New probes for QGP: quarkonium polarisation at LHC

- A long standing puzzle
- General remarks on the measurement procedure
- A rotation-invariant formalism to measure vector polarizations
- Quarkonium polarization measurements
- Heavy Ion applications

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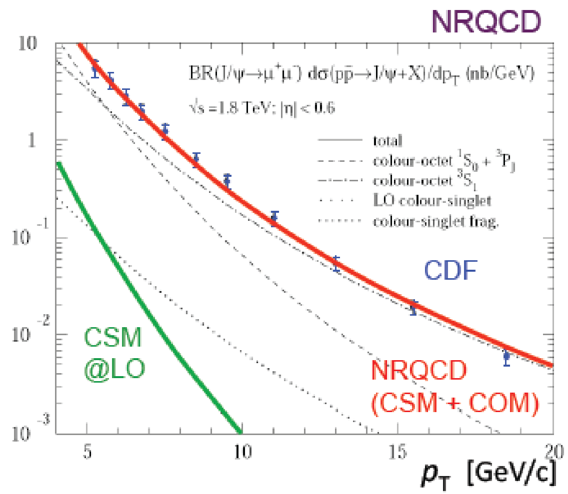
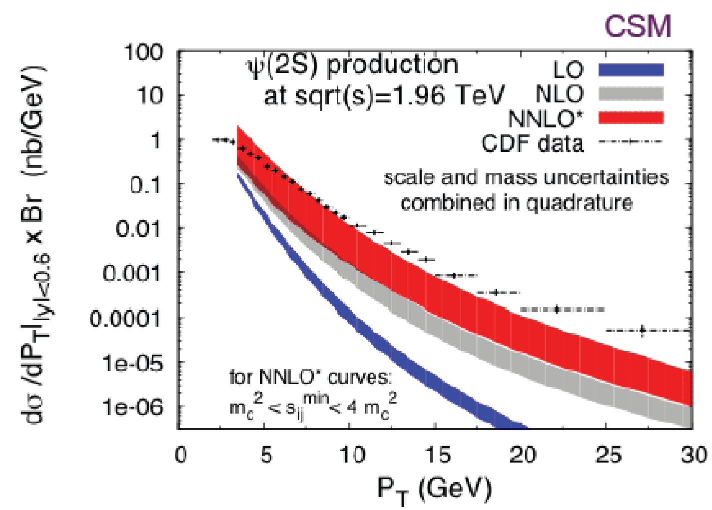
A long standing problem

One assumes that the production of **quark-antiquark states** can be described using **perturbative QCD**, as long as we "**factor out**" long-distance bound-state effects

An inescapable prediction of the semi-perturbative approach (NRQCD) is that "high" p_T quarkonia come from fragmenting gluons and are fully tranversely polarized

Despite good success in describing cross sections...



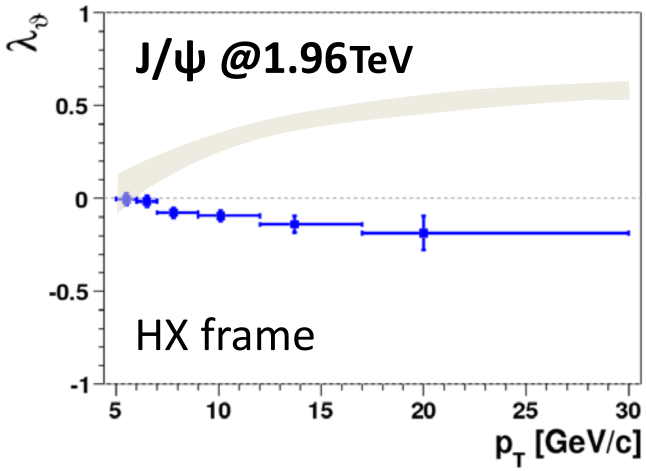


A long standing problem

One assumes that the production of **quark-antiquark states** can be described using **perturbative QCD**, as long as we "**factor out**" long-distance bound-state effects

An inescapable prediction of the semi-perturbative approach (NRQCD) is that "high" p_T quarkonia come from fragmenting gluons and are fully tranversely polarized

The first comparisons with data were not promising…



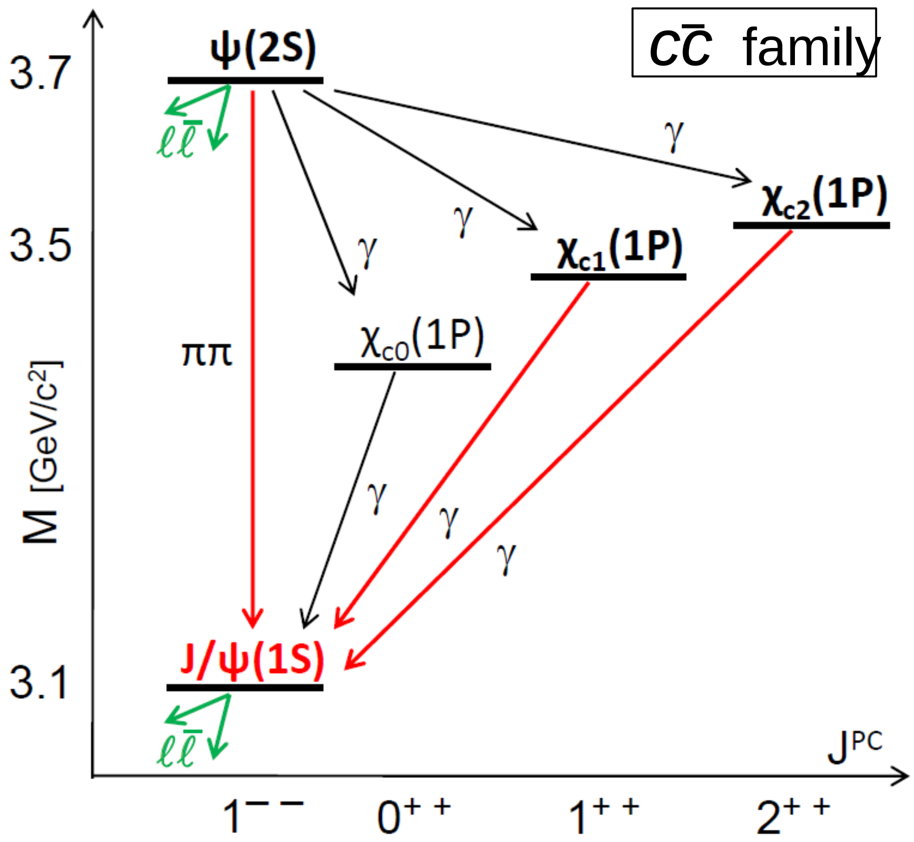
NRQCD factorization
Braaten, Kniehl & Lee, PRD62, 094005 (2000)

CDF Run II
CDF Coll., PRL 99, 132001 (2007)

But:

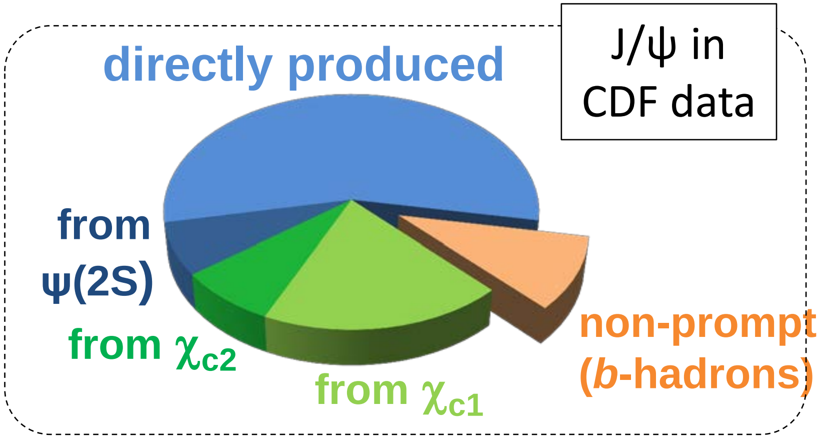
- the current experimental situation is contradictory and incomplete, as it was emphasized in Eur. Phys. J. C69, 657 (2010)
  → improve drastically the quality of the experimental information
- maybe the theory is only valid at asymptotically high p_T
  → extend measurements to $p_T >> M$
- contributions of intermediate *P*-wave states have not been fully calculated yet and are still unknown experimentally
  → measure polarizations of *directly* produced states, *ψ'* and Υ(3S)
  → measure polarizations of *P*-wave states, χ_c and χ_b, and their feeddown to *S* states

Strongly interrelated measurements

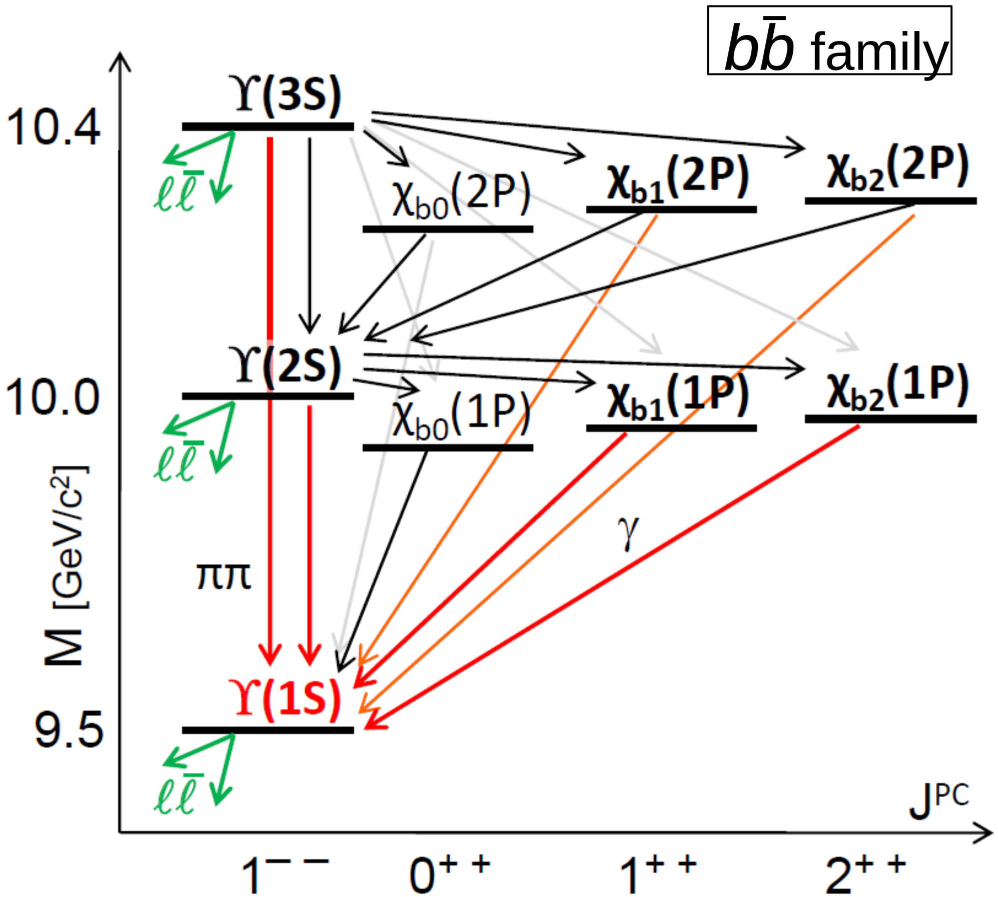


Measuring the properties of all family members is essential to fully understand quarkonium production

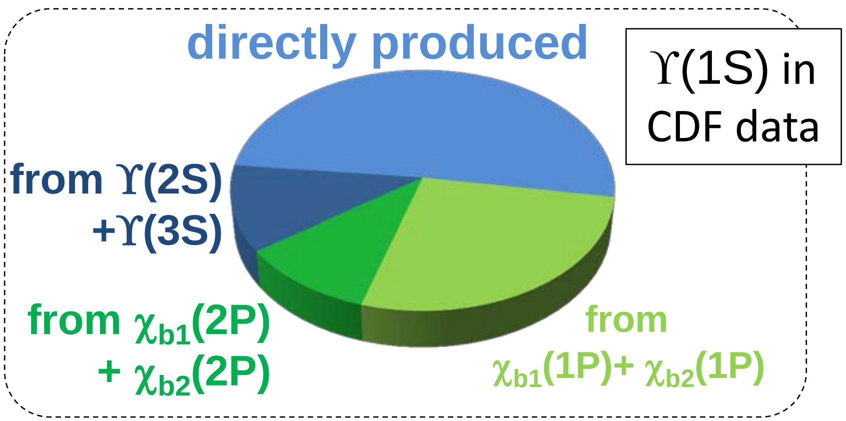
For example, the ***observed prompt J/ψ*** embodies production properties of all charmonium states in a global "average":



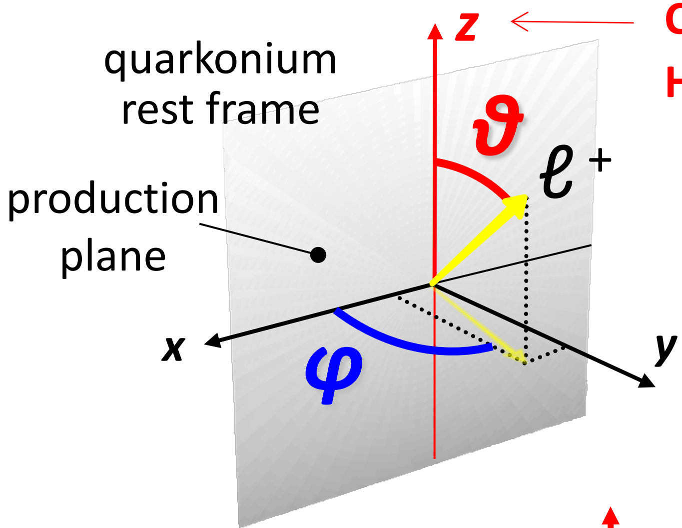
Strongly interrelated measurements



Composition of the ***observed* Υ(1S)**:



Frames and parameters



Collins-Soper axis (CS): ≈ dir. of colliding partons

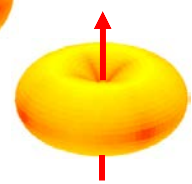
Helicity axis (HX): dir. of quarkonium momentum

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$

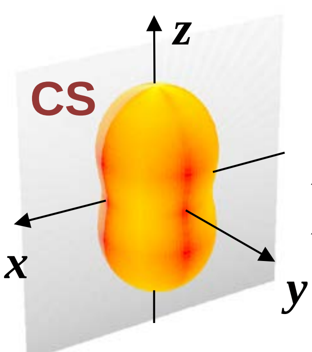
$\lambda_\vartheta = +1$
$\lambda_\varphi = \lambda_{\vartheta\varphi} = 0$

$J_z = \pm 1$ → λ_ϑ **= +1 : "transverse"** (= photon-like) pol.

$\lambda_\vartheta = -1$
$\lambda_\varphi = \lambda_{\vartheta\varphi} = 0$



$J_z = 0$ → λ_ϑ **= –1 : "longitudinal"** pol.

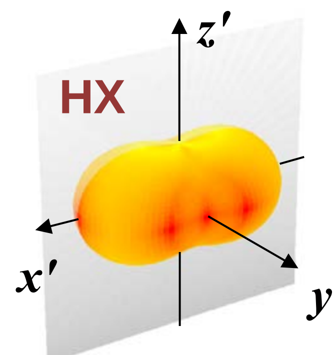


CS ⊥ HX for mid rap. / high p_T

$\lambda_\vartheta = +1$
$\lambda_\varphi = 0$

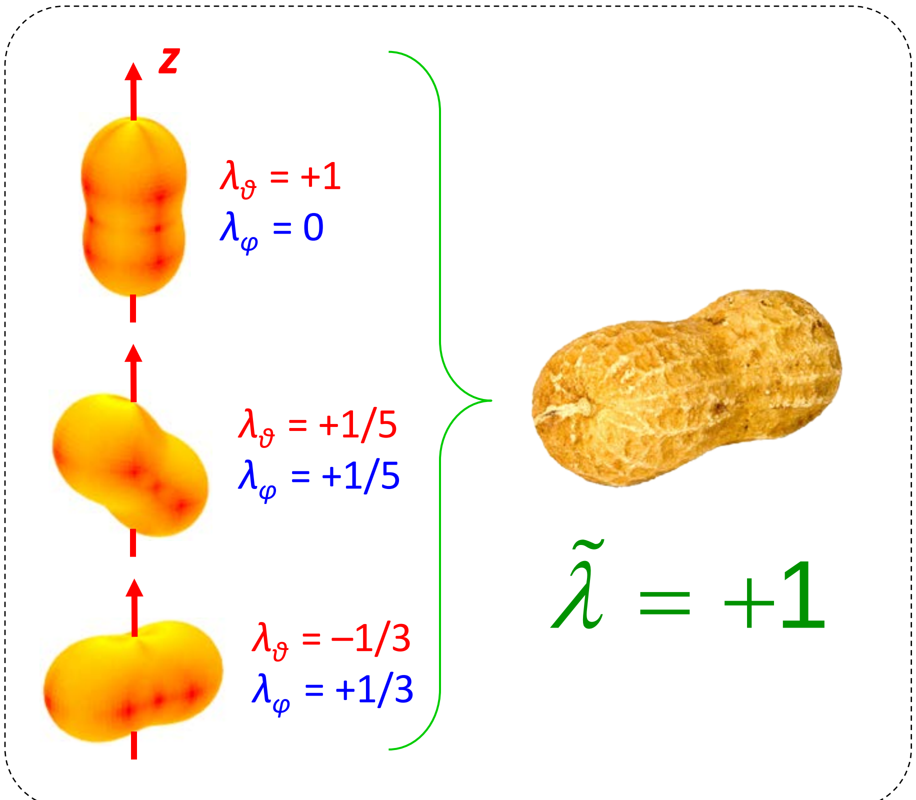
90º

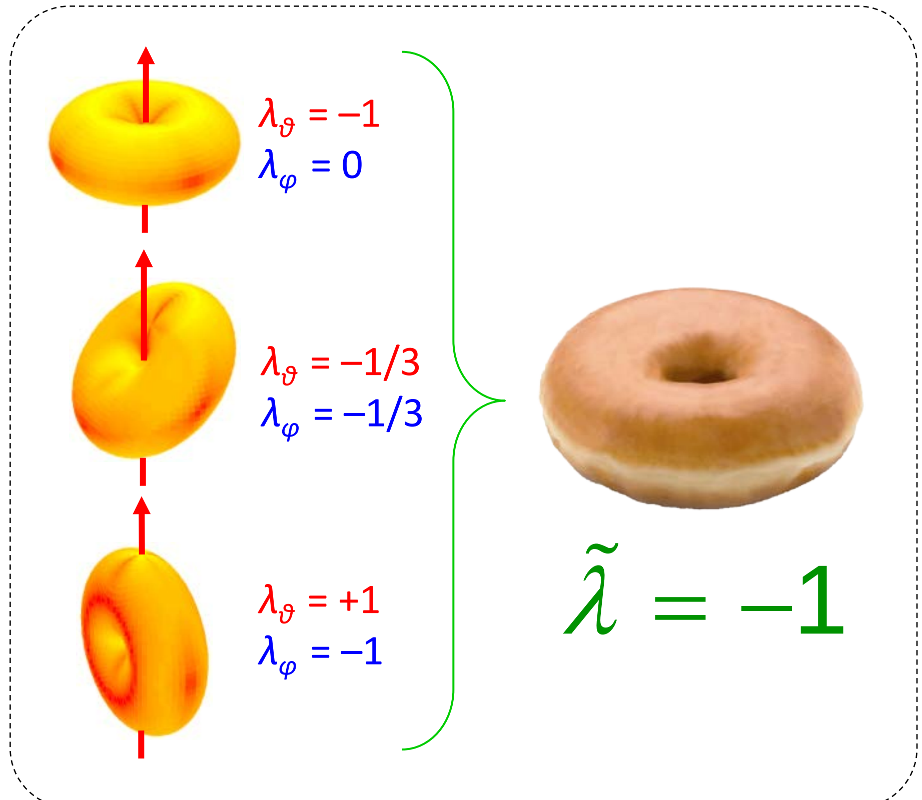
$\lambda_\vartheta = -1/3$
$\lambda_\varphi = +1/3$



Frame-independent polarization

The ***shape*** of the distribution is obviously frame-invariant.
→ it can be characterized by a frame-independent parameter, e.g. $\tilde{\lambda} = \dfrac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$





FLSW, PRL 105, 061601; PRD 82, 096002; PRD 83, 056008

J=1 states are intrinsically polarized

Single elementary subprocess: $|\psi\rangle = a_{-1}\,|1,\mathbf{-1}\rangle + a_0\,|1,\mathbf{0}\rangle + a_{+1}\,|1,\mathbf{+1}\rangle$

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi + \ldots$$

$$\lambda_\theta = \frac{1-3|a_0|^2}{1+|a_0|^2} \qquad \lambda_\varphi = \frac{2\,\mathrm{Re}\,a_{+1}^* a_{-1}}{1+|a_0|^2} \qquad \lambda_{\theta\varphi} = \frac{\sqrt{2}\,\mathrm{Re}[a_0^*(a_{+1}-a_{-1})]}{1+|a_0|^2}$$

There is no combination of a_0, a_{+1} and a_{-1} such that $\lambda_\vartheta = \lambda_\varphi = \lambda_{\vartheta\varphi} = 0$

The angular distribution is never intrinsically *isotropic*

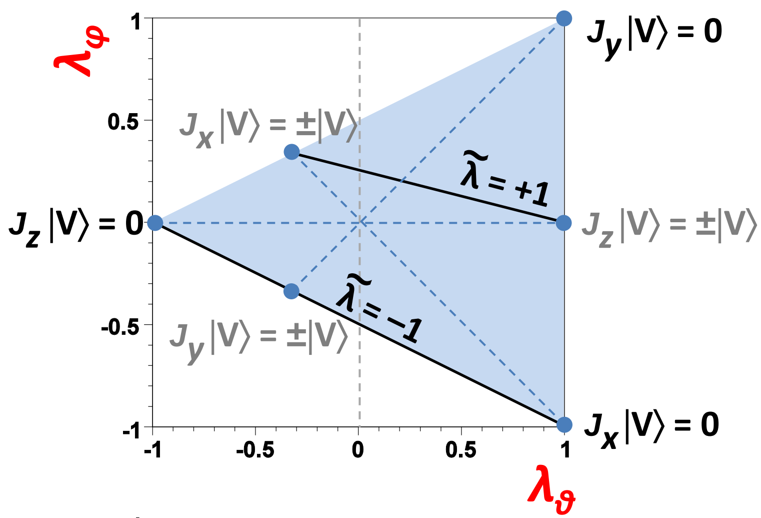
Only a "fortunate" ***mixture of subprocesses***
(or randomization effects)
can lead to a cancellation of ***all three*** observed
anisotropy parameters

To measure zero polarization
would be (in fact, is) an exceptionally interesting result...

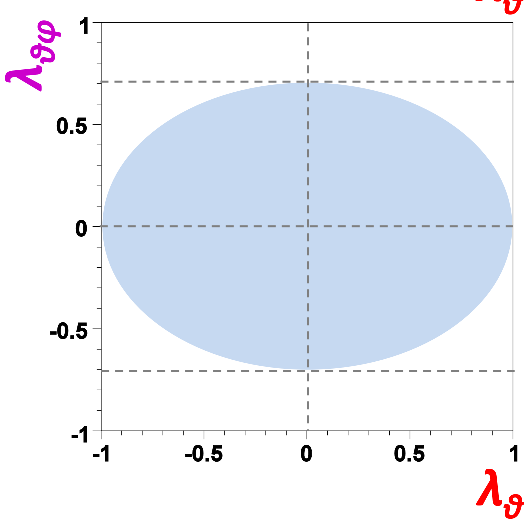
Positivity constraints for dilepton distributions

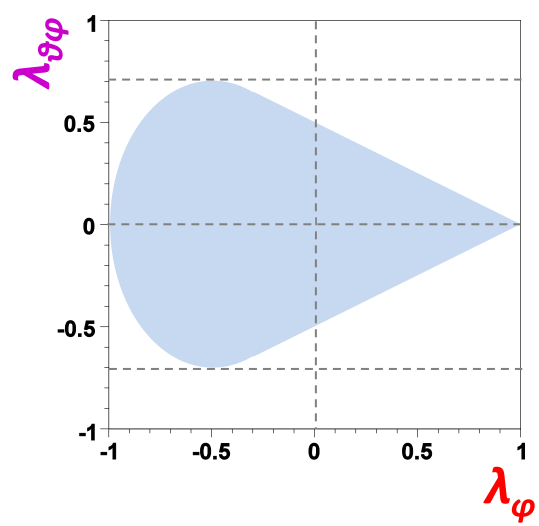
P. F., C.L., J.S., Phys. Rev. D 83, 056008 (2011)

- General and frame-independent constraints on the anisotropy parameters of vector particle decays



$$|\lambda_\varphi| \le \frac{1}{2}(1+\lambda_\vartheta), \quad \lambda_\vartheta^2 + 2\lambda_{\vartheta\varphi}^2 \le 1,$$
$$|\lambda_{\vartheta\varphi}| \le \frac{1}{2}(1-\lambda_\varphi),$$
$$(1+2\lambda_\varphi)^2 + 2\lambda_{\vartheta\varphi}^2 \le 1 \quad \text{for} \quad \lambda_\varphi < -1/3$$

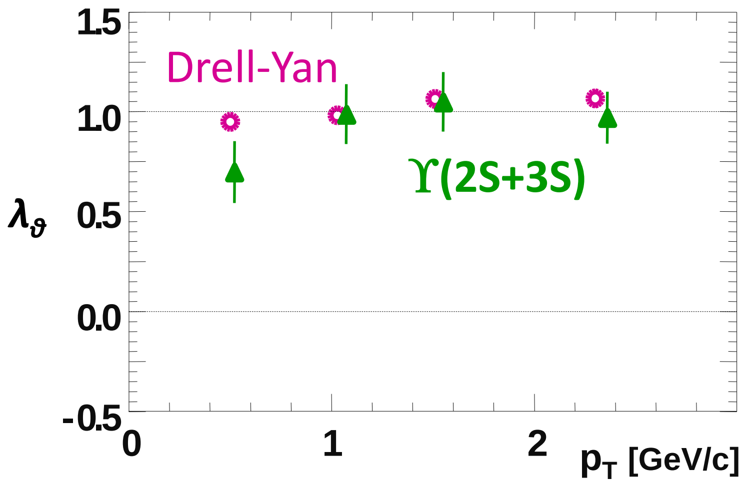




physical domain

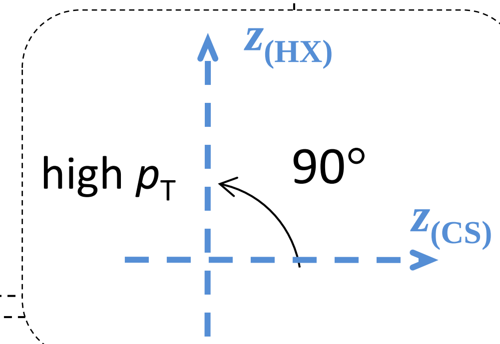
Which polarization axis?

1) helicity conservation (at the *production* vertex)
→ J =1 states produced in *fermion-antifermion annihilations* (q-$\bar{q}$ or e^+e^-) at Born level have transverse polarization along the
relative direction of the colliding fermions ***(Collins-Soper axis)***

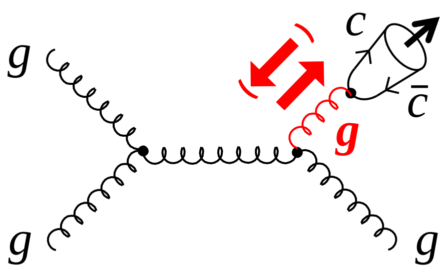


Drell-Yan is a paradigmatic case
but not the only one

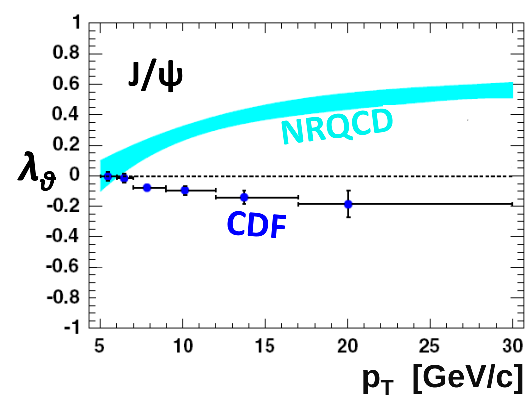
E866 (p-Cu)
Collins-Soper frame



2)



NRQCD → at *very large* p_T, quarkonium produced from the fragmentation of an on-shell gluon, inheriting its natural spin alignment

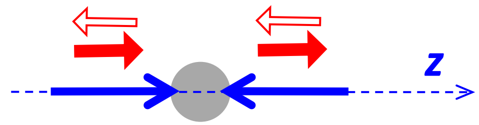


→ large, transverse polarization along the ***$Q\bar{Q}$*** *(=gluon)* **momentum** (***helicity axis***)

Example: Drell-Yan, *Z* and *W* polarization

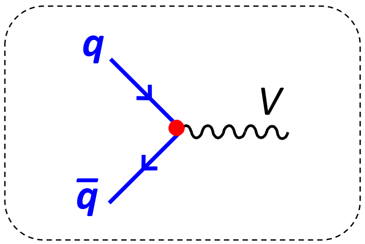
- *always fully transverse* polarization
- but with respect to a *subprocess-dependent quantization axis*

$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex, $J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction z

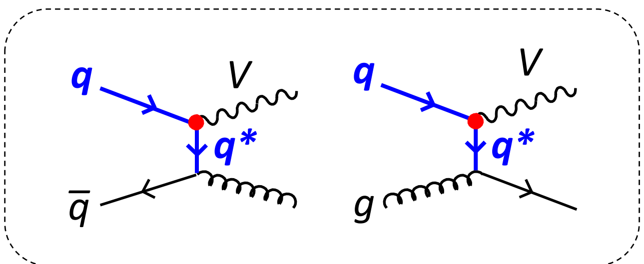
$O(\alpha_S^0)$



z = relative dir. of incoming q and $q$bar (**Collins-Soper**)

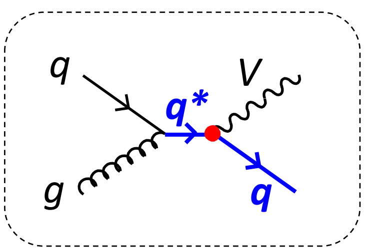
$\tilde{\lambda} = +1$

$O(\alpha_S^1)$ QCD corrections



z = dir. of *one* incoming quark (**Gottfried-Jackson**)

$\tilde{\lambda} = +1$



z = dir. of outgoing q (**cms-helicity**)

$\tilde{\lambda} = +1$

$\tilde{\lambda} = +1$

[FLS, PRL 105, 061601 (2010)]

Note: $$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \quad \boxed{\lambda_\vartheta + 4\lambda_\varphi = 1}$$

the *Lam-Tung relation* simply derives from rotational invariance + helicity conservation!

Advantages of “frame-invariant” measurements

Gedankenscenario:

Consider this (purely hypothetic) mixture of subprocesses for Υ production:

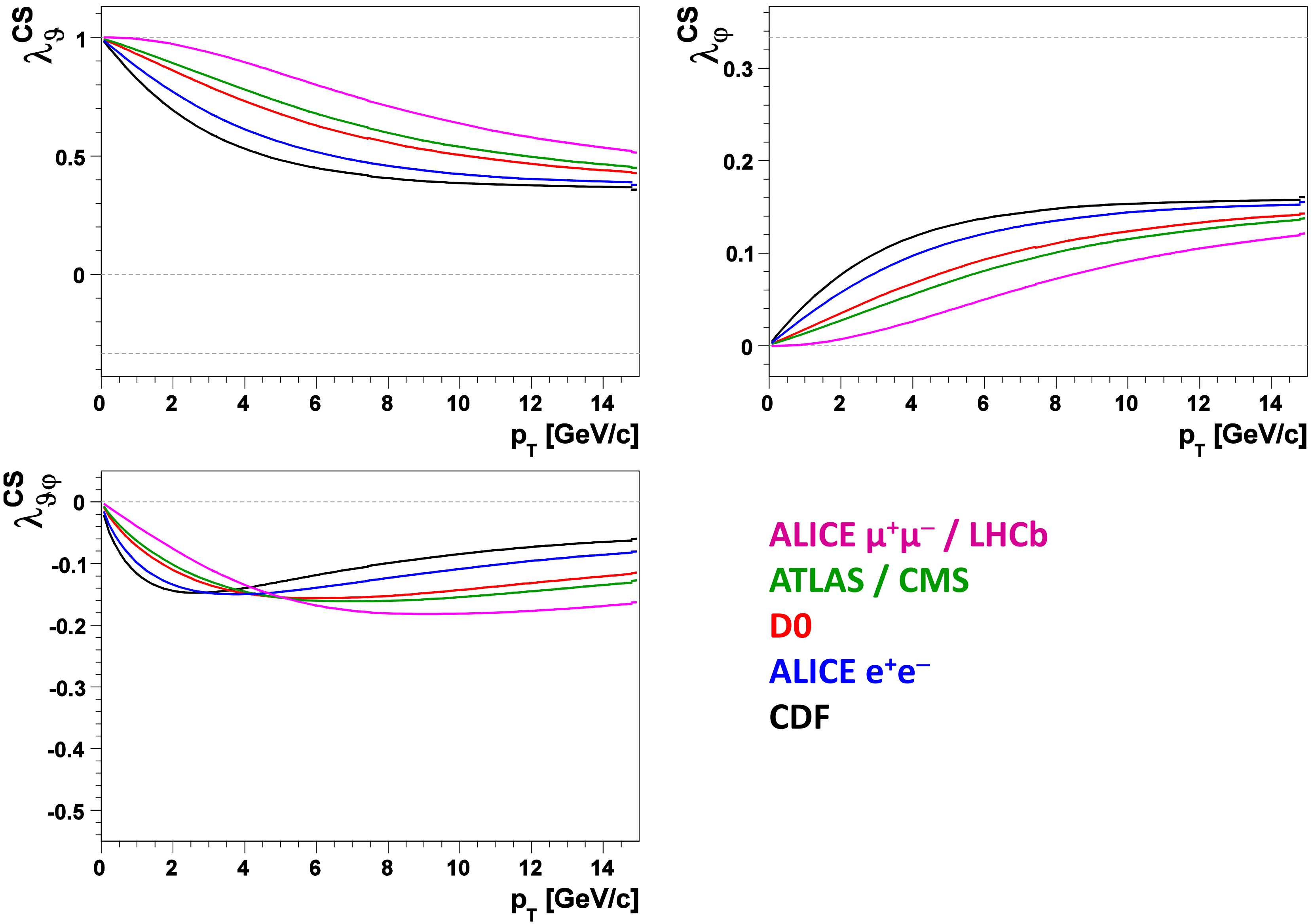
- **60%** of the events have a natural **transverse** polarization in the **CS** frame
- **40%** of the events have a natural **transverse** polarization in the **HX** frame

As before:

CDF	$\lvert y \rvert < 0.6$
D0	$\lvert y \rvert < 1.8$
ATLAS & CMS	$\lvert y \rvert < 2.5$
ALICE e^+e^-	$\lvert y \rvert < 0.9$
ALICE $\mu^+\mu^-$	$-4 < y < -2.5$
LHCb	$2 < y < 5$

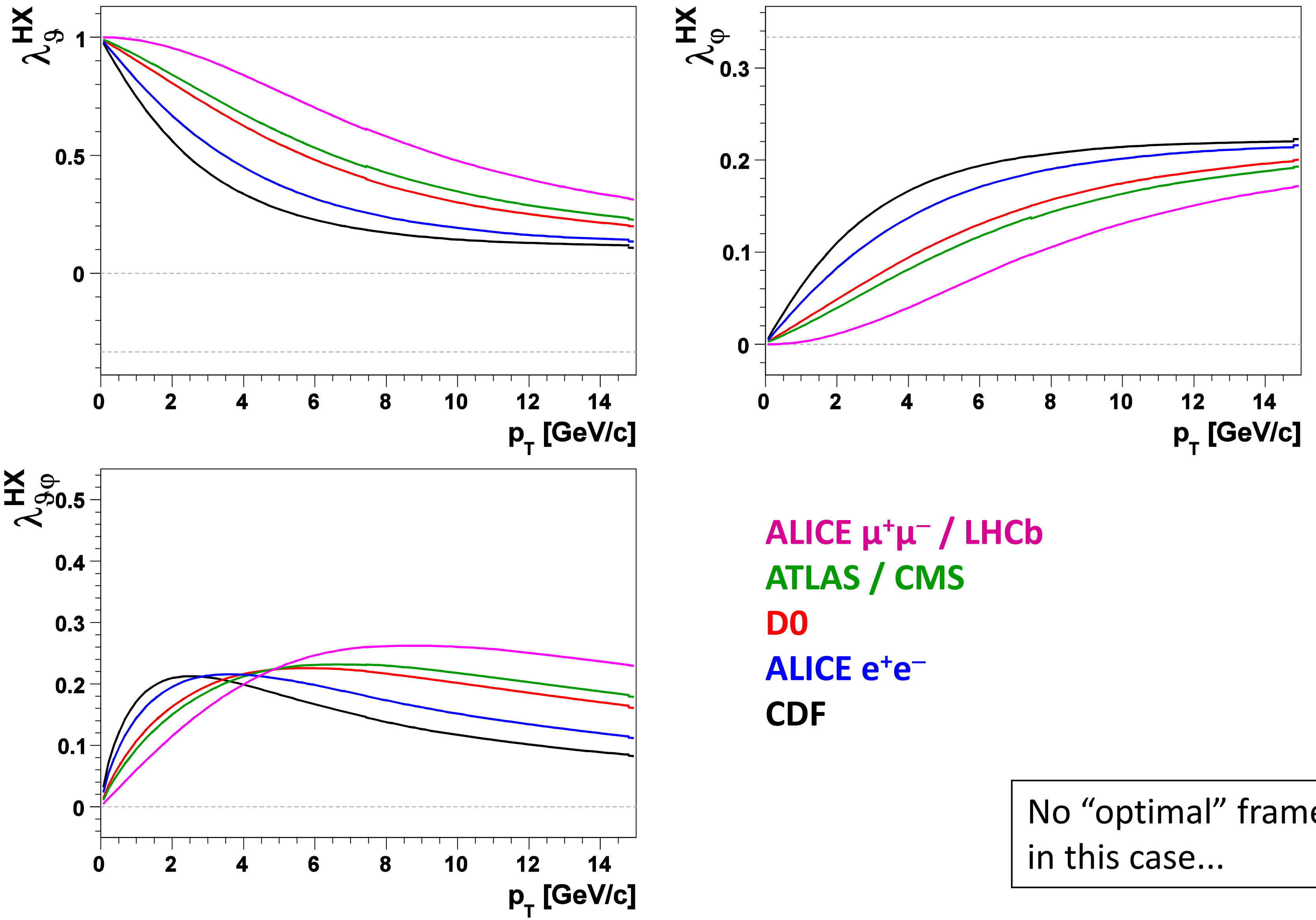
Frame choice 1

All experiments choose the CS frame



Frame choice 2

All experiments choose the HX frame



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

D0

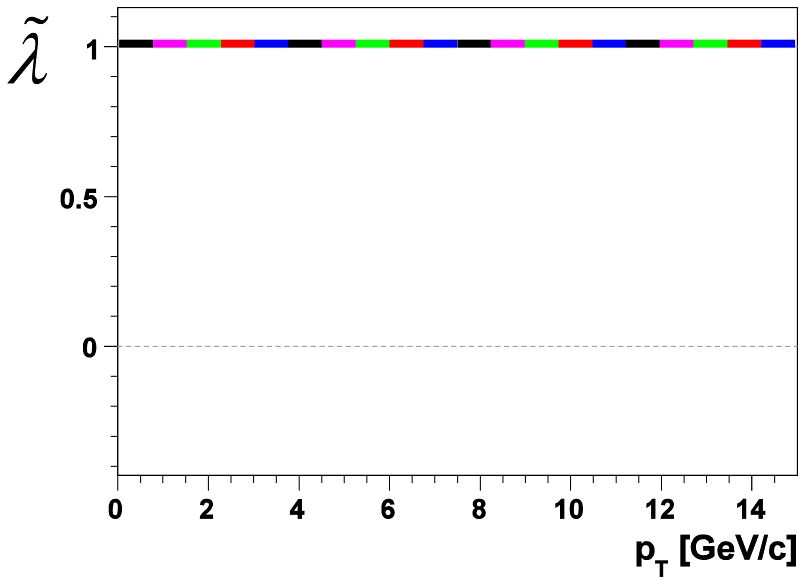
ALICE e^+e^-

CDF

No "optimal" frame in this case...

Any frame choice

The experiments measure an invariant quantity, for example $\tilde{\lambda} = \dfrac{\lambda_\vartheta + 3\,\lambda_\varphi}{1 - \lambda_\varphi}$



ALICE $\mu^+\mu^-$ / LHCb
ATLAS / CMS
D0
ALICE e^+e^-
CDF

Using $\tilde{\lambda}$ we measure an "intrinsic quality" of the polarization (always transverse and kinematics-independent, in this case)

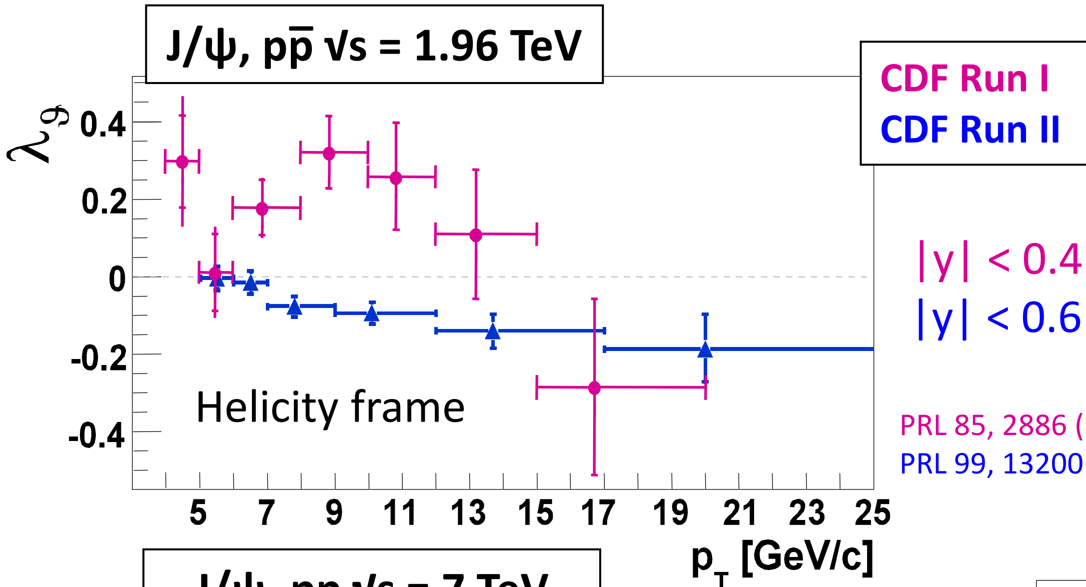
Frame-invariant quantities

- are immune to "extrinsic" kinematic dependencies
- minimize the acceptance-dependence of the measurement
- facilitate the comparison between experiments, and between data and theory
- can be used as a cross-check: is the measured $\tilde{\lambda}$ identical in different frames?
  (not trivial: spurious anisotropies induced by the detector do not have the qualities of a J = 1 decay distribution)

[FLSW, PRD 81, 111502(R) (2010), EPJC 69, 657 (2010)]

Quarkonium polarization: a "puzzle"

- J/ψ: Measurements at Tevatron , LHC (ALICE)



$|y| < 0.4$

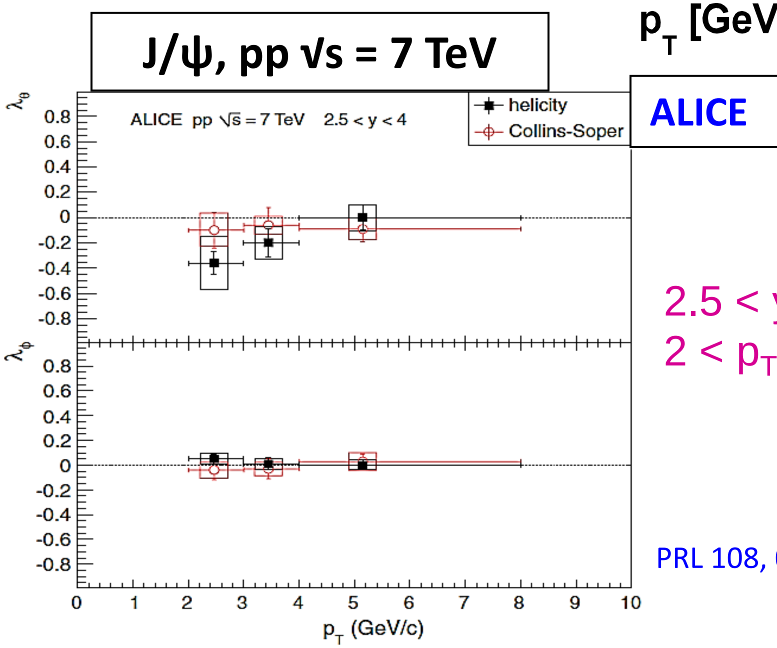
$|y| < 0.6$

PRL 85, 2886 (2000)

PRL 99, 132001 (2007)

- Only λ_θ measured
- Only one frame used (**HX**)

- **CDF II** vs **CDF I** → not known what caused the change



$2.5 < y < 4$,

$2 < p_T < 8$ GeV/c

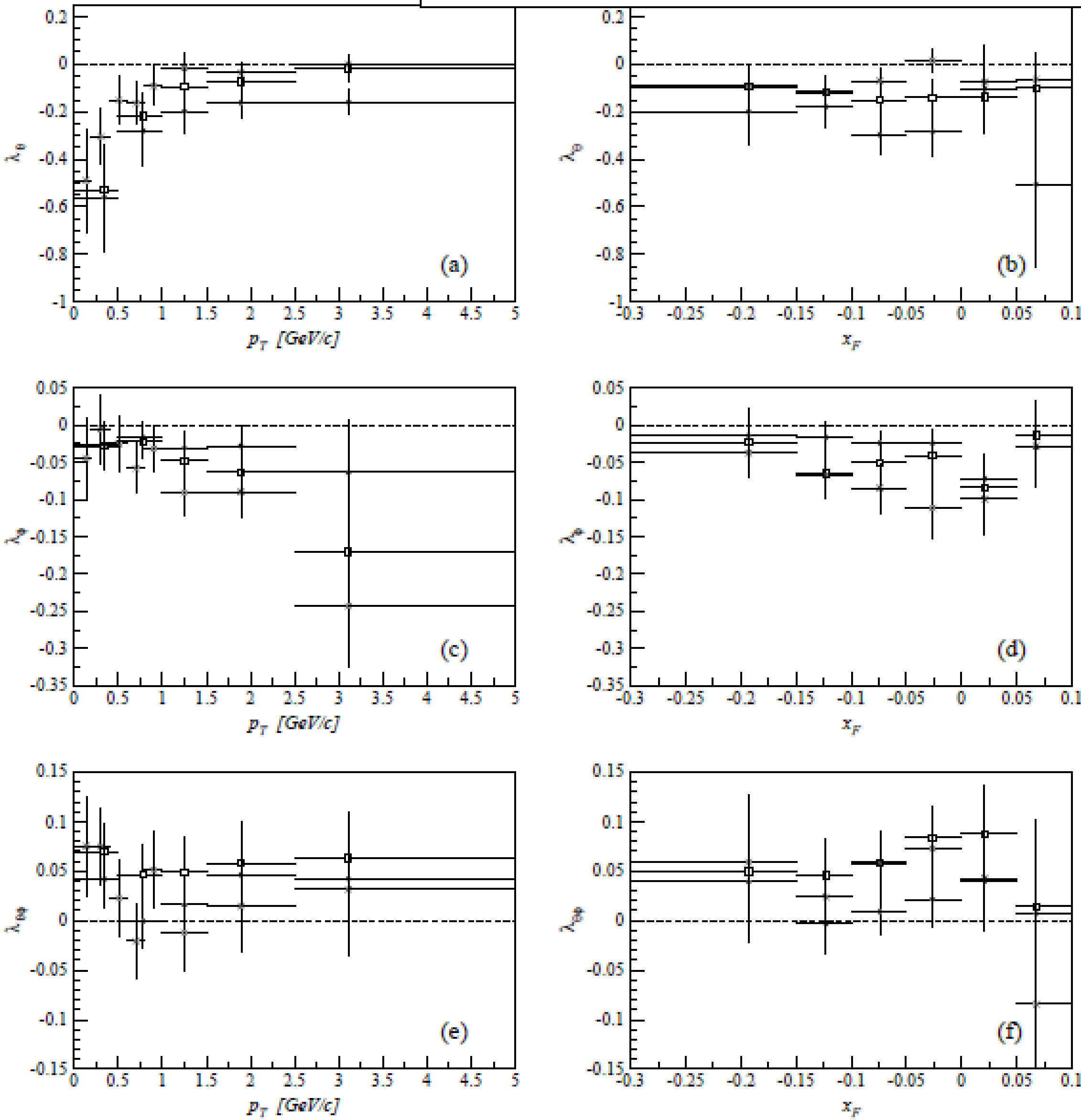
PRL 108, 082001 (2012)

- λ_θ and λ_ϕ separately measured
- Two frames used (**HX & CS**)
- $|\cos\theta|$ & $|\phi|$ dist. fit imposing $\tilde{\lambda}$ to be invariant in the two frames

Quarkonium polarization: a "puzzle"

- J/ψ: HERA-B

J/ψ, p-Cu and p-W √s = 41.6 GeV



- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured by single variable projections
- Three frames used (**HX, GJ & CS**)

$-1.5 < y < 0.8$
$-0.34 < x_F < 0.14$
$0 < p_T < 5.4$ GeV/c

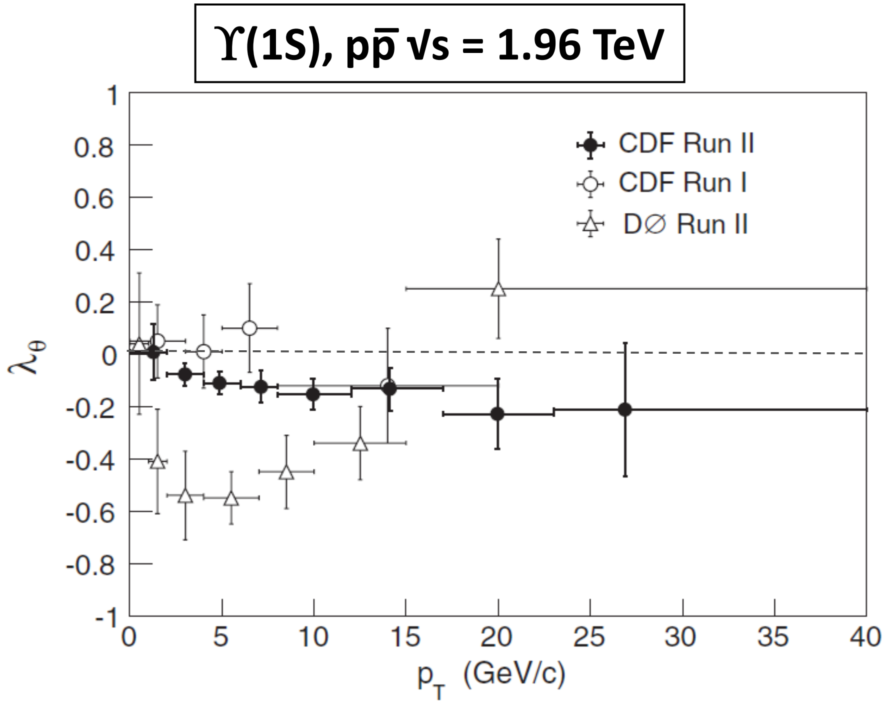
EPJ C60 517 (2009)

Quarkonium polarization: a "puzzle"

- Υ(nS): Measurements at Tevatron (**2002-2012**)

CDF+D∅ (2002)
- Only λ_θ measured
- Only one frame used (**HX**)

CDF (2012)
- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
- $\tilde{\lambda}$ checked
- Two frames used (**HX & CS**)



|y| < 0.4 **√s = 1.8 TeV** (2002)

|y| < 0.6

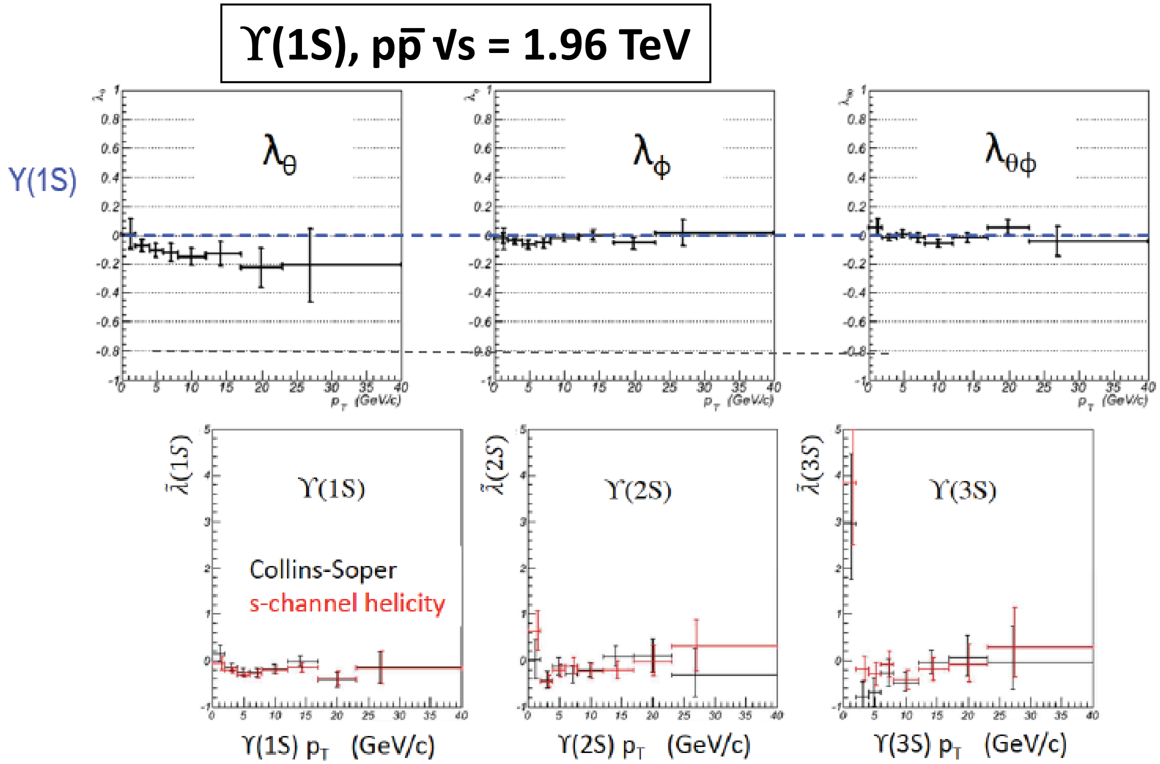
|y| < 1.8

PRL 88, 161802 (2002)
PRL 108, 151802 (2012)
PRL 101, 182004 (2008)

CDF vs **D0** :
Can a strong *rapidity dependence* justify the discrepancy?

Quarkonium polarization: a "puzzle"

- Υ(nS): Measurements at Tevatron (**2002-2012**)



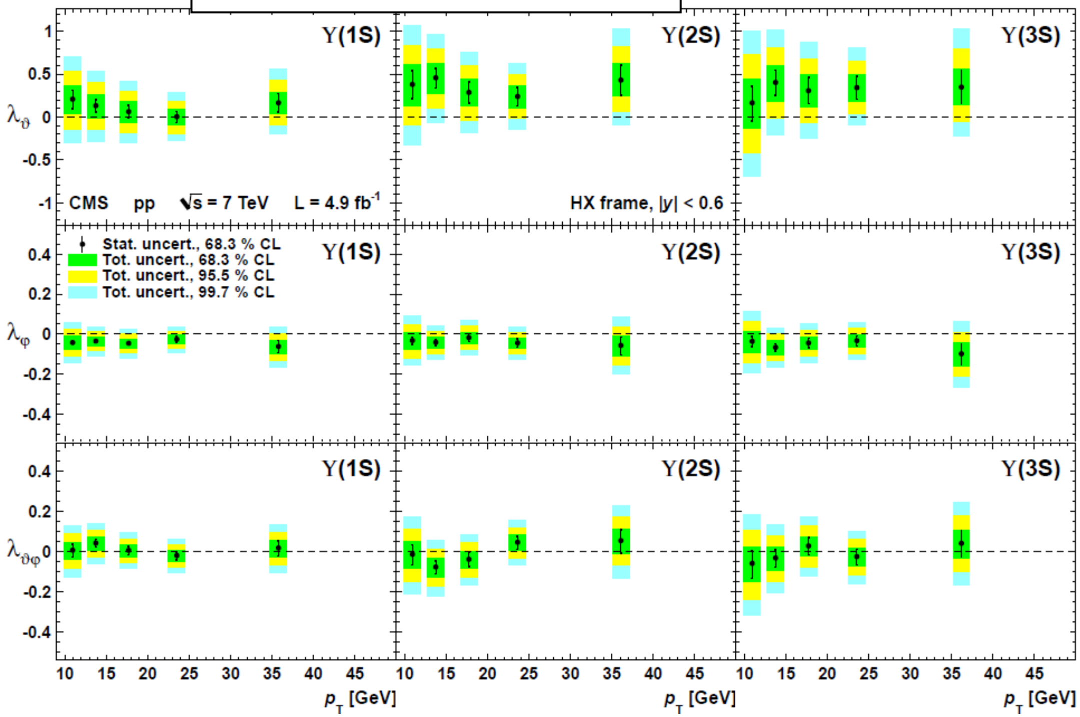
CDF+D∅ (2002)
- Only λ_θ measured
- Only one frame used (**HX**)

CDF (2012)
- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
- $\tilde{\lambda}$ checked
- Two frames used (**HX & CS**)

Quarkonium polarization: a “puzzle”

- Υ(nS): Measurements at LHC (**CMS**)

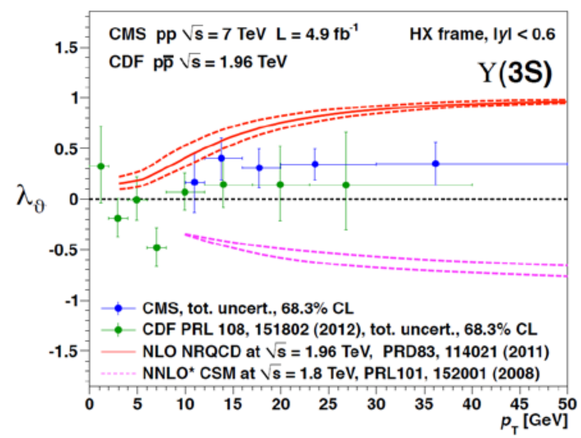
ϒ(nS), pp √s = 7 TeV



Phys Rev Lett. 2013 110(8):081802

- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
- Three frames used (**HX, CS, PX**)
- $\tilde{\lambda}$ checked
- Fully multidimensional

|y| < 0.6
0.6<|y| < 1.2
10 <p_T< 40 GeV/c

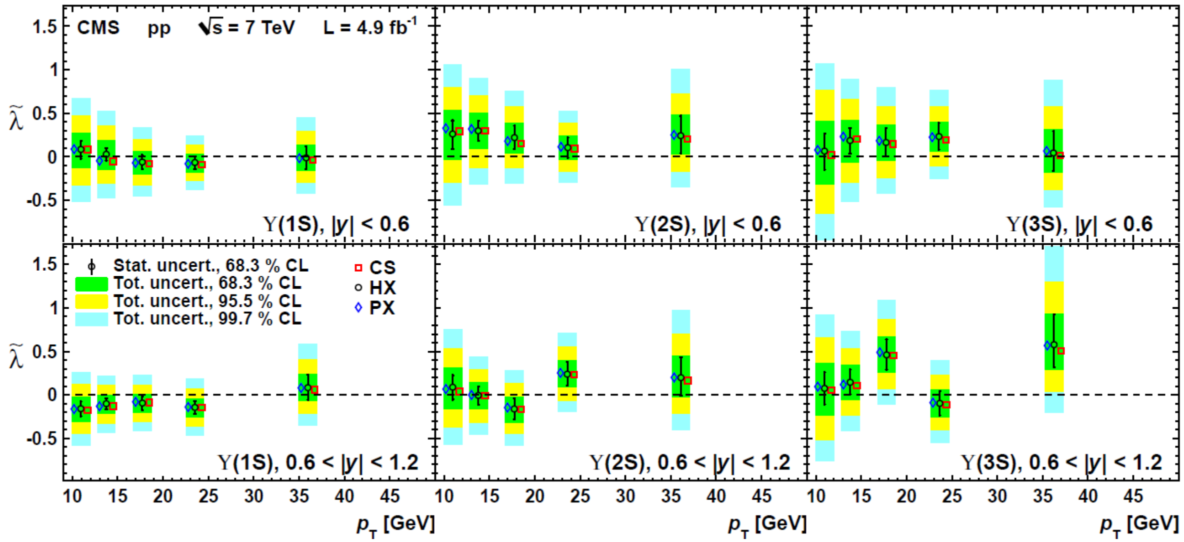


Comparison with CDF results

Quarkonium polarization: a "puzzle"

- Υ(nS): Measurements at LHC (**CMS**)

ϒ(nS), pp √s = 7 TeV



- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
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Quarkonium polarization: a "puzzle"

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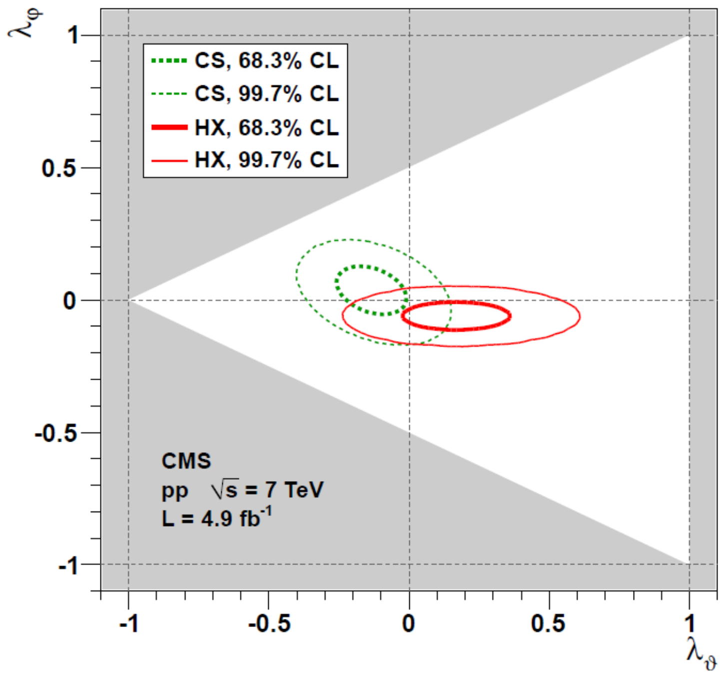
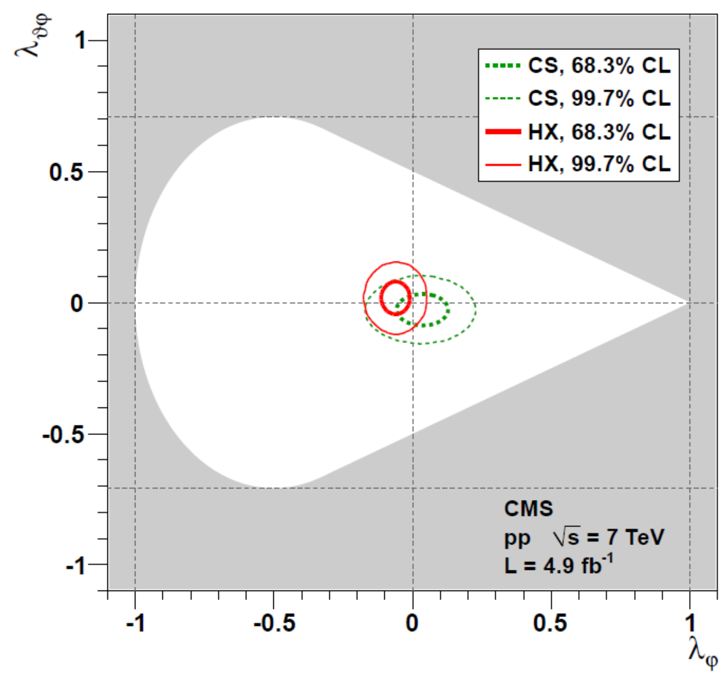


- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
- Three frames used (HX, CS, PX)
- $\tilde{\lambda}$ checked
- Fully multidimensional



Quarkonium polarization: a "puzzle"

- Υ(nS): E866/NuSea

Most reasonable explanation is that most $\Upsilon(1S)$ come from χ_b and have very different polarization

Υ(nS), p-Cu √s = 38.8 GeV

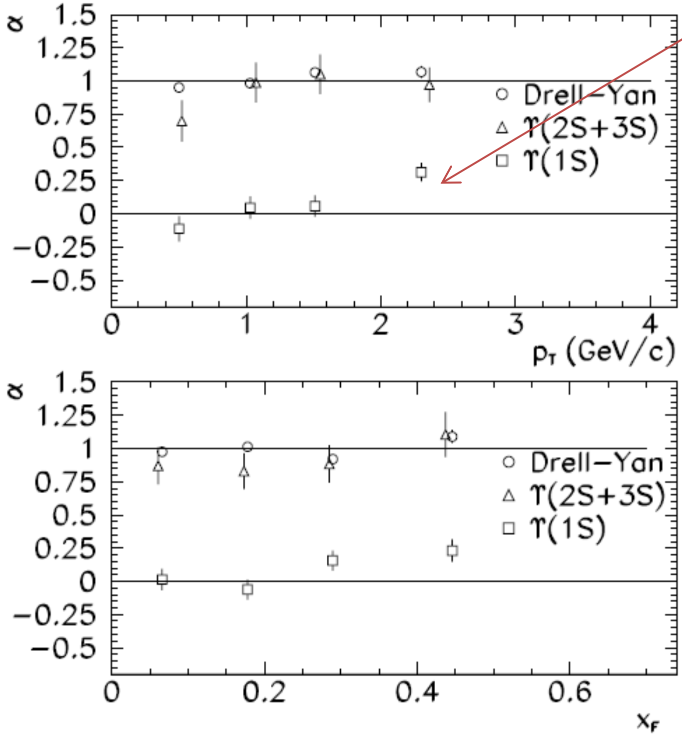
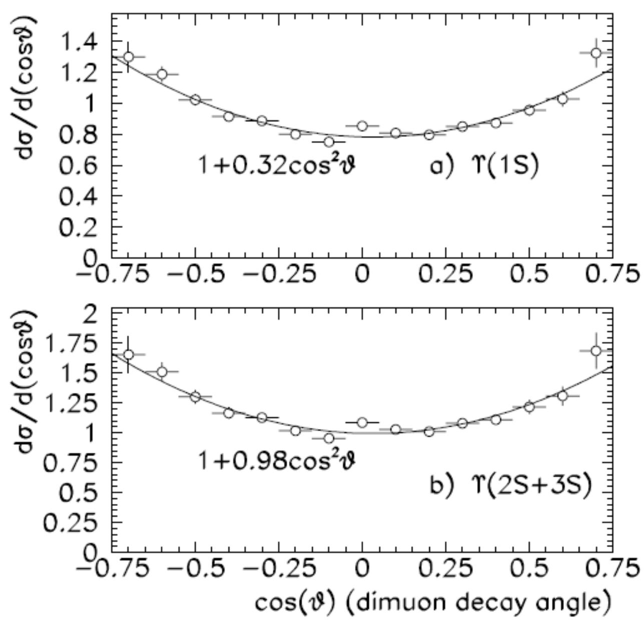


FIG. 4. (a) α versus p_T for the Drell-Yan sidebands ($8.1 < m_{\mu^+\mu^-} < 8.45$ GeV and $11.1 < m_{\mu^+\mu^-} < 15.0$ GeV), $\Upsilon(1S)$ ($8.8 < m_{\mu^+\mu^-} < 10.0$ GeV), and $\Upsilon(2S+3S)$ ($10.0 < m_{\mu^+\mu^-} < 11.1$ GeV). (b) α versus x_F for the same mass regions. The errors shown are statistical; there is an additional systematic error not shown of 0.02 in α for Drell-Yan polarizations and 0.06 in α for onium polarizations.

- Υ(nS) measured
- λ_θ measured
- One frame used (**CS**)



$p_T > 1.8$ GeV/c

Direct vs. prompt J/ψ

The direct-J/ψ polarization (cleanest theory prediction) can be derived from the prompt-J/ψ polarization measurement of CDF knowing

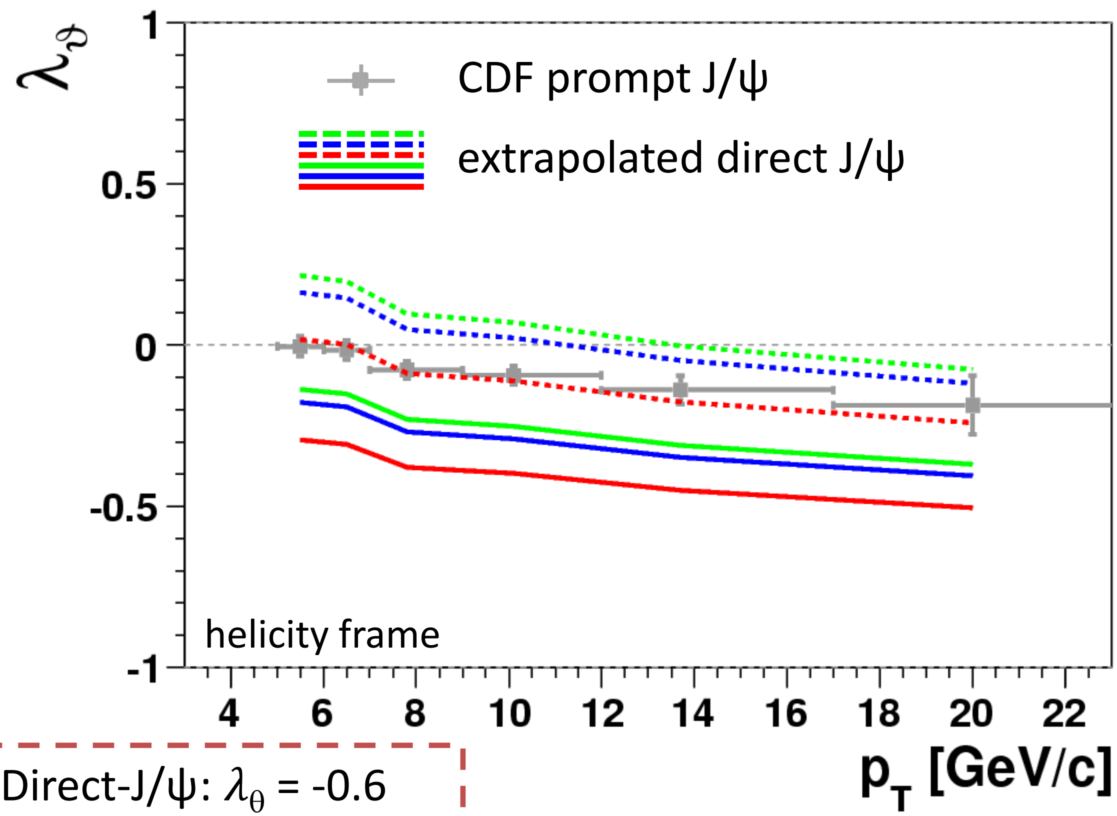
- the **χ_c-to-J/ψ feed-down fractions**
- the **χ_c polarizations**

CDF data

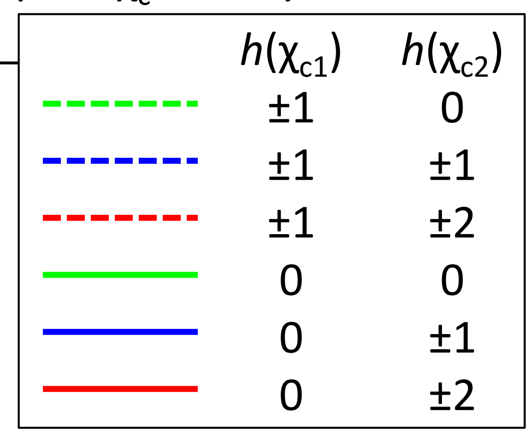
$R(\chi_{c1})+R(\chi_{c2}) = 30 \pm 6$ %

$R(\chi_{c2})/R(\chi_{c1}) = 40 \pm 2$ %

taking central values



possible combinations of pure χ_c helicity states



Direct-J/ψ: λ_θ = -0.6

From χ_c: λ_θ = +1

χ_c measurements are crucial !

Direct vs. prompt J/ψ

The direct-J/ψ polarization (cleanest theory prediction) can be derived from the prompt-J/ψ polarization measurement of CDF knowing

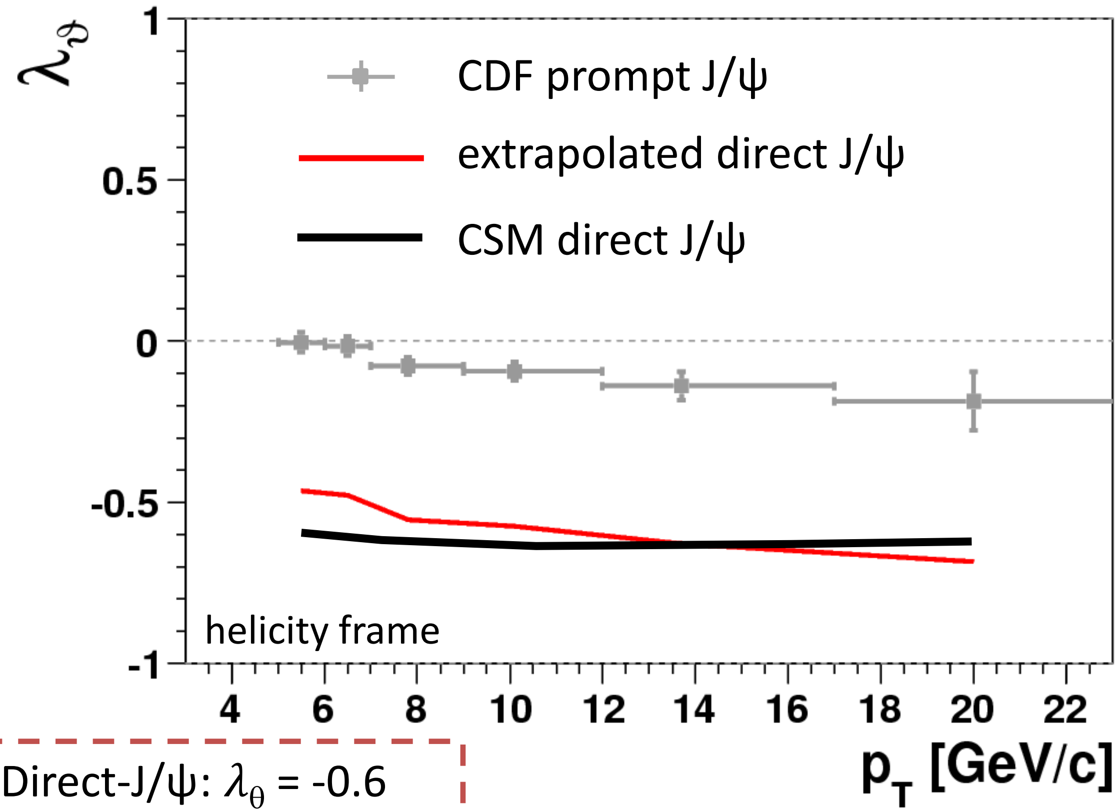
- the **χ_c-to-J/ψ feed-down fractions**
- the **χ_c polarizations**

CDF data

$R(\chi_{c1})+R(\chi_{c2})$ = 30 ± 6 %

$R(\chi_{c2})/R(\chi_{c1})$ = 40 ± 2 %

taking



using the values
$R(\chi_{c1})+R(\chi_{c2})$ = 42 % (+2σ)
$R(\chi_{c2})/R(\chi_{c1})$ = 38 % (-1σ)

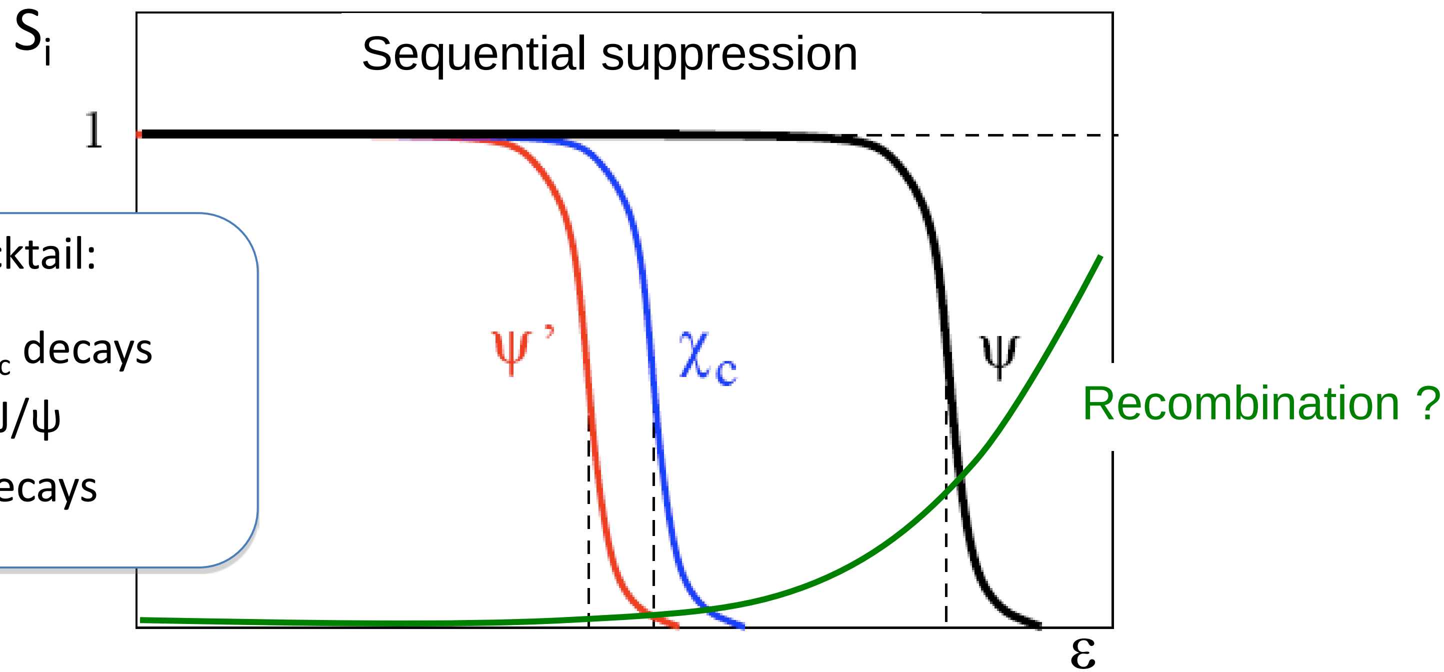
the CSM prediction of direct-J/ψ polarization agrees very well with the CDF data *in the scenario* *h*(χ_{c1}) = 0 and *h*(χ_{c2}) = ±2

	0	±1
	0	±2

Direct-J/ψ: λ_θ = -0.6
From χ_c: λ_θ = +1

χ_c measurements are crucial !

J/ψ polarization as a signal of colour deconfinement?

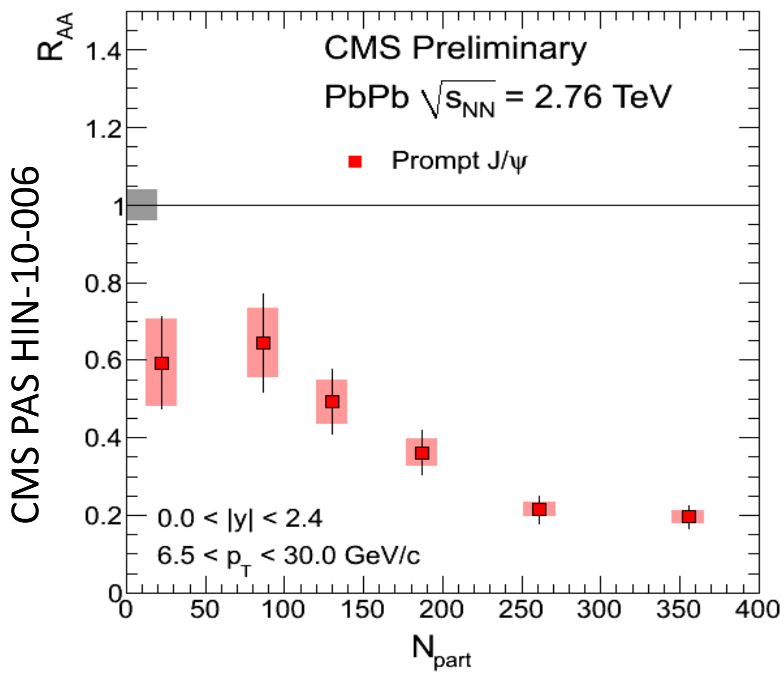


J/ψ cocktail:

≈ 30% from χ_c decays
≈ 70% direct J/ψ
+ ψ' decays

- As the χ_c (and ψ') mesons get dissolved by the QGP, λ_ϑ should *change to its direct value*

J/ψ polarization as a signal of sequential suppression?



P. Faccioli, JS, PRD 85, 074005 (2012)

CMS data:

- up to 80% of J/ψ's disappear from pp to Pb-Pb
- more than 50% ($\gtrsim$ fraction of J/ψ's from ψ' and χ_c) disappear from peripheral to central collisions

→ **sequential suppression** gedankenscenario: in central events ***ψ' and χ_c are fully suppressed*** and all J/ψ's are *direct*

It may be impossible to test this directly: measuring the χ_c yield (reconstructing χ_c radiative decays) in PbPb collisions is prohibitively difficult due to the huge number of photons

However, a ***change of prompt-J/ψ polarization*** must occur from pp to central Pb-Pb!

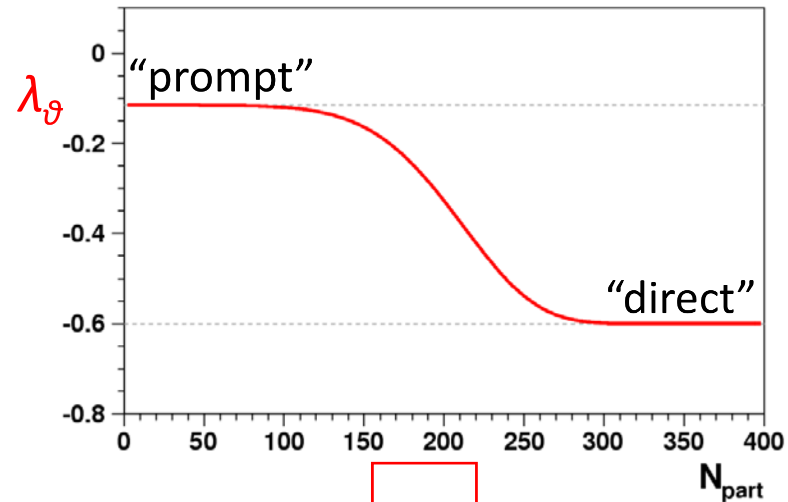
Reasonable sequence of measurements:

1) prompt J/ψ polarization in pp
2) χ_c-to-J/ψ fractions in pp
3) χ_c polarizations in pp
4) prompt J/ψ polarization in PbPb

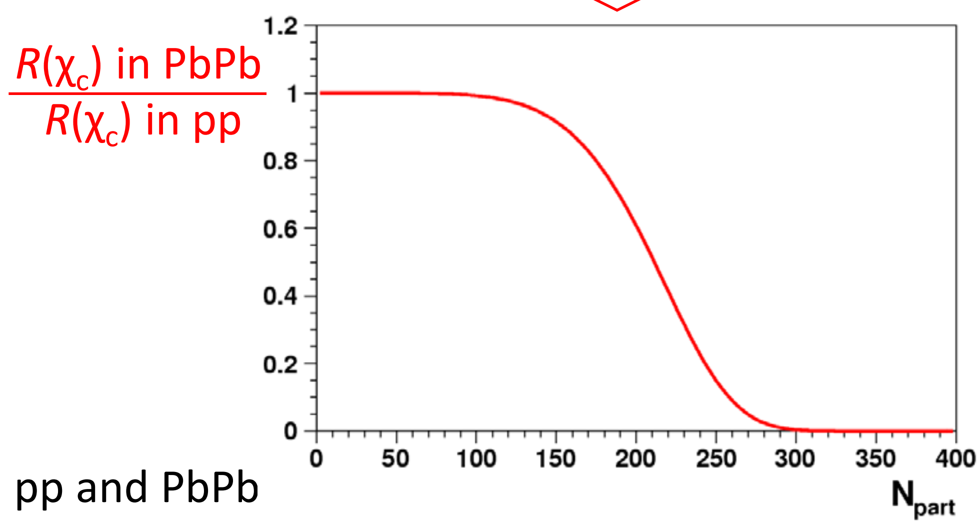
→ χ_c suppression in PbPb!

J/ψ polarization as a signal of sequential suppression?

If we measure a change in prompt polarization like this...



... we are observing the disappearance of the χ_c relative to the J/ψ



Simplifying assumptions:

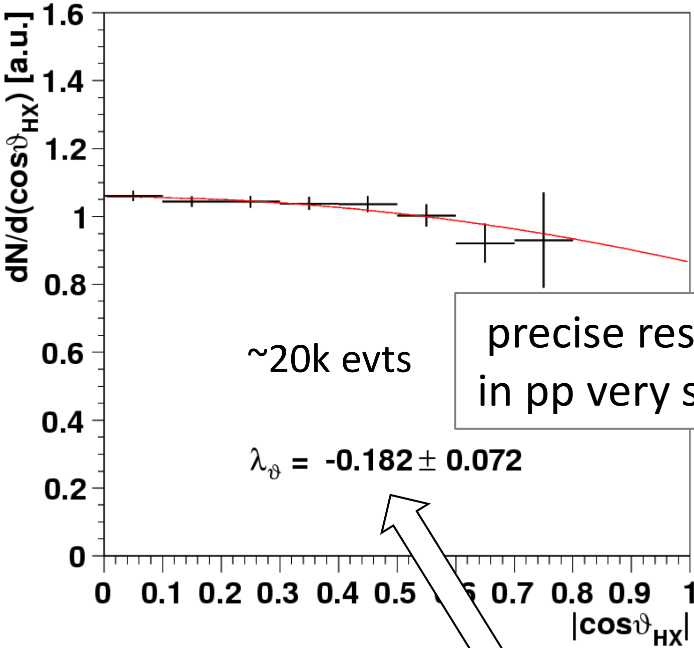
- direct-J/ψ polarization is the same in pp and PbPb
- *normal* nuclear effects affect J/ψ and χ_c in similar ways
- χ_{c1} and χ_{c2} are equally suppressed in PbPb

J/ψ polarization as a signal of sequential suppression?

When will we be sensitive to an effect like this?

CMS-like toy MC with $p_T(\mu) > 3$ GeV/c, $6.5 < p_T < 30$ GeV/c, $0 < |y| < 2.4$

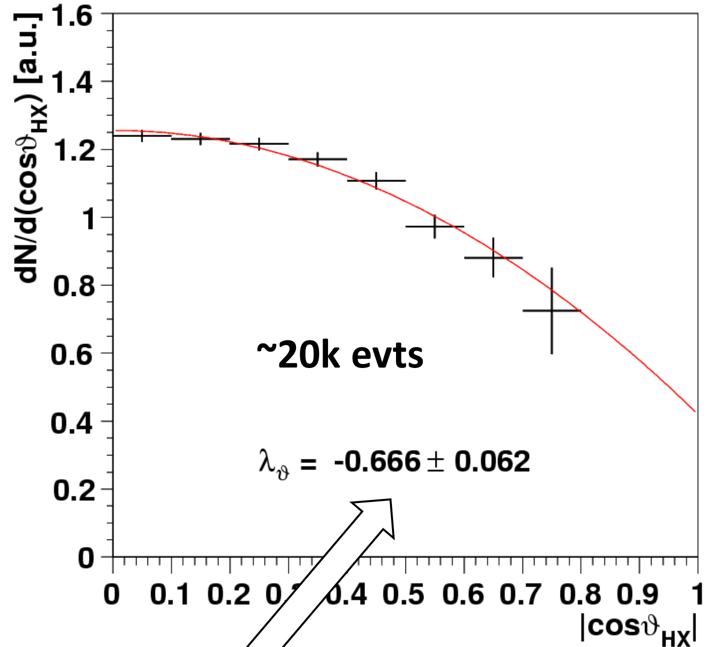
prompt-J/ψ polarization as observed in **pp** (and peripheral PbPb)



precise results in pp very soon

efficiency-corrected $|\cos\vartheta_{HX}|$ distribution

prompt-J/ψ polarization as observed in **central PbPb**

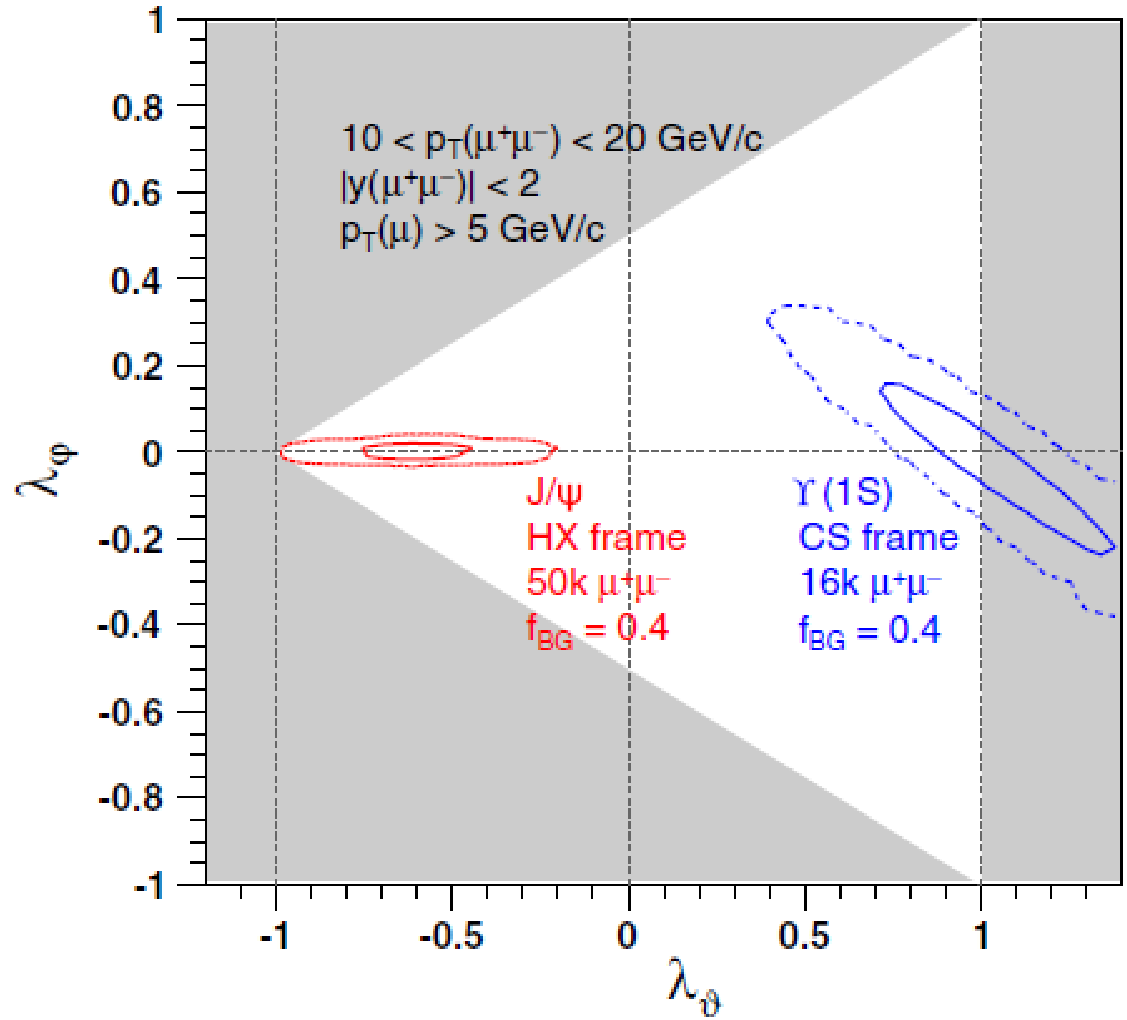


In this scenario, the χ_c disappearance is measurable at ~5σ level with ~20k J/ψ's in central Pb-Pb collisions

J/ψ polarization as a signal of sequential suppression?

When will we be sensitive to an effect like this?

CMS-like toy MC



Summary

- The new quarkonium polarization measurements have many improvements with respect to previous analyses

  Will we are starting to (experimentally) solve an old puzzle

- General advice: do not throw away physical information!
  (azimuthal-angle distribution, rapidity dependance, ...)

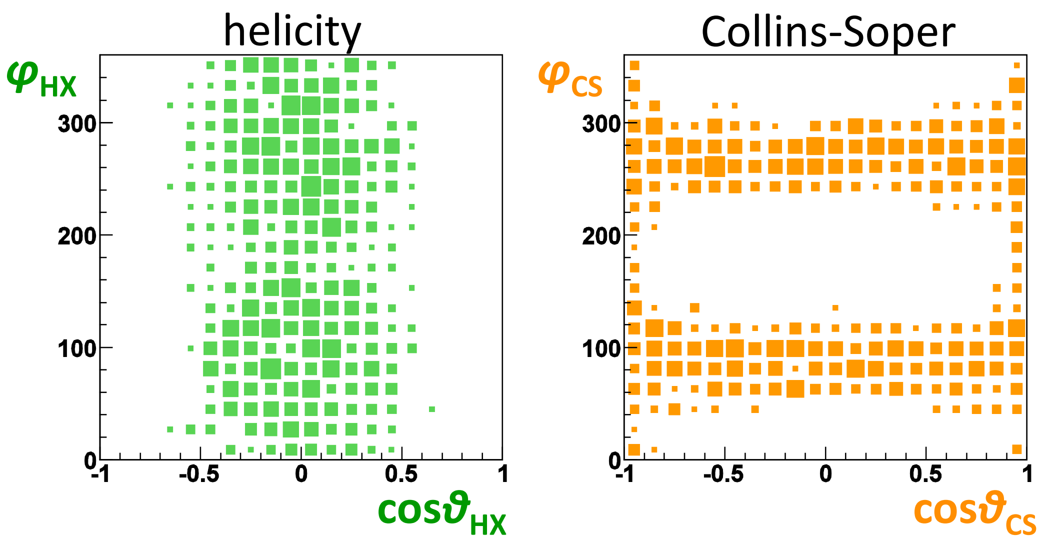
- A new method based on rotation-invariant observables gives several advantages in the measurement of decay distributions and in the use of polarization information

- Quarkonium polarization can be used to probe QGP formation

Backup slides

Some remarks on methodology

- Measurements are challenging
  - A typical collider experiment imposes p_T cuts on the single muons; this creates zero-acceptance domains in decay distributions from "low" masses:



Toy MC with
$p_T(\mu) > 3$ GeV/c (both muons)

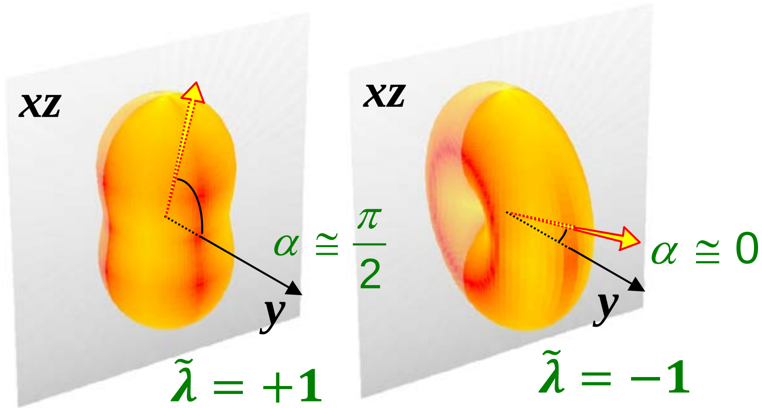
Reconstructed
unpolarized Υ(1S)

$p_T(\Upsilon) > 10$ GeV/c, $|y(\Upsilon)| < 1$

  - This spurious "polarization" must be accurately taken into account.
  - Large holes strongly reduce the precision in the extracted parameters

- In the analyses we must avoid simplifications that make the present results sometimes difficult to be interpreted:
  - only λ_θ measured, azimuthal dependence ignored
  - one polarization frame "arbitrarily" chosen *a priori*
  - no rapidity dependence

Frame-independent angular distribution

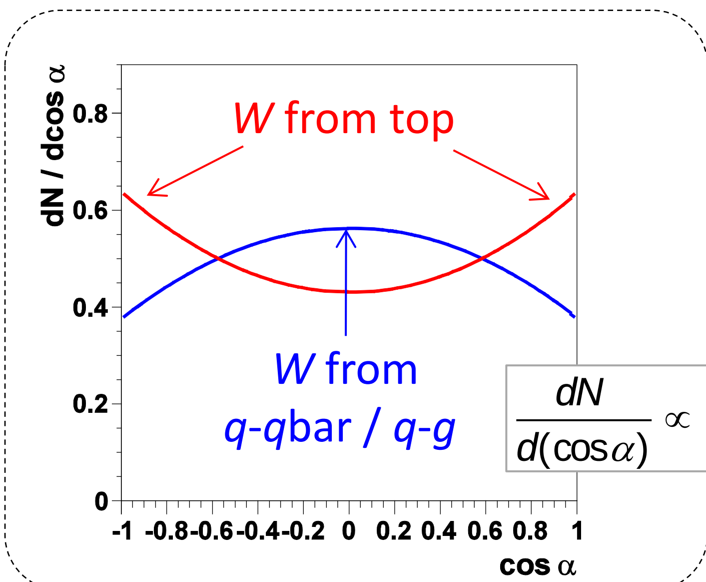


(transverse) (longitudinal)

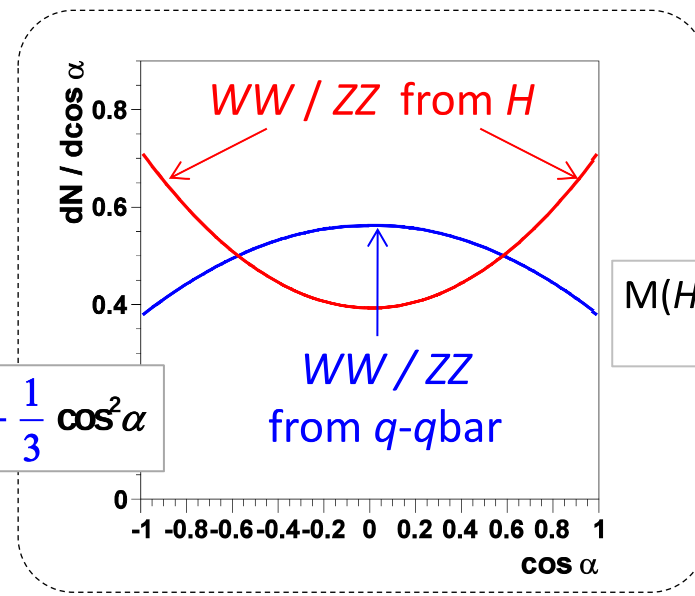
$\tilde{\lambda}$ determines the event distribution of the angle α of the lepton w.r.t. the y axis of the polarization frame:

$$\frac{dN}{d(\cos\alpha)} \propto 1 - \frac{\tilde{\lambda}}{2+\tilde{\lambda}} \cos^2\alpha$$

Example:
lepton emitted at small cos α is more likely to come from directly produced W / Z



$$\frac{dN}{d(\cos\alpha)} \propto 1 - \frac{1}{3}\cos^2\alpha$$



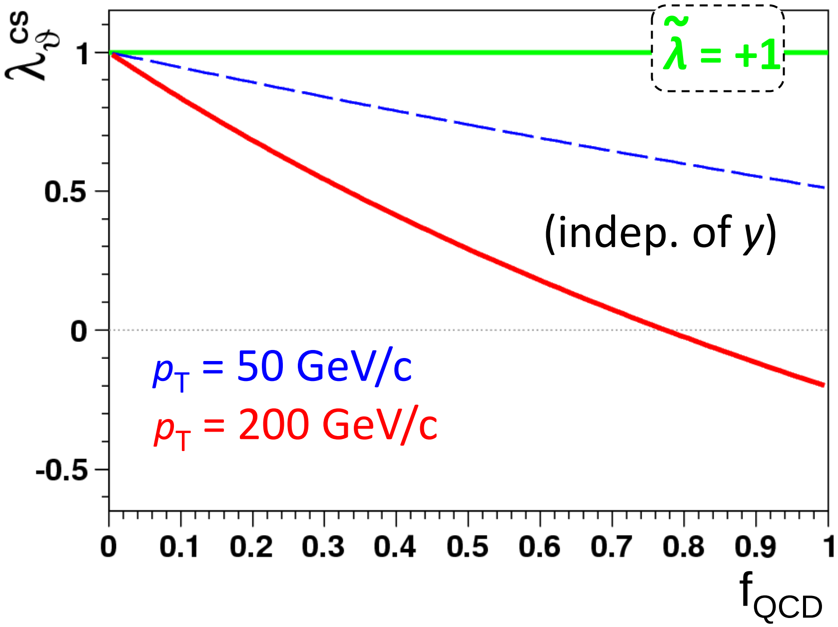
M(H) = 300 GeV/c^2

independent of W/Z kinematics

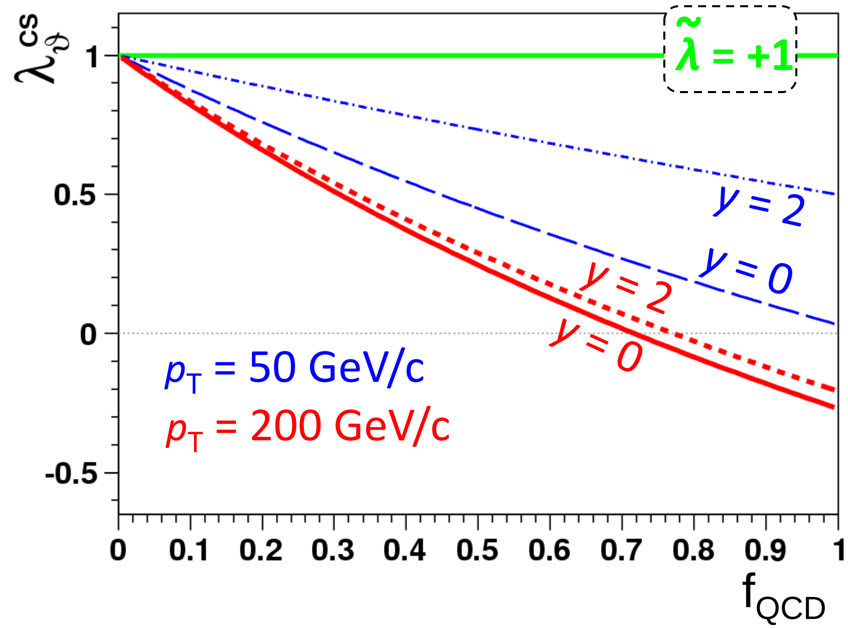
λ_ϑ(CS) vs $\tilde{\lambda}$

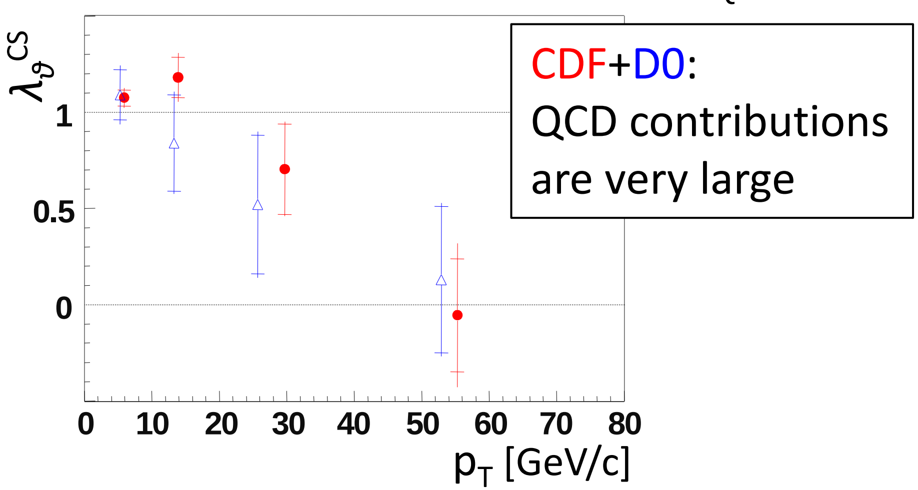
Example: *W* polarization as a function of contribution of LO QCD corrections, p_T and *y*:

case 1: dominating ***q-qbar*** QCD corrections



case 2: dominating ***q-g*** QCD corrections

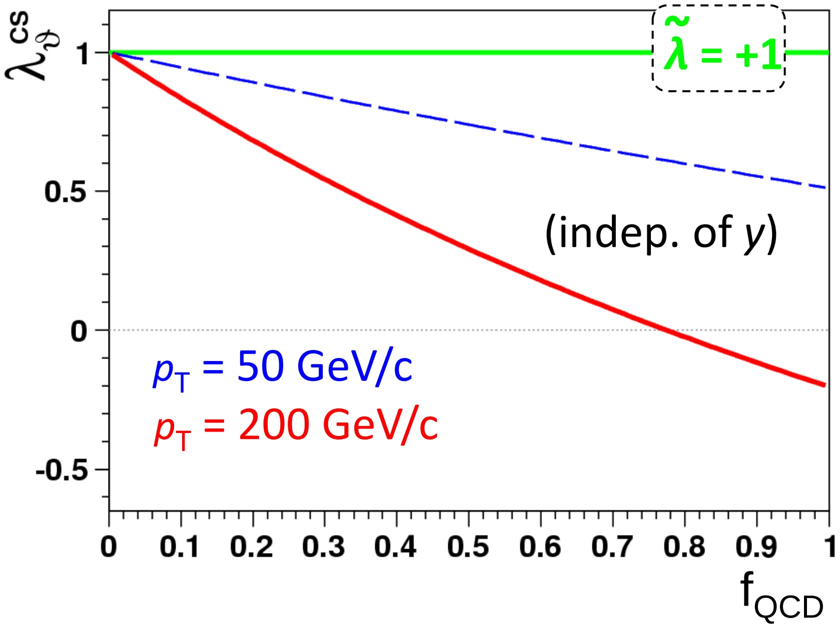




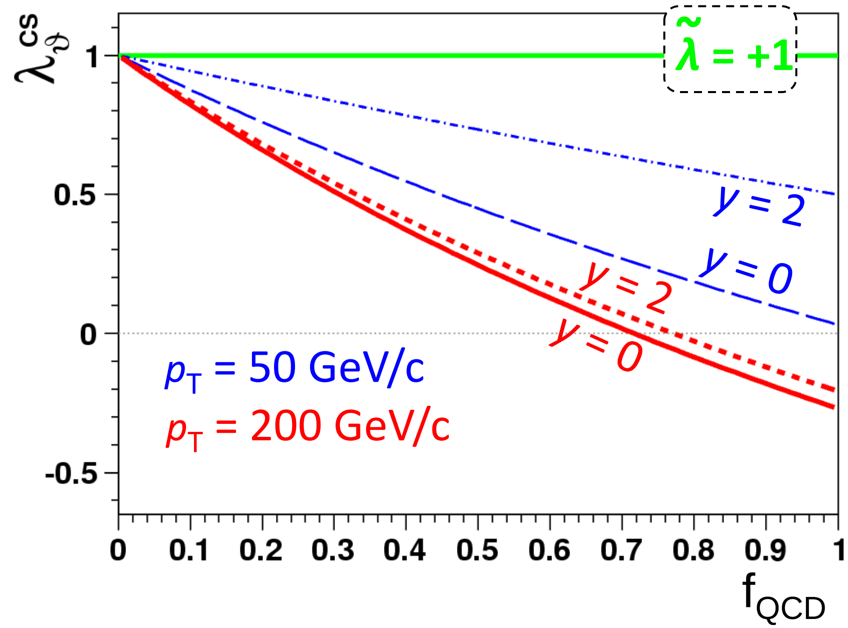
$\tilde{\lambda}$, constant, maximal and independent of the process mixture,
gives a simpler and more significant representation of the polarization information

λ_ϑ(CS) vs $\tilde{\lambda}$

case 1: dominating ***q-qbar*** QCD corrections



case 2: dominating ***q-g*** QCD corrections



On the other hand, $\tilde{\lambda}$ forgets about the direction of the quantization axis.
In this case, this information is crucial if we want to disentangle the *qg* contribution, the only one giving maximum spin-alignment along the boson momentum, resulting in a *rapidity-dependent* λ_ϑ

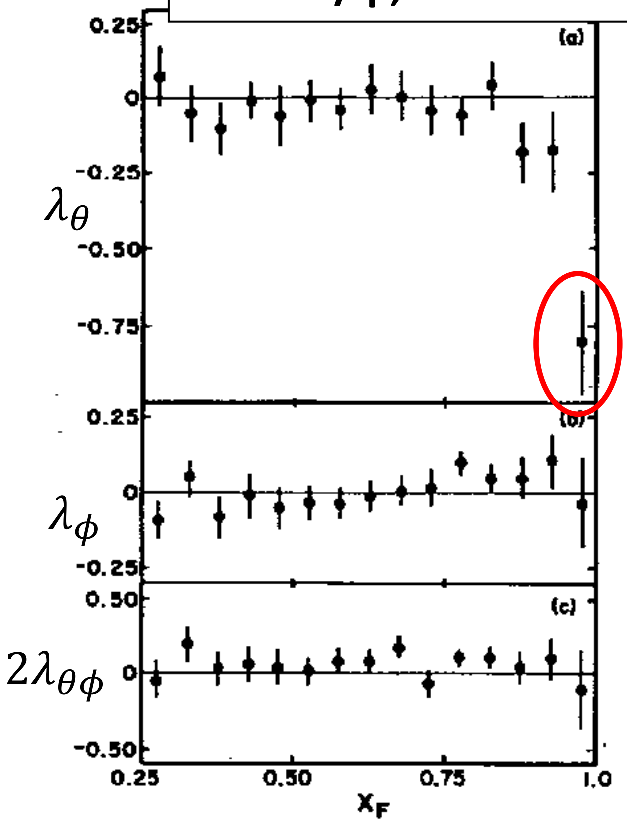
Measuring λ_ϑ(CS) *as a function of rapidity* gives information on the gluon content of the proton!

Quarkonium polarization: a "puzzle"

- J/ψ: Other fixed target experiments

Experiment	reaction	$\sqrt{S}$	x_F range	λ_θ
E537	$\overline{p}$ + W	15.3 GeV	$x_F > 0$	-0.115 ± 0.061
E537	π^- + W	15.3 GeV	$x_F > 0$	0.028 ± 0.004
E672/706	π^- + Be	31.5 GeV	$0.1 < x_F < 0.8$	-0.01 ± 0.08
E771	p + Si	38.8 GeV	$-0.05 < x_F < 0.25$	-0.09 ± 0.12
E866	p + Cu	38.8 GeV	$0.25 < x_F < 1.0$	$0.069 \pm 0.004 \pm syst.$

J/ψ, 252 GeV π on W



Chicago-Iowa-Princeton Coll.

- λ_θ, λ_ϕ and $\lambda_{\theta\phi}$ measured
- One frame used (**GJ**)
- Violates Lam-Tung relation

PRL 58, 2523 (1987)

Using polarization to identify processes

If the polarization depends on the specific production process (in a known way), it can be used to characterize “signal” and “background” processes.

In certain situations the rotation-invariant formalism can allow us to
- estimate relative contributions of signal and background in the distribution of events
- attribute to each event a likelihood to be signal or background (work in progress)

Example n. 1: *W* from top ↔ *W* from *q*-*q*bar and *q*-*g*

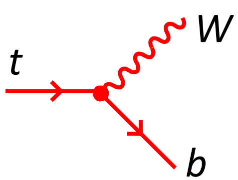
Hypothetical, illustrative experimental situation:

- selected *W*'s come either from top decays or from direct production (+jets)
- we want to estimate the relative contribution of the two types of W, using polarization

longitudinally polarized:

$\lambda_\vartheta^{SM} \cong -0.65$
$\lambda_\varphi^{SM} \cong 0$

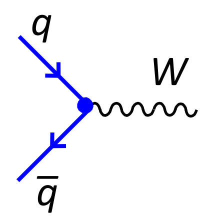
wrt *W direction in the top rest frame* (top-frame helicity)



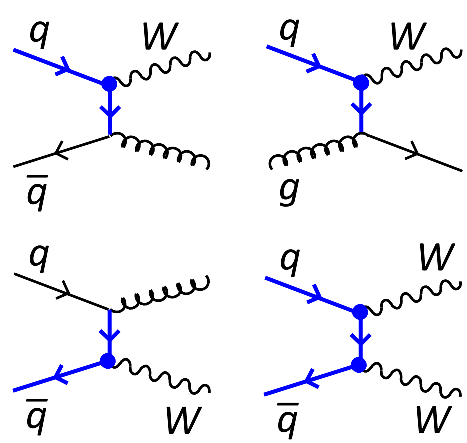
independently of top production mechanism

The top quark decays almost 100% of the times to *W*+*b*
→ the longitudinal polarization of the W is a signature of the top

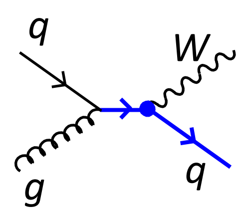
transversely polarized,
$\lambda_\vartheta = +1$ & $\lambda_\varphi = 0$ *wrt 3 different axes:*



relative direction of *q* and *q*bar (Collins-Soper)



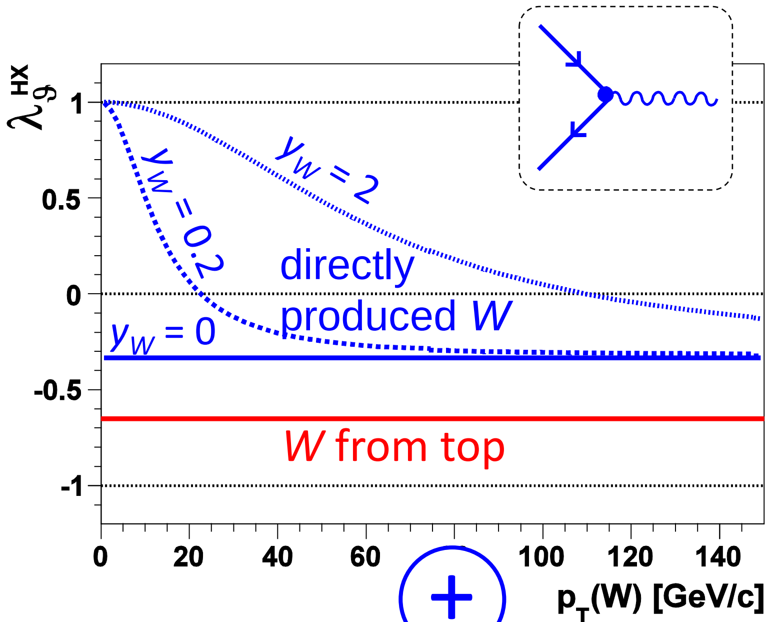
direction of *q* or *q*bar (Gottfried-Jackson)

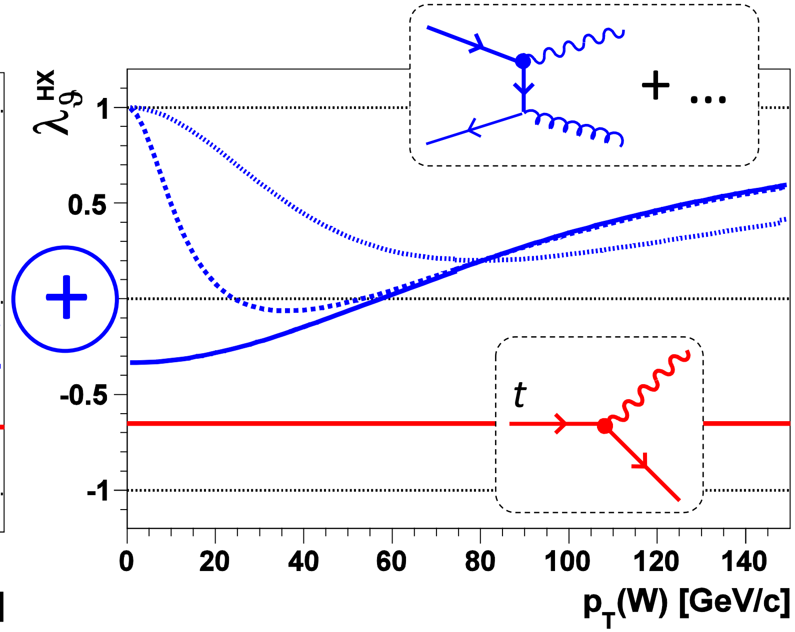


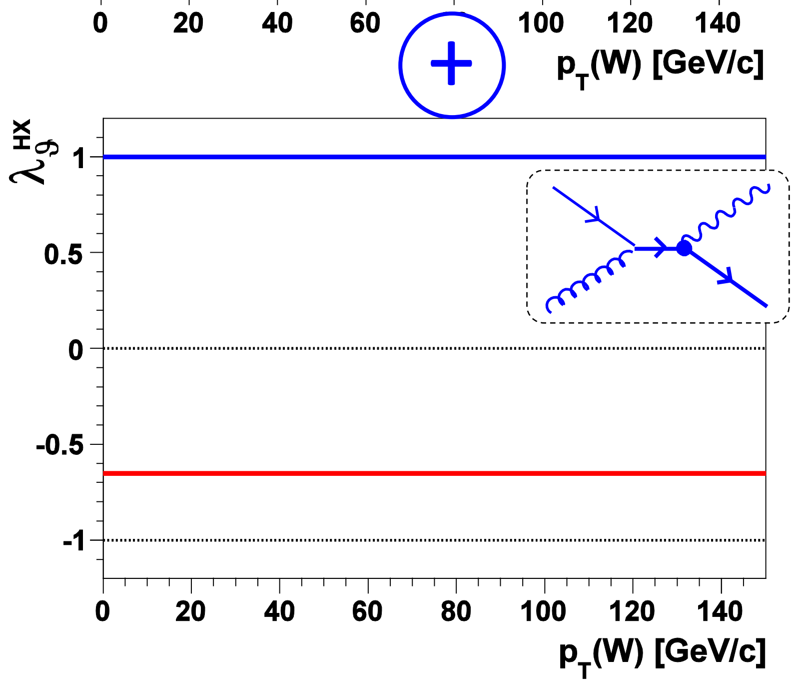
direction of outgoing *q* (cms-helicity)

a) Frame-dependent approach

We measure λ_ϑ choosing the helicity axis defined wrt the top rest frame





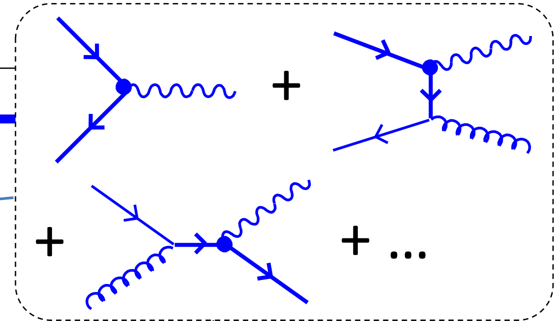


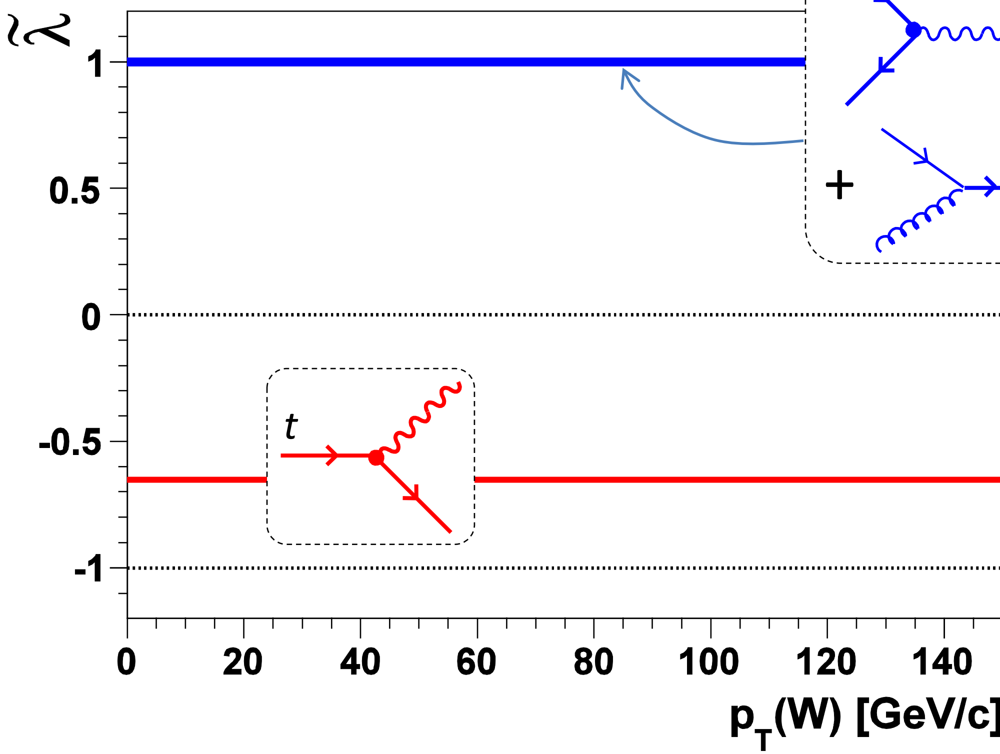
The polarization of *W* from *q*-*qbar* / *q*-*g*

- depends on the actual mixture of processes
  $\rightarrow$ we need pQCD and PDFs to evaluate it
- depends on p_T and y
  $\rightarrow$ if we integrate we lose discriminating power
- is generally far from being maximal
  $\rightarrow$ we should measure also λ_φ for a sufficient discrimination

b) Rotation-invariant approach

We measure $\tilde{\lambda}$, choosing any frame defined using beam directions (cms-HX, CS, GJ...)



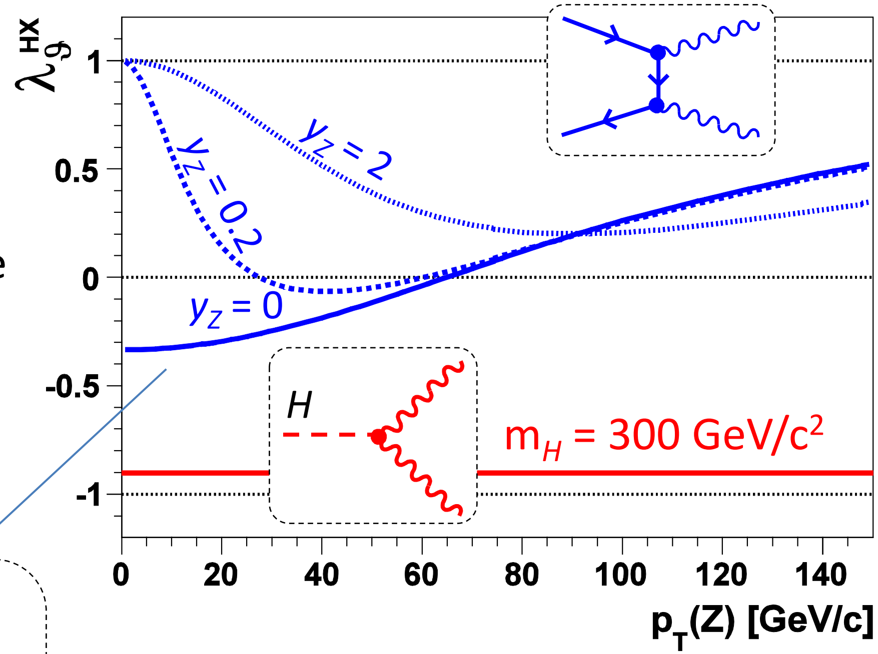


- same $\tilde{\lambda}$ for all "background" processes
  $\rightarrow$ no need of theory calculations
- no dependence on p_T and y
  $\rightarrow$ we can use a larger event sample
- difference wrt signal is maximized
  $\rightarrow$ more significant discrimination

From the measured overall $\tilde{\lambda}$ we can deduce the fraction $f_{top} = \frac{N(W \text{ from } t)}{N_{tot}(W)} = \frac{1-\tilde{\lambda}}{3+\tilde{\lambda}} \frac{3+\tilde{\lambda}_{top}}{1-\tilde{\lambda}_{top}}$

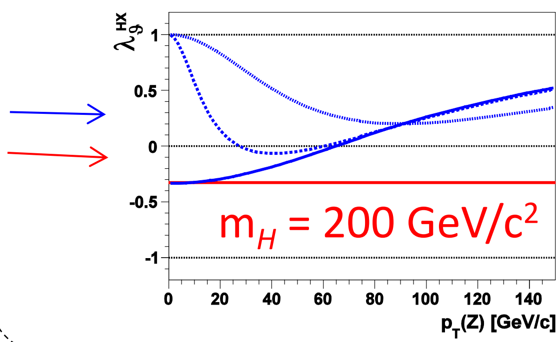
E.g. $\tilde{\lambda} = 0.0 \pm 0.1 \overset{\tilde{\lambda}_{top} \cong -0.65}{\Rightarrow} f_t = (50 \pm 7)\%$

Example n. 2: Z (W) from Higgs ↔ Z (W) from q-qbar

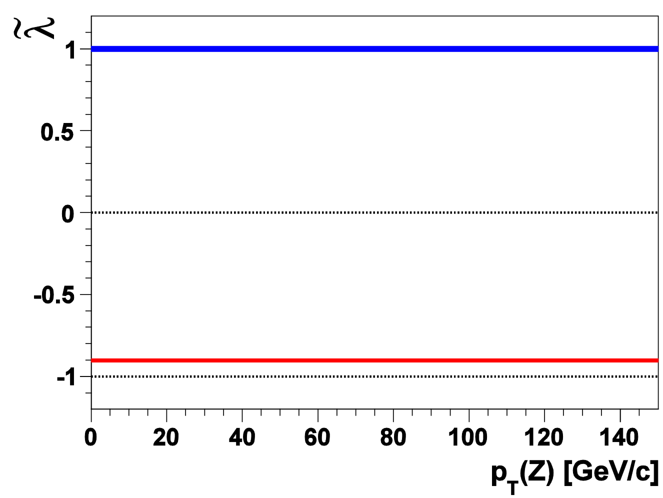


Z bosons from $H \to ZZ$ are **longitudinally** polarized. The polarization is stronger for heavier *H*

even larger overlap if the Higgs is lighter:



$\tilde{\lambda}$ is better than λ_ϑ to discriminate between signal and background:



Rotation-invariant parity asymmetry

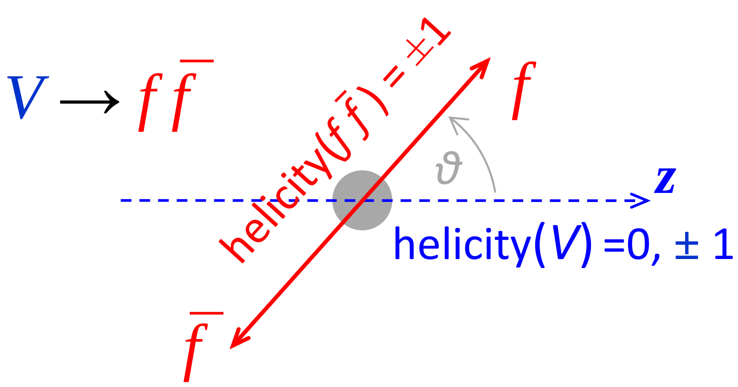
parity-violating terms

$$\frac{dN}{d\Omega} \propto 1 + \ldots + 2A_\theta \cos\theta + 2A_\varphi \sin\theta\cos\varphi + 2A_\varphi^\perp \sin\theta\sin\varphi$$

$$\tilde{\mathcal{A}} = \frac{4}{3+\lambda_\vartheta}\sqrt{A_\theta^2 + A_\varphi^2 + A_\varphi^{\perp 2}}$$

is invariant under *any* rotation

It represents the magnitude of the *maximum observable parity asymmetry*, i.e. of the *net* asymmetry as it can be measured along the polarization axis that maximizes it (which is the one minimizing the helicity-0 component)



$$\tilde{\mathcal{A}} = \max_z \frac{P(\pm 1, \pm 1) - P(\pm 1, \mp 1)}{P(\pm 1, \pm 1) + P(\pm 1, \mp 1)}$$

[PRD 82, 096002 (2010)]

Frame-independent "forward-backward" asymmetry

The rotation invariant parity asymmetry can also be written as

$$\tilde{\mathcal{A}} = \frac{4}{3}\sqrt{\mathcal{A}^{2}_{\cos\theta} + \mathcal{A}^{2}_{\cos\varphi} + \mathcal{A}^{\perp 2}_{\sin\varphi}}$$

$$\mathcal{A}_{\cos\theta} = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N_{tot}} = \mathcal{A}_{FB}$$

- *Z* "forward-backward asymmetry"
- (related to) *W* "charge asymmetry"

experiments usually measure these in the Collins-Soper frame

$$\mathcal{A}_{\cos\varphi} = \frac{N(\cos\varphi > 0) - N(\cos\varphi < 0)}{N_{tot}}$$

$$\mathcal{A}_{\sin\varphi} = \frac{N(\sin\varphi > 0) - N(\sin\varphi < 0)}{N_{tot}}$$

$\tilde{\mathcal{A}}$ can provide a better measurement of parity violation:

- it is not reduced by a non-optimal frame choice
- it is free from extrinsic kinematic dependencies
- it can be checked in two "orthogonal" frames

$\mathcal{A}_{FB}$(CS) vs $\tilde{\mathcal{A}}$

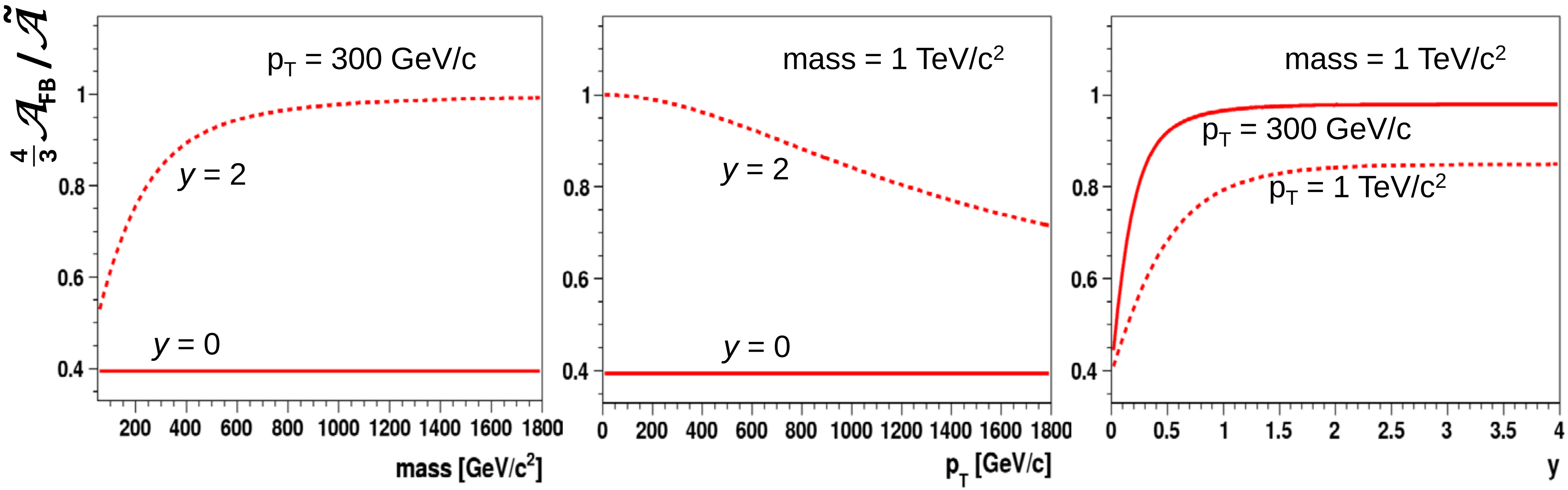
In general, we lose significance when measuring only the azimuthal "projection" of the asymmetry ($\mathcal{A}_{FB}$) wrt some chosen axis

This is especially relevant if we do not know a priori the optimal quantization axis

Example: imagine an unknown massive boson

70% polarized in the HX frame and 30% in the CS frame

By how much is $\mathcal{A}_{FB}$(CS) smaller than $\tilde{\mathcal{A}}$ if we measure in the CS frame?



Larger loss of significance for smaller mass, higher p_T , mid-rapidity

ψ'→ J/ψ x-section ratio experimental parameters

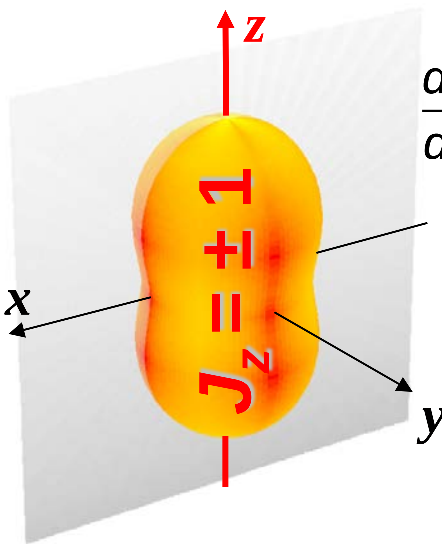
Experiment	Collision system	$E_{\rm beam}$ [GeV]	Phase space	$\langle x_{\rm F}\rangle$
E331 [5]	p-C	225	$0 < x_{\rm F} < 0.7$	$\simeq 0.3$
E444 [6]	p-C	225	$0 < x_{\rm F} < 0.9$	$\simeq 0.35$
E705 [7]	p-Li	300	$-0.1 < x_{\rm F} < 0.5$	$\simeq 0.2$
E288 [8]	p-Be	400	$-0.6 < x_{\rm F} < 0.8$	$\simeq 0.1$
NA38 [9]	p-W/U p-C/Al/Cu/W	200 450	$-0.4 < y_{\rm cm} < 0.6$	$\simeq 0$
NA51 [10]	p-H/D	450	$-0.4 < y_{\rm cm} < 0.6$	$\simeq 0$
NA50 96/98 [11]	p-Be/Al/Cu/ Ag/W	450	$-0.5 < y_{\rm cm} < 0.5$	$\simeq 0$
NA50 2000 [12]	p-Be/Al/Cu/ Ag/W/Pb	400	$-0.425 < y_{\rm cm} < 0.575$	$\simeq 0$
E771 [13]	p-Si	800	$-0.05 < x_{\rm F} < 0.25$	$\simeq 0.1$
E789 [14]	p-Au	800	$-0.03 < x_{\rm F} < 0.15$	$\simeq 0.06$
E866 [15]	p-Be/Fe/W	800	$-0.1 < x_{\rm F} < 0.8$	$\simeq 0.3$
HERA-B [16]	p-C/Ti/W	920	$-0.35 < x_{\rm F} < 0.1$	-0.065
WA39 [17]	$\pi^{\pm}$-W	39.5	$-0.5 < x_{\rm F} < 0.8$	$\simeq 0.2$
E537 [18]	π^{-}-W	125	$0 < x_{\rm F} < 1$	$\simeq 0.3$
WA11 [19]	π^{-}-Be	150	$-0.4 < x_{\rm F} < 0.9$	$\simeq 0.3$
E331 [5]	π^{+}-C	225	$0 < x_{\rm F} < 0.9$	$\simeq 0.35$
E444 [6]	$\pi^{\pm}$-C	225	$0 < x_{\rm F} < 1$	$\simeq 0.4$
E615 [20]	π^{-}-W	253	$0.3 < x_{\rm F} < 1$	$\simeq 0.6$
E705 [7]	$\pi^{\pm}$-Li	300	$-0.1 < x_{\rm F} < 0.5$	$\simeq 0.2$
E672-706 [21]	π^{-}-Be	515	$0.1 < x_{\rm F} < 0.8$	$\simeq 0.4$
Experiment	Collision system	$\sqrt{s}$ [GeV]	Phase space	$\langle x_{\rm F}\rangle$
ISR [22]	pp	58 (avg.)	$y_{\rm cm} \simeq 0$	0

$R(\chi_c)$ experimental parameters

Experiment	Collision system	E_{beam} [GeV]	Phase space	$\langle x_F \rangle$
E369-610-673 [23]	p-Be	225 (avg.)	$0.1 < x_F < 0.6$	0.32
E705 [24]	p-Li	300	$-0.1 < x_F < 0.5$	$\simeq 0.2$
E771 [25]	p-Si	800	$-0.05 < x_F < 0.25$	$\simeq 0.1$
HERA-B 2000 [26]	p-C/Ti	920	$-0.25 < x_F < 0.15$	-0.035
HERA-B 2003 [27]	p-C/W	920	$-0.35 < x_F < 0.15$	-0.065
SERPUKHOV-140 [28]	π^--H	38	$0.3 < x_F < 0.8$	$\simeq 0.5$
WA11 [29]	π^--Be	185	$-0.4 < x_F < 0.9$	$\simeq 0.3$
E369-610-673 [23]	π^--Be (mostly)	209 (avg.)	$0 < x_F < 0.8$	0.43
E705 [24]	$\pi^\pm$-Li	300	$-0.1 < x_F < 0.5$	$\simeq 0.2$
E672-706 [30]	π^--Be	515	$0.1 < x_F < 0.8$	$\simeq 0.4$
Experiment	Collision system	$\sqrt{s}$ [GeV]	Phase space	$\langle x_F \rangle$
ISR [22, 31]	pp	58 (avg.)	$y_{\text{cm}} \simeq 0$	0
CDF [32]	p$\bar{\text{p}}$	1800	$\|y_{\text{cm}}\| < 0.6$	0

The azimuthal anisotropy is not a detail

Case 1: natural transverse polarization

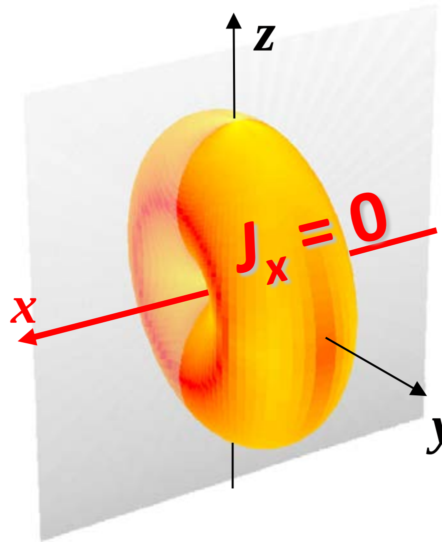


$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

$\lambda_\vartheta = +1$

$\lambda_\varphi = 0$

Case 2: natural longitudinal polarization, observation frame ⊥ to the natural one



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

$\lambda_\vartheta = +1$

$\lambda_\varphi = -1$

- Two very different physical cases
- Indistinguishable if λ_φ is not measured (integration over φ)

Basic meaning of the frame-invariant quantities

Let us suppose that, in the collected events, *n* different elementary subprocesses yield angular momentum states of the kind

$$\left|\psi^{(i)}\right\rangle = a_{-1}^{(i)}\,|1,-1\rangle + a_{0}^{(i)}\,|1,0\rangle + a_{+1}^{(i)}\,|1,+1\rangle, \qquad i = 1,2,\ldots n$$

(wrt a given quantization axis), each one with probability $f^{(i)}$ $(\sum f^{(i)} = 1)$.

The **rotational properties of J=1 angular momentum states** $\left[d^{1}_{+1,M}(\theta) + d^{1}_{-1,M}(\theta) = \delta_{|M|,1}\right]$ imply that

the combinations $a_{+1}^{(i)} + a_{-1}^{(i)}$ are independent of the choice of the quantization axis

The quantity

$$\mathcal{F} = \sum_{i=1}^{n} f^{(i)}\mathcal{F}^{(i)} = \frac{1}{2}\sum_{i=1}^{n} f^{(i)} \left|a_{+1}^{(i)} + a_{-1}^{(i)}\right|^{2} \qquad (0 \le \mathcal{F} \le 1)$$

is therefore frame-independent. It can be shown to be equal to

$$\mathcal{F} = \frac{1 + \lambda_{\vartheta} + 2\lambda_{\varphi}}{3 + \lambda_{\vartheta}}$$

note: $\mathcal{F} = \frac{1+\tilde{\lambda}}{3+\tilde{\lambda}}$

In other words, there always exists *a calculable frame-invariant relation* of the form

$$(1 - \mathcal{F})\lambda_{\vartheta} + 2\lambda_{\varphi} = 3\mathcal{F} - 1$$

Simple derivation of the Lam-Tung relation

Another consequence of rotational properties of angular momentum eigenstates:

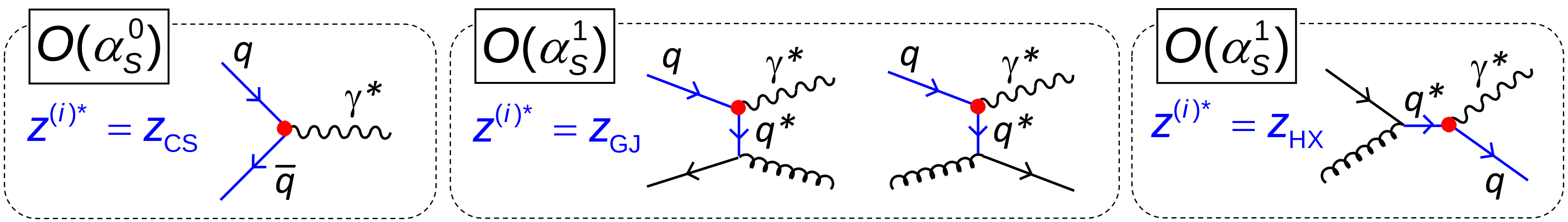
for each state $\left|\psi^{(i)}\right\rangle = a_0^{(i)}\,|0\rangle + a_{+1}^{(i)}\,|+1\rangle + a_{-1}^{(i)}\,|-1\rangle$

there exists a quantization axis $z^{(i)*}$ wrt which $a_0^{(i)*} = 0$

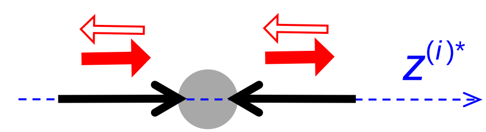
→ dileptons produced in each single elementary subprocess have a distribution of the type

$$\lambda_\vartheta^{(i)*} = +1, \quad \lambda_\varphi^{(i)*} = 2\mathcal{F}^{(i)} - 1, \quad \lambda_{\vartheta\varphi}^{(i)*} = 0$$ wrt its specific "$a_0^{(i)*} = 0$" axis.

DY:



Due to helicity conservation at the q-$\bar{q}$-γ^* (q-q^*-γ^*) vertex,
$J_{z^{(i)*}} = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction $z^{(i)*}$



→ for each diagram $W^{(i)*} \propto 1 + \cos^2\vartheta^{(i)*} \Rightarrow \mathcal{F}^{(i)} = ½$

sum independent of spin alignment directions!

$$\mathcal{F} = \sum f^{(i)}\mathcal{F}^{(i)} = \frac{1}{2} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}$$

[FLS, PRL 105, 061601 (2010)] → $\lambda_\vartheta + 4\lambda_\varphi = 1$ Lam-Tung identity

Essence of the LT relation

1. The *existence* (*and frame-independence*) of the LT relation is the *kinematic* consequence of the rotational properties of $J = 1$ angular momentum eigenstates
2. Its *form* derives from the *dynamical* input that all contributing processes produce a *transversely* polarized ($J_z = \pm 1$) state (wrt whatever axis)

More generally:

- Corrections to the Lam-Tung relation (parton-k_T, higher-twist effects) should continue to yield *invariant* relations.
  In the literature, deviations are often searched in the form

  $$\lambda_\vartheta + 4\lambda_\varphi = 1 - \Delta$$

  But this is not a frame-independent relation. Rather, corrections should be searched in the invariant form

  $$\mathcal{F} = 1/2\,(1 - \Delta_{\text{inv}}) \quad \rightarrow \quad \lambda_\vartheta(1 + \Delta_{\text{inv}}) + 4\lambda_\varphi = 1 - 3\Delta_{\text{inv}}$$

- For *any* superposition of processes, concerning *any* $J = 1$ particle
  (even in parity-violating cases: W, Z),
  we can always calculate a *frame-invariant* relation analogous to the LT relation.

A lot of measurements to do...

- Measurement of χ_{c0}(1P), χ_{c1}(1P) and χ_{c2}(1P) production cross sections
- Measurement of χ_b (1P), χ_b(2P) and χ_b(3P) production cross sections;
- Measurement of the relative production yields of J = 1 and J = 2 χ_b states
- Measurement of the χ_{c1} (1P) and χ_{c2}(1P) polarizations versus p_T and rapidity
- Measurement of the χ_b (1P) and χ_b (2P) polarizations
- …