TOWARDS SIMPLICITY IN MULTI-JETS

LISHEP 2013

Rio de Janeiro

Erik Gerwick

March 19th, 2013

Georg-August-Universität Göttingen

OUTLINE

- Exclusive multi-jets in LHC analyses
- The concept of idealized scaling
- Relevance to experimental analyses
- Conclusions

Based on:

EG, Schumann, Gripaios, Webber; "QCD Jet Rates with the Inclusive Generalized kt Algorithms" JHEP, arXiv:1212.5235.

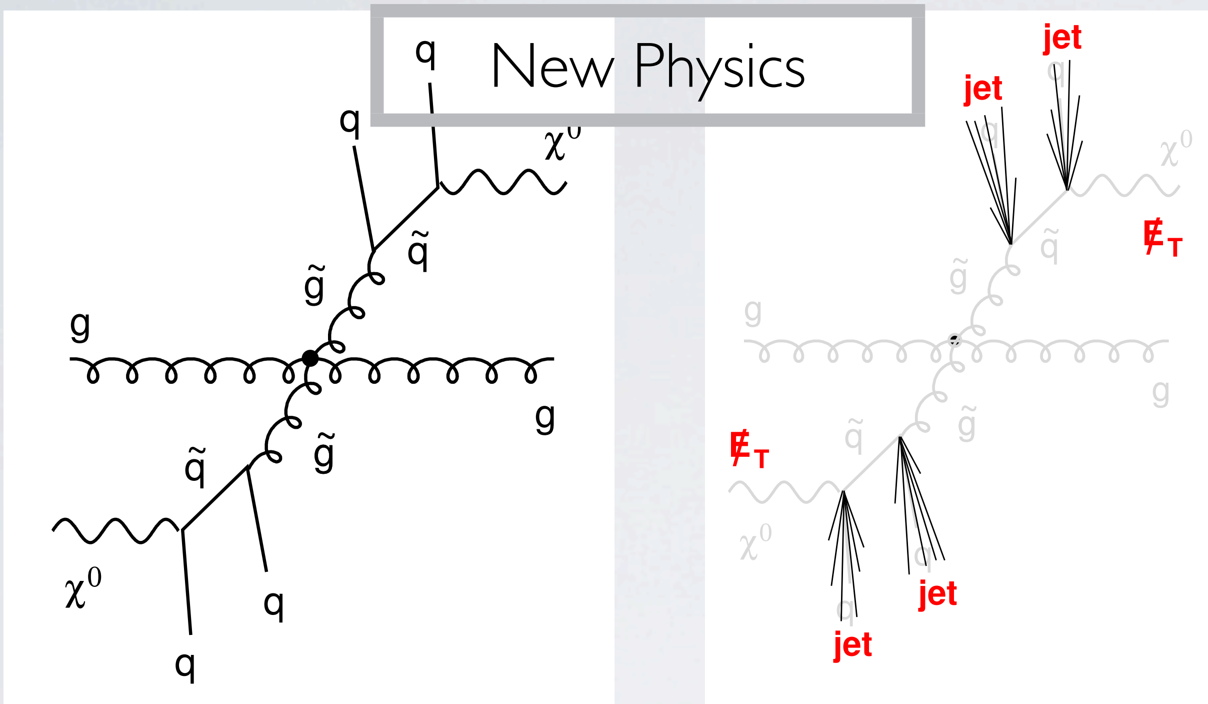
EG, Plehn, Schumann and Schichtel; "Scaling patterns for QCD jets", JHEP **1210**, 162 (2012), arXiv:1208.3676

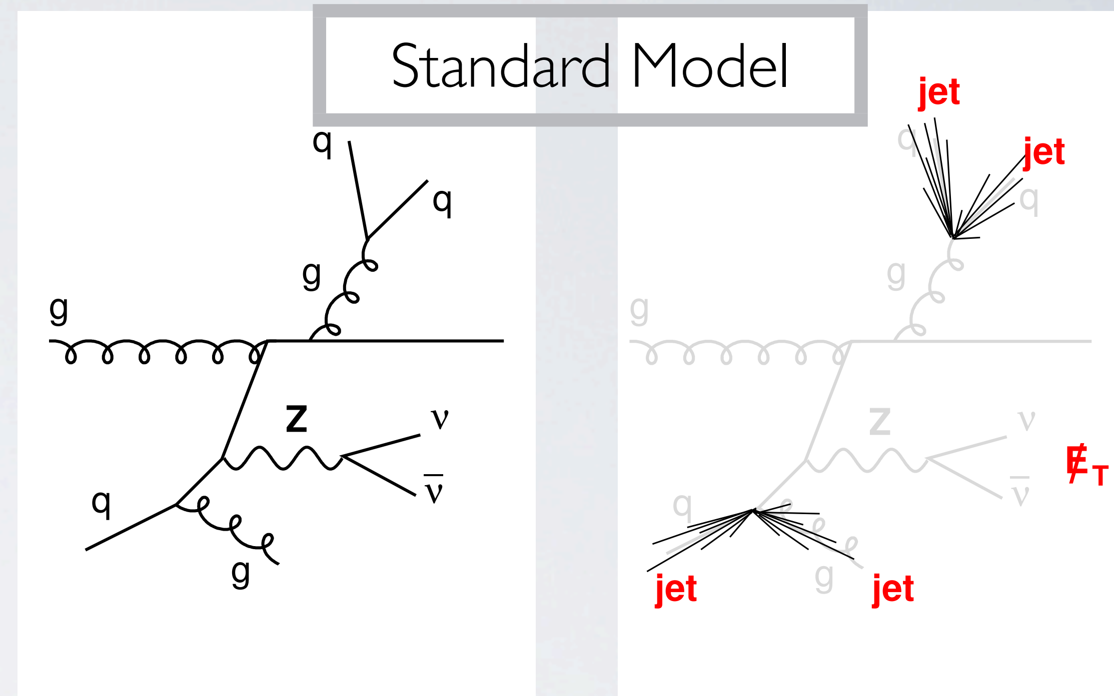
EG, Plehn and Schumann; "Understanding Jet Scaling and Jet Vetos in Higgs Searches", Phys. Rev. Lett. **108**, 032003 (2012) arXiv:1108.3335

MULTI-JET RATES AT THE LHC

Jet intensive final states and the search for new physics

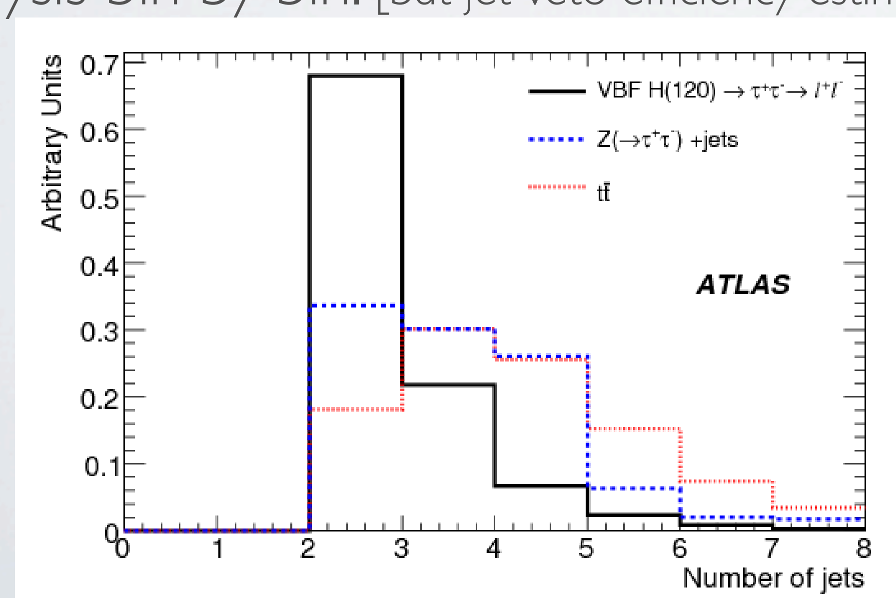
- Multi-jet final states an interesting and challenging search for new physics.





Exclusive jet bins in analyses ⟺ jet vetos

- Many analyses rely on dividing the event sample into jet multiplicity bins and optimize analysis bin by bin. [but jet veto efficiency estimate can and often does introduce large uncertainties]



Analysis type	Excl. jet bin
Higgs WW*	0,1 jet
Higgs WBF	2 jet
Di-boson	0,1 jet
Top mass	4 jet
New physics	4,8,n? jet

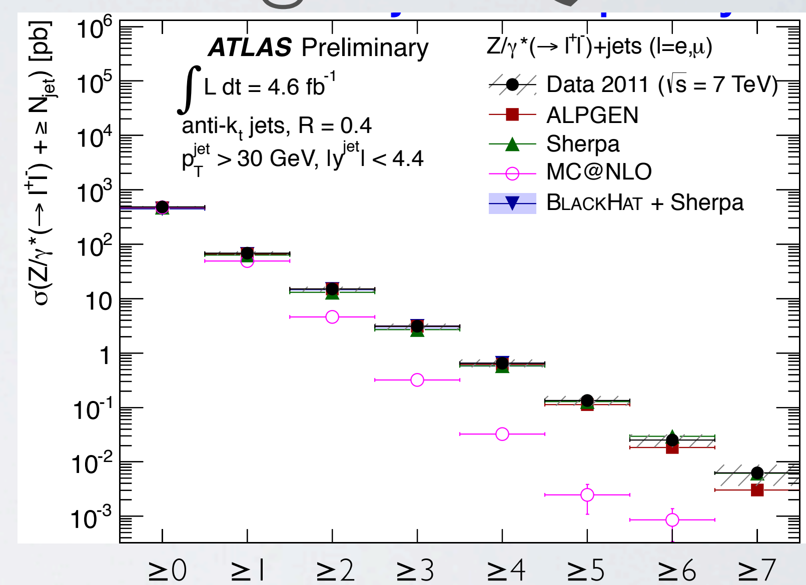
RATIOS OF EXCLUSIVE JET RATES

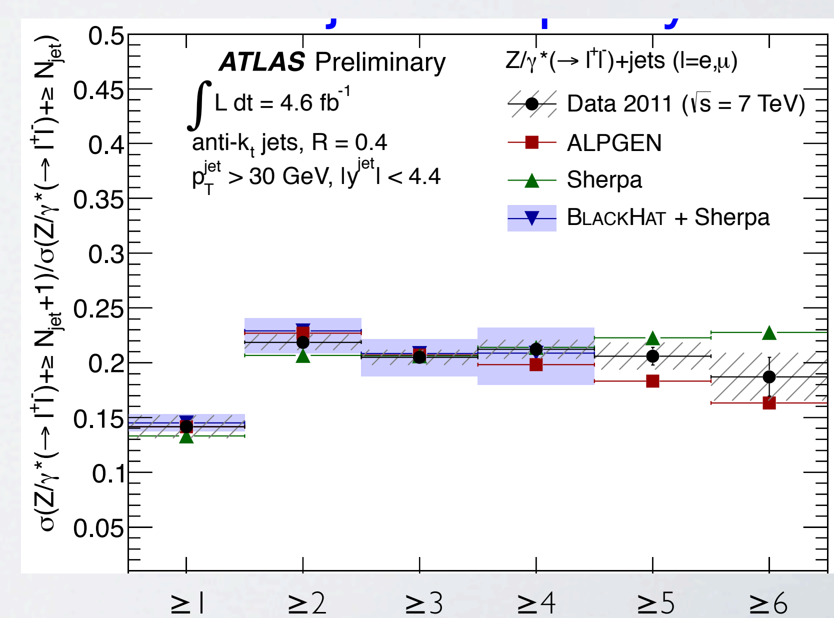
Status of theoretical prediction

- For exclusive rate predictions need resummed jet rates. [analytic resummation vs. parton shower]
- But rather than take a direct approach we will study the ratio of exclusive jet cross-sections.

$$R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n}$$

Hassani @ Moriond QCD 2013





Reasons for studying ratio instead of rate

- Experimentally: systematics tend to cancel.
- Theoretically: scale uncertainties are generally weaker.
- Most importantly: visually much easier to see patterns...rates not so interpretable.

TOWARDS SIMPLICITY

IDEALIZED N_{JET} DISTRIBUTIONS

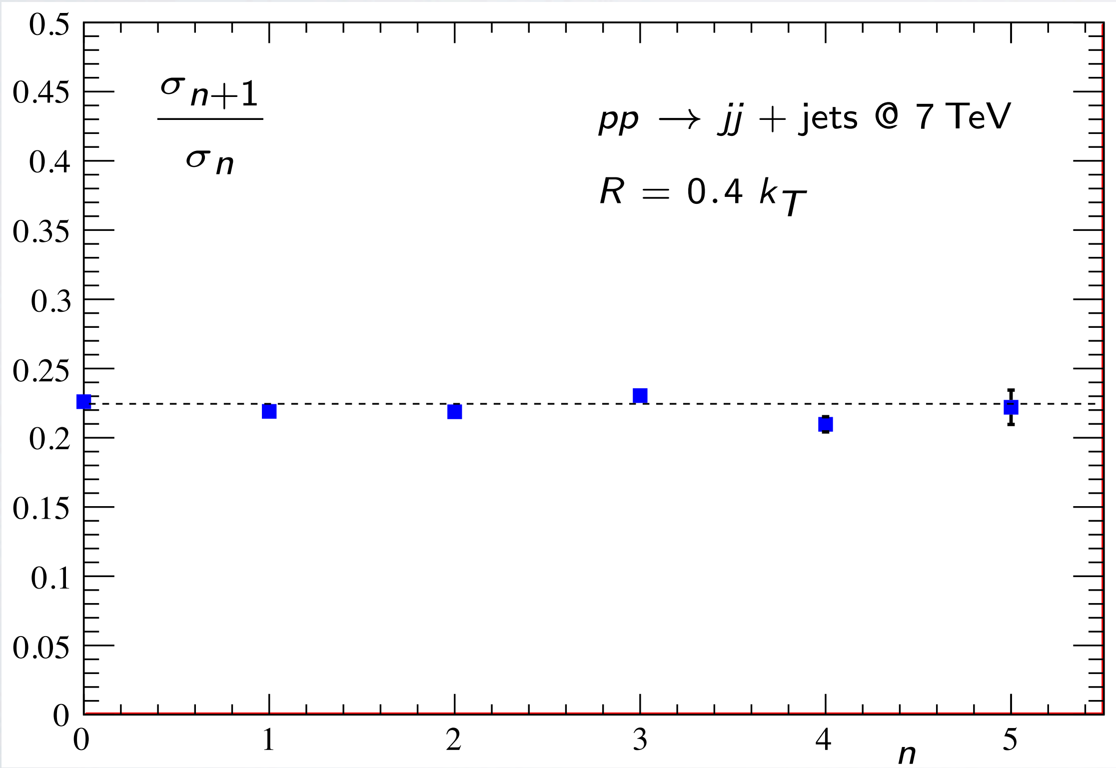
Staircase [Steve Ellis, Kleiss, Stirling (1985), Berends, Giele (1989)]

$$\sigma_n^{\text{excl}} = R_{1/0}^n \equiv e^{-bn}$$

- Ratios are constant (geometric)

$$\frac{\sigma_{n+1}}{\sigma_n} = e^{-b}$$

- Observed: UA1, Tevatron, LHC



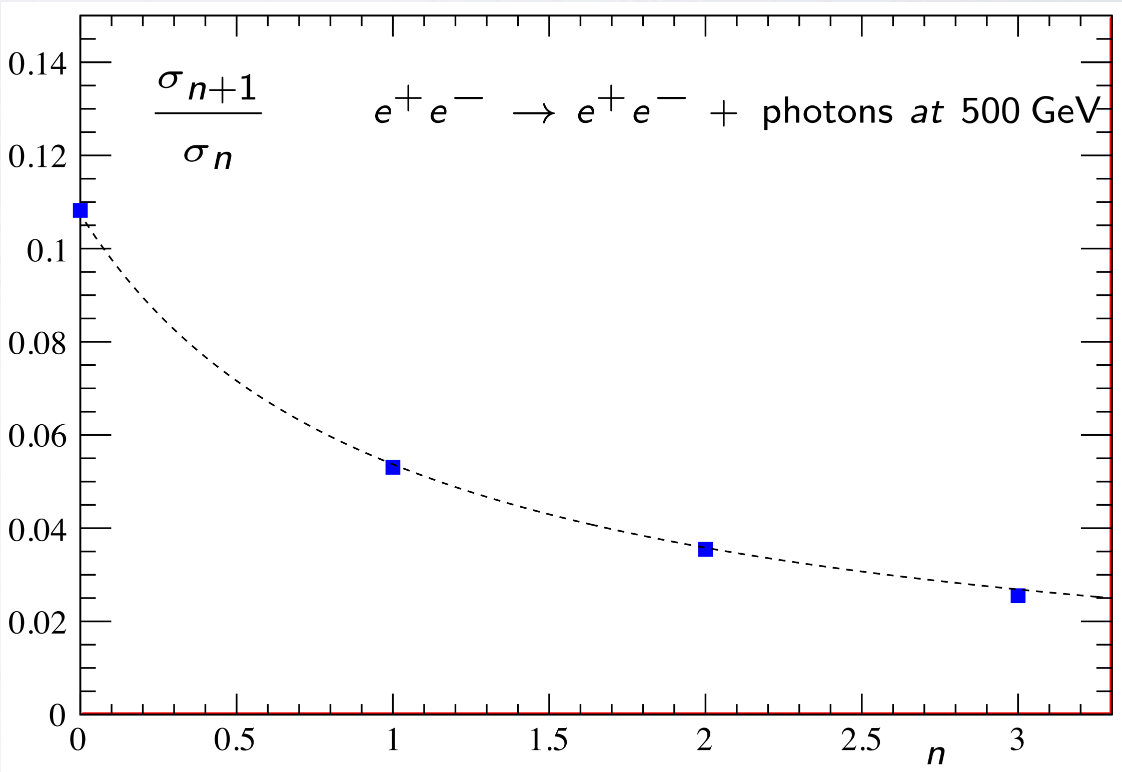
Poisson [Rainwater, Zeppenfeld (1997); EG, Plehn, Schumann (2011)]

$$\sigma_n^{\text{excl}} = \frac{e^{-\bar{n}}\bar{n}^n}{n!}$$

- Ratios are not constant

$$\frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1}$$

- Observed: WBF like cuts, γ's at LEP



Plots made with SHERPA

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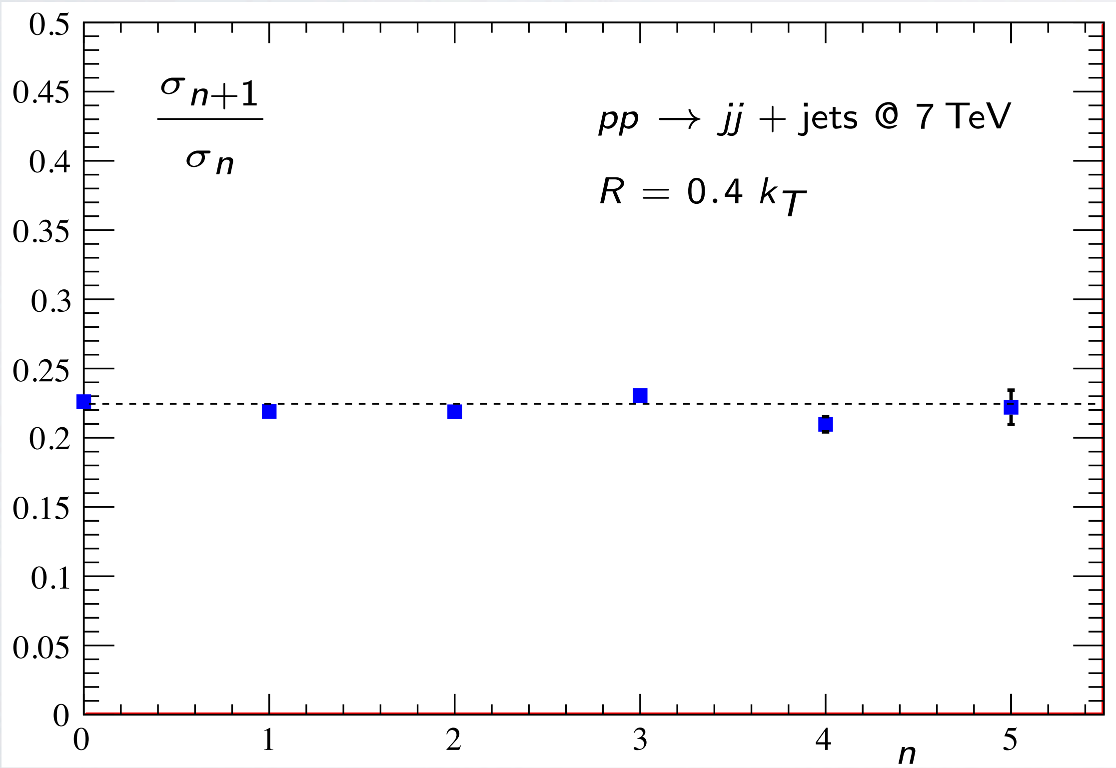

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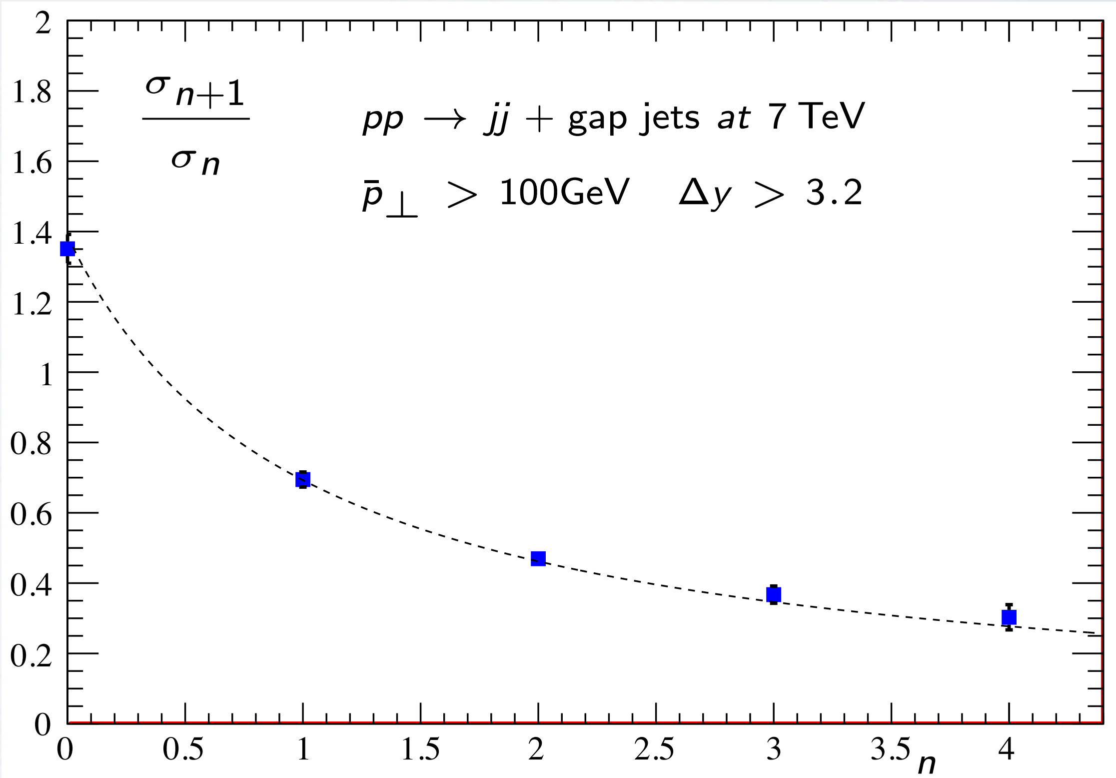


Plots made with SHERPA

SCALING BOTTOM LINE

Definition:

The shape of the ratio distribution defines the scaling, a recipe for extrapolating the ratios. [i.e. we don't worry about normalization of the ratios]

Dominant causes of scaling:

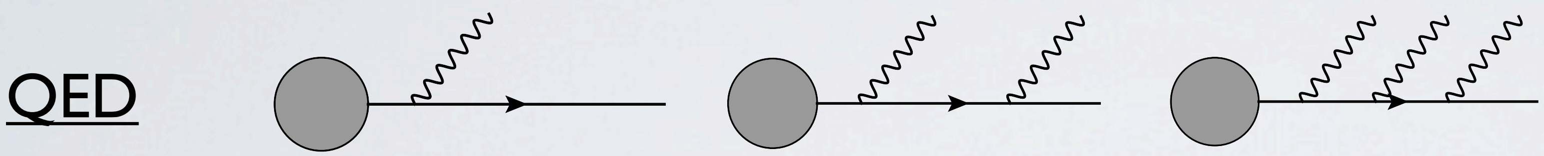
1. The relative contribution of the primary and secondary splitting component.
2. PDF "effect".

BASIS OF SCALING

Explanation from primary versus secondary splittings

- Old result that even in Eikonal limit gluons violate Poisson statistics [Kuraev, Fadin 1978]
- Consider massless gauge boson emission in the Durham jet algorithm for limit α_S $\mathbf{L^2} \ll \mathbf{1}$

[L=log(1/y_{cut})]



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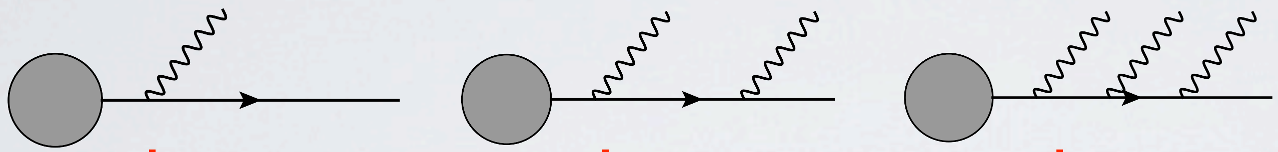

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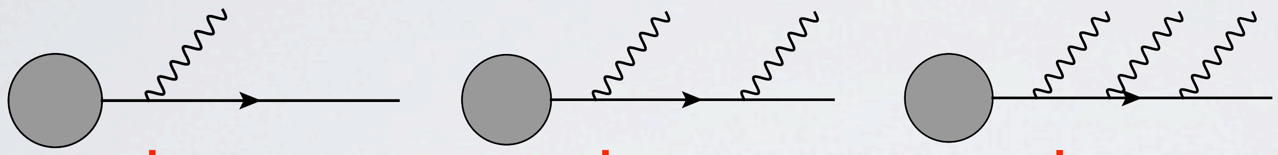

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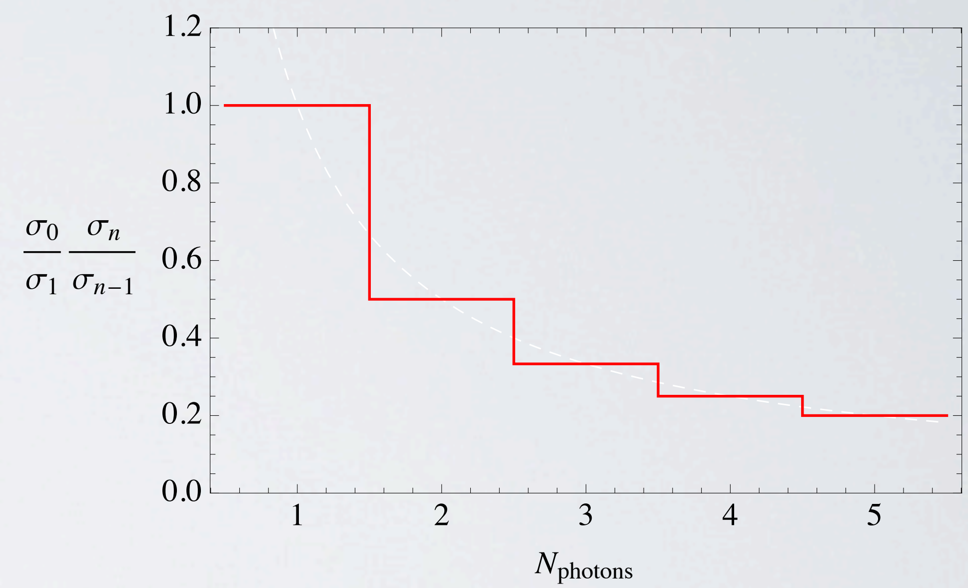

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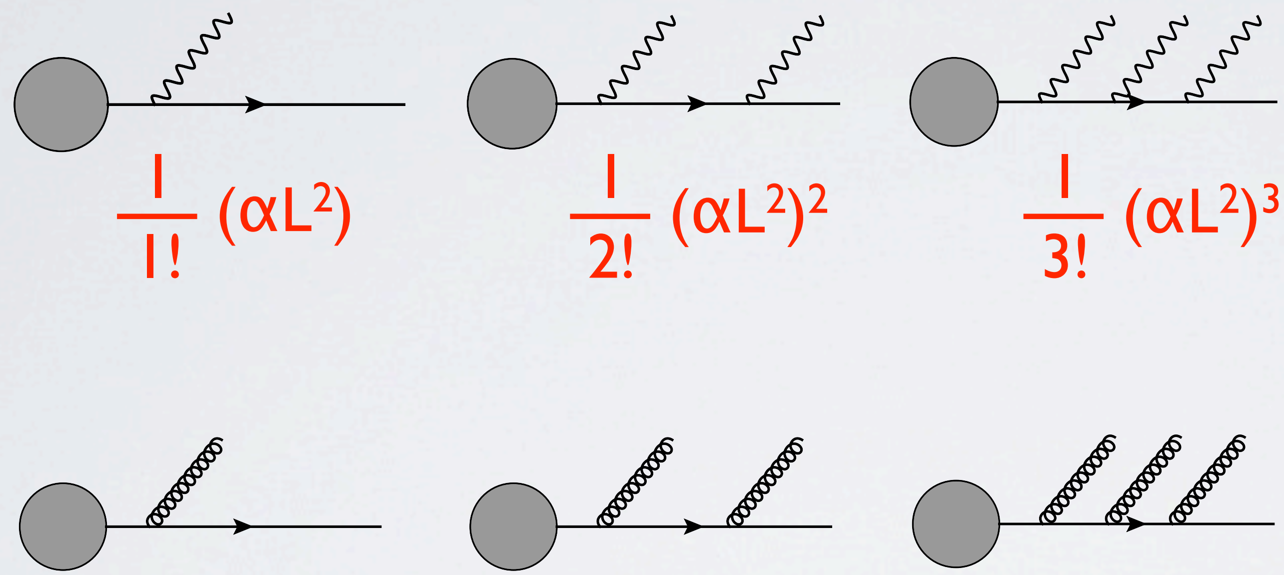

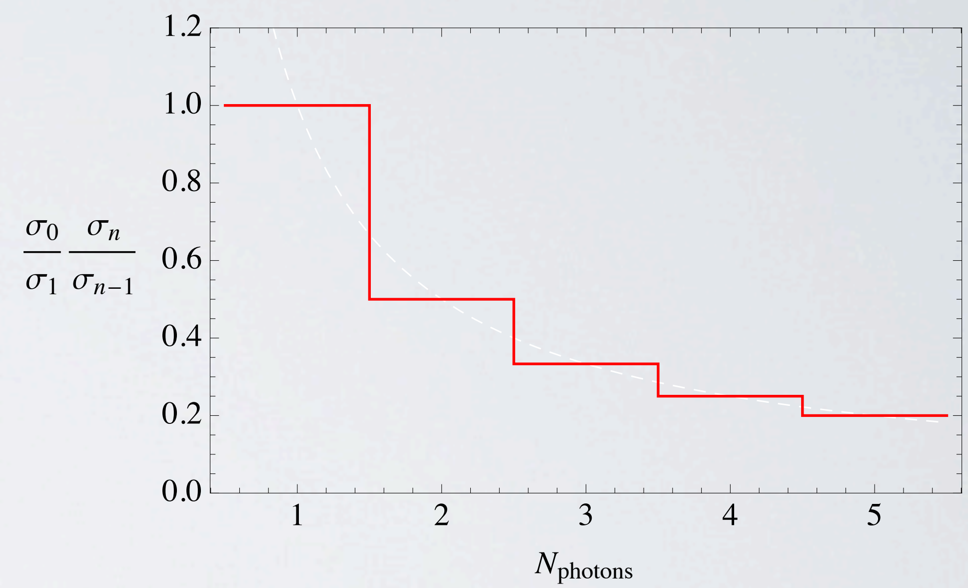

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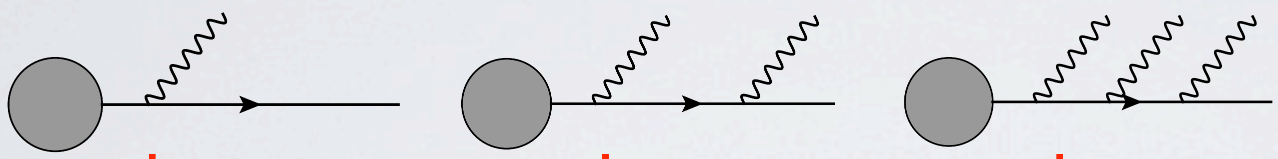

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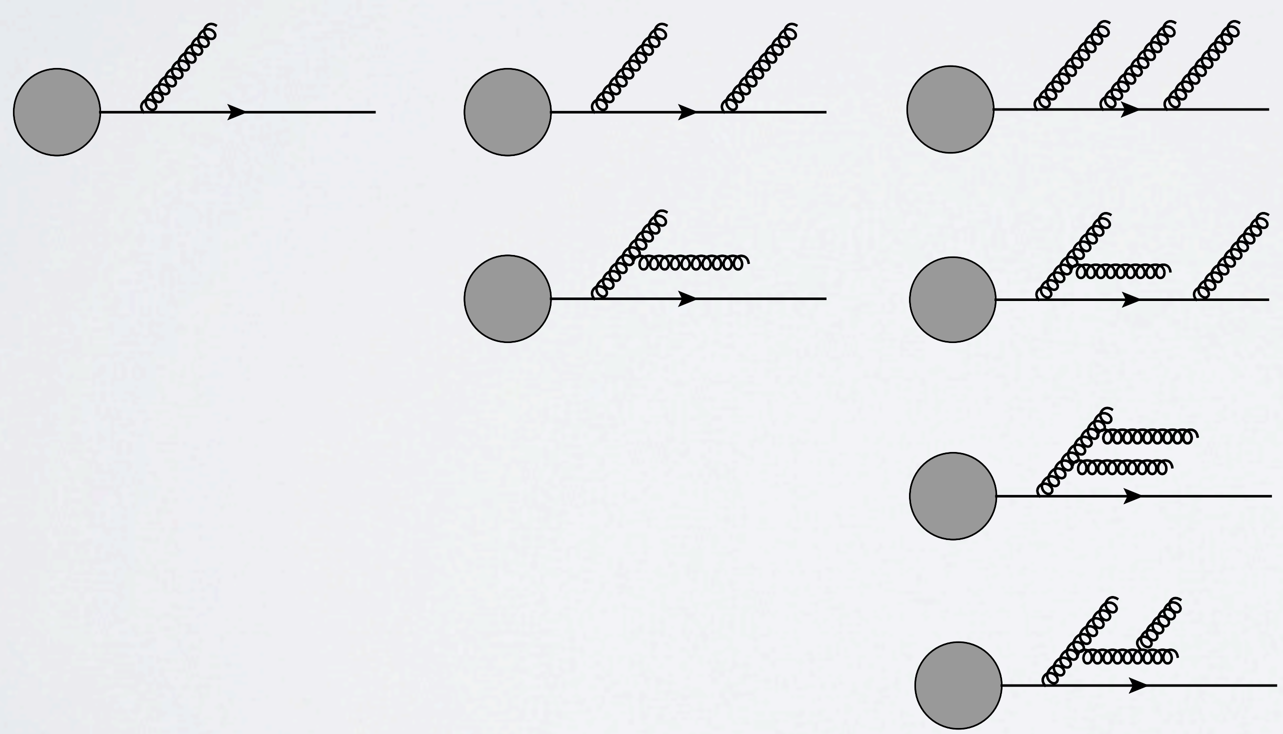

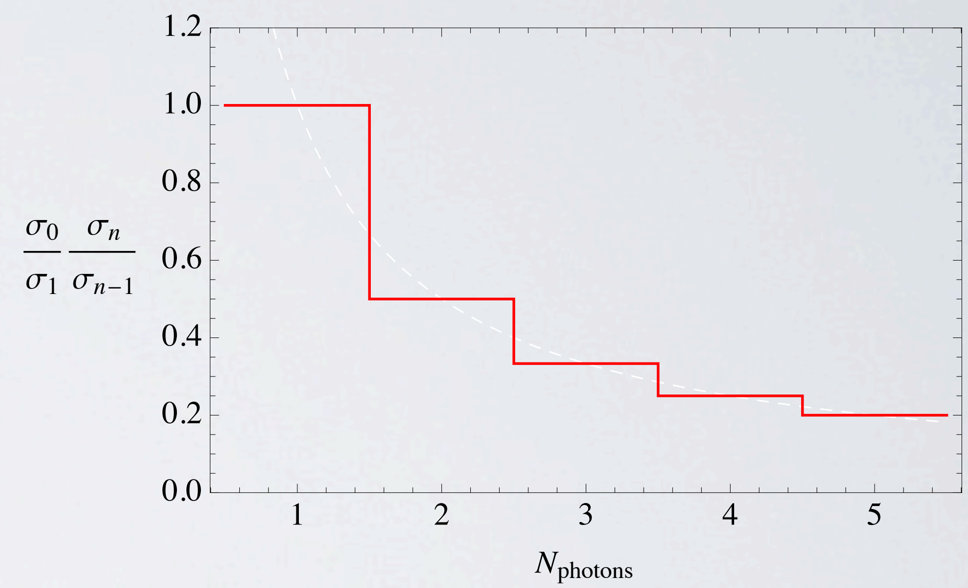

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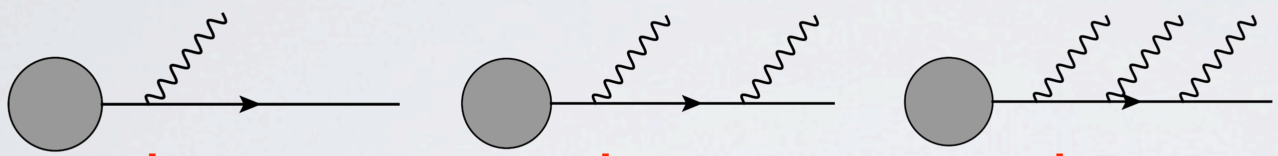

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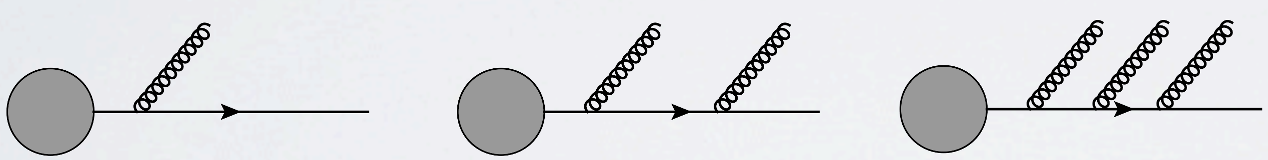
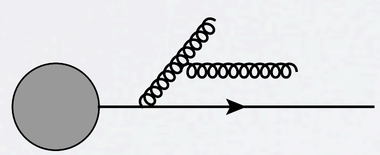
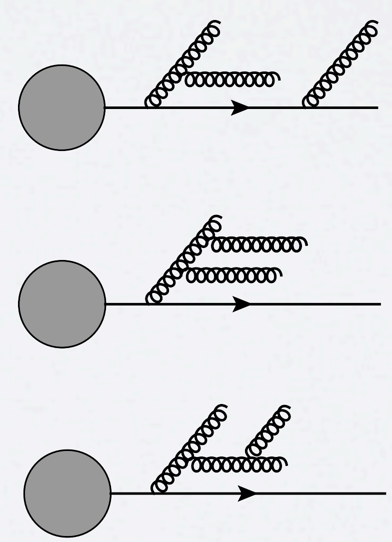


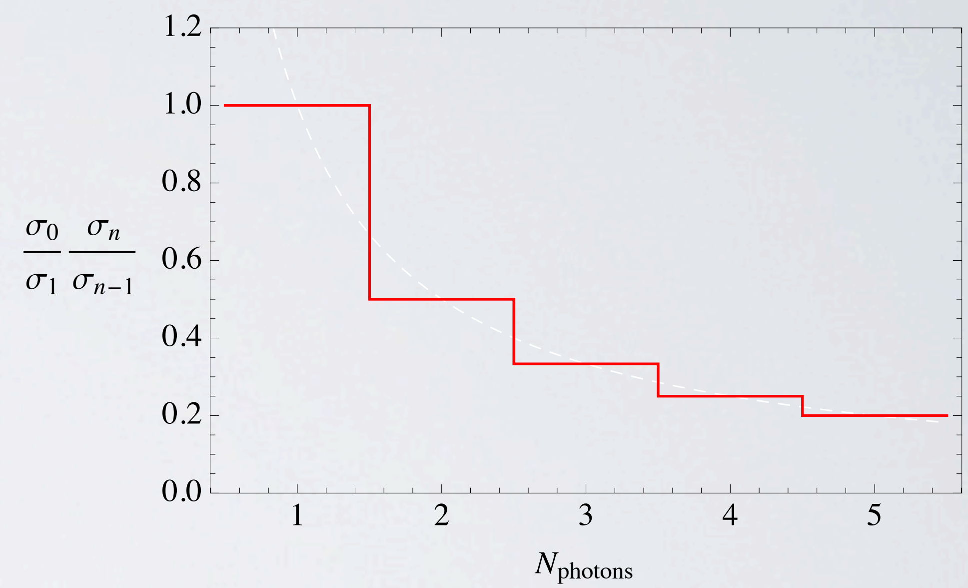
$\frac{1}{1!}(\alpha_S L^2)$





$\left(\frac{3}{16}+\frac{1}{2!}\right)(\alpha_S L^2)^2$

$\left(\frac{39}{160}+\frac{1}{3!}\right)(\alpha_S L^2)^3$



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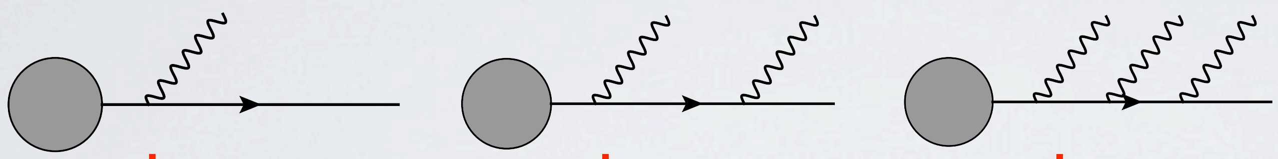

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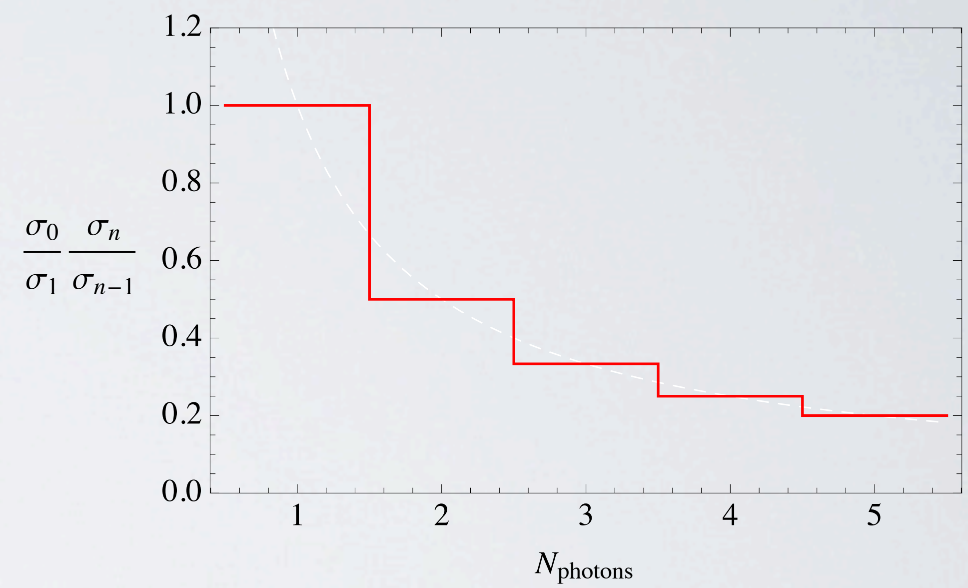

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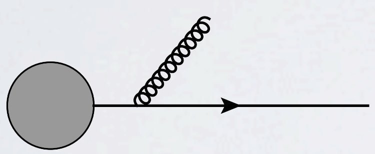

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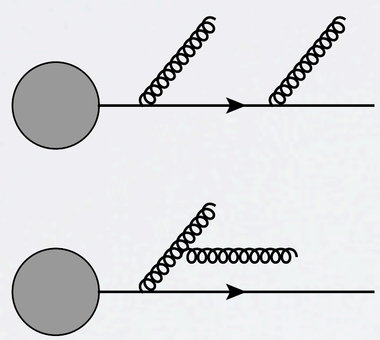

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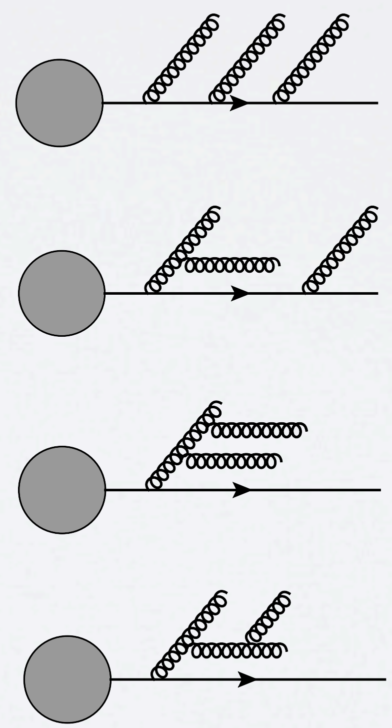

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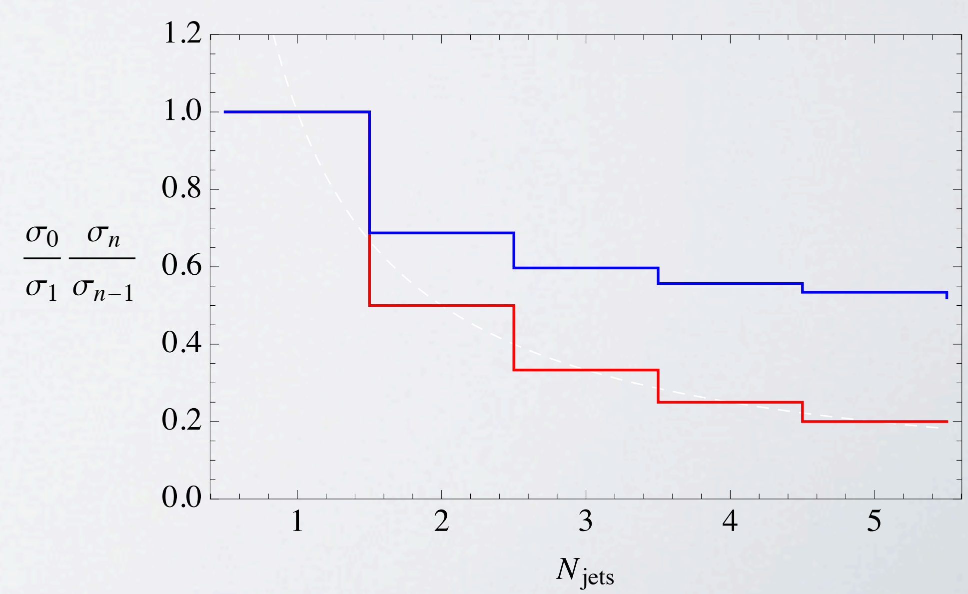
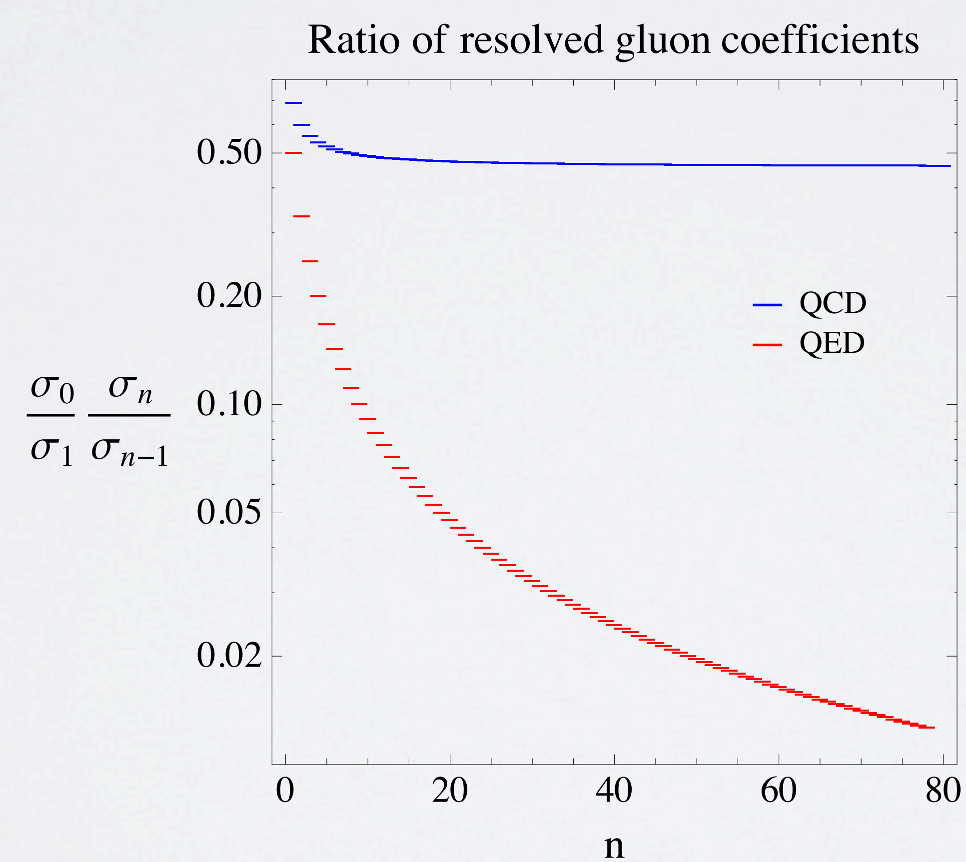


RESOLVED GLUONS COEFFICIENTS

Formally correspond to the rates in the limit $\alpha_S \to \mathbf{0}$ and $\mathbf{L} \to \infty$ with $\alpha_S\, \mathbf{L^2} \ll \mathbf{1}$

$$\text{Res}_{21} = \Big(\frac{1}{5109094217170944000} + \frac{352552873457246307069012458671 C_A^{20}}{669959124447288464805194190141921792000000000 C_F^{20}} + \frac{128471285759108523856950109 31 C_A^{19}}{15845300799935869086301710335950848000000000 C_F^{19}}$$
$$+ \frac{176296228168954465833426997187 C_A^{18}}{316906015998717381726034206719016960000000 C_F^{18}} + \frac{650706424921354998859 09 C_A^{17}}{284085812858856623799486382080000000 C_F^{17}} \frac{940578795190149234230 2697 C_A^{16}}{14677766997707592229640129740800000000 C_F^{16}}$$
$$+ \frac{4504523017310813079367783 C_A^{15}}{3453592234754727583444736409600000000 C_F^{15}} + \frac{972023286763341454373 C_A^{14}}{482277926931256470247833600000000 C_F^{14}} + \frac{2831888384397273694 3 C_A^{13}}{11641191339719983764602880000000 C_F^{13}} +$$
$$\frac{816675669069438582 59 C_A^{12}}{3492357401915995129380864000000 0 C_F^{12}} + \frac{278500541108600453 C_A^{11}}{153510215468834950742016000000 C_F^{11}} + \frac{10783218219383569 C_A^{10}}{941255501759857557504000000 0 C_F^{10}} +$$
$$\frac{723754949844653 C_A^{9}}{122363215228781482475520000 0 C_F^{9}} + \frac{51087188239 C_A^{8}}{20444981658944274432000 0 C_F^{8}} + \frac{560949853 C_A^{7}}{651433239133028352000 0 C_F^{7}} + \frac{5477491037 C_A^{6}}{22800163369655992320000 C_F^{6}} +$$
$$\frac{159707 C_A^{5}}{2982752926433280000 0 C_F^{5}} + \frac{5903 C_A^{4}}{632705166213120000 0 C_F^{4}} + \frac{233 C_A^{3}}{189811549863936000 0 C_F^{3}} + \frac{53 C_A^{2}}{460970906812416000 0 C_F^{2}} + \frac{C_A}{1459741204905984000 C_F}\Big)$$



Tuesday, March 19, 13

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$$
\begin{aligned}
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& + \frac{1762962281689544658334269971 87C_A^{18}}{31690601599871738172603420671901696000000000C_F^{18}} + \frac{650706424921354998859 09C_A^{17}}{284085812858856623799486382080000000C_F^{17}} \frac{9405787951901492342302697C_A^{16}}{146777669977075922296401297408000000000C_F^{16}} \\
& + \frac{450452301731081307936778 3C_A^{15}}{3453592234754727583444736409600000000C_F^{15}} + \frac{9720232867633414543 73C_A^{14}}{482277926931256470247833600000000C_F^{14}} + \frac{28318883843972736943C_A^{13}}{11641191339719983764602880000000C_F^{13}} + \\
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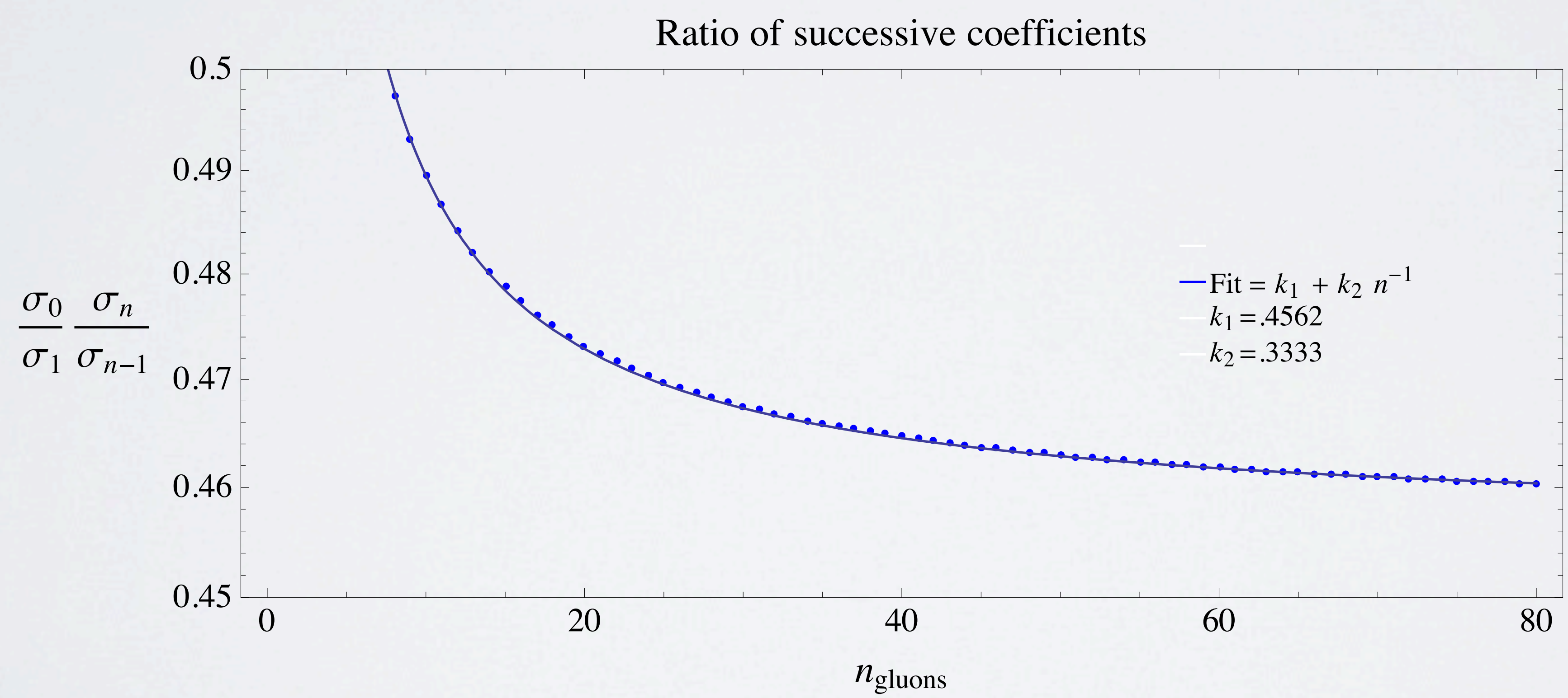
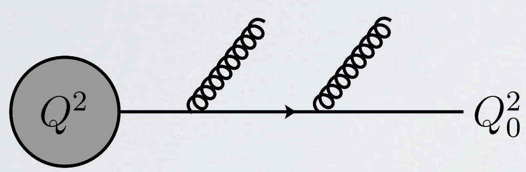
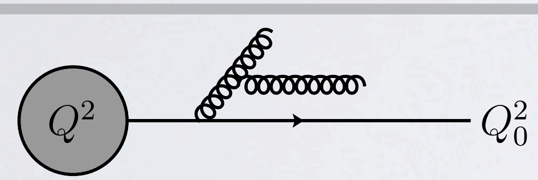
\end{aligned}
$$

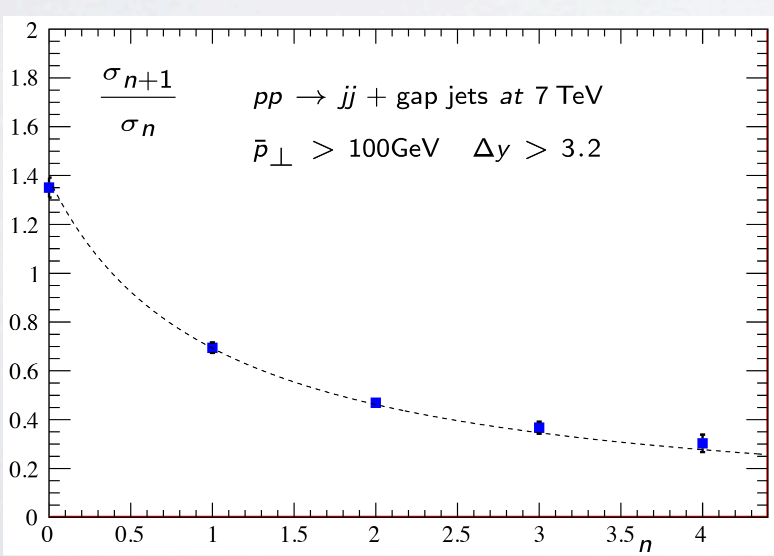


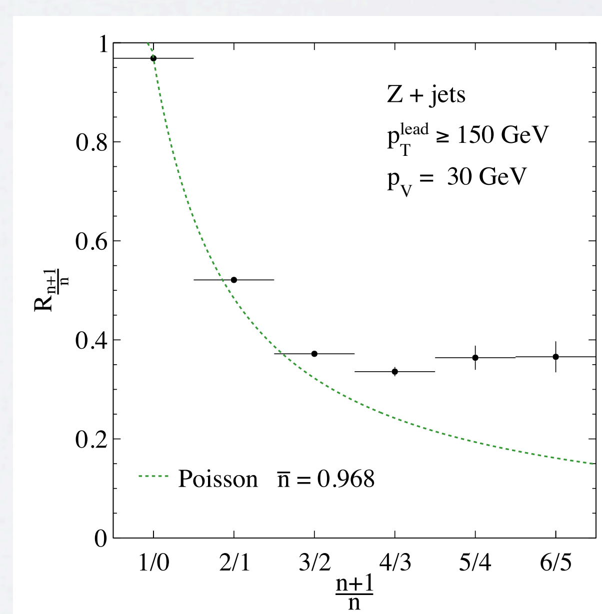
RESUMMED JET RATES

Resummed rates from Generating Functional formalism

- Key point: the primary emissions are dynamically enhanced over the secondaries in the resummed limit corresponding to $\alpha_S L^2 \gg 1$.

Diagram	Resummed expression	In kinematic limit $\frac{\alpha_s}{\pi}\log^2\frac{Q}{Q_0} \gg 1$
	$\sigma^{\text{primary}}(Q^2,Q_0^2) = c^{\text{primary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2,t)\Delta_g(t) \int_{Q_0^2}^{Q^2} dt'\, \Gamma(Q^2,t')\Delta_g(t')$	$\frac{\alpha_s}{C_A}\log^2\frac{Q}{Q_0}$
	$\sigma^{\text{secondary}}(Q^2,Q_0^2) = c^{\text{secondary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2,t)\Delta_g(t) \int_{Q_0^2}^{t} dt'\, \Gamma(t,t')\Delta_g(t')$	$\sqrt{\frac{\alpha_s}{C_A^3}}\log\frac{Q}{Q_0}$



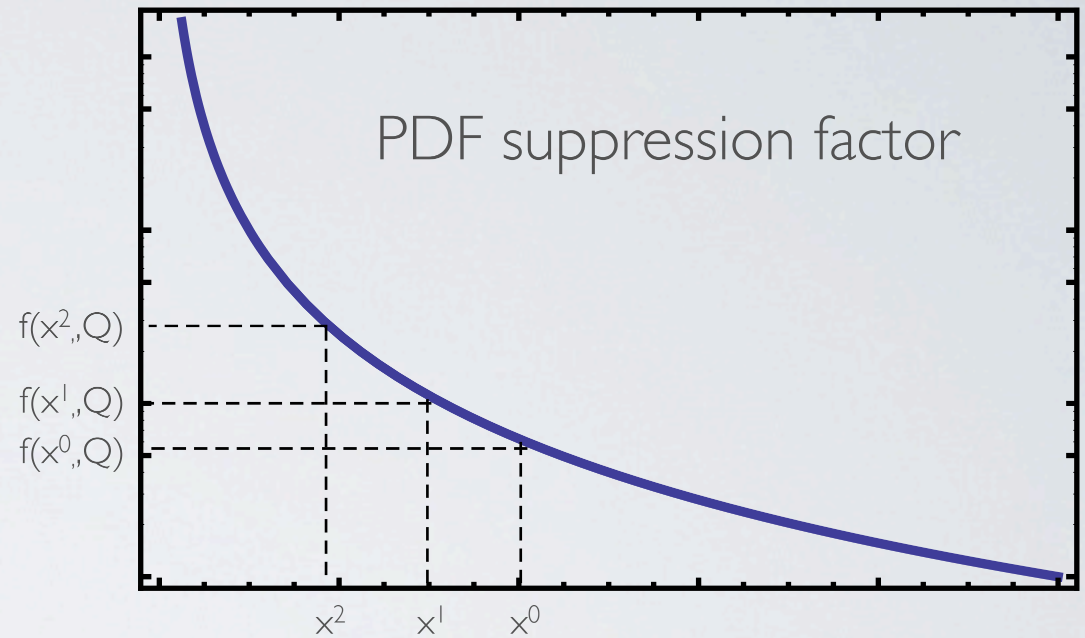


PDF EFFECTS ON SCALING

Factorization at leading logarithmic level

- Generating functional for initial state evolution [Catani, Webber, Dokshitzer (1993)].

- We can apply our leading log rate calculation as long as we take into account the kinematic effect of the PDFs.



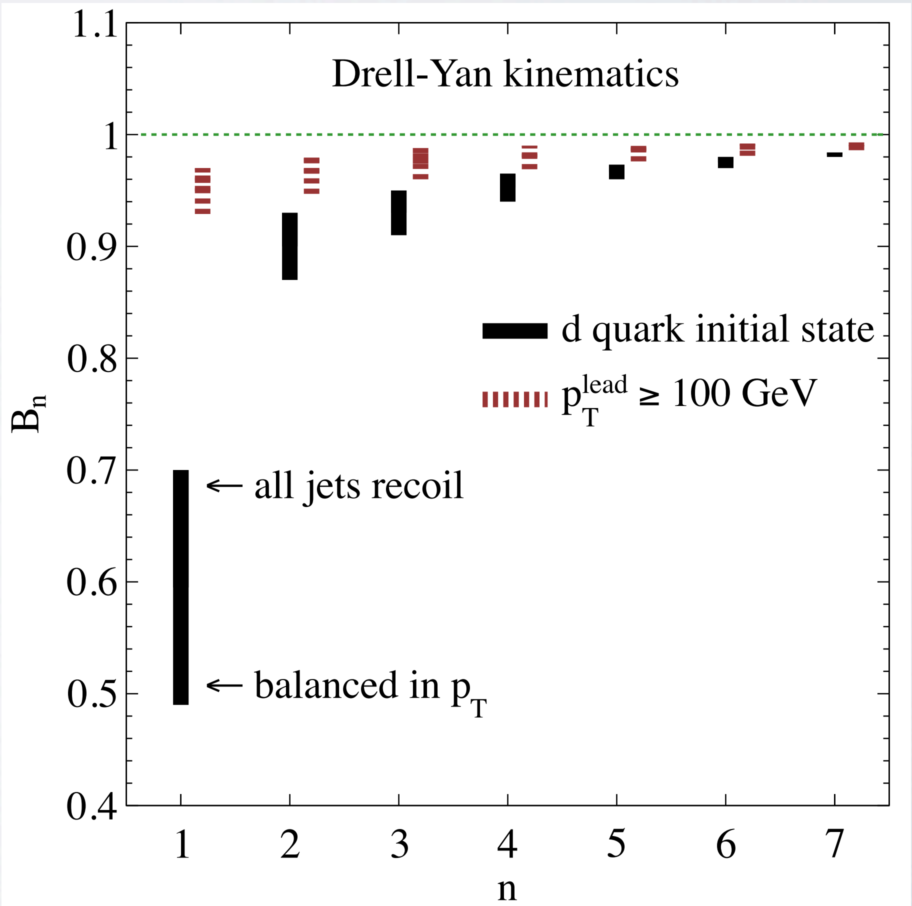
Estimating the PDF suppression

- Assume threshold kinematics on additional jet.

$$R_{(n+1)/n} \sim \left| \frac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)} \right|^2$$

- Effect on scaling essentially discretized second derivative with respect to x.

$$B_n = \left| \frac{\dfrac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)}}{\dfrac{f(x^{(n+2)}, p_V)}{f(x^{(n+1)}, p_V)}} \right|^2$$

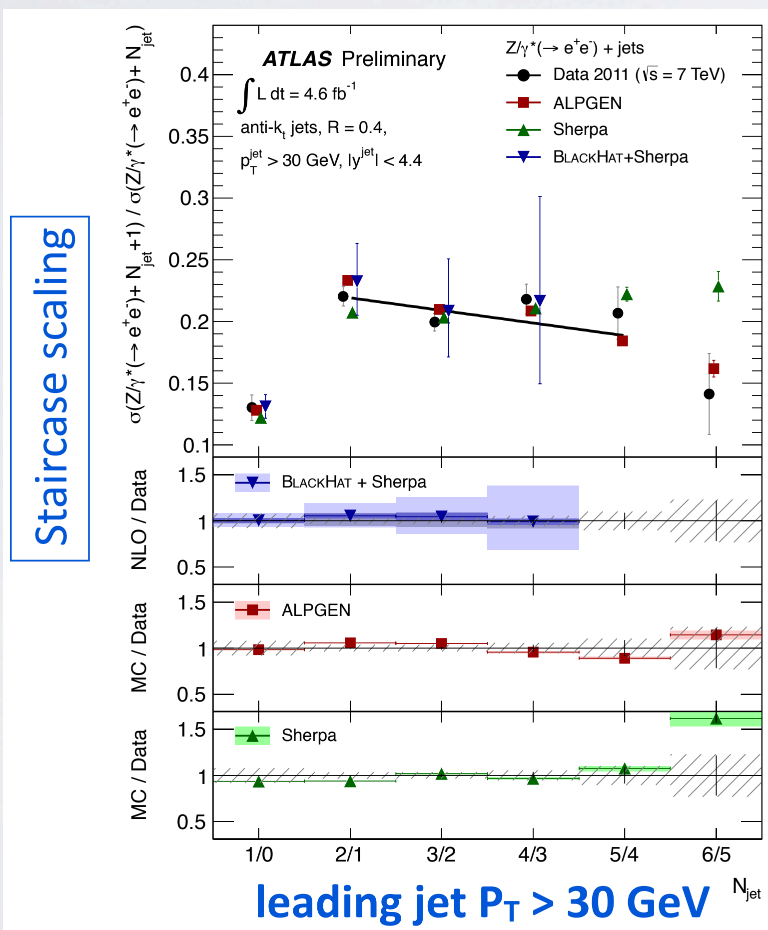


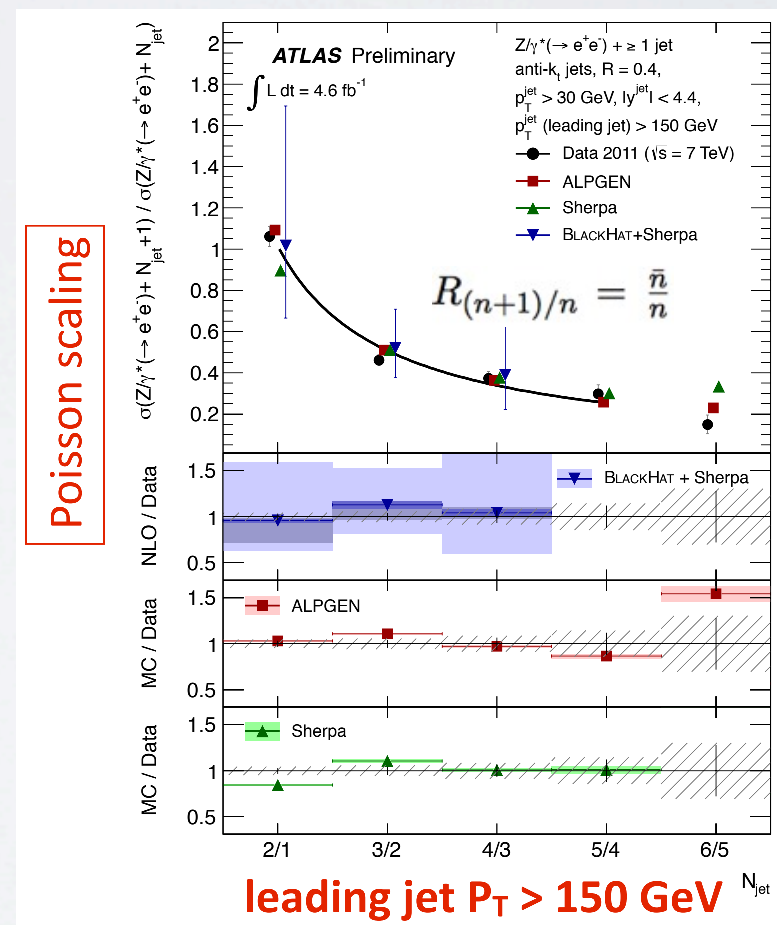
EXPERIMENTAL TESTS FOR SCALING

Proposal to look for both Staircase and Poisson scaling in exclusive jets

- Proposed selections to look for both Staircase and Poisson scaling in Z/W + exclusive jet events.
- Also in Gap jet studies (WBF kinematics).

ATLAS PRELIMINARY **[from Theofilotas @ Moriond QCD 2013]**





SCALING IN BFKL REGIME

Scaling in the context of BFKL-like dynamics

- Generating functional for small x in the Multi-Regge-Kinematics limit [Webber]

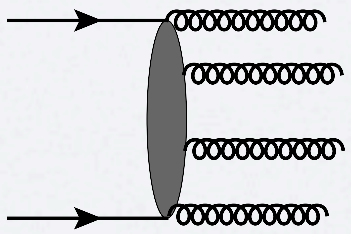
$$\Phi(Q^2, p_V^2)_{\rm BFKL} = \exp\left(-\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)\left[1+(1-u)\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right]^{u/(1-u)}$$

where again we have $f_{n-1} = \frac{1}{n!}\frac{d^n}{du^n}\Phi\Big|_{u=0}$

$$\frac{1}{w} \longleftrightarrow \log\frac{1}{x}$$ (related by Mellin transformation)

In kinematic limit	Emission pattern behaves as	Scaling pattern
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \gg 1$	$\sigma_n \approx \frac{1}{n!}\log^n\left(1+\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)$	Poisson
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \ll 1$	$\sigma_n \approx \left(\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)^n$	Geometric

- Poisson emerges very slowly.
- Exact Staircase (Geometric) scaling does not appear from secondary emissions...?



CONCLUSIONS

- Better knowledge of exclusive jet rates can help searches/measurements in and beyond the SM.

- The idealized scaling patterns are good approximations to LHC physics.

- This provides a new source for data driven extractions of multi-jet backgrounds.

- Theoretical origins of scaling still of on-going...an interesting field on its own.

PREDICTIONS FOR EXCLUSIVE JET RATES

Fixed order calculation

- Lots of progress in NLO (and NNLO) fixed order calculations.

[Z/W + 5 jets, 4-jet production @ NLO Blackhat et al.]

- Only reliable as an exclusive rate if $\sigma_n \gg \sigma_{n+1}$.

Analytic resummation

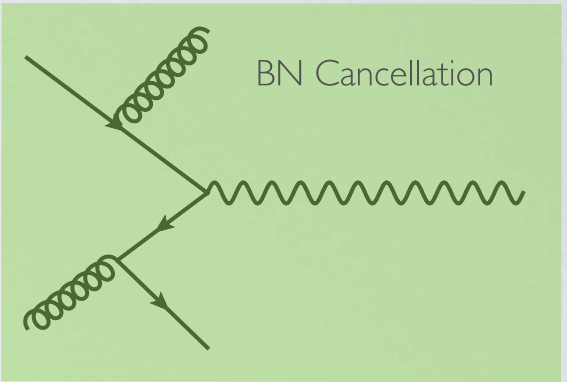
- Very useful for predictions for 0-jet exclusive cross sections (e.g. H + 0 jets) [Banfi, Salam, Zanderighi; Becher, Neubert ; Stewart, Tackmann, Walsh]
- No current method for general process/multiplicity.

Parton Shower

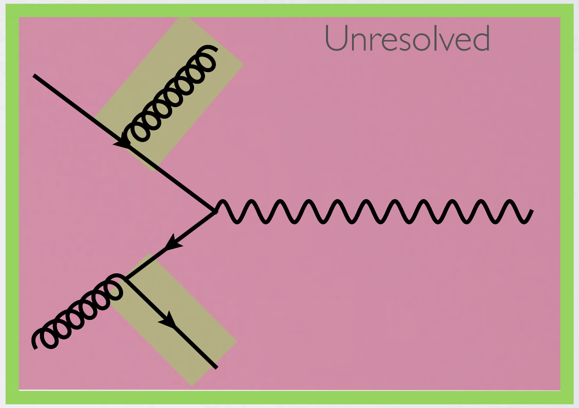
- In principle predictions for jet rates to arbitrarily high multiplicity.
- Accuracy limited (for some observables leading logarithm), computational time a factor.

Bloch-Nordsieck theorem

2 -jet inclusive



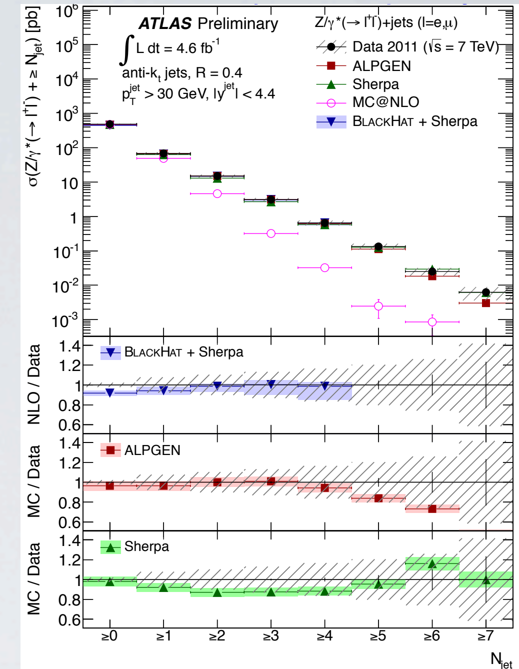
2 -jet exclusive



No approach is optimal...for high multiplicity exclusive rates we are limited to the parton shower (potentially also with ME matching).

THE N_{JET} DISTRIBUTION

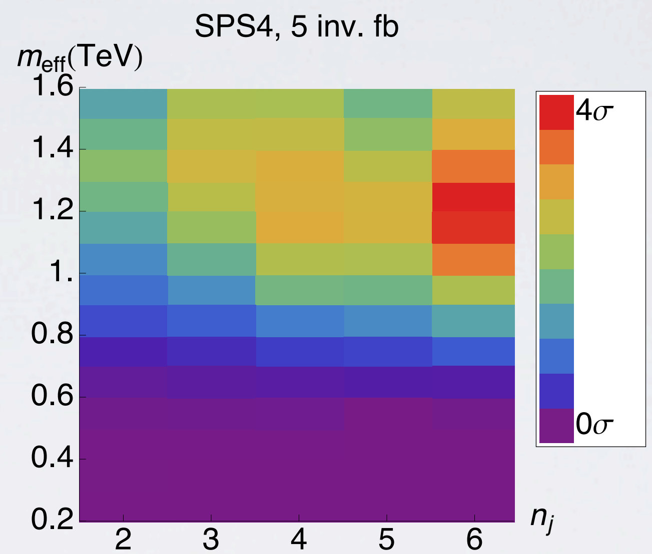
[From Hassani Moriond 2013]



New Physics searches in all hadronic channel

- N_{jet} distribution potentially strong discriminator over QCD background [Englert, Plehn, Schichtel, Schumann]

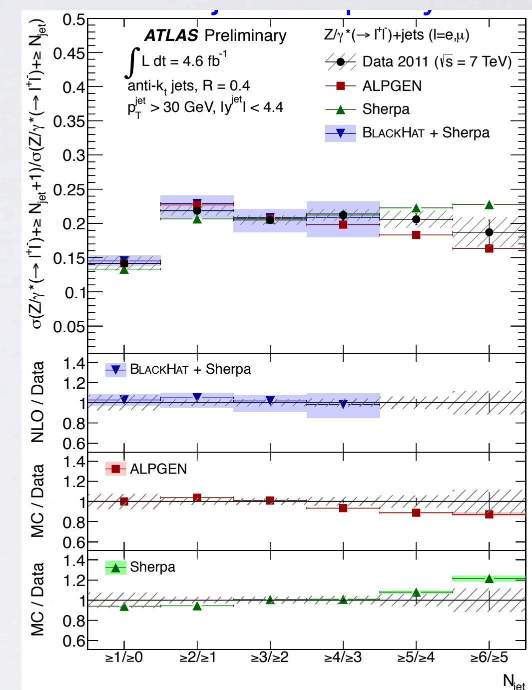
- Signals often appearing at high multiplicity (and high m_{eff}) → exactly where predictions are hardest.



Jet rates in generic processes

We want to understand jet rates which are:

1. Exclusive
2. Radiated
3. Not MonteCarlo



Idealized notion of jet scaling → Observable of interest is jet ratios

$$R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n}$$

GENERATING FUNCTIONAL FORMALISM

- Derivatives with respect to "source" u at u=0 produce (resummed) exclusive multiplicities.

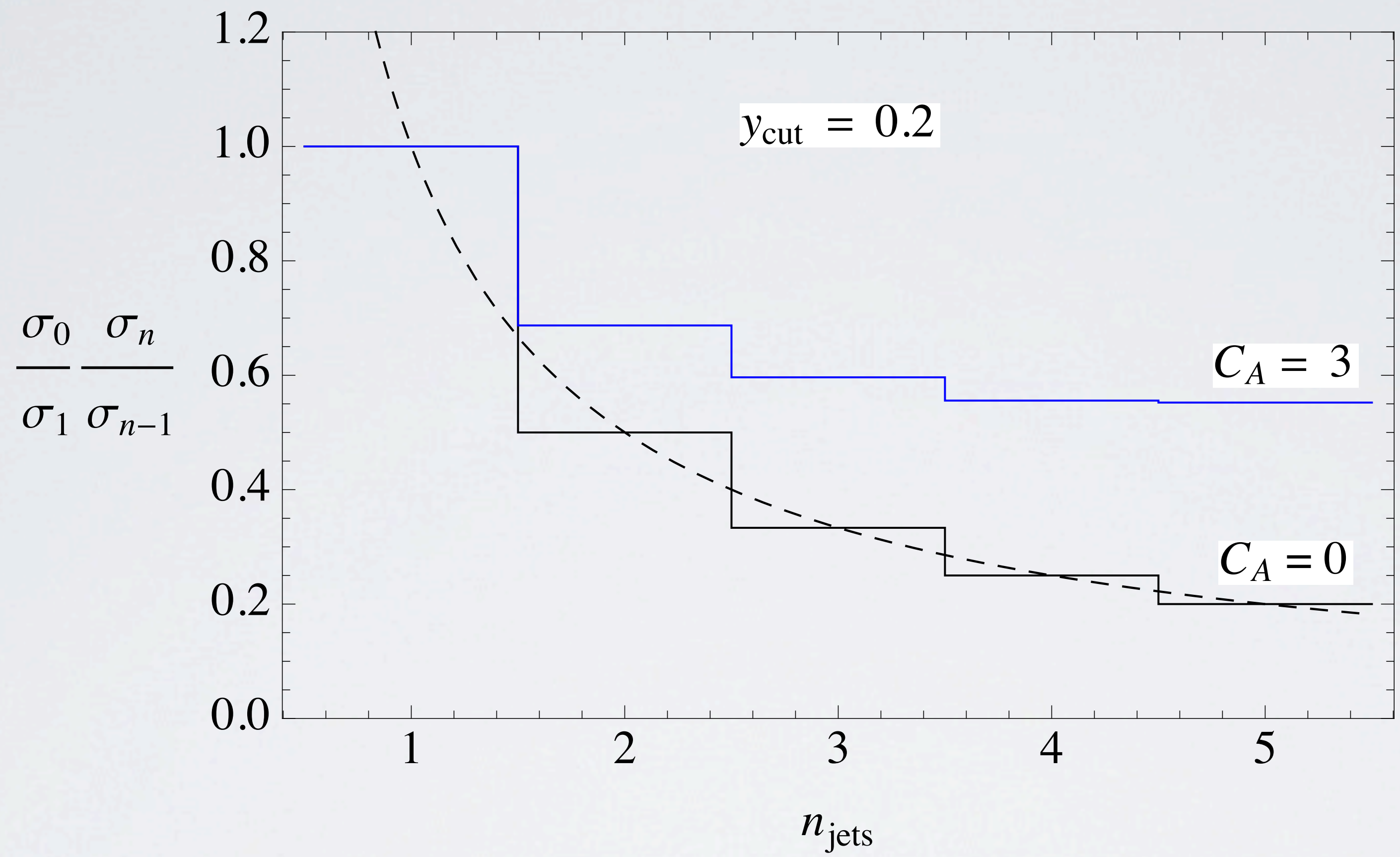
$$f_{n-1} = \frac{1}{n!}\frac{d^n}{du^n}\Phi\Big|_{u=0}$$

- Jet rates include the unresolved components to all-orders (valid even for $\sigma_{n+1} \gg \sigma_n$).

	Durham Algorithm [Catani, Dokshitzer, Olsson, Turnock, Webber (1991)]	Generalized K_T Algorithm [EG, Gripaios, Schumann, Webber (2012)]
	$\Phi_q(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\ \Gamma_q(Q^2,t)\,(\Phi_g(t)-1)\right]$	$\Phi_q(u,E,\xi) = u\exp\left\{\frac{\alpha_S}{2\pi}\int_{\xi_R}^{\xi}\frac{d\xi'}{\xi'}\int_{E_R/E}^{1} dz\, P_{gq}(z)\,[\Phi_g(u,zE,\xi')-1]\right\}$
	$\Phi_g(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\left(\Gamma_g(Q^2,t)\,(\Phi_g(t)-1)+\Gamma_f(t)\left(\frac{\Phi_q^2(t)}{\Phi_g(t)}-1\right)\right)\right]$	$\Phi_g(u,E,\xi) = u\Delta_g(E,\xi)+\frac{\alpha_S}{2\pi}\int_{\xi_R}^{\xi}\frac{d\xi'}{\xi'}\frac{\Delta_g(E,\xi)}{\Delta_g(E,\xi')}\int_{E_R/E}^{1} dz\{P_{gg}(z)\Phi_g(u,E,\xi')\Phi_g(u,zE,\xi')$ $+P_{qg}(z)[\Phi_q(u,E,\xi')]^2\}$

THE POISSON DISTRIBUTION IN QCD

Comparison of fixed order analytic results for jet fractions



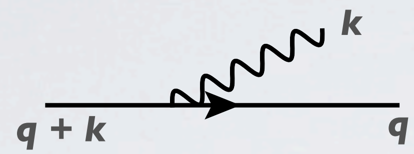
Remarks from the fixed order calculations

1. At high multiplicities subsequent splittings take-over and seem to lead to geometric scaling.
2. Lower multiplicities neither Poisson or staircase like behavior.
3. Doesn't really answer our questions in either regime! (need resummed rates)

BASIS OF THE POISSON DISTRIBUTION

Prototypical example: Soft-photon radiation in QED

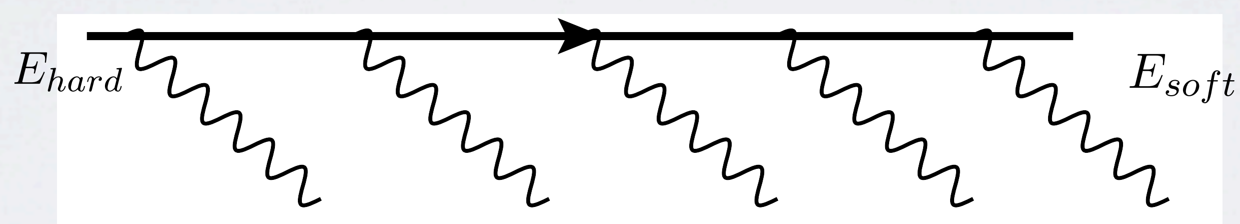
- Fully factorized form of the matrix element (eikonal approximation) [e.g. Peskin and Schroeder]



$$\gamma^\mu \frac{\not{q} + \not{k}}{(q+k)^2} \quad \to \quad \frac{q^\mu}{q \cdot k}$$

- Integrating over phase space, including 1/n! for identical bosons in the final state,

$$\Rightarrow \sigma_n \sim \frac{L^n}{n!} e^{-L} \qquad \text{with} \qquad L \sim \frac{\alpha}{\pi} \log^2\left(\frac{E_{hard}}{E_{soft}}\right)$$



- Adding together (independent) Poisson processes generates another Poisson process (rate parameters simply add together)

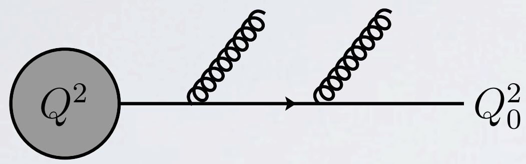
- Matrix element corrections of course important for the rates (unless very log enhanced), but small effect on the scaling.

RESUMMED RATES IN QCD: SOME FORMALISM

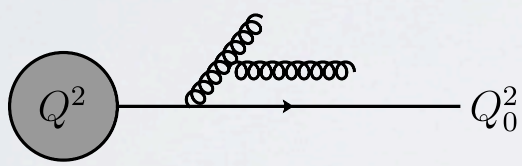
Resummed 2-gluon emission

$$\frac{1}{4!}\frac{d}{du}|\Phi_u|^2 \;=\; 2[\Delta_q(Q)]^2\left[\left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\right)^2 + \left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(q')\right)\right]$$

- From a scaling point of view even the primary emission provide a Poisson pattern.



$$\sigma^{\text{primary}}(Q^2,Q_0^2) = c^{\text{primary}}\int_{Q_0^2}^{Q^2} dt\;\Gamma(Q^2,t)\Delta_g(t)\int_{Q_0^2}^{Q^2} dt'\;\Gamma(Q^2,t')\Delta_g(t')$$



$$\sigma^{\text{secondary}}(Q^2,Q_0^2) = c^{\text{secondary}}\int_{Q_0^2}^{Q^2} dt\,\Gamma(Q^2,t)\Delta_g(t)\int_{Q_0^2}^{t} dt'\,\Gamma(t,t')\Delta_g(t')$$

- Key point is that the primary emissions are enhanced wrt secondary emissions as the size of the overall logarithm grows (effect completely missing in the fixed order calculation)

In kinematic limit	Primary emission	Secondary emissions
$\frac{\alpha_s}{\pi}\log^2\frac{Q}{Q_0} \ll 1$	$c^{\text{primary}}\;\log^4\frac{Q}{Q_0}$	$c^{\text{secondary}}\;\log^4\frac{Q}{Q_0}$
$\frac{\alpha_s}{\pi}\log^2\frac{Q}{Q_0} \gg 1$	$\frac{\alpha_s}{C_A}\log^2\frac{Q}{Q_0}$	$\sqrt{\frac{\alpha_s}{C_A^3}}\log\frac{Q}{Q_0}$

DURHAM GENERATING FUNCTIONAL

All multiplicity proof for the scaling patterns

- For Poisson this is more or less simple; the integral for large Q is dominated by region in *t* space close to Q_0.

$$\Phi_j(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\ \Gamma_j(Q^2,t)\,(u-1)\right] = \frac{u\,\Delta_j(Q^2)}{\Delta_j(Q^2)^u}$$

Gives Poisson ratios: $R_{(n+1)/n} = \frac{|\log \Delta_j(Q^2)|}{n+1}$

- In pure YM keeping leading powers of $(Q - Q_0)/Q$, corresponds to not too large single emission probability (still need log enhancement).

$$\frac{d\Phi_g(Q^2)}{dQ^2} \approx \Phi_g(Q^2)\,\tilde{\Gamma}_g(Q^2,Q_0^2)\,(\Phi_g(Q^2)-1) \longrightarrow \Phi_g(Q^2) = \frac{1}{1+\frac{(1-u)}{u\tilde{\Delta}_g(Q^2)}}$$

Gives Staircase ratios: $R_{(n+1)/n} = 1 - \tilde{\Delta}_g(Q^2)$

- High multiplicity proof of Staircase tale in large emission probability limit an empirical fact, sets in at $n_{\rm trans} \approx \bar{n}$ (number of Poisson breaking terms grows as a function of n).

OBSERVED SCALING PATTERNS

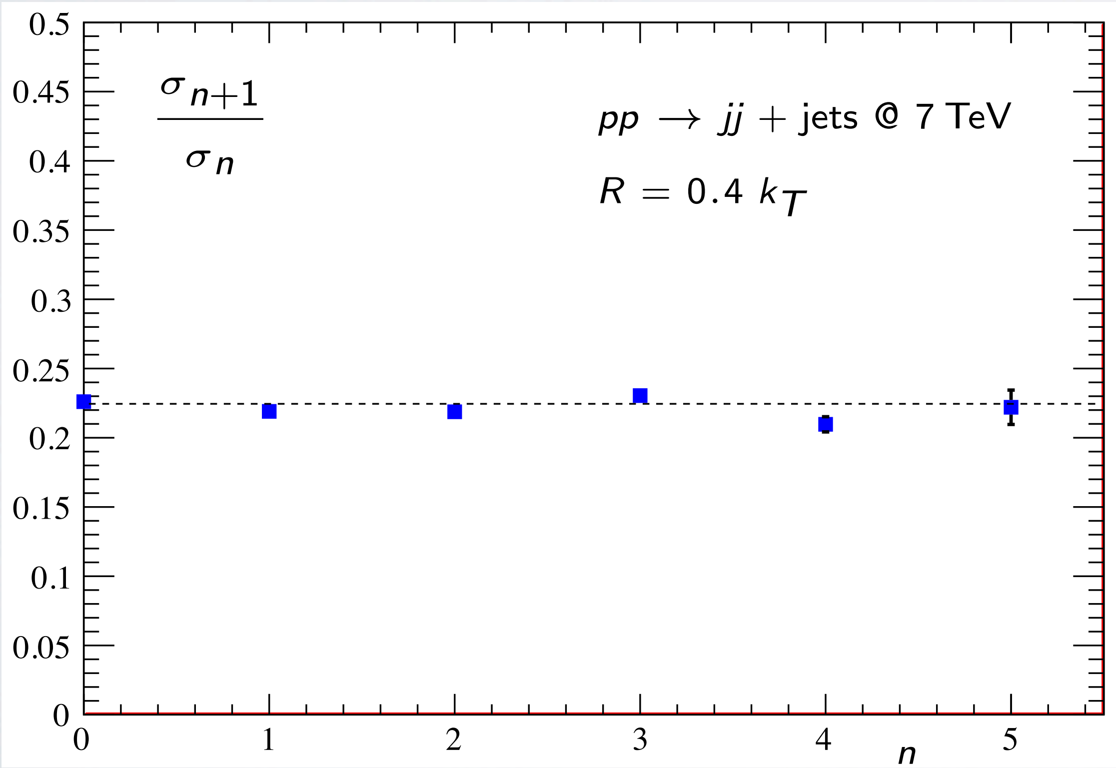
Staircase [Steve Ellis, Kleiss, Stirling (1985), Berends (1989)]

$$\sigma_n^{\mathrm{excl}} = R_{1/0}^n \equiv e^{-bn}$$

- Ratios are constant (geometric)

$$\frac{\sigma_{n+1}}{\sigma_n} = e^{-b}$$

- Observed: UA1, Tevatron, LHC



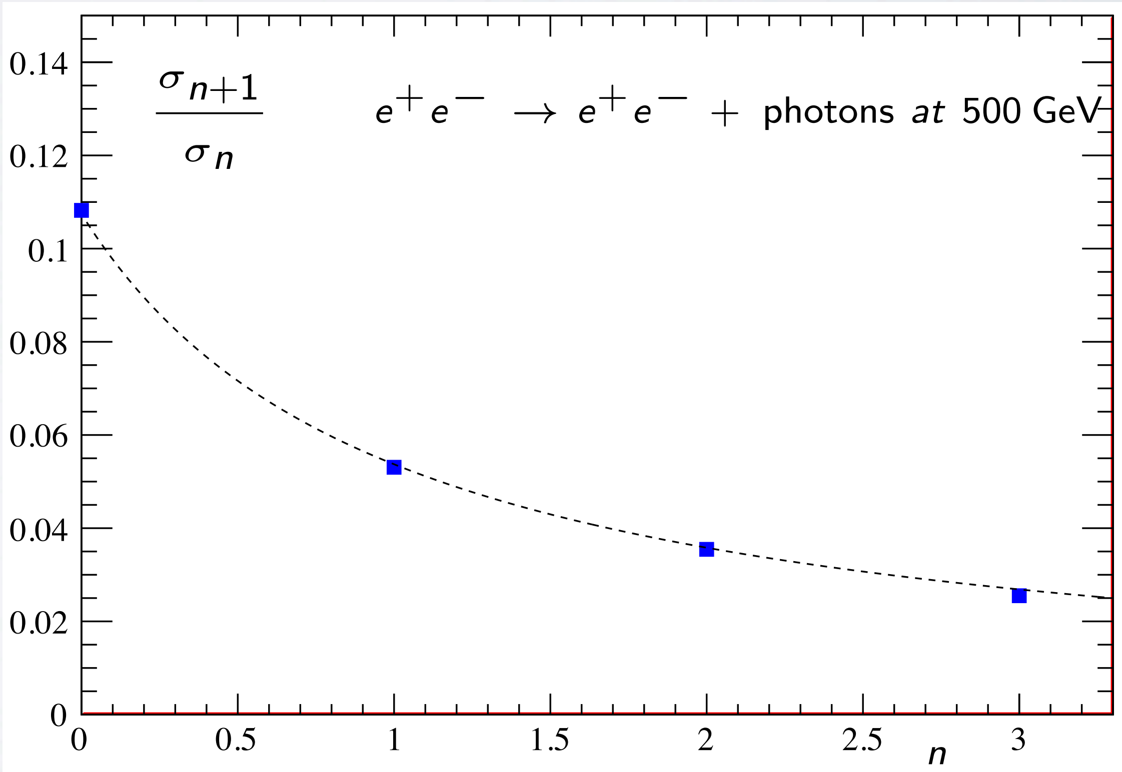
Poisson [Peskin & Schroder; Rainwater, Zeppenfeld (1997)]

$$\sigma_n^{\mathrm{excl}} = \frac{e^{-\bar{n}}\bar{n}^n}{n!}$$

- Ratios are not constant

$$\frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1}$$

- Observed: Photons at LEP, LHC



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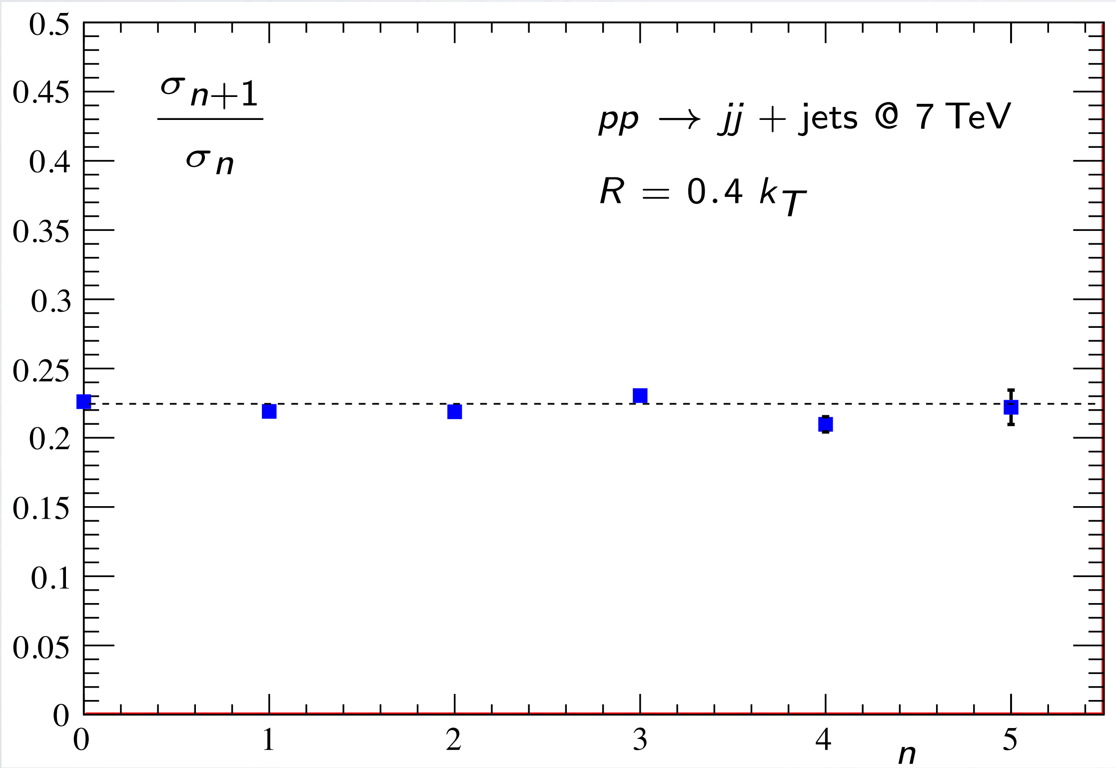

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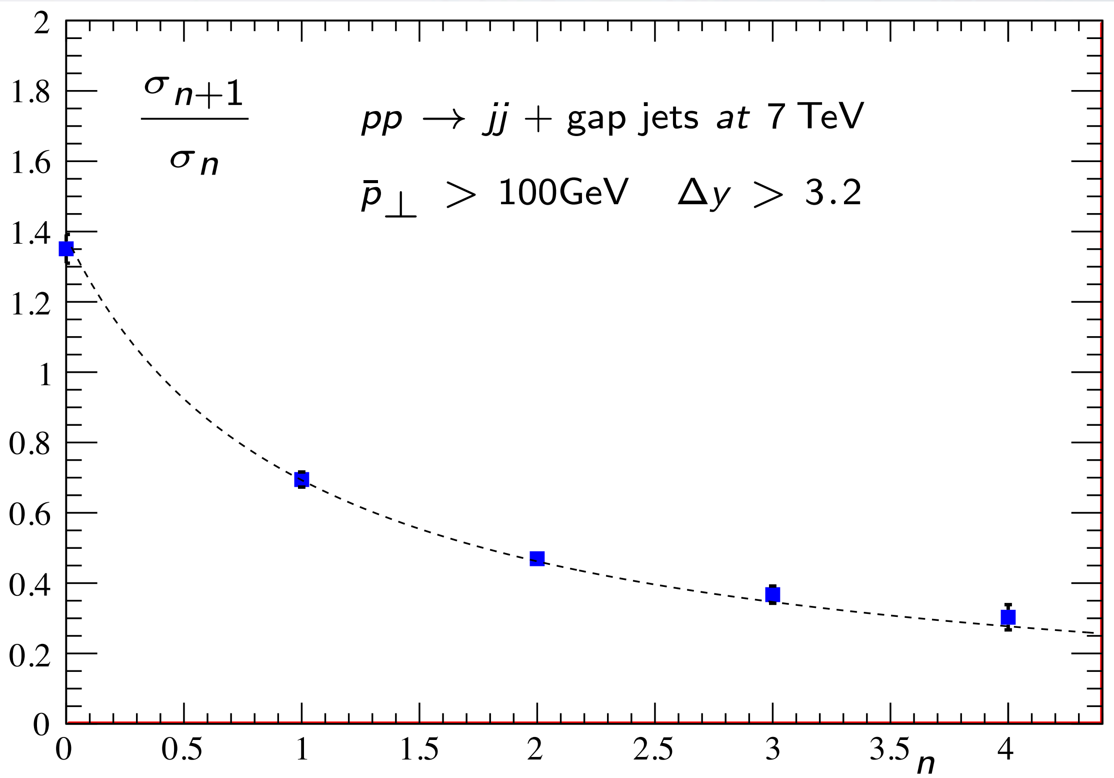

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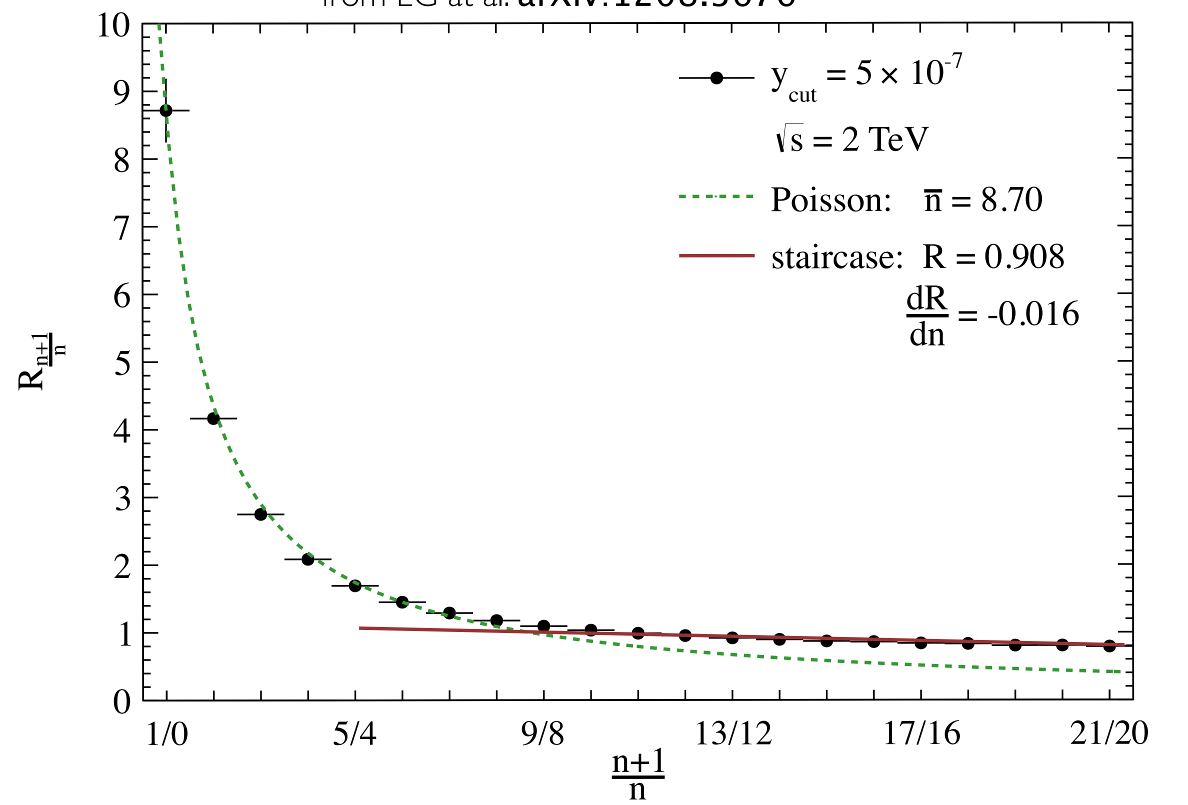
from EG at al. arXiv:1208.3676



Tuesday, March 19, 13

SUMMARY: RATES AND GEN. FUNCTIONAL

Fixed-order

- Fixed order calculation tells us that Poisson distribution altered by secondary emissions.
- At high multiplicity start to see the on-set of geometric scaling.

Resummed jet rates

- From the 2-jet rate (2 gluon emission) able to see how the (Poisson-making) primary comments are dynamically enhanced with respect to the secondaries.

Generating Functional

- Able to derive the desired patterns in two opposing limits, in the case of the staircase limit only able to solve the PDE analytically for pure YM.

However, the Durham algorithm is somewhat special, in that there is no resolution scale in physical energy or angle. Therefore, we would like to study an algorithm which mimics LHC relevant jet. i.e Generalized kt algorithm.

GENERALIZED K_T ALGORITHM

Generating functional

- For the Generalized class of algorithms (more analogous to LHC algorithms of choice) [EG et al.]

$$
\begin{aligned}
d_{ij} &= \min\{E_i^{2p}, E_j^{2p}\} \frac{(1-\cos\theta_{ij})}{(1-\cos R)} \\
&\equiv \min\{E_i^{2p}, E_j^{2p}\} \xi_{ij}/\xi_R \,,
\end{aligned}
$$

- Generating functional solved via iteration for the rates, can be compared with Parton shower.

$$
\Phi_q(u,E,\xi) = u \exp\left\{ \frac{\alpha_S}{2\pi} \int_{\xi_R}^{\xi} \frac{d\xi'}{\xi'} \int_{E_R/E}^{1} dz\, P_{gq}(z)\, [\Phi_g(u,zE,\xi') - 1] \right\}
$$

$$
\Phi_g(u,E,\xi) = u\Delta_g(E,\xi) + \frac{\alpha_S}{2\pi} \int_{\xi_R}^{\xi} \frac{d\xi'}{\xi'} \frac{\Delta_g(E,\xi)}{\Delta_g(E,\xi')} \int_{E_R/E}^{1} dz \{P_{gg}(z)\Phi_g(u,E,\xi')\Phi_g(u,zE,\xi') + P_{qg}(z)[\Phi_q(u,E,\xi')]^2\}
$$

- Splitting apart of soft (energy) and collinear (angle) singularity leads to important differences (already evident looking at the Sudakov form factor).

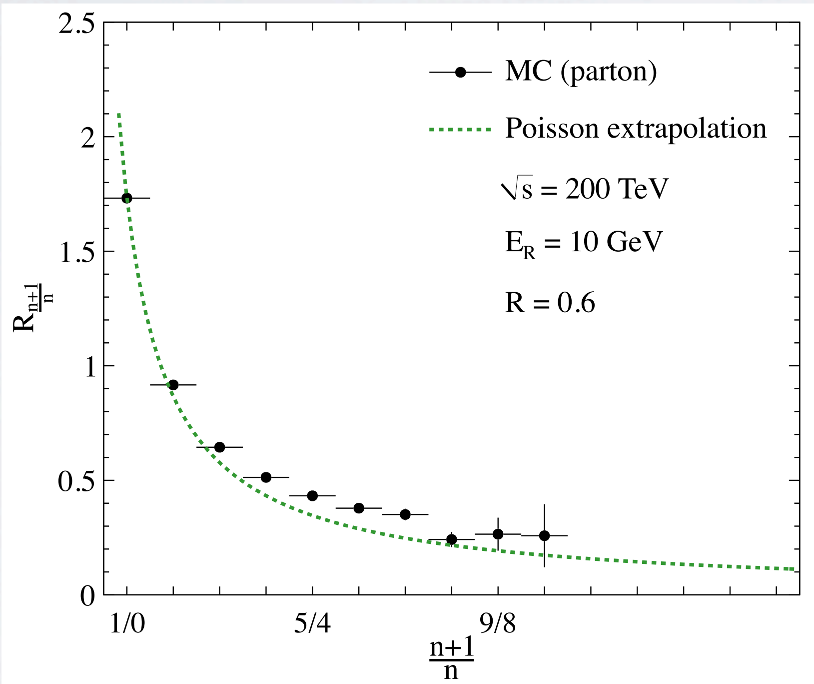
$$
\Delta_g(E,\xi) = \exp\left\{ -\frac{\alpha_S}{\pi} \ln\left(\frac{\xi}{\xi_R}\right) \left[C_A \ln\left(\frac{E}{E_R}\right) - \frac{11C_A - 2n_f}{12} \right] \right\}
$$

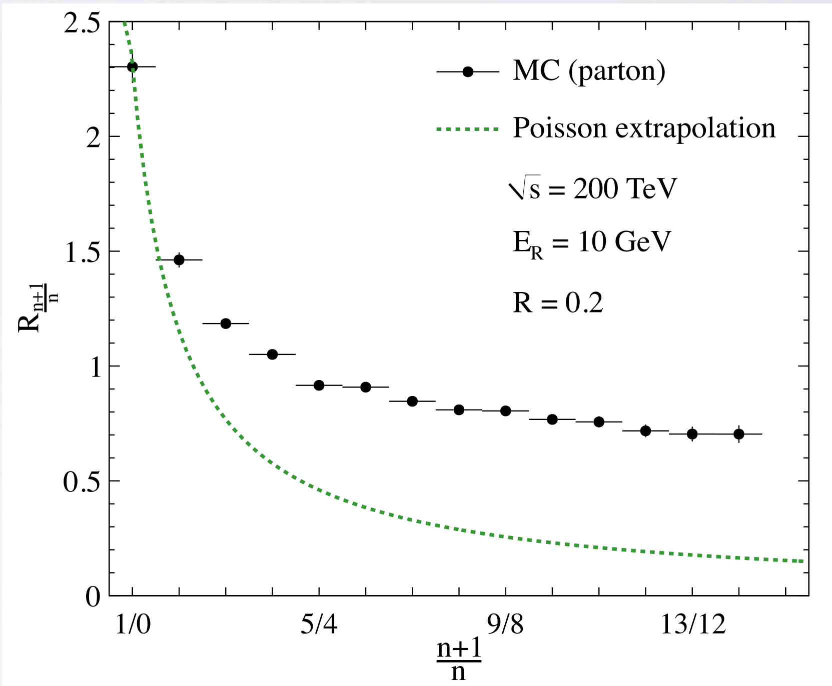
GENERALIZED K_T ALGORITHM

Scaling as a function of the jet area

- With the Durham measure, smaller average jets (i.e smaller y_{cut}) raised the size of the size of the overall logarithms, and increases the goodness of the Poisson fit.

- However, with the generalized algorithm, overall logarithm again increases, although the goodness of the Poisson fit is significantly worse.





Angular dependence of the emission types

- The resummed jet rates (and gen func.) contain no phase space dependence of the two emission types (primary vs secondary), but the parton shower does (through kinematics).

CONCLUSIONS ON e+e- SCALING

We expect the following for the distribution of jet rates in e+e-

Small emission probability (small log):

1. Staircase tail, not a good Poisson at low multiplicity.

Large emission probability (large log):

1. Increasingly good Poisson fit at multiplicities up to <N>.
2. Staircase tail sets in after <N>.
3. Strong deviation from 1. for small jet sizes (in Gen. k_T).

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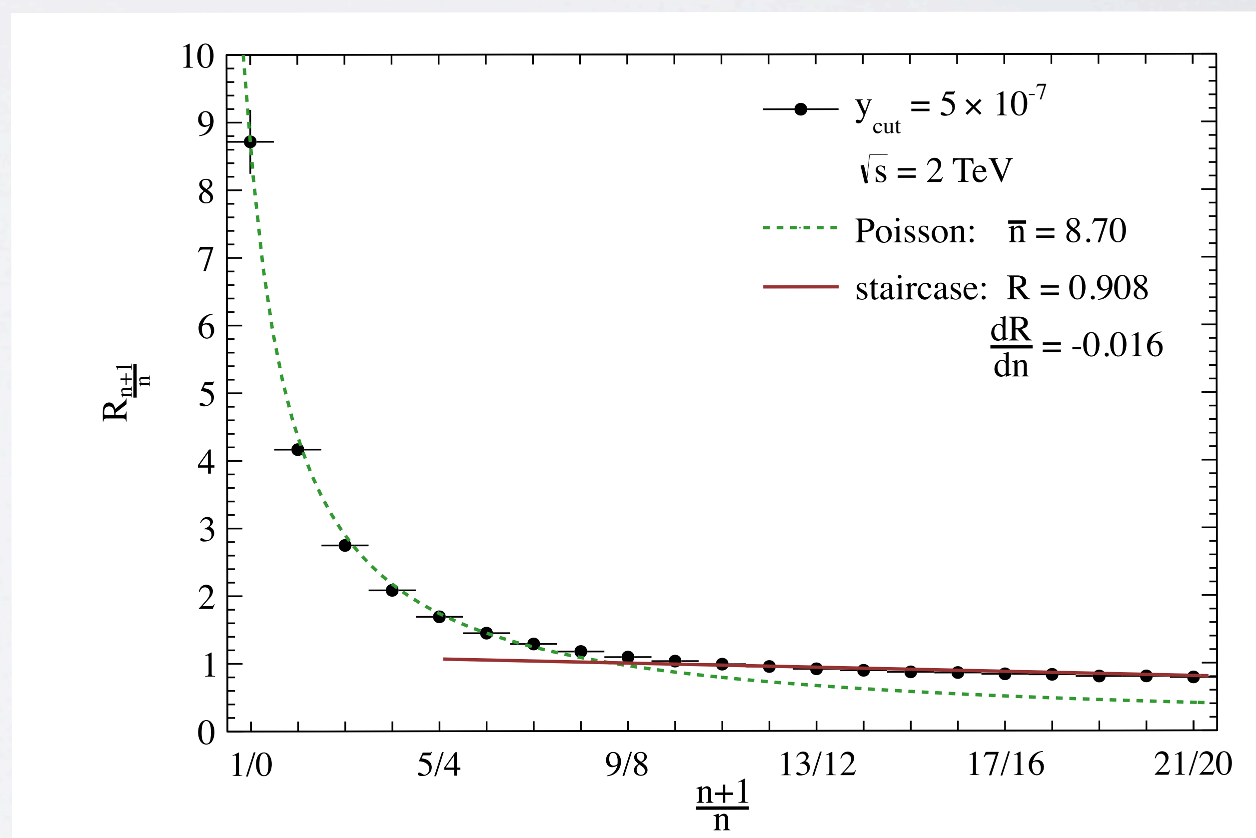


PDF EFFECTS ON SCALING

Estimating the threshold kinematics

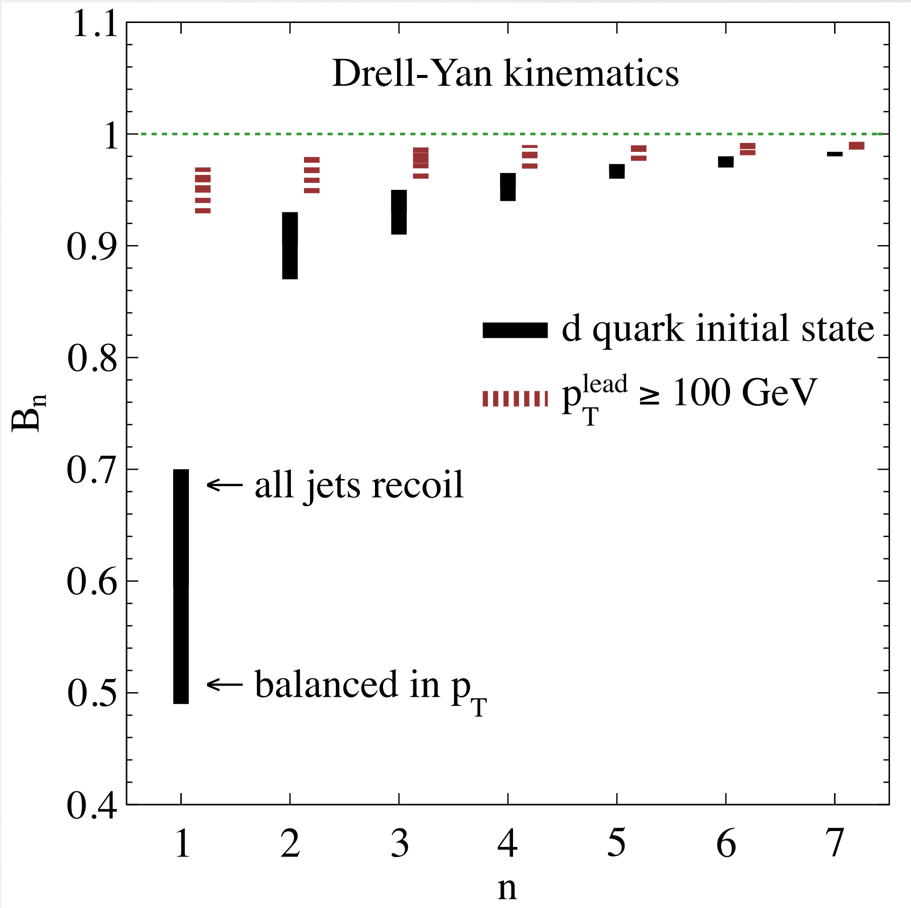
- Most naive estimate not sufficient for good agreement with the data.
- Slightly more sophisticated choice is to include the recoil of the boson from $x^{(0)} \longrightarrow x^{(1)}$

$$x^{(0)} \approx \frac{m_Z}{2E_{\text{beam}}} \quad \longrightarrow \quad x^{(1)} = \frac{\sqrt{m_Z^2 + 2\,(p_V\sqrt{p_V^2 + m_Z^2} + p_V^2)}}{2\,E_{\text{beam}}}$$

Estimating the PDF suppression

- Assume threshold kinematics on additional jet.

$$B_n = \left| \frac{\frac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)}}{\frac{f(x^{(n+2)}, p_V)}{f(x^{(n+1)}, p_V)}} \right|^2$$



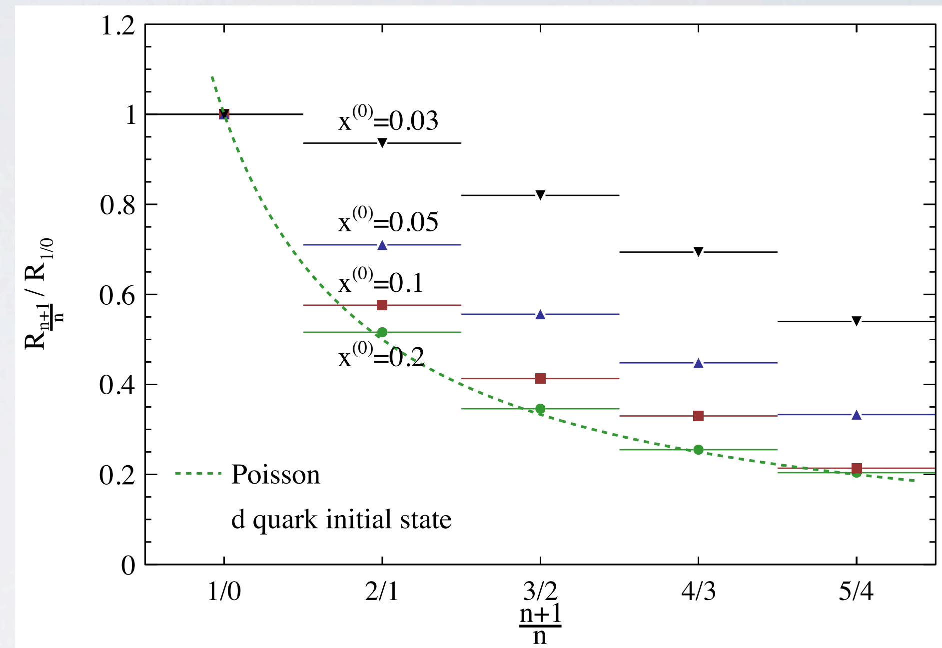
- Ambiguity starting at the kinematics of the second jet (recoil against the Z-boson or not)
- Almost identical effect for ggH (gluons vs m_{higgs})
- Key result: PDFs push down the lower multiplicity jet ratios ⟶ more staircase-like.

OTHER EFFECTS IN PP

Scaling from the backward evolution form-factor

- Forward evolution Sudakov always gives Poisson along a single line. For backward evolution depends on starting value for x.

$$\Pi(t_1, t_2; x) = \exp\left\{-\int_{t_1}^{t_2} \frac{dt}{t} \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{q\to qg}(z) \frac{f_q(x/z, t)}{f_q(x, t)}\right\}$$



Scaling in the context of BFKL dynamics

- Generating functional in the small x regime in the Multi-Regge-Kinematics limit [Webber 1998]

$$\Phi(Q^2, p_V^2)_{\mathrm{BFKL}} = \exp\left(-\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)\left[1 + (1-u)\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right]^{u/(1-u)}$$

where again we have $f_{n-1} = \frac{1}{n!}\frac{d^n}{du^n}\Phi\Big|_{u=0}$

In kinematic limit	Emission pattern behaves as	Scaling pattern
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \gg 1$	$\sigma_n \approx \frac{1}{n!}\log^n\left(1 + \frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)$	Poisson
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \ll 1$	$\sigma_n \approx \left(\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)^n$	Geometric

CONCLUSIONS ON SCALING AT PP

- Final state radiation effects from e+e- carry-over and give Poisson in large-log limit with staircase tail as usual
- PDF effect suppresses the lower multiplicity rates, flattening out the overall distribution.

“APPLICATIONS”

Deserted island physics

DESERTED ISLAND PHYSICS

Task: Calculate the normalized jet ratios for Drell-Yan at the LHC.

Direct Approach

1. Find favorite MonteCarlo
2. Wait a while (days/weeks?)
3. Count rates in each bin
4. Divide to obtain ratios

Scaling Approach

1. Everything starts of as Poisson
2. Add first inhomogenous term [from g->gg splitting]

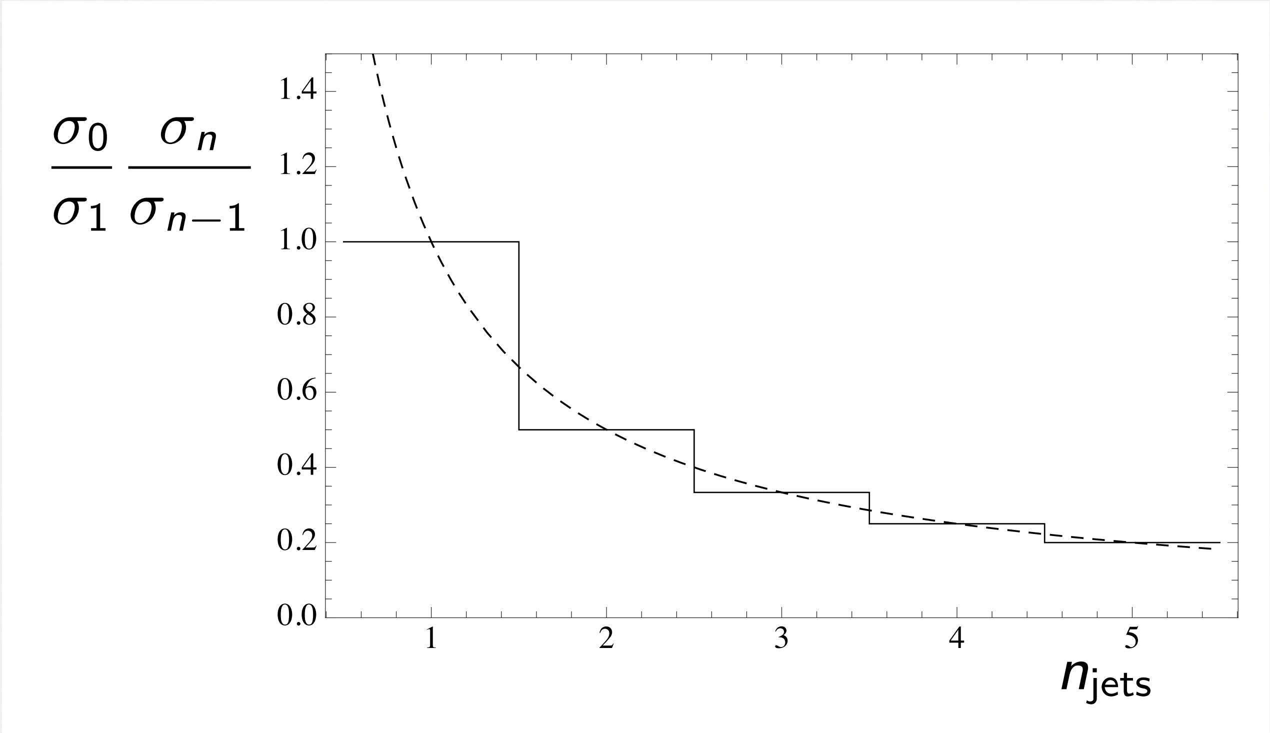
$$\bar{n} \sim 1 \qquad \bar{n}' \sim \frac{C_A}{12 C_F}$$

3. Evaluate PDF function B
4. Fold together!

DESERTED ISLAND PHYSICS

Task: Calculate the normalized jet ratios for Drell-Yan at the LHC.

1. Everything starts of as Poisson
2. Add first inhomogenous term
3. Fold in PDF effect (strong suppression of first bin)



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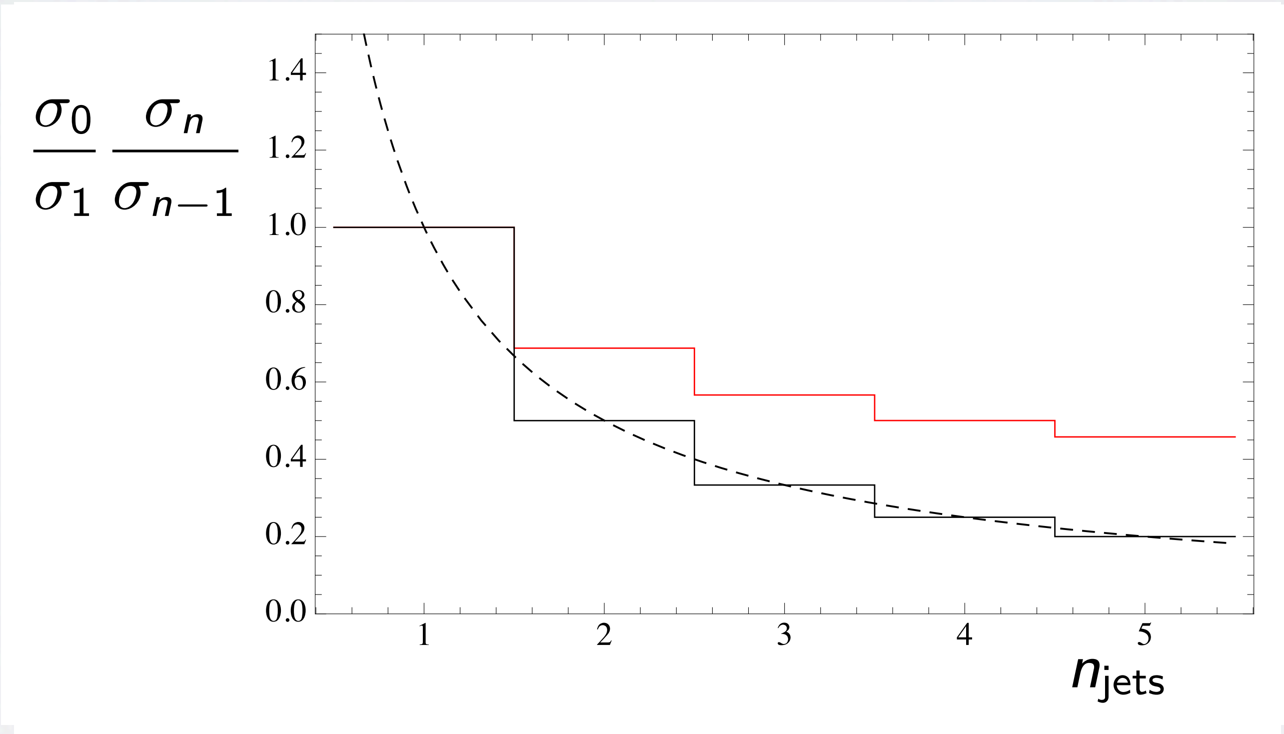

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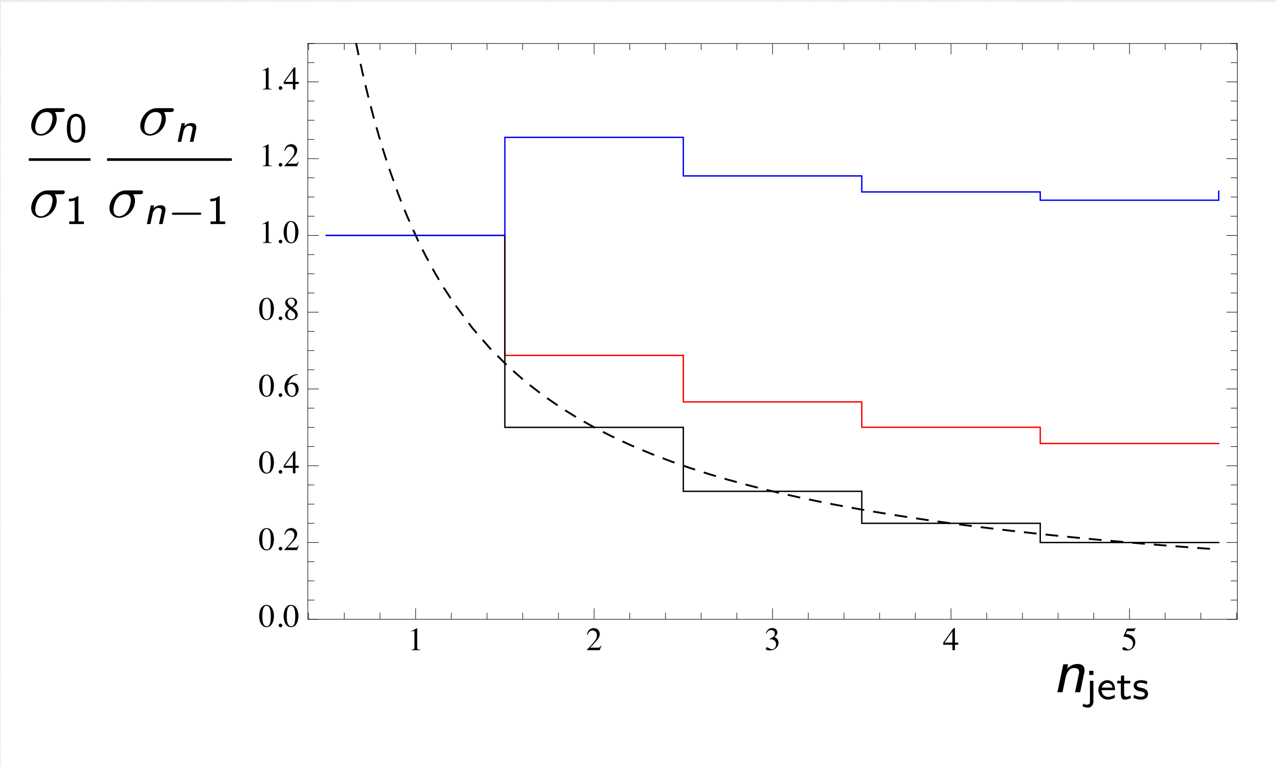
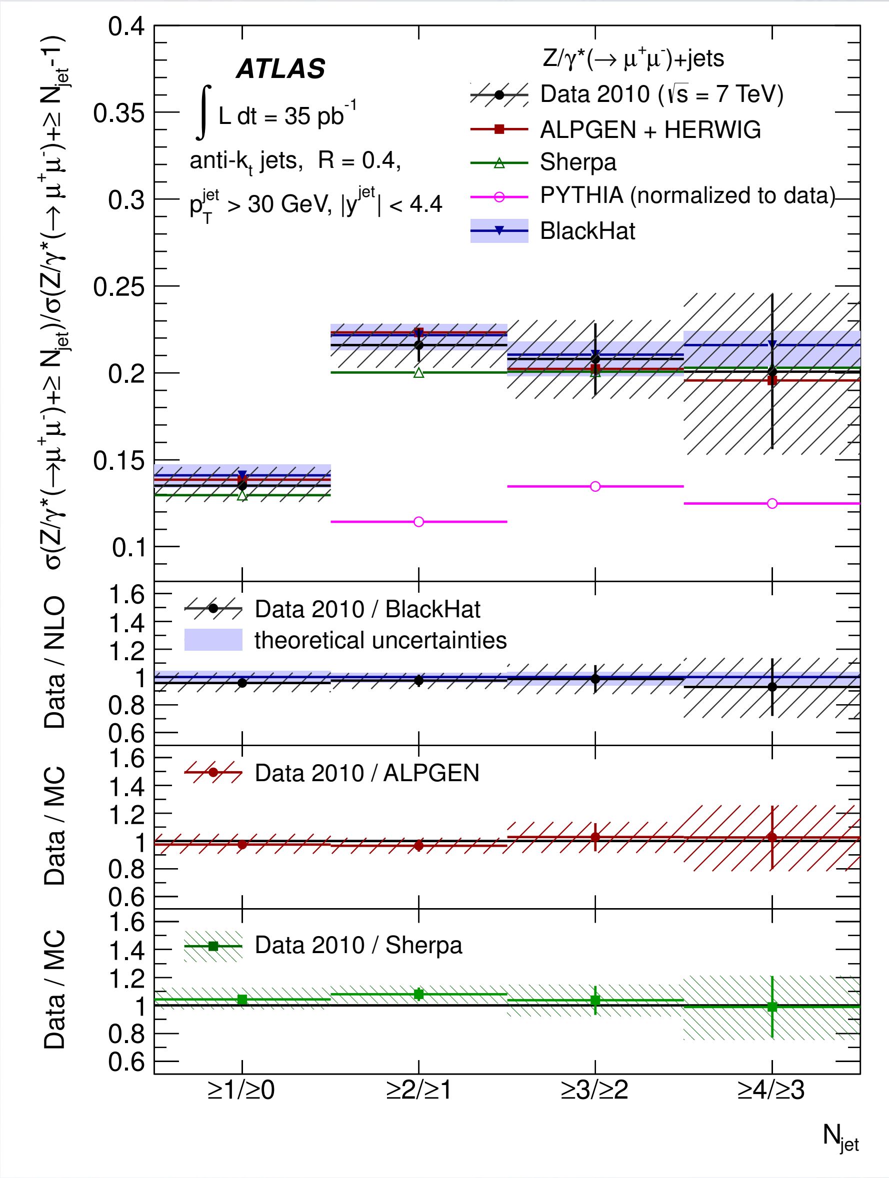
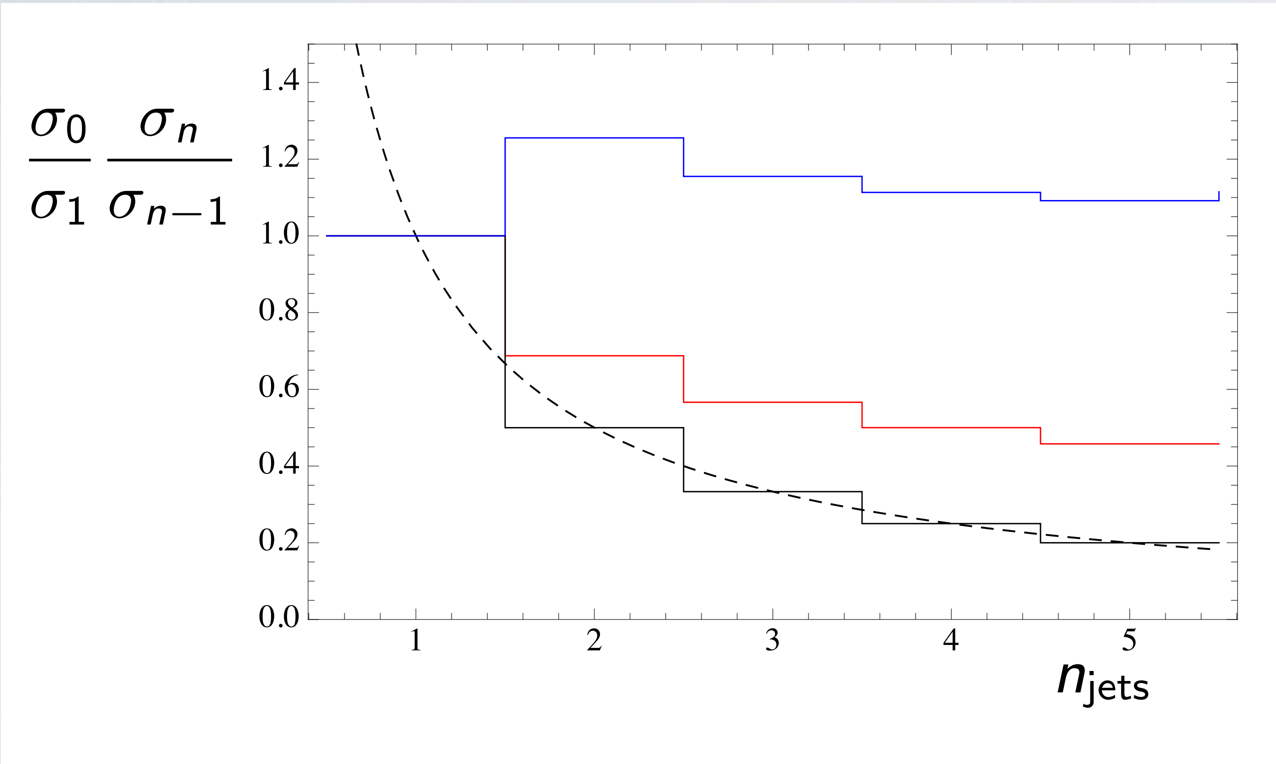
2. Add first inhomogenous term
3. Fold in PDF effect (strong suppression of first bin)

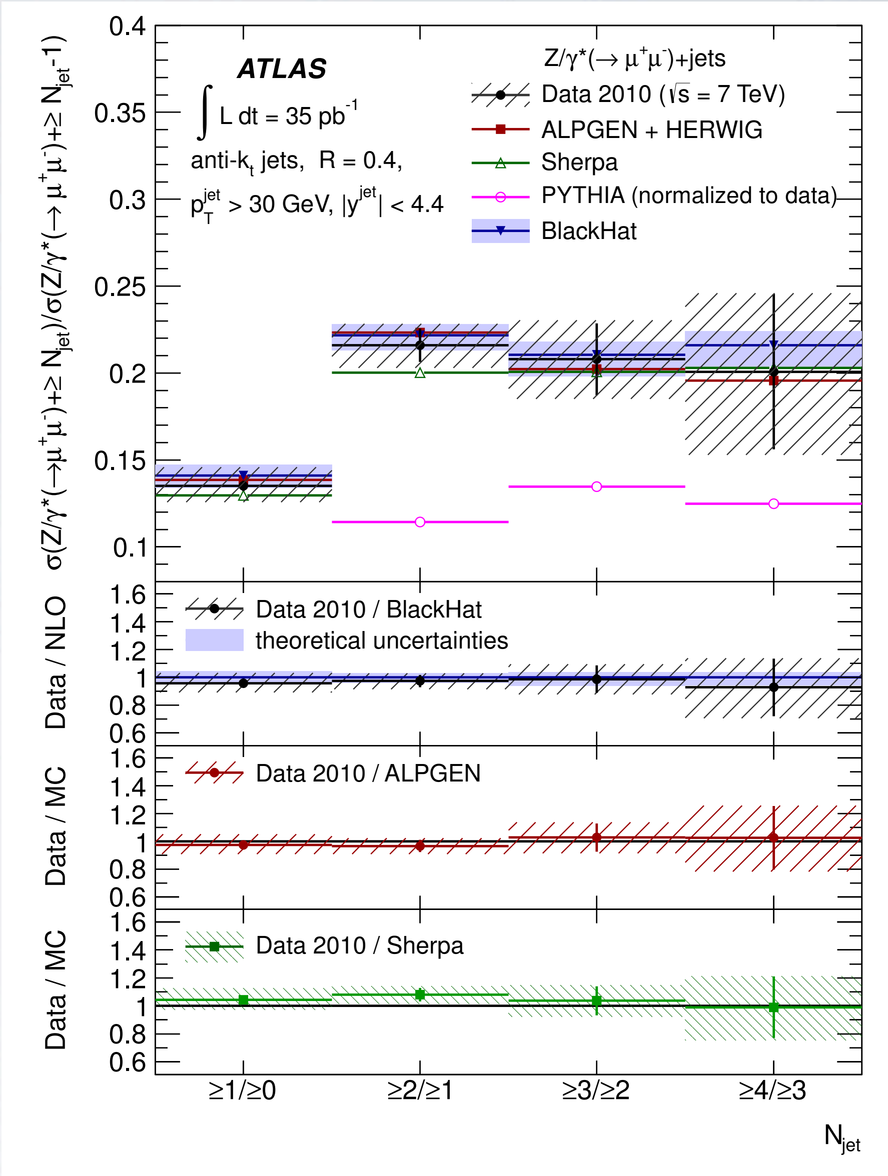


DESERTED ISLAND PHYSICS



DESERTED ISLAND PHYSICS





APPLICATIONS

Automation of analyses

- Proposal by CMS to automate NP search via automation ~100,000 different observables and direct comparison to MonteCarlo.
- Limitation is generating enough (high statistic) MonteCarlo, and my personal opinion is that there will be prohibitively many false positive.

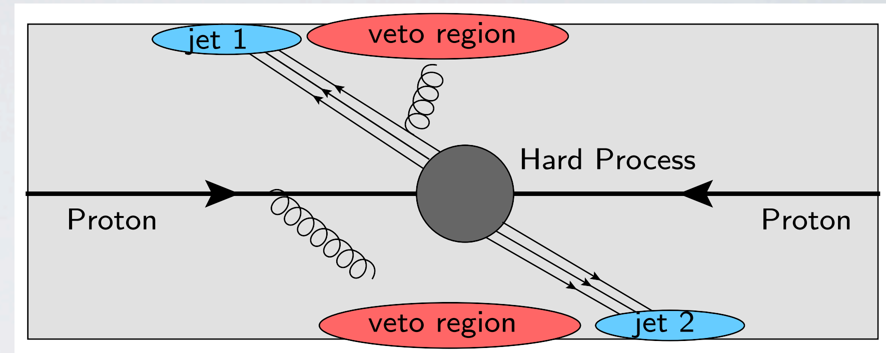
General searches via a scaling hypothesis

- QCD continuum background produces staircase scaling ratios (no structure)
- Many models of new physics produce an excess of jets starting at a certain multiplicity.
- Seeing an excess in e.g. 8 jet bin via automated MonteCarlo a very tedious task. Many models of new physics produce an excess of jets starting at a certain multiplicity.

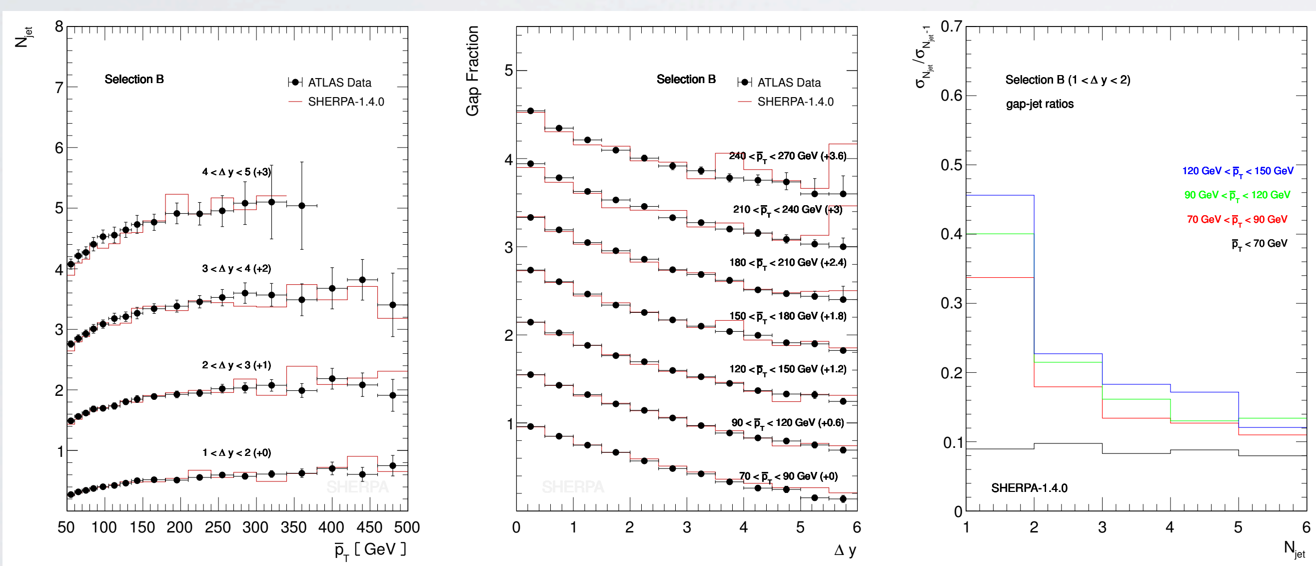
PREDICTIONS IN DI-JET GAPS

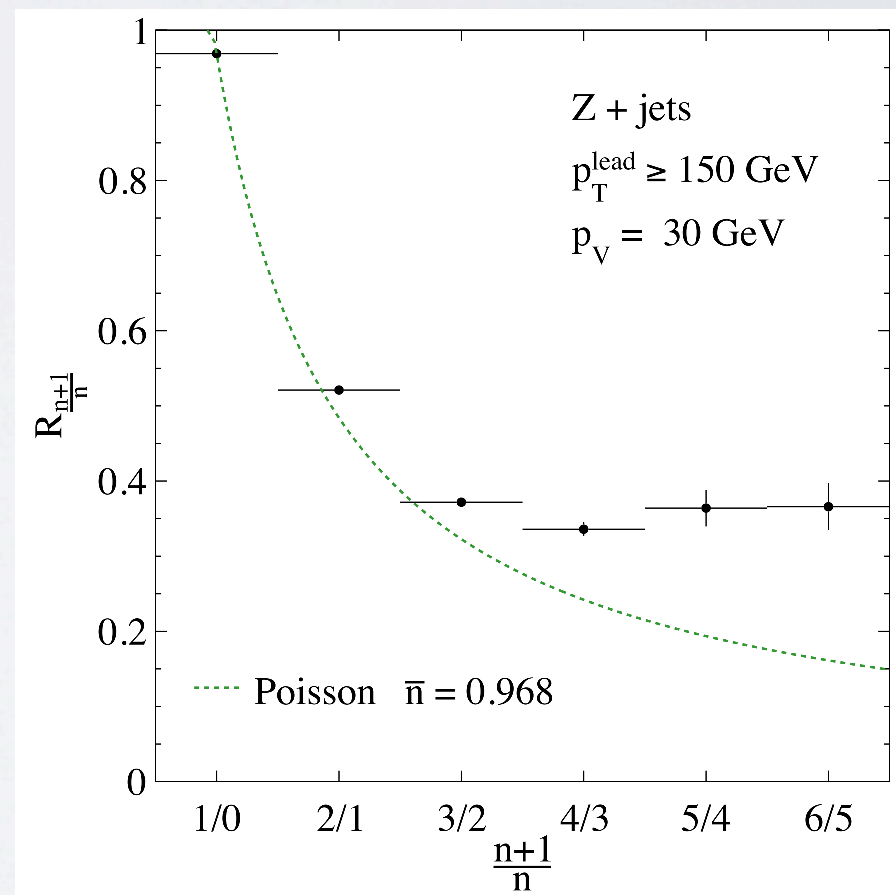
- Atlas public analysis on jet activity in rapidity gaps between "tagging" jets (ATLAS)

- Gap fraction observable sensitive to many different types of QCD effects.



- Ideal testing ground for tools to predict veto efficiencies in Higgs WBF process.





LHC CONFIRMATION OF SCALING

Measurements by ATLAS in Z+jets (awaiting measurement in di-jets)

- 1st step: for our purposes exclusive jet multiplicities more interesting observable.
- Different selections expected to interpolate between the patterns.

More on Staircase

Theoretical Basis

- Black-hat + Sherpa NLO Z and W + jets (Anti-kt; $R = .4$; $E_T > 30$ GeV) [Berger et al.]
- Staircase improves for NLO versus LO
- No solid theoretical motivation

$R_n = \sigma_n/\sigma_{n-1}$	LO	NLO
R_2	.2805	.235
R_3	.2483	.223
R_4	.2394	.226

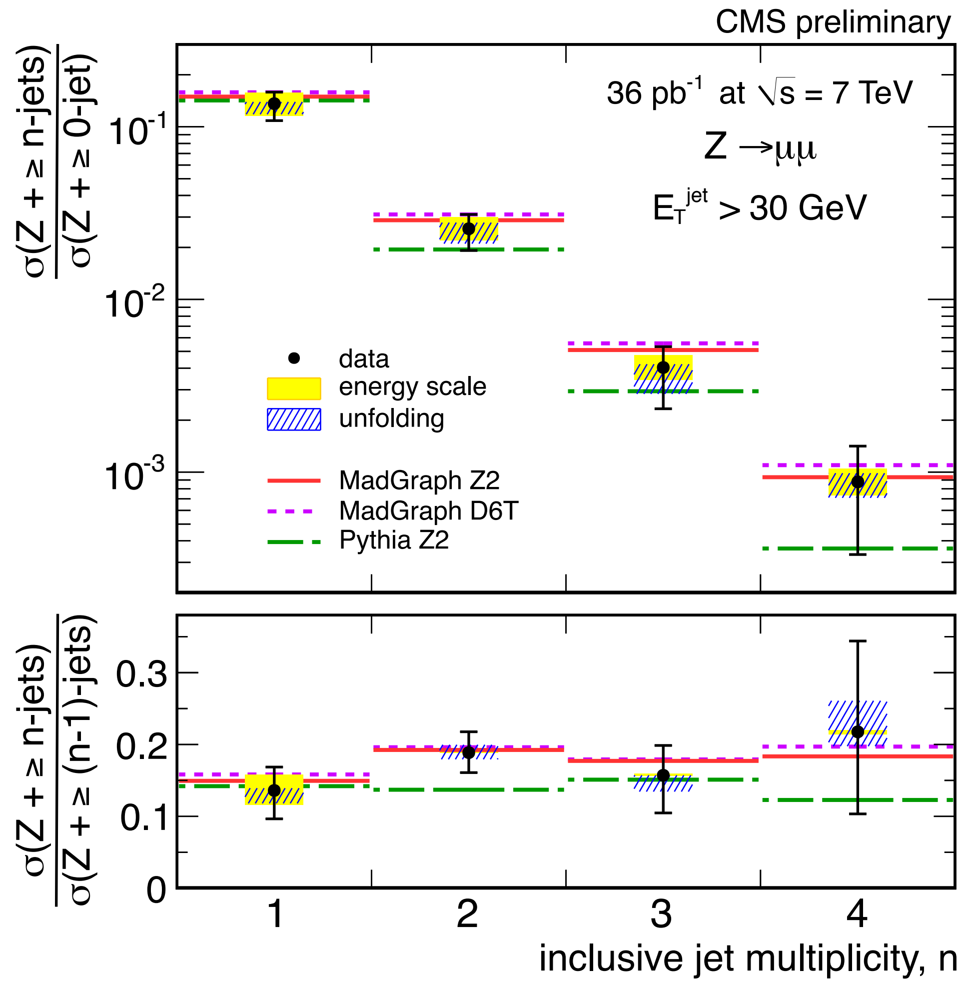
Experimental Observations

- Inclusive = exclusive ratios (for perfect staircase)

$$R_{incl} = \frac{\hat{\sigma}_{n+1}}{\hat{\sigma}_n} = \frac{\sigma_{n+1}\sum R^j_{excl}}{\sigma_n + \sigma_{n+1}\sum R^j_{excl}}$$

$$= \frac{R_{excl}\sigma_n}{(1 - R_{excl})\sigma_n + R_{excl}\sigma_n}$$

$$= R_{excl}$$



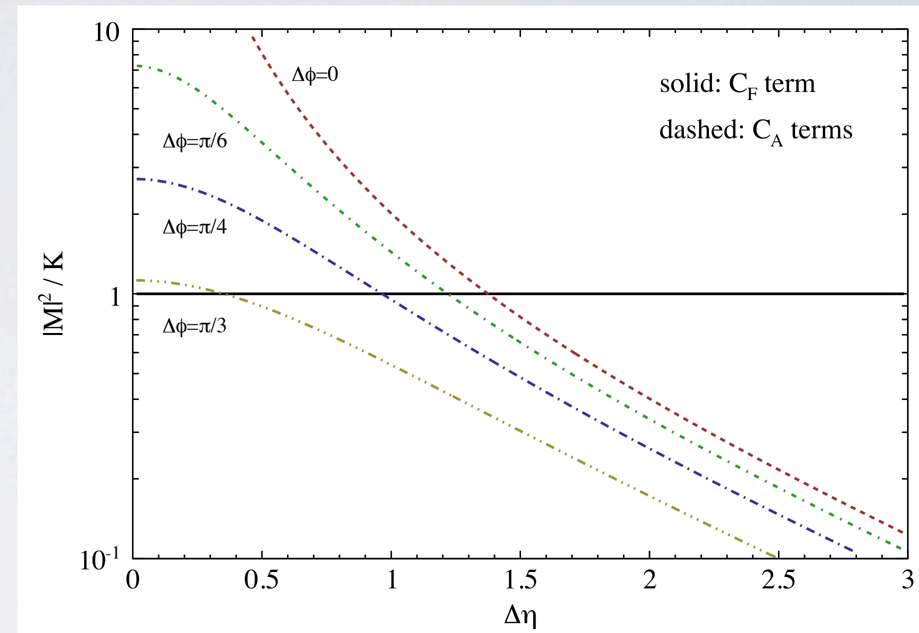
Tuesday, March 19, 13

GENERALIZED K_T ALGORITHM

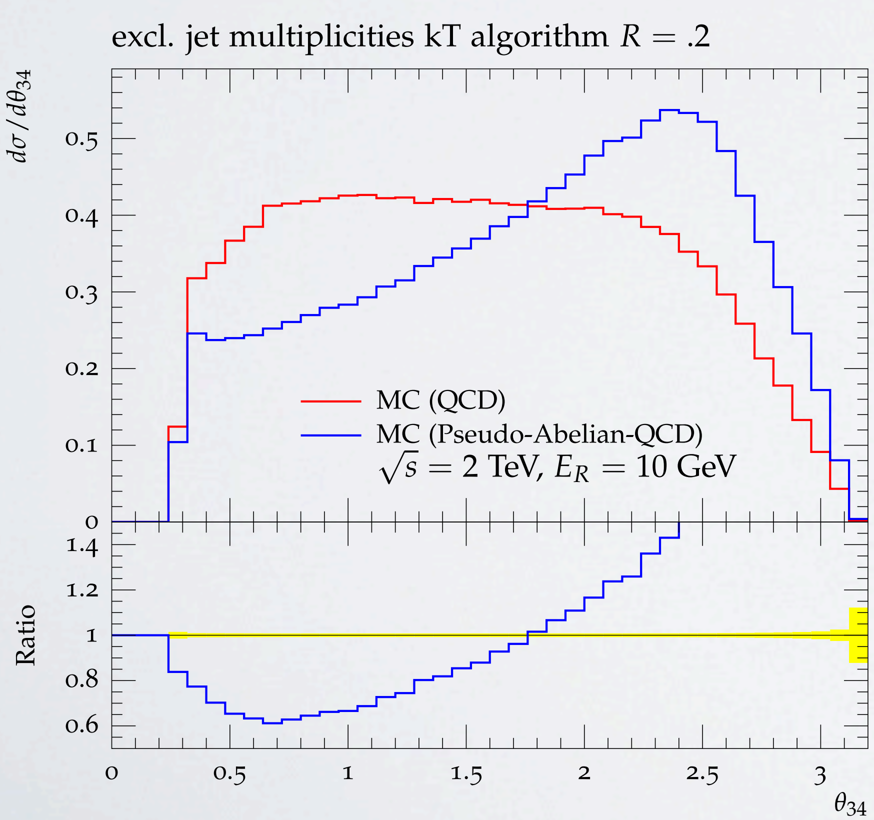
Angular dependence on the emission type

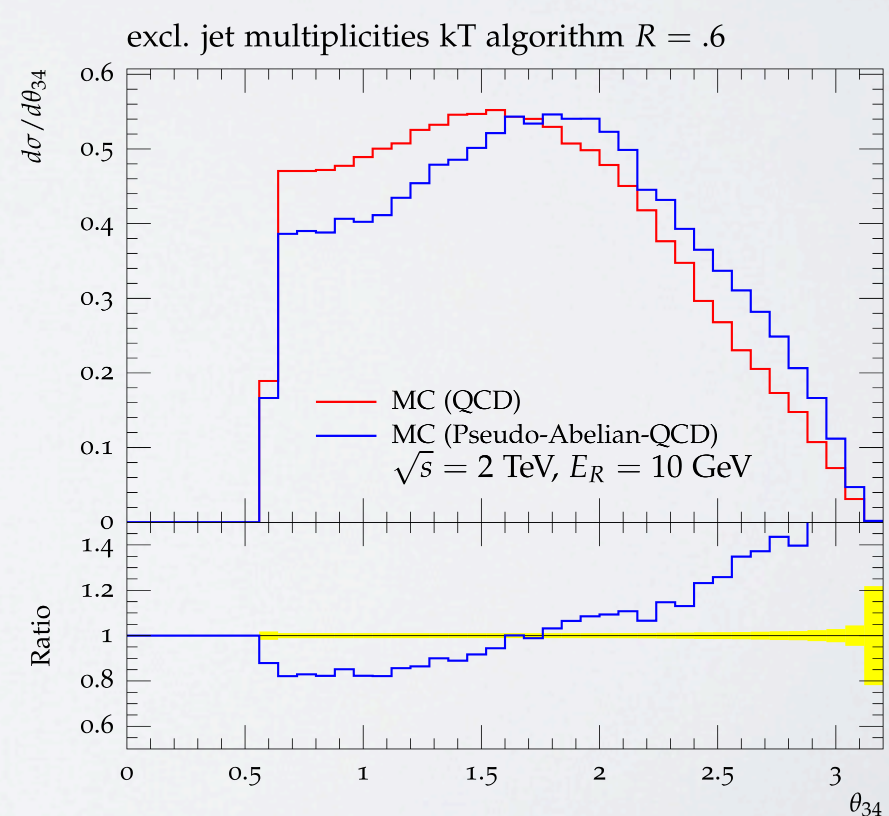
- Squared matrix element (still in the eikonal limit) tells us that secondary emission tend to be closer together in angular space.

$$|\mathcal{M}(p_1,p_2)|^2 = \frac{32C_F}{p_{T,1}p_{T,2}}\left[C_A\left(\frac{\cosh(\eta_1-\eta_2)}{\cosh(\eta_1-\eta_2)-\cos(\phi_1-\phi_2)}-1\right)+2\,C_F\right]$$



- Secondary emission for large (small) jets correspond to intra-jet (jet) evolution.





OTHER EFFECTS ON SCALING FROM e+e-

Phase space suppression

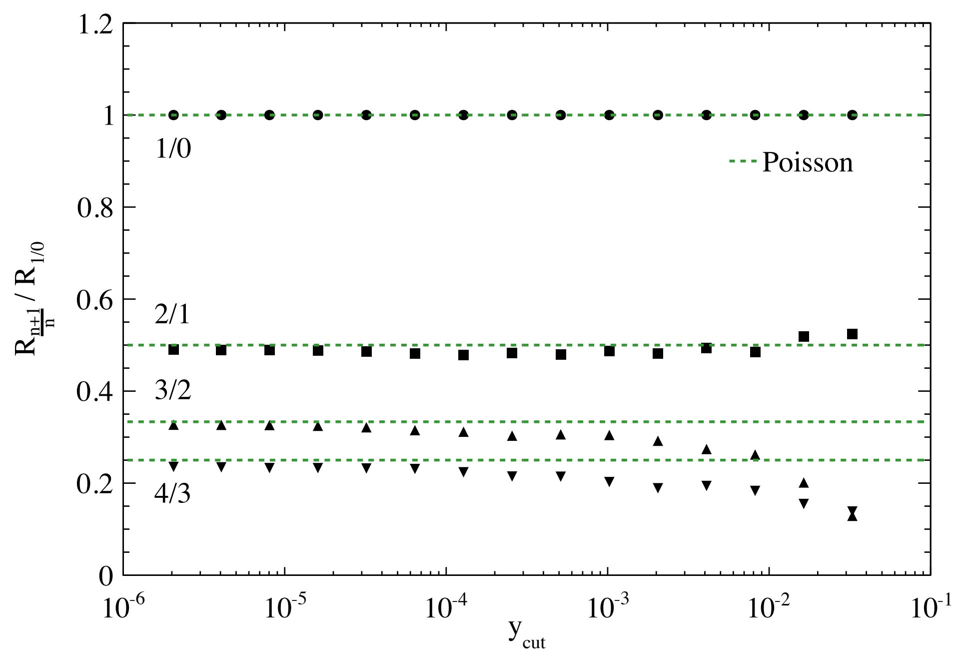
- Clearly there is a maximal number of jets which are energetically or geometrically possible in a certain process and selection.
- Most naive estimate of phase space suppression too small to explain the tilting in the staircase tail. (e.g. for R=.4, 0% over 20 multiplicities)

$$\frac{dR}{dn} \sim \frac{1-(n+1)R/4\pi}{1-nR/4\pi}$$

- But jets are preferentially emitted in the direction of the emitter, real phase space suppression much larger (though hard to estimate)

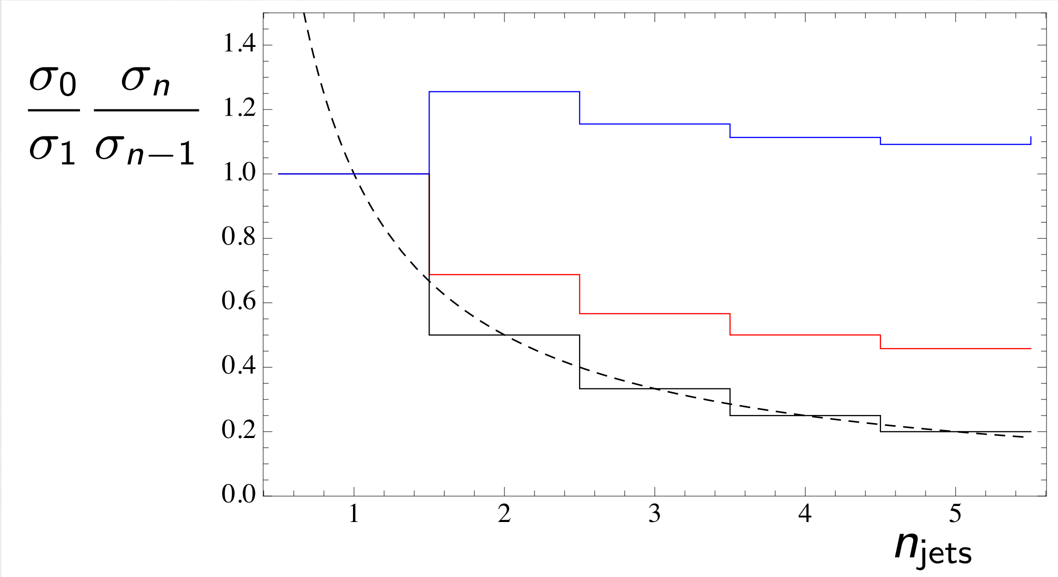
Matrix element corrections

- The full matrix elements of course do not have a reason to follow an exact scaling pattern. However, while the rates and ratios depend on these, the scaling (shape of the ratios in general does not).

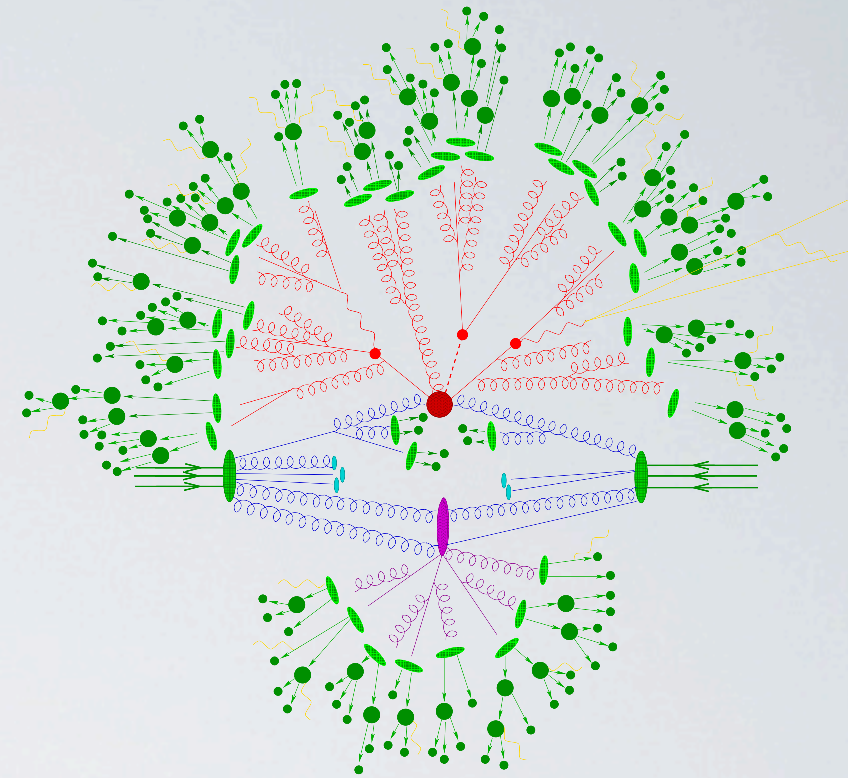


"Applications"

Desert Island Physics



JETS @ LHC

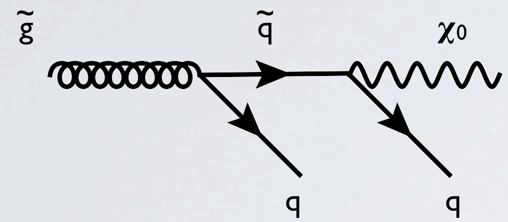


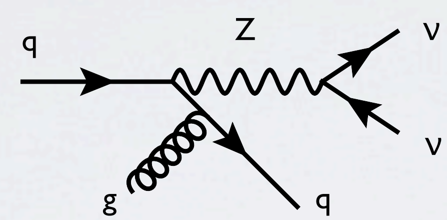
QCD and Jet intensive final states

- Jets are relevant for nearly every analysis at the LHC

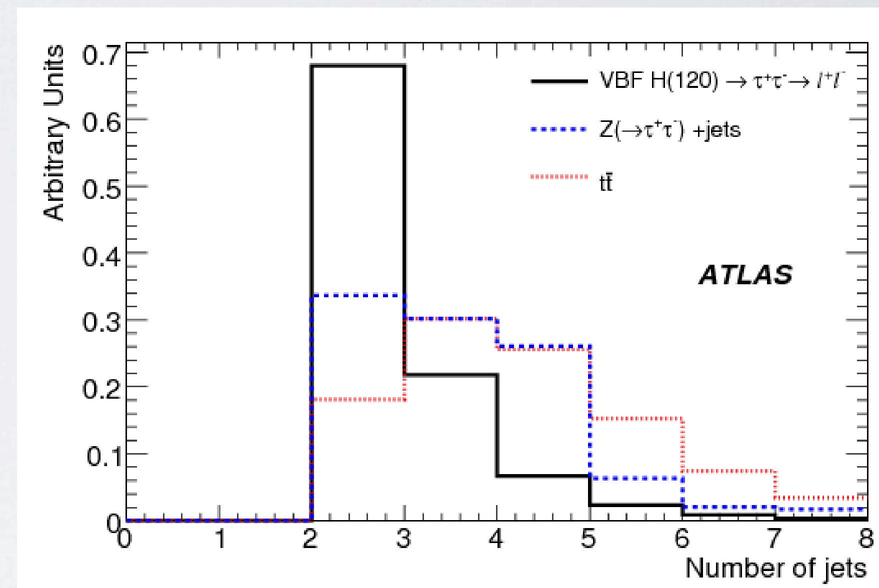
[also average multiplicity increases with energy, more jets at 13 TeV].

- Classified in this talk as either <u>decay</u> or <u>radiated</u> jets.





> The better we understand radiated jets the more sensitive we are to decay jets (new physics)



Exclusive jet bins in analyses ⟺ n_{jet} distribution

- Many analyses rely on dividing the event sample into jet multiplicity bins and optimize analysis bin by bin.
- But potentially large uncertainty on jet veto efficiency.

Analysis type	**Excl. jet bin**
Higgs WW*	0,1 jet
Higgs WBF	2 jet
Di-boson	0,1 jet
Top mass	4 jet
New physics	4,8,n? jet