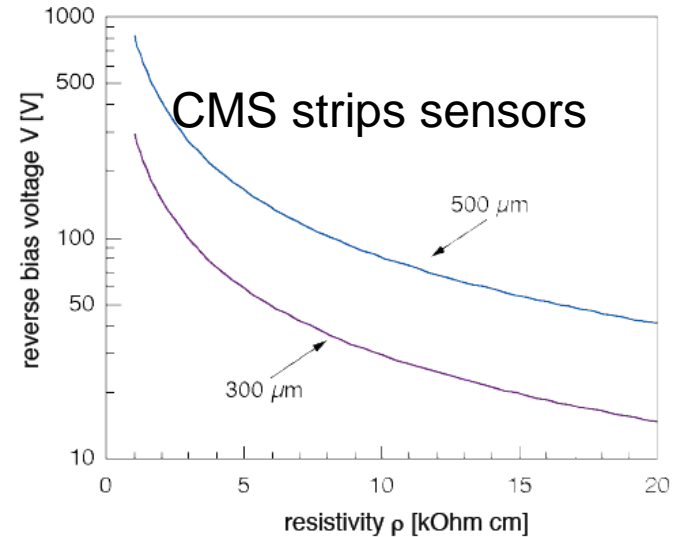
- Minimum voltage for which the device is fully depleted
- Normally one works slightly over-depleted

$$V_{depletion} = \frac{qN_D W^2}{2e}$$

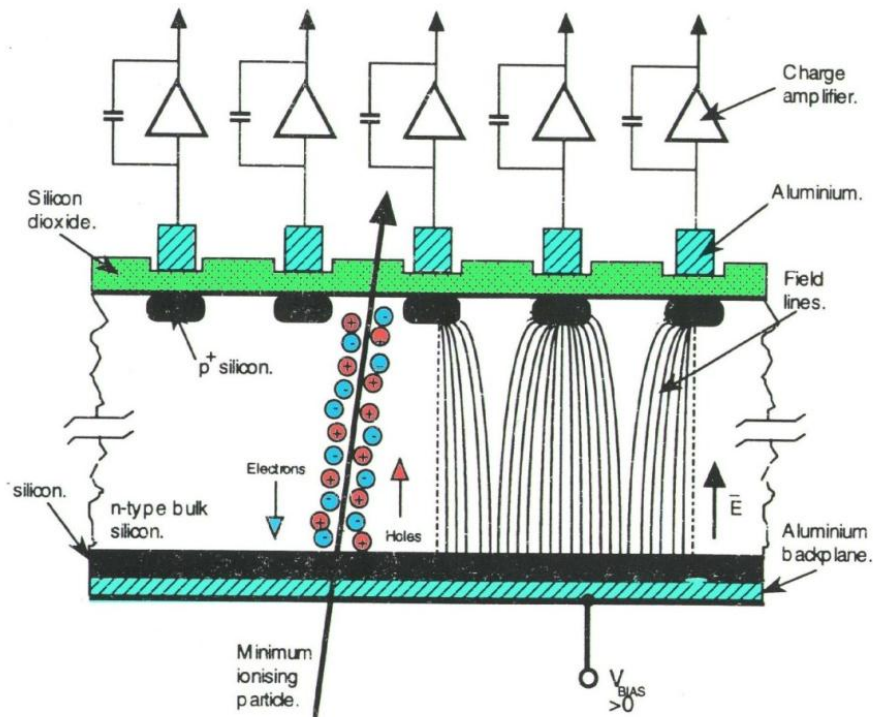
- Low doping of the bulk → High resistivity
→ Low depletion voltage

- **Leakage current**

- Dominated by the e/h pairs generated thermally
- They get separated by the E field and move to the electrodes
- It depends on the quality of the silicon, on the process and on the damages from radiation

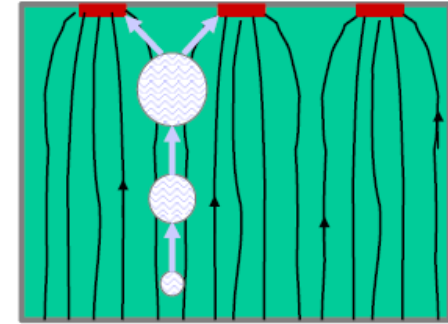


Si strip detector

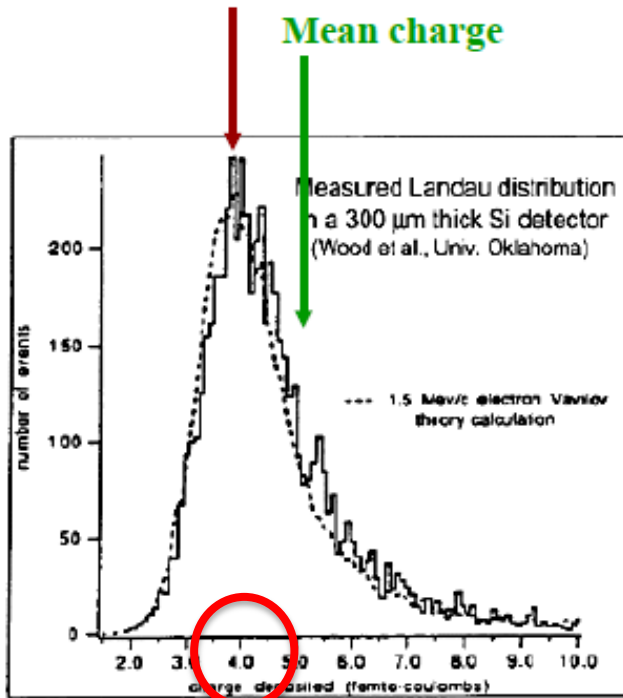


- Microstrip Si detector
 - A MIP releases 24000 e/h pair for a Si thickness of 300 μ
 - The pairs in the the depletion region drift in the E field creating the signal
 - The signal is small $\approx 4fC$ and need to be amplified
 - An amplifier is connected to *each* strip
 - From the signal on the strips one measures the position of the particle
 - Similar to a MWPC, but no internal amplification
 - MWPC: $100e^- \times 10^5 = 10^7 e^-$

Si strips signal



Most probable charge $\approx 0.7 \times$ mean



- Charge released in 300 μm
 - 32500 $e^- \approx 5.2\text{fC}$ (mean)
 - 24000 $e^- \approx 3.8\text{fC}$ (most probable)

- Collection time and diffusion

$$t = \frac{d}{v} = \frac{d}{mE} = \frac{d^2}{mV}$$

- With $d=300\mu\text{m}$, $E=2.5\text{kV/cm}$

$$t_e = 9\text{ns} \quad \text{fast}$$

$$t_h = 27\text{ns}$$

- While drifting the charge diffuses

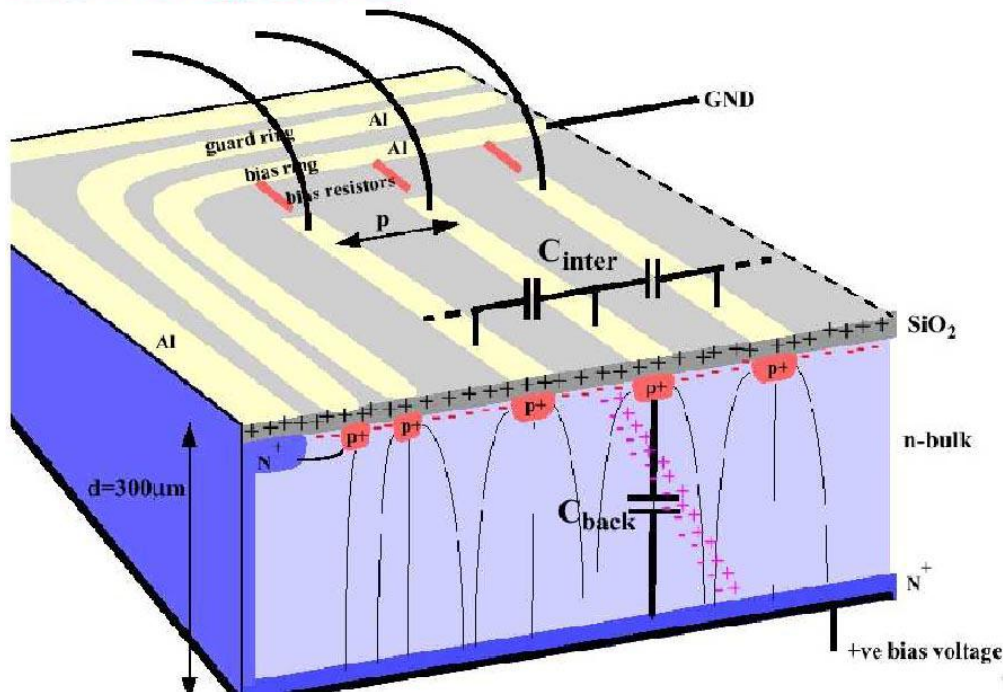
$$S_D = \sqrt{2Dt}$$

$$D = \frac{kT}{q} m$$

- Typical value $\sigma_D=6\mu\text{m}$

Si strips sensor

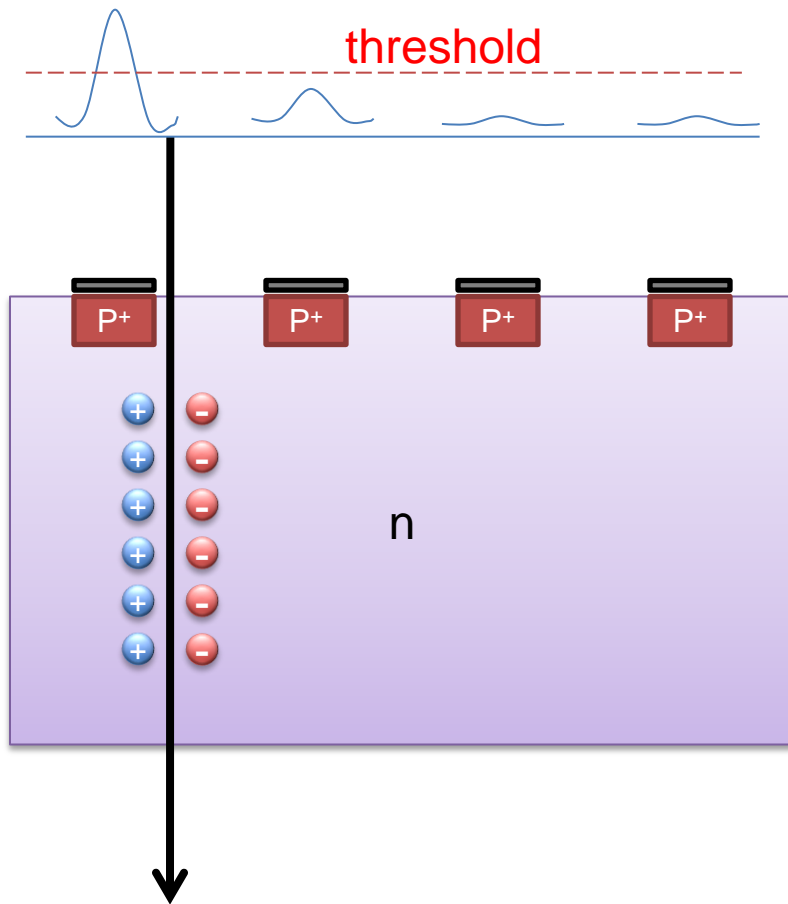
Sensor Design Baseline



Typical parameters

- Strip pitch 25-250 μ
- Thickness 300 μ
- DC or AC coupling of the strips
- P⁺n (n doped bulk)
 - $N_a \approx 10^{15} \text{ cm}^{-3}$
 - $N_d \approx 10^{12} \text{ cm}^{-3}$
 - $\rho > 2\text{k}\Omega$
- V 100V ($E=3\text{kV/cm}$)

Si strips resolution



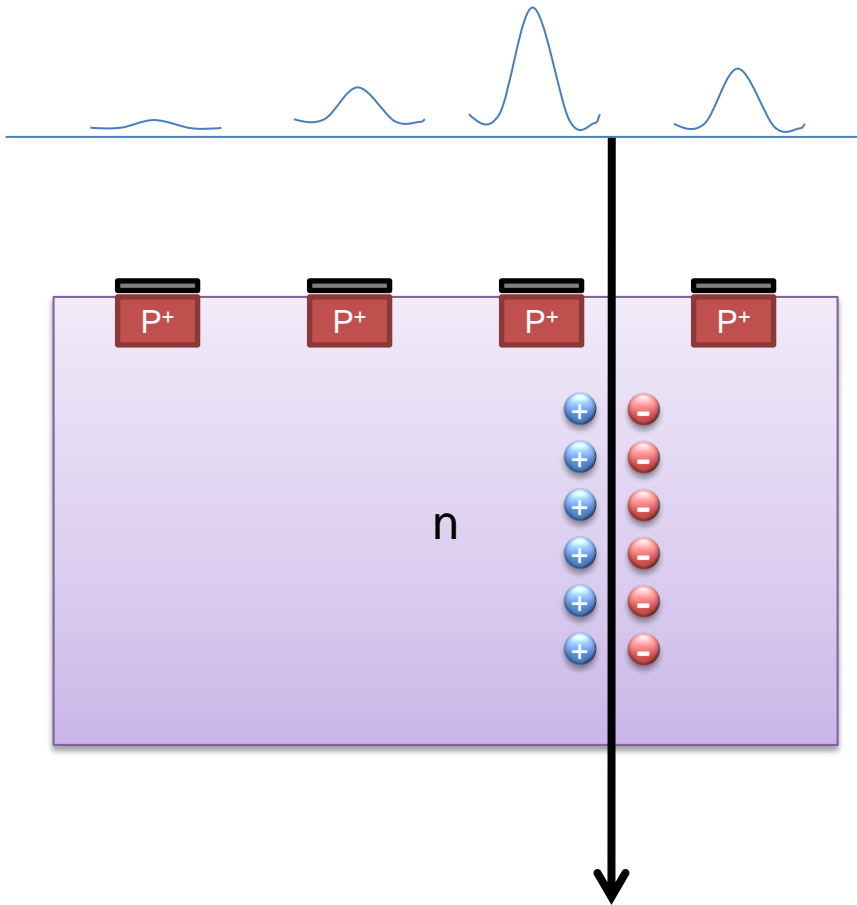
Binary readout

- Position = centre of the strip
- Resolution
 - If strip pitch = p

$$S_x \gg \frac{p}{\sqrt{12}}$$

- If $p = 50\mu\text{m} \rightarrow \sigma = 14\mu\text{m}$

Si strips resolution



Analog readout

- Position = centroid of the signal

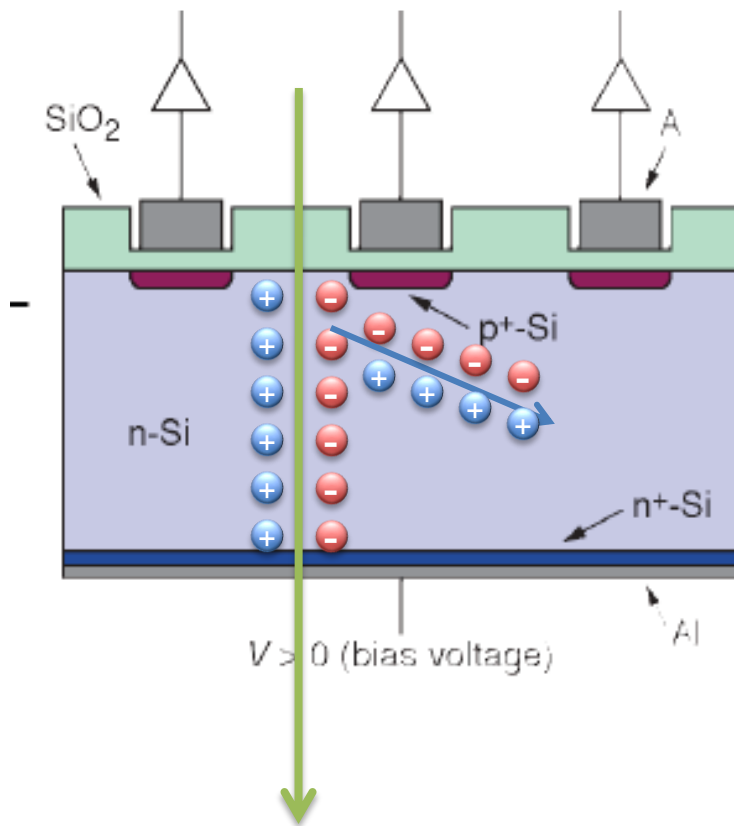
$$x = \frac{h_1 x_1 + h_2 x_2}{h_1 + h_2}$$

- Resolution

$$S_X \gg \frac{p}{SNR}$$

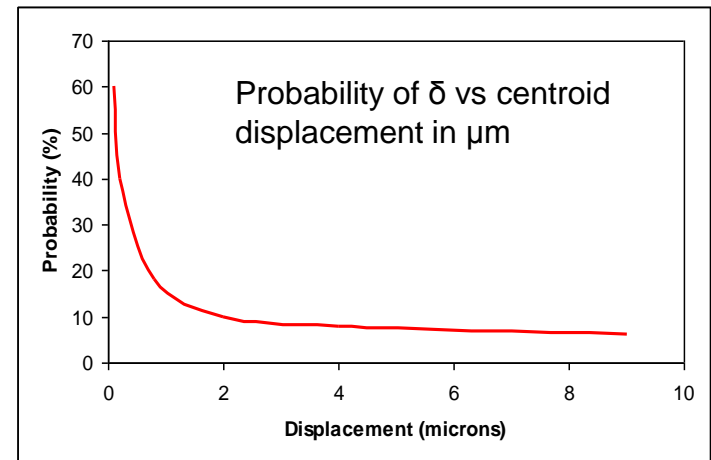
- $\sigma < 10\mu\text{m}$

Si strips resolution



δ rays can affect the position reconstruction

- Shift of the centroid by few μm

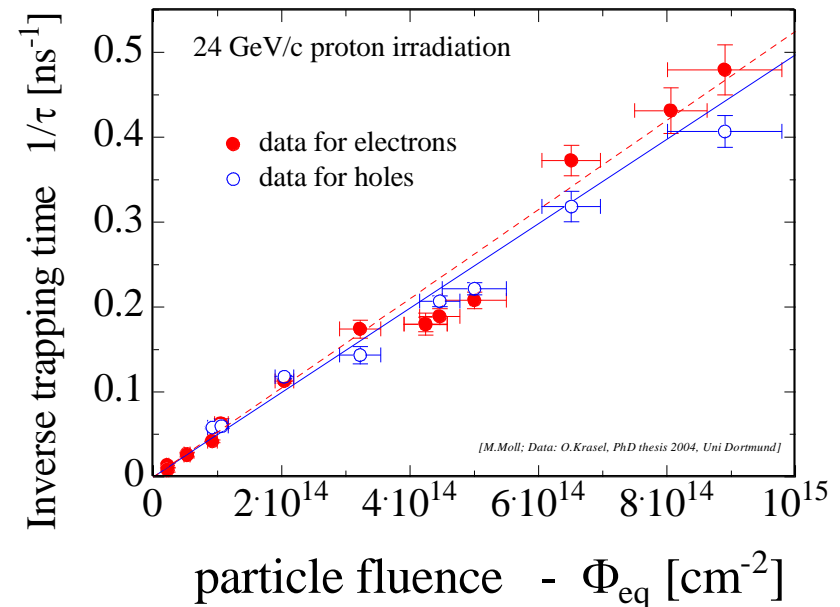


- Charge diffusion can instead help to increase the charge sharing between strip, better analogue resolution

Si radiation damage

- Lattice damage (Non Ionizing Energy Loss)
 - Decrease of charge collection efficiency
 - Changes in depletion voltage
 - Larger V , not full depletion
 - Increase of leakage current
- Surface damage (Ionizing Energy Loss)
 - Trapping of charges is the SiO_2 layers
 - Noise, breakdown

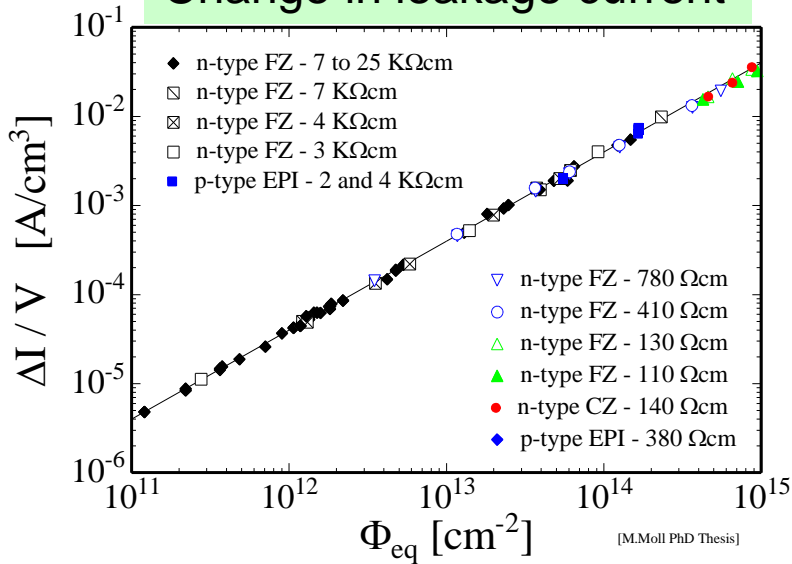
Deterioration in Q collection



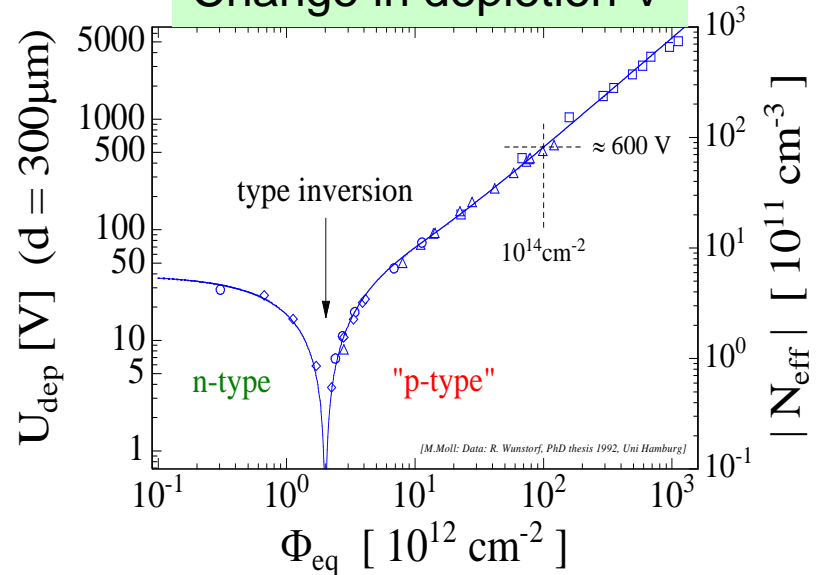
Caused by trapping from radiation induced defects

Si radiation damage

Change in leakage current



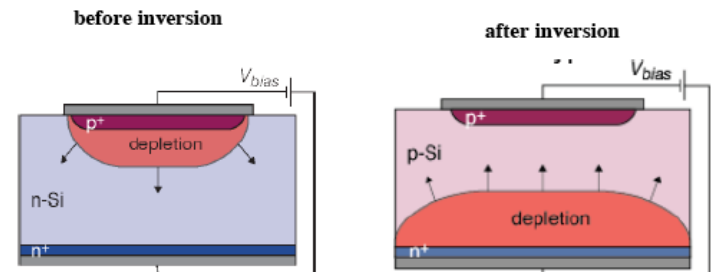
Change in depletion V



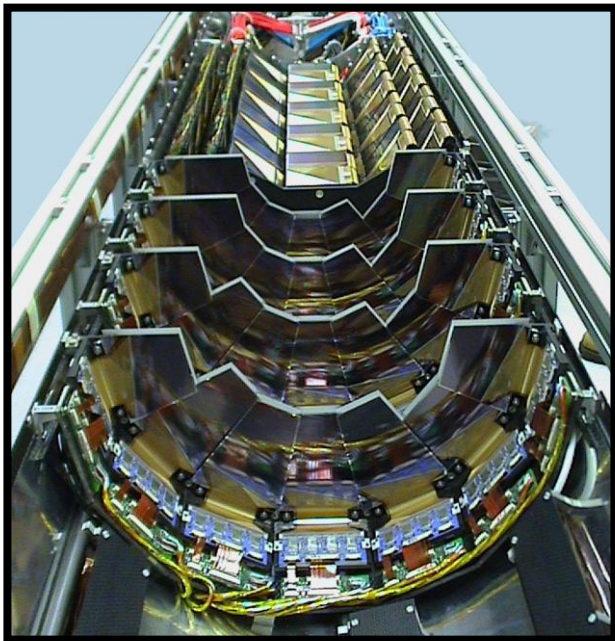
$$a = \frac{DI}{V \cdot F_{eq}}$$

Damage parameter a

- Change of leakage current per unit of volume and fluence
- Constant over many order of magnitudes of fluence

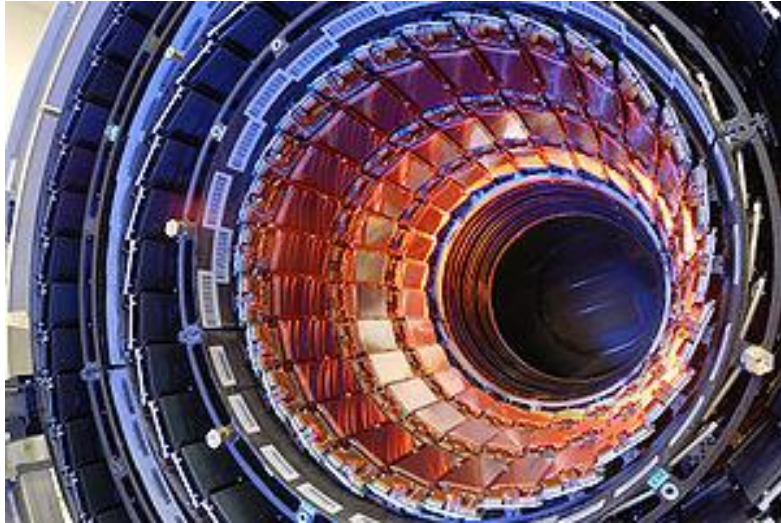


Example: ZEUS MVD (yr 2000)

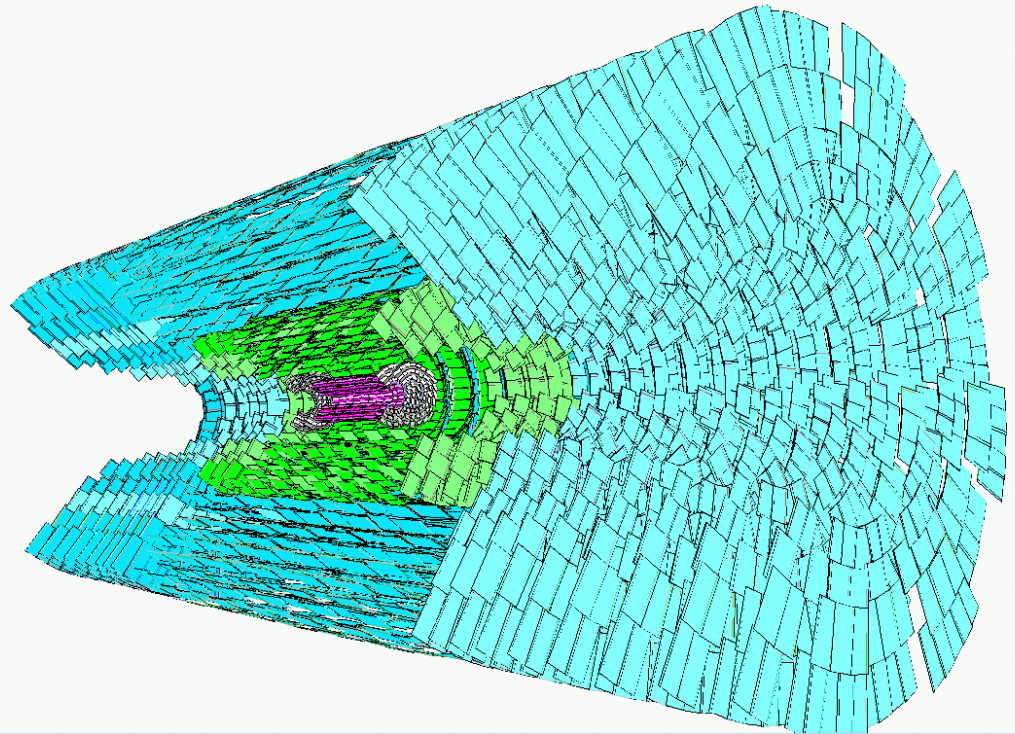


- 3 barrel layers, 2 forward wheels
 - Outer diameter 25cm
 - Length $\approx 1\text{m}$
 - Resolution $\approx 15\mu$ for normal tracks
 - $\approx 3\%$ X_0 per layer
 - $\approx 2.5\text{m}^2$ of Si planes

Example: CMS tracker

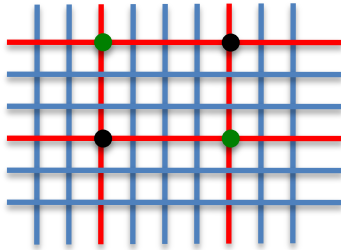


- 10 barrel layers and 2x9 end cap layers
- 223m² of Si sensors
 - 600 thin (300 μ) sensors,
 - 20000 thick (500 μ) sensors
- 10 millions channels



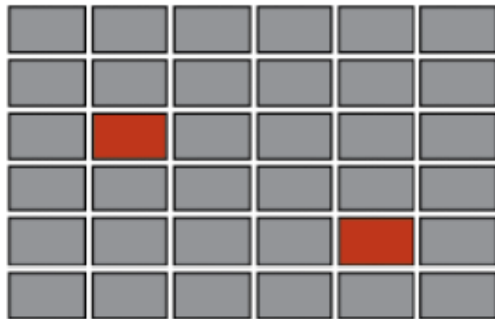
Pixel detectors

true



ghosts

- In case of bi-dimensional x-y readout, high hit density generates ghost hits
- Pixel detectors solve this ambiguity



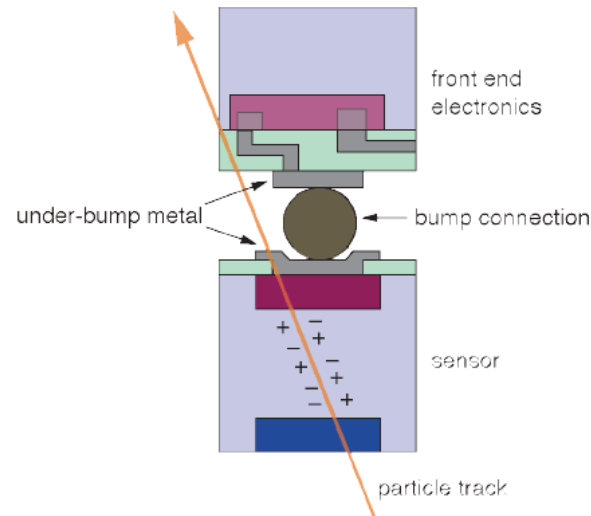
real tracks

Advantages

- Small area → small capacity → large SNR
- Small volume small dark current/channel

Disadvantages

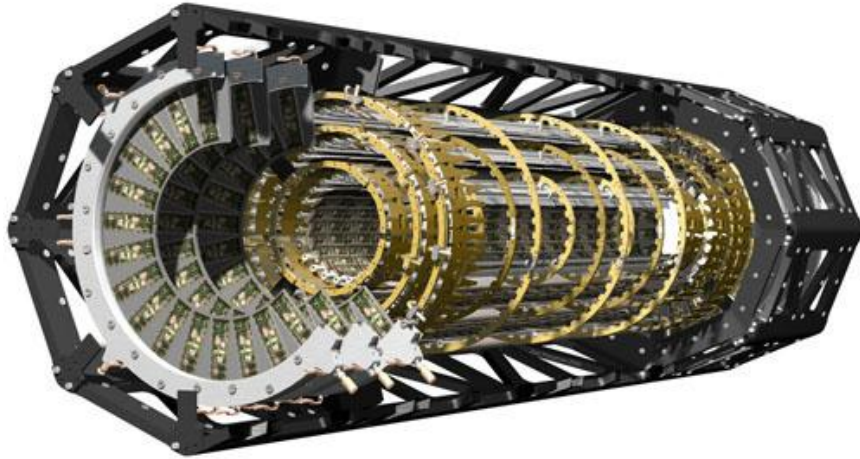
- Large number of channels (N^2 compared to strip readout)
- Large number of electrical connections and amplifiers
 - Big power dissipation



Bump-bonding to electronics

- Expensive
- Limit pixel size
- Increases material budget (X_0)

Pixel detectors

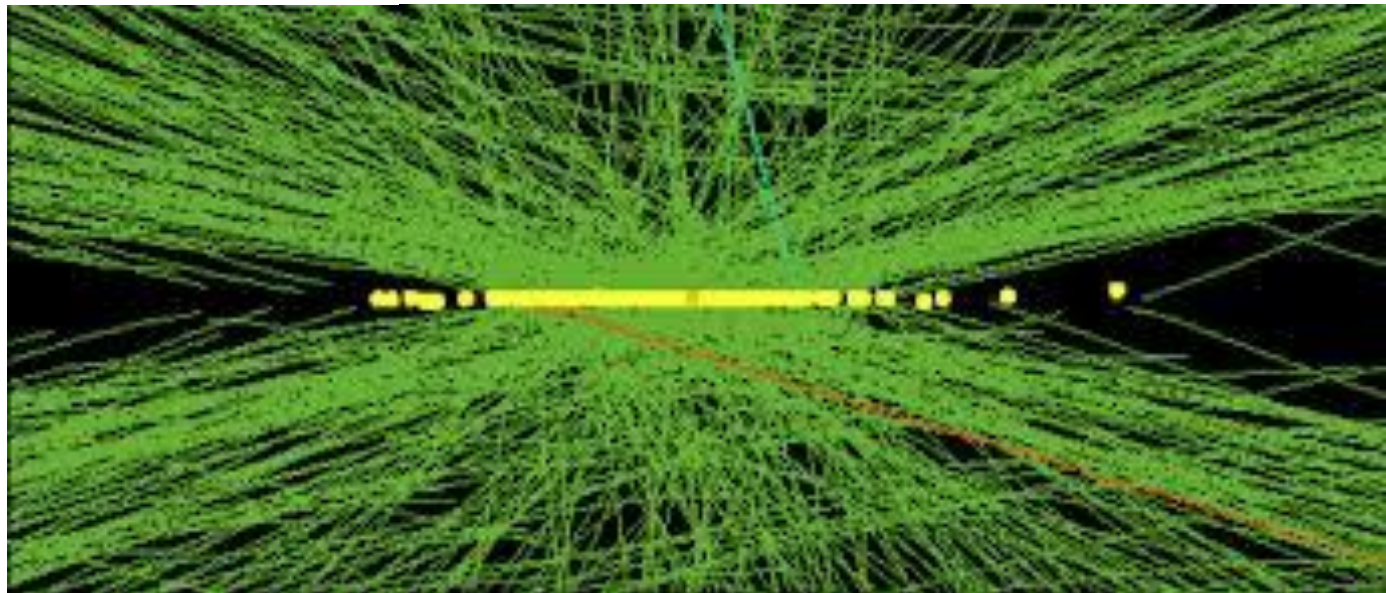


Atlas pixel detector

- 80 million channels
- 1.7 m²

Largely used in the central regions of the LHC experiments

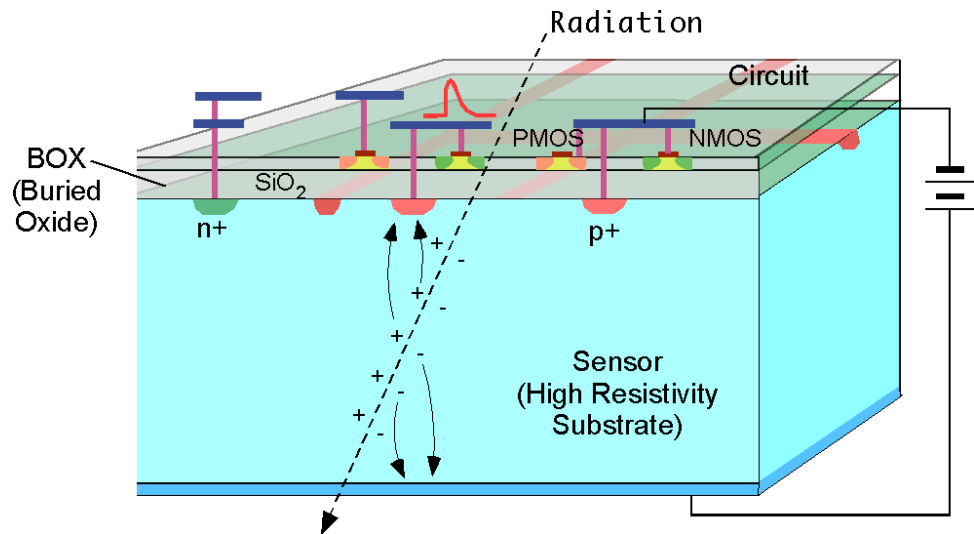
- Very high density of particles close to the interaction point
- In 2012, pile-up (number of overlapping event) up to 35 average



Developments: monolithic pixel detectors

Is it possible to integrate on the same Si the sensor and its electronics?

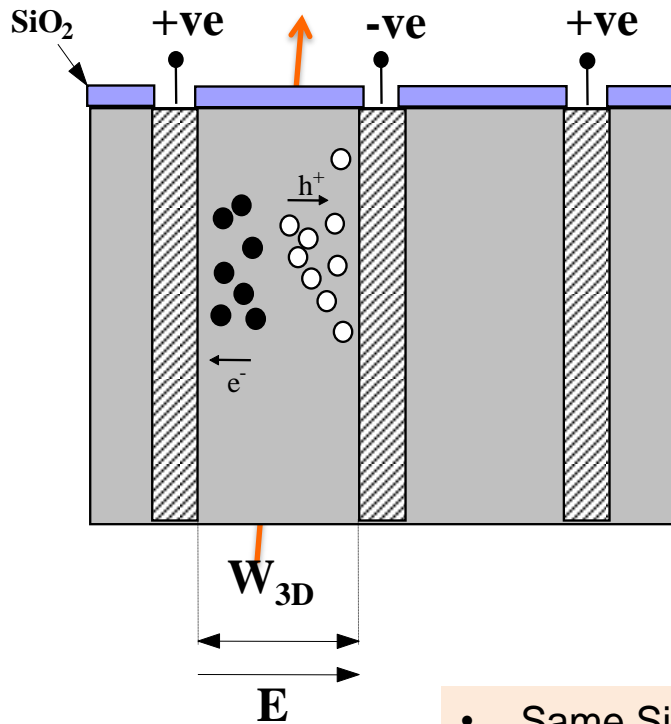
- **Detectors** → need large signals, large depletion regions → **high resistivity** (low doping)
- **Electronics** → large integration in small spaces → small junctions → **low resistivity** (high doping)



MAPS SOI (Silicon On Insulator)

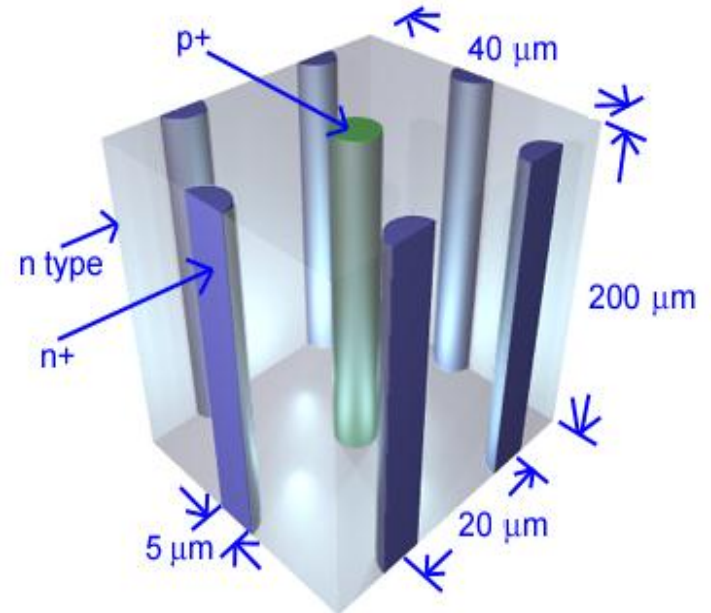
- Commercial process, electronics separated from the wafer by a small (200nm) layer of SiO₂
- High-resistive substrate, holes through the oxide, P⁺ implants → apply depletion V and collect charges
- **Problems**
 - Coupling between electronics and depletion voltage
 - Sensitive to ionizing radiation (charge trapped in the SiO₂ layers)

3D detectors



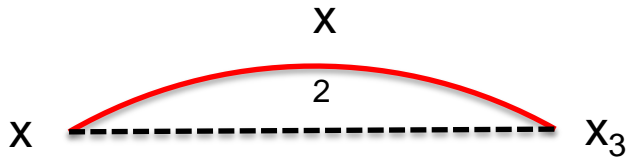
Candidate for the new inner barrel layer of Atlas pixel detector

Bulk



- Same Si thickness of the 2D detectors
 - Same signal
- Carriers move laterally
 - Low bias V and fast collection time if electrodes are close
 - Detector thickness becomes an independent parameter
- **More complex fabrication process**

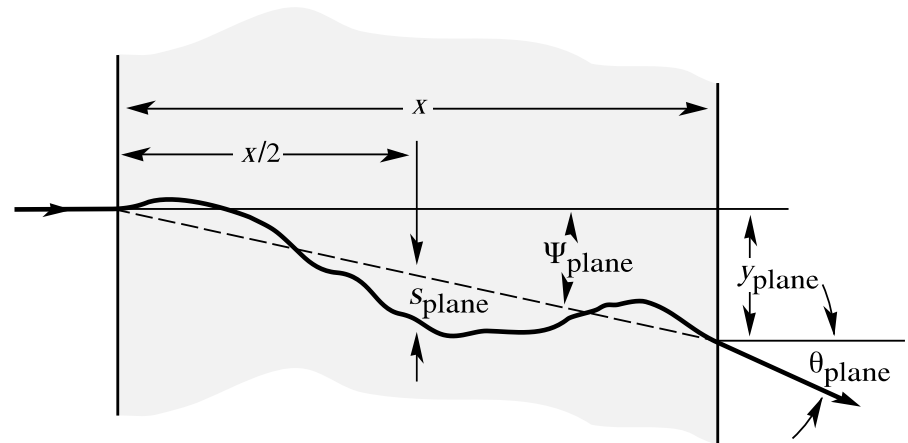
Multiple scattering



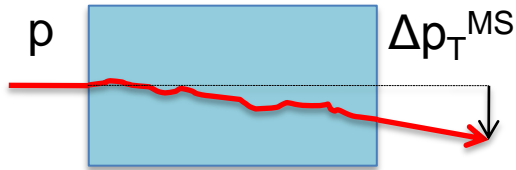
$$\frac{S(p)}{p} = \frac{8\sqrt{\frac{3}{2}}S(x)}{0.3BL^2} p$$

- Many choices of tracking detectors with resolution from $\approx 1\text{mm}$ to $\leq 10\mu\text{m}$
 - Is it all we need to take into account?

- **No**
 - Even at infinite detector resolution, the momentum determination is limited by the effect of scattering of the particle in the detector



Multiple scattering



$$q_{RMS}(L) \approx \frac{14 \text{ MeV}}{p \cdot cb} \sqrt{\frac{L}{X_0}} \longleftrightarrow q_{Bending} = \frac{0.3}{p} \int B dl$$

considering $b \approx 1$

$$Dp_T^{MS} \approx p q_{RMS} \approx 14 \sqrt{\frac{L}{X_0}} \text{ MeV} / c$$

$$\left. \frac{\sigma(p)}{p} \right|^{MS} = \frac{Dp_T^{MS}}{Dp_T^{Bending}} = \frac{14 \sqrt{\frac{L}{X_0}} \text{ MeV} / c}{p q_{bending}} = \frac{14 \sqrt{\frac{L}{X_0}} \text{ MeV} / c}{0.3 \int B dl}$$

- The contribution of the MS to $\sigma(p)/p$ is a constant term, does not depend on p
 - So it limits the resolution at low p
 - Is very important when bending is in iron (muon detectors)

Examples

In Iron

Fe: $X_0 = 1.76\text{cm}$ $B = 1.8\text{T}$

$$\left. \frac{S(p)}{p} \right|^{MS} = 0.19 \frac{1}{\sqrt{L[m]}}$$

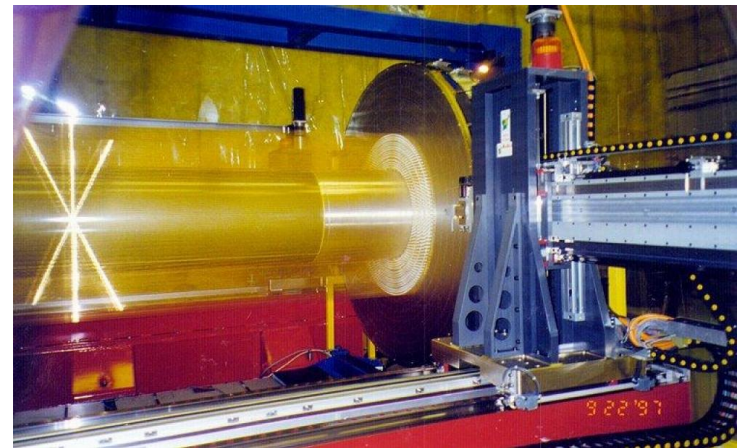
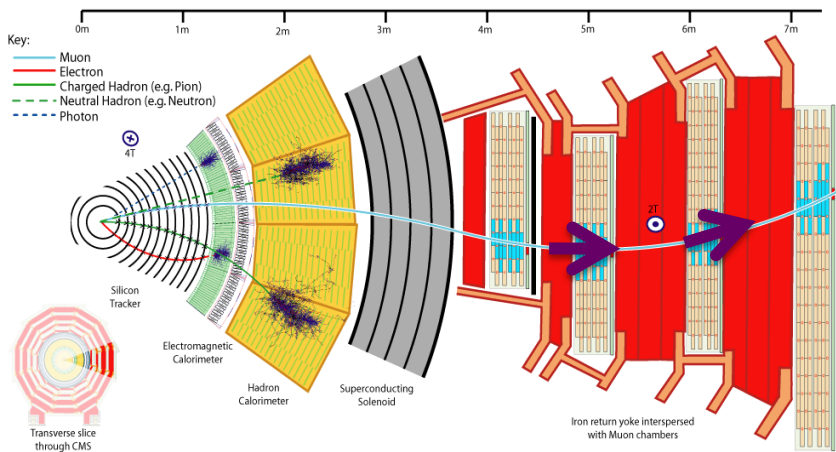
$$L = 3\text{m} \rightarrow \left. \frac{S(p)}{p} \right|^{MS} = 11\%$$

In gas

Air: $X_0 \approx 300\text{m}$ $B = 1.8\text{T}$

$$\left. \frac{S(p)}{p} \right|^{MS} = 1.4 \cdot 10^{-3} \frac{1}{\sqrt{L[m]}}$$

$$L = 1\text{m} \rightarrow \left. \frac{S(p)}{p} \right|^{MS} = 0.14\%$$



Tracking resolution summary

- In general the resolution of a tracking detector is the sum of two terms
 - For example, for the central drift chamber of ZEUS it was $\frac{s(p)}{p} = 0.005 p \oplus 0.007$
- Depending on the momentum range, one can optimize
 - For low momentum, optimize the radiation length
 - E.g. Babar used helium as noble gas
 - At high momenta the term proportional to p dominates
 - Need to increase B , lever arm and resolution
 - E.g. CMS is using a full-silicon central detector, certainly not optimized for dead material

