B Mixing in the Standard Model and Beyond

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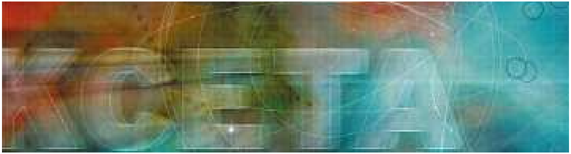

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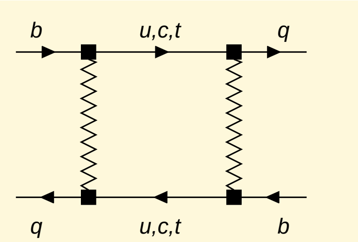

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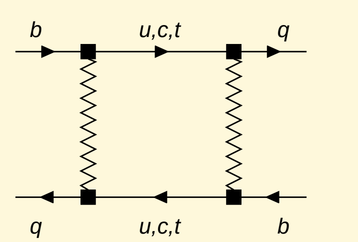

3 physical quantities in $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing:

$$\left|M_{12}^q\right|, \quad \left|\Gamma_{12}^q\right|, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$$

The two eigenstates found by diagonalising $M - i\,\Gamma/2$ differ in their masses and widths:

$$\begin{array}{lccl} \text{mass difference} & \Delta m_q & \simeq & 2|M_{12}^q|, \\ \text{width difference} & \Delta\Gamma_q & \simeq & 2|\Gamma_{12}^q|\cos\phi_q \end{array}$$

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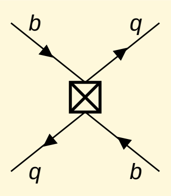
CP asymmetry in flavor-specific decays (semileptonic CP asymmetry):

$$a_{\mathrm{fs}}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q$$

Δm_s and Δm_d

Operator Product Expansion:



$$M_{12} = |V_{tq}^* V_{tb}|^2 \, CQ$$

Local Operator:

$$Q \;=\; \overline{q}_L \gamma_\nu b_L \, \overline{q}_L \gamma^\nu b_L$$

Theoretical uncertainty of Δm_q dominated by matrix element:

$$\langle \mathrm{B_q} | Q | \overline{\mathrm{B}}_q \rangle \;=\; \frac{2}{3} M_{B_q}^2 \, f_{B_q}^2 \, B_{B_q}$$

Standard Model: $C = C(m_t, \alpha_s)$ is well-known.

$\mathrm{B_s - \overline{B}_s}$ mixing: CKM unitarity fixes $|V_{ts}| \simeq |V_{cb}|$. Use lattice results for $f_{B_q}^2\, B_{B_q}$ to confront $\Delta m_s^{\rm exp}$ with the Standard Model:

$$\Delta m_s = \left(18.8 \pm 0.6_{V_{cb}} \pm 0.3_{m_t} \pm 0.1_{\alpha_s} \right) \mathrm{ps}^{-1} \, \frac{f_{B_s}^2\, B_{B_s}}{(220\,\mathrm{MeV})^2}$$

Here $\overline{\rm MS}$-NDR scheme for B_{B_q} at scale m_b.

Often used: scheme-invariant $\widehat{B}_{B_q} = 1.51 B_{B_q}$.

Recall:

$$\Delta m_s = \left(18.8 \pm 0.6_{V_{cb}} \pm 0.3_{m_t} \pm 0.1_{\alpha_s} \right) \text{ps}^{-1} \, \frac{f_{B_s}^2 \, B_{B_s}}{(220\,\text{MeV})^2}$$

CKMfitter lattice averages (1203.0238):

$$f_{B_s} = (229 \pm 2 \pm 6)\,\text{MeV}, \qquad B_{B_s} = 0.85 \pm 0.02 \pm 0.02$$

means $f_{B_s}^2 B_{B_s} = (211 \pm 9)\,\text{MeV}$ and

$$\Delta m_s = (17.3 \pm 1.5)\,\text{ps}^{-1}$$

complying with LHCb/CDF average

$$\Delta m_s^{\text{exp}} = (17.731 \pm 0.045)\,\text{ps}^{-1}$$

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But also:

population of Cincinnati:	296943
population of Karlsruhe:	294761
ratio:	1.0074

With recent preliminary Fermilab/MILC result (1112.5642),

$$f_{B_s}^2 B_{B_s} = 0.0559(68)\,\mathrm{GeV}^2 \simeq [(237 \pm 14)\,\mathrm{MeV}]^2\,,$$

one finds

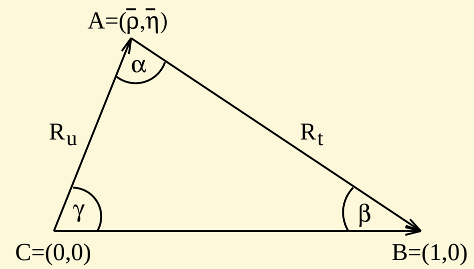
$$\Delta m_s = (21.7 \pm 2.6)\,\mathrm{ps}^{-1}$$

Δm_d

$|V_{cb}|$, short-distance coefficient and some hadronic uncertainties drop out from the ratio $\Delta m_d / \Delta m_s$:

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{|V_{td}|^2}{|V_{ts}|^2} \propto R_t^2$$



Usual way to probe the Standard Model with Δm_d: Global fit to unitarity triangle.

Easier way:
Determine R_t from Δm_d:

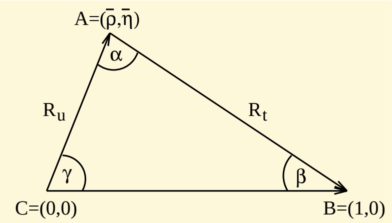
$$R_t = 0.880 \frac{\xi}{1.16} \sqrt{\frac{\Delta m_d}{0.49\,\text{ps}^{-1}}} \sqrt{\frac{17\,\text{ps}^{-1}}{\Delta m_s}} \frac{0.22}{|V_{us}|} (1 + 0.050\bar{\rho})$$

and compare with indirect determination of R_t from angles:

$$R_t = \frac{\sin\gamma}{\sin\alpha} = \frac{\sin(\alpha+\beta)}{\sin\alpha}$$

$$\beta = 21.4^\circ \pm 0.8^\circ, \;\; \alpha = 88.7^{\circ}{}^{+4.6^\circ}_{-4.2^\circ}$$

$$\Rightarrow \;\; R_t = 0.939 \pm 0.027$$



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Fermilab/MILC (1205.7013): $\xi = 1.268 \pm 0.063$ implying

$$R_t = 0.942 \pm 0.047_{\xi} \pm 0.006_{\mathrm{rest}}$$

agrees well with $R_t = 0.939 \pm 0.027$ from angles.

CKMfitter (Sep 27, 2012) global fit result:

$$R_t = 0.926^{+0.027}_{-0.028}$$

QCD sum rule result $\xi = 1.16 \pm 0.04$ challenged by data.

Decay matrix

The calculation Γ_{12}^q, $q = d, s$, is needed for

the width difference $\Delta\Gamma_q \simeq 2|\Gamma_{12}^q| \cos\phi_q$

and the semileptonic CP asymmetry $a_{\rm fs}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q$

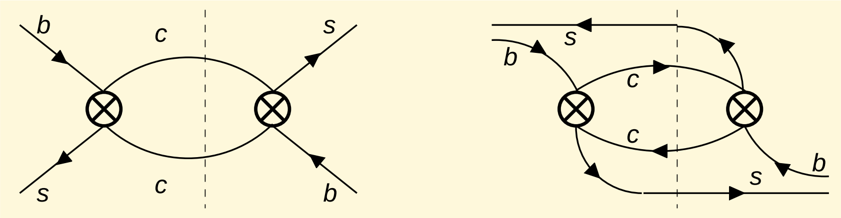
In the Standard Model

$$\phi_s = 0.22^\circ \pm 0.06^\circ \quad \text{and} \quad \phi_d = -4.3^\circ \pm 1.4^\circ.$$

Recalling $\phi_q = \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$, a new physics contribution to $\arg M_{12}^q$ may deplete $\Delta\Gamma_q$ and enhance $|a_{\rm fs}^q|$ to a level observable at current experiments.

But: Precise data on CP violation in $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ preclude large NP contributions to $\arg\phi_d$ and $\arg\phi_s$.

Leading contribution to Γ_{12}^{s}:



Γ_{12}^{s} stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays, sizable new-physics contributions are impossible.

Updated Standard-Model prediction for $\Delta\Gamma_s/\Delta m_s$ in terms of hadronic parameters:

$$\frac{\Delta\Gamma_s}{\Delta m_s}\Delta m_s^{\rm exp} = \left[0.082 + 0.019\frac{\widetilde{B}'_{S,B_s}}{B_{B_s}} - 0.025\frac{B_R}{B_{B_s}}\right] \text{ps}^{-1}$$

Here

$$\langle B_s|\bar{s}_L^\alpha b_R^\beta\, \bar{s}_L^\beta b_R^\alpha|\overline{B}_s\rangle = \frac{1}{12}M_{B_s}^2\, f_{B_s}^2\, \widetilde{B}'_{S,B_s}$$

and $B_R = 1 \pm 0.5$ parametrises the size of higher-dimension operators.

With preliminary Fermilab/MILC result (1112.5642),

$$\frac{\widetilde{B}'_{S,B_s}}{B_{B_S}} = 1.23 \pm 0.24$$

find:

$$\frac{\Delta\Gamma_s}{\Delta m_s}\Delta m^{\rm exp} = \left[0.075 \pm 0.015_{B_R/B} \pm 0.012_{\rm scale} \pm 0.004_{\widetilde{B}/B}\right] \text{ps}^{-1}$$

complies well with

$$\Delta\Gamma_s = \left[0.116 \pm 0.018_{\rm stat} \pm 0.006_{\rm syst}\right] \text{ps}^{-1}$$

New physics

Trouble maker:

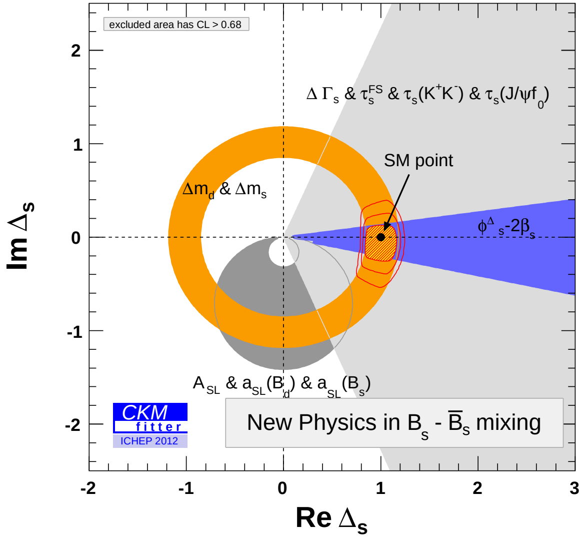
$$
\begin{aligned}
A_{\mathrm{SL}} &= (0.532 \pm 0.039) a_{\mathrm{fs}}^d + (0.468 \pm 0.039) a_{\mathrm{fs}}^s \\
&= (-7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3} \qquad \text{DØ 2011}
\end{aligned}
$$

Define the complex parameters Δ_d and Δ_s through

$$
M_{12}^q \equiv M_{12}^{\mathrm{SM,q}} \cdot \Delta_q \,, \qquad \Delta_q \equiv |\Delta_q| e^{i\phi_q^\Delta} .
$$

In the Standard Model $\Delta_q = 1$. Use $\phi_s = \phi_s^{\mathrm{SM}} + \phi_s^\Delta \simeq \phi_s^\Delta$.

CKMfitter September 2012 update of 1203.0238:



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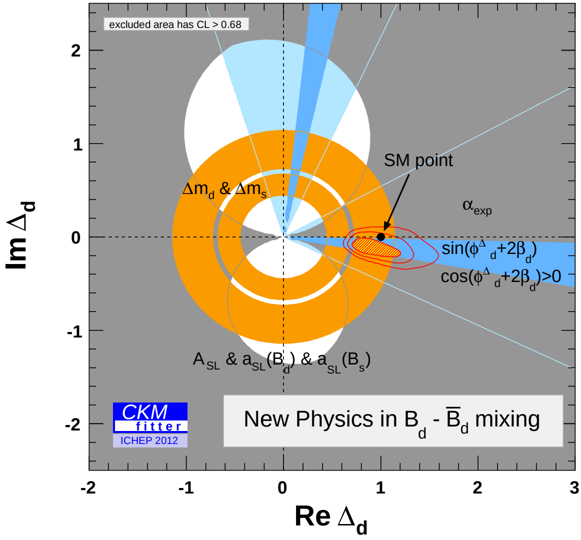

$A_{\rm SL}$ and WA for $B(B \to \tau\nu)$ prefer small $\phi_d^\Delta < 0$.

Plots courtesy of Jérôme Charles

Pull value for A_{SL}: 3.3σ

$\Rightarrow$ Scenario with NP in M_{12}^q only cannot accomodate the DØ measurement of A_{SL}.

The Standard Model point $\Delta_s = \Delta_d = 1$ is disfavoured by 1σ, down from the 2010 value of 3.6σ.

New physics in Γ_{12}^q?

The LHCb measurement of Γ_s implies

$$\frac{\Gamma_d}{\Gamma_s} = \frac{\tau_{B_s}}{\tau_{B_d}} = 0.997 \pm 0.013$$

in excellent agreement with the SM prediction
$\tau_{B_s}/\tau_{B_d} = 0.998 \pm 0.003$.
Changing the Cabibbo-favoured tree-level quantity $|\Gamma_{12}^s|$ by opening new enhanced decay channels such as $B_s \to \tau^+\tau^-$ will spoil this ratio.

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Phenomenologically, new physics in the doubly Cabibbo-suppressed quantity Γ_{12}^d is still allowed, but requires somewhat contrived models of new physics.

Conclusions

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- Δm_s and $\Delta\Gamma_s$ comply with the Standard-Model expectation.
- Scenarios with new physics only in $M_{12}^{d,s}$ can only marginally improve A_{SL}, through $\phi_d^\Delta < 0$.
- New physics in Γ_{12}^s from yet undiscovered B_s decay modes is not viable, but maybe new physics in Γ_{12}^d is worthwhile to look at.