B-meson decay constants and mixing on the lattice

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1. Introduction

Discrepancies between some flavour observables and SM predictions at the $2-3\sigma$ level:

* Unitarity triangle fits: driven by tension between $B(B\to\tau\nu)$ and $\sin(2\beta)$ **UTfit**, 1010.5089, **CKMfitter**, 1203.0238.

* Like-sign dimuon charge asymmetry A_{SL} (3.9σ): **D0**, 1106.6308.

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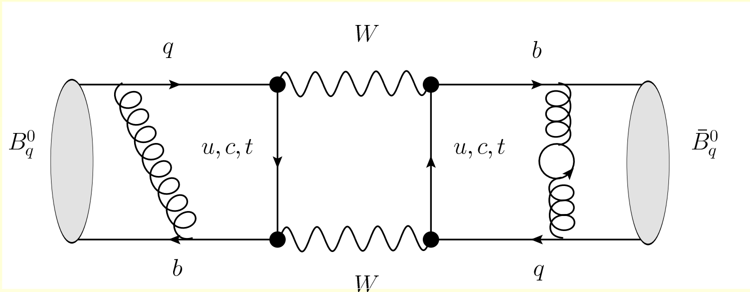
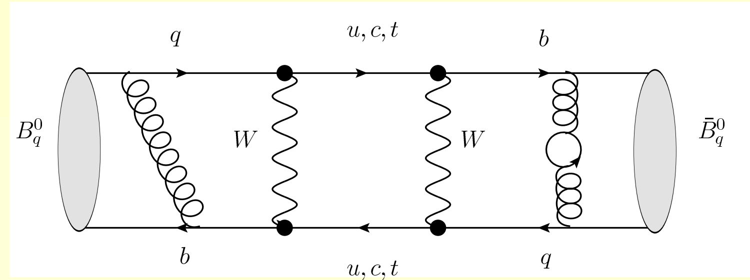
It has been argued that these discrepancies may be due to BSM effects in neutral B mixing.

Lenz, Nierste, CKMfitter, 1203.0238 **Laiho, Lunghi, Van de Water**, 1102.3917

1. Introduction





- B_0 mixing parameters determined by the off diagonal elements of the mixing matrix

$$i\frac{d}{dt}\begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix} = \left(M^q - \frac{i}{2}\Gamma^q\right)\begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix}$$

$$\Delta M_q \propto |M_{12}^q| \qquad \Delta\Gamma_q \propto |\Gamma_{12}^q| \qquad \phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$$

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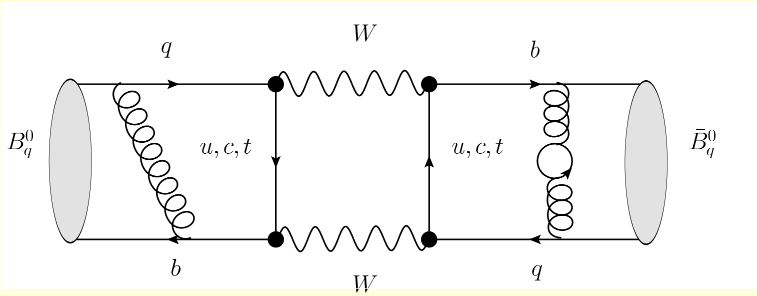

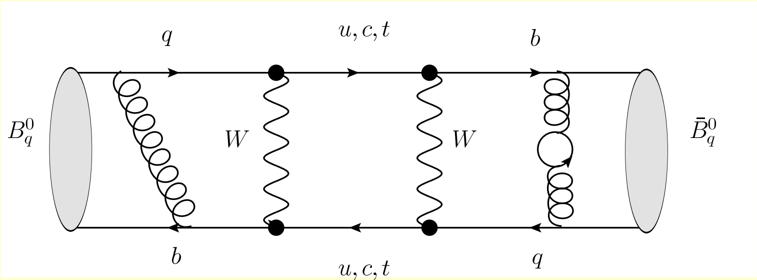

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New physics in M_{12}^q and/or Γ_{12}^q?

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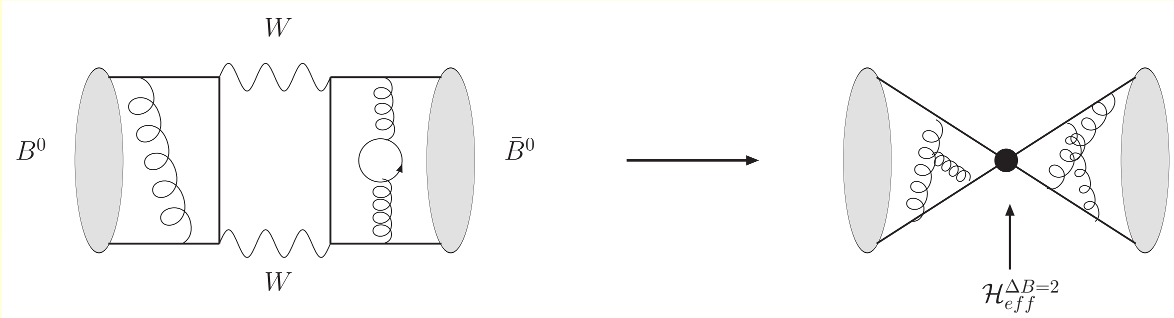

The most general Effective Hamiltonian describing $\Delta B = 2$ processes is

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = (\bar{b}^\alpha \gamma_\mu L\, q^\alpha) \left(\bar{b}^\beta \gamma^\mu L\, q^\beta\right)$$ **SM**

$$Q_2^q = (\bar{b}^\alpha\, L\, q^\alpha) \left(\bar{b}^\beta\, L\, q^\beta\right) \qquad Q_3^q = \left(\bar{b}^\alpha\, L\, q^\beta\right) \left(\bar{b}^\beta\, L\, q^\alpha\right)$$

$$Q_4^q = (\bar{b}^\alpha\, L\, q^\alpha) \left(\bar{b}^\beta\, R\, q^\beta\right) \qquad Q_5^q = \left(\bar{b}^\alpha\, L\, q^\beta\right) \left(\bar{b}^\beta\, R\, q^\alpha\right)$$

where $L = (1-\gamma_5)\,, R = (1+\gamma_5)$.

$\tilde{Q}_{1,2,3} = Q_{1,2,3}$ with the replacement $L(R) \to R(L)$

1. Introduction: B mixing parameters in the SM

Mass differences:

$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

** Non-perturbative input $\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1^q | B_q^0 \rangle (\mu)$

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Decay width differences: **Lenz and Nierste**, JHEP0706 (2007) 072

$$\Delta\Gamma_q = C_1 \langle Q_1^q \rangle + C_3 \langle Q_3^q \rangle + \left(C_{R_0} \langle R_0 \rangle + C_{R_2} \langle R_2 \rangle + C_R \langle R \rangle \right)_{1/m_b}$$

with $R_0 = Q_2 + \alpha_1(\alpha_s) Q_3 + 1/2 \alpha_2(\alpha_s) Q_1$,

$R_2 = \frac{1}{m_b^2} \left(\bar{q}_\alpha \overleftarrow{D}_\rho \gamma_\mu L D^\rho b_\alpha \right) \left(\bar{q}_\beta \gamma_\mu L b_\beta \right)$, and R labels other $1/m_b$ contr.

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→ Most interesting for phenomenology (unitarity triangle fits):

$$f_{B_q} \sqrt{\hat{B}_{B_q}}^{\,*}, \qquad \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}, \qquad B_q^0, \qquad \frac{B_s^0}{B_d^0}$$

* In particular as the role of $|V_{cb}|$ (reaching is ultimate theoretical accuracy) in unitarity triangle analyses is being replaced by ΔM_{B_s} and $B \to \tau\nu$.

1. Introduction: B mixing parameters BSM

Constraining NP models:

SM predictions + BSM contributions = experiment

→ constraints on BSM building

see, e.g., **Dobrescu and Krnjaic**, 1104.2893; **Altmannshofer and Carena**, 1110.0843; **Buras and Girrbach**, 1201.1302, **Lenz, Nierste and CKMfitter**, 1008.1593 ...

* Need matrix elements of all the operators in $\mathcal{H}_{eff}^{\Delta B=2}$: $\langle Q_{1\ldots5}^q \rangle$

... In conjunction with experimental measurements (**HFAG 12**)

$$\Delta M_d|_{exp.} = (0.507 \pm 0.004) ps^{-1} \qquad \Delta M_s|_{exp.} = (17.719 \pm 0.043) ps^{-1}$$

$$sign(Re\lambda_{CP}) \frac{\Delta\Gamma_d}{\Gamma_d} = 0.015 \pm 0.018 \qquad \Delta\Gamma_s = 0.095 \pm 0.014$$

1. Introduction: B^0 mixing on the lattice

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Unquenched calculations

* Quenching the strange quark could have an error as large as 5% and an error estimate is not possible without $N_f = 2+1$
  → want $N_f = 2+1$

* Neglecting sea charm has effects $\mathcal{O}(1\%)$ (can be estimated with HQET). **Starting to need sea charm effects,** $N_f = 2+1+1$

ETMC, MILC

1. Introduction: B^0 mixing on the lattice

1.2. Heavy quarks

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

- * **Effective theories:** Need to include multiple operators matched to full QCD (NRQCD,HQET,RHQ,static). `B-physics` $\surd$

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* **Relativistic (improved) formulations:**

  ** Allow accurate results for `charm` (especially twisted mass, Hisq (Highly improved staggered quarks)).

  ** Also for `bottom`: Results for $m_h = m_c \cdots < m_b$ and extrapolation to m_b using HQET-based expresions (twisted mass, HISQ).

  ** Advantages of having the same formulation for light and heavy: ratios light/heavy, PCAC for heavy-light, ... Also simpler tuning of masses.

One could get the same precision for D as for K

2. B and B_s decay constants

The measured value of $Br(B \to \tau\nu)$ suffers from a tension with the SM at the $2-3\sigma$ level **Laiho,Lunghi,Van de Water**, 1204.0791

Needed in theoretical predictions for processes potentially very sensitive to NP, for example, $Br(B_q \to \mu^+\mu^-)$.

2. B and B_s decay constants on the lattice

Simple matrix element $\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B_q(p)\rangle = if_{B_q}p_\mu$ → precise calculations

(caveat: am_b discr. errors)

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Simple matrix element $\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p_\mu$ $\rightarrow$ precise calculations

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$N_f = 2+1$ **determinations** (all use MILC configurations)

Collaboration	FNAL/MILC 2011 **old data**	HPQCD 2011 **relativistic**	HPQCD 2012 **non-relativistic**
Light Action (staggered)	asqtad	HISQ	HISQ
Heavy Action	Fermilab	HISQ	NRQCD
Error	$\sim 4.5\%$	$\sim 2\%$	$\sim 4.5\%$

FNAL/MILC old data: $a = 0.12, 0.09$ fm, $\sim 500-800$ config.

* HPQCD relativistic: Using relativistic actions reduce the error by half.

** Cross-checks: $m_b^{\overline{MS}}$, $m_{B_s} - m_{\eta_b}/2$

** First empirical evidence for $1/\sqrt{m_{B_s}}$ depende predicted by HQET.

2. B and B_s decay constants on the lattice

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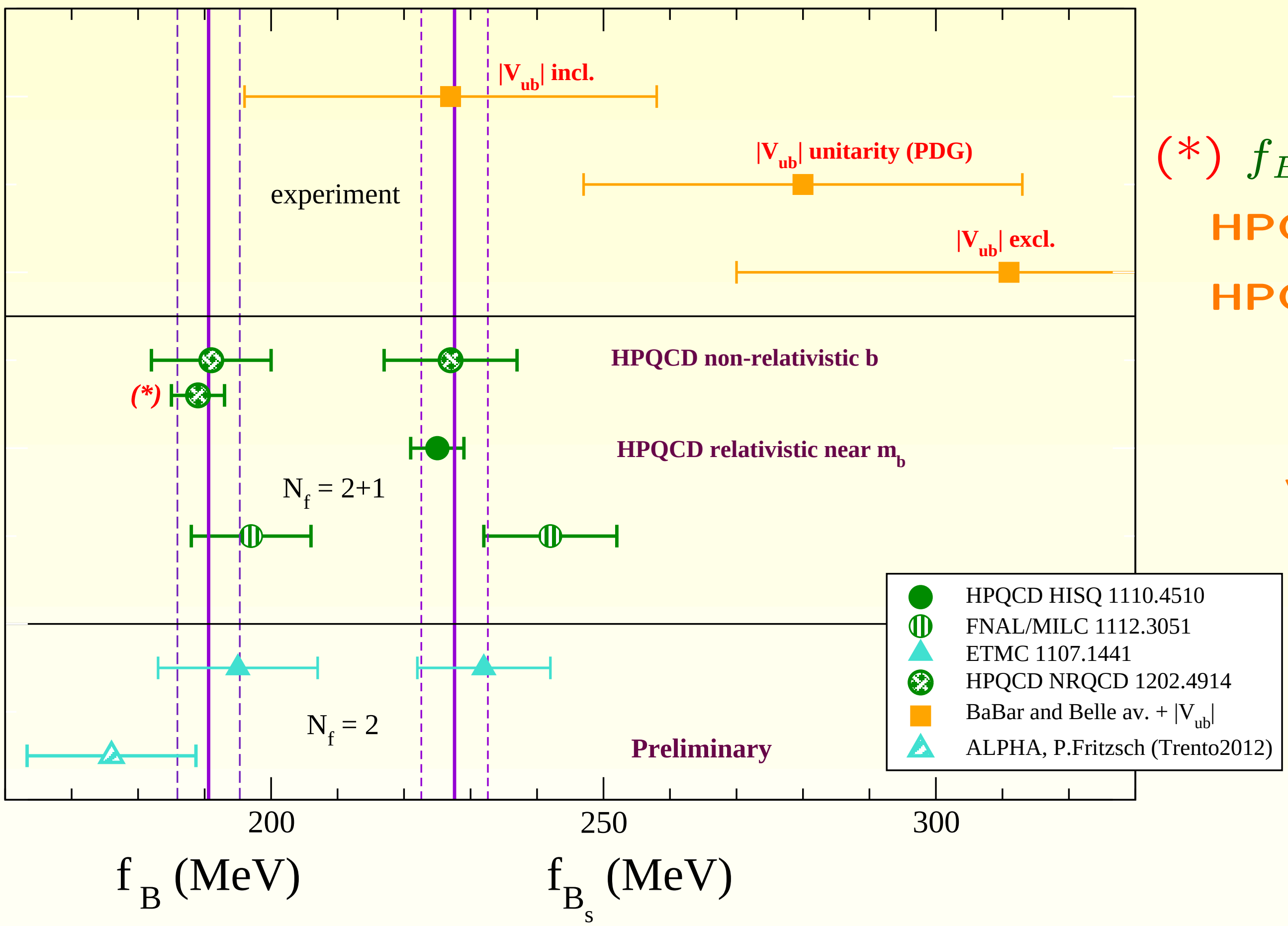

(*) f_B obtained from f_{B_s}/f_B HPQCD non-relativistic and f_{B_s} HPQCD relativistic.

LLV = Laiho, Lunghi, Van de Water,

www.latticeaverages.org

$$f_B^{\mathbf{LLV}} = (190.6 \pm 4.7)\ MeV \qquad f_{B_s}^{\mathbf{LLV}} = (227.6 \pm 5.0)\ MeV$$

Exper. average **Rosner and Stone**, 1201.2401: $\mathcal{B}r(B \to \tau\nu) = (1.68 \pm 0.31) \cdot 10^{-4}$

2. B and B_s decay constants on the lattice

From **HPQCD**, 1202.4914 (NRQCD), **FNAL/MILC**, 1107.1441, and **RBC/UKQCD**, 1001.2023, the average of the ratio of decay const. is **Laiho, Lunghi, Van de Water**

$$\boxed{f_{B_s}/f_{B_d} = 1.201 \pm 0.017}$$

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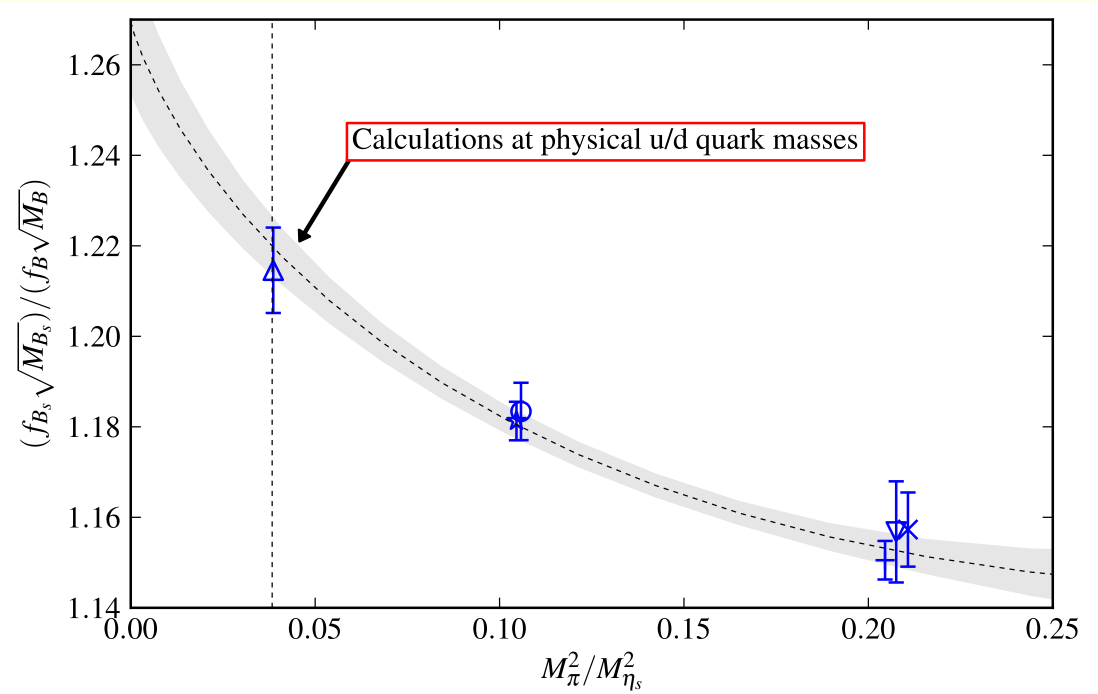
Preliminary results from **R. Dowdall** *et al* **[HPQCD]**



* Radiatively improved NRQCD + HISQ.

* Data at the physical quark masses.

* $N_f = 2 + 1 + 1$, including charm quarks on the sea.

2. B and B_s decay constants on the lattice

In progress:

ALPHA ($N_f = 2$, improv. Wilson+HQET);

RBC/UKQCD ($N_f = 2 + 1$, domain wall+RHQ);

FNAL/MILC new data ($N_f = 2 + 1$, staggered+Fermilab):
$a = 0.12, 0.09, 0, 06, 0, 045$ fm,$\sim 1000 - 2000$ configurations, improved techniques/analysis ...

HPQCD ($N_f = 2 + 1 + 1$, staggered+NRQCD)

ETMC ($N_f = 2 + 1 + 1$, twisted mass)

3. B^0 mixing: $N_f = 2 + 1$ lattice calculations

COMPLETE CALCULATIONS

- **HPQCD**: E. Gámiz *et al.*, Phys.Rev.D80:014503 (2009)
  - * Configurations: MILC staggered.
  - * Light quarks: Improved staggered (Asqtad)
  - * Heavy quarks: NRQCD
- **FNAL/MILC old data**: A. Bazavov *et al.*, Phys.Rev.D86:034503 (2012)
  - * Configurations: MILC staggered.
  - * Light quarks: Improved staggered (Asqtad)
  - * Heavy quarks: Fermilab $\rightarrow$ it can also be used for c quarks.
- **RBC/UKQCD**: C. Albertus Phys.Rev.D82:014505 (2010) **exploratory**
  - * Configurations: RBC/UKQCD domain wall.
  - * Light quarks: Domain wall.
  - * Heavy quarks: Static.

3. B^0 mixing: $N_f = 2+1$ lattice calculations

ON-GOING CALCULATIONS

• **FNAL/MILC new data**: Same strategy, actions, and configurations as before but:

higher statistics, more ensembles, smaller lattice spacing, smaller light-quark mass, full $\Delta B = 2$ basis, bag parameters ...

• **ETMC**:

* Configurations: ETMC twisted mass ($N_f = 2$, $N_f = 2+1+1$).

* Light quarks: twisted mass.

* Heavy quarks: twisted mass.

• **RBC/UKQCD**:

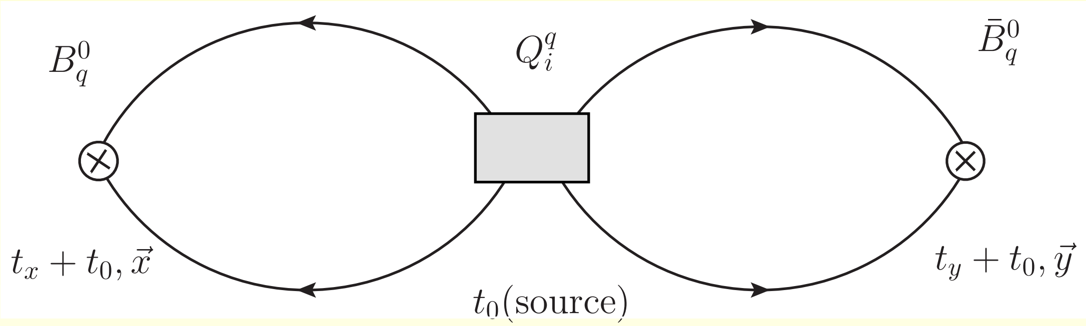
* Configurations: RBC/UKQCD domain wall.

* Light quarks: Domain wall.

* Heavy quarks: RHQ.

3. B^0 mixing: $N_f = 2+1$ lattice calculations

Need 3-point and 2-point correlators



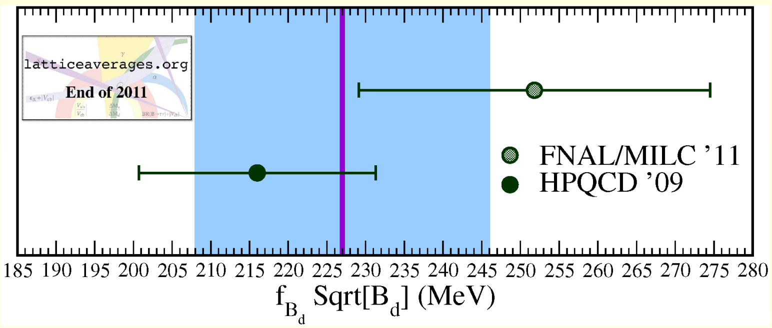
$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) \left[\hat{Q}_i^q \right] (0) \Phi^\dagger_{\bar{B}_q}(\vec{x}_2, -t_2) | 0 \rangle$$

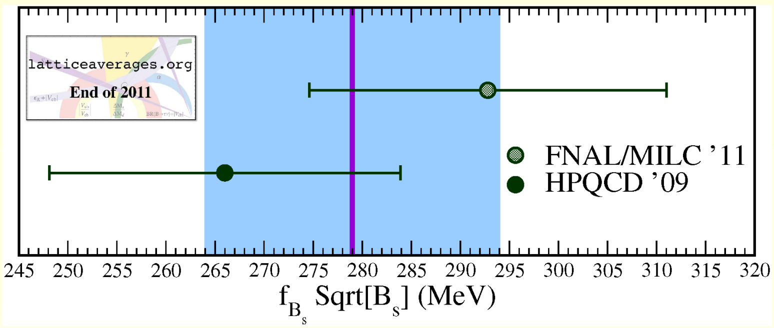
$$C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi^\dagger_{\bar{B}_q}(\vec{0}, 0) | 0 \rangle$$

- $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$ is an interpolating operator for the B_q^0 meson.

3. B^0 mixing: SM parameters

Two results for $\sqrt{f_B \hat{B}_B}$ using MILC $N_f = 2+1$ but different description of heavy quarks (FNAL/MILC 2011 new data is **preliminary**).





$$f_{B_s}\sqrt{\hat{B}_{B_s}}^{\,\mathrm{LLV}} = 279(15)\mathrm{MeV}$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}}^{\,\mathrm{LLV}} = 227(19)\mathrm{MeV}$$

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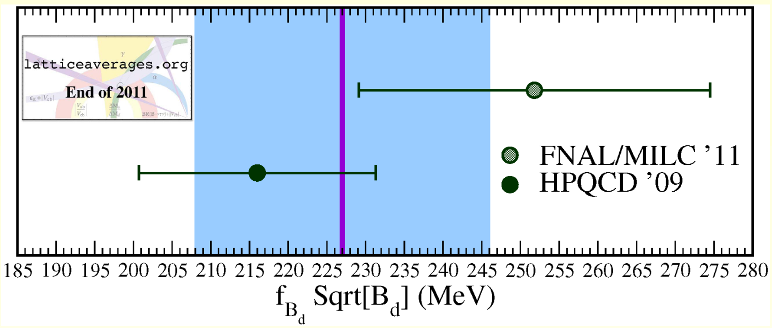

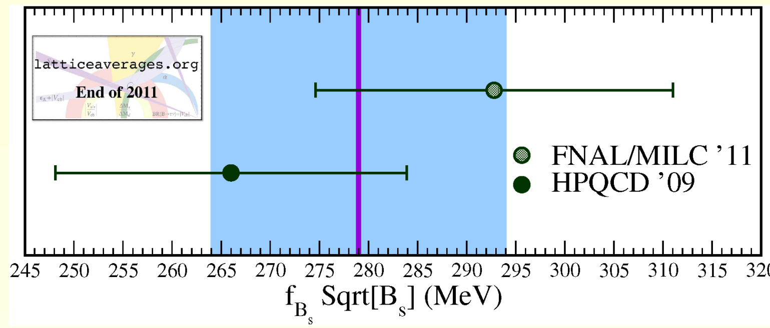

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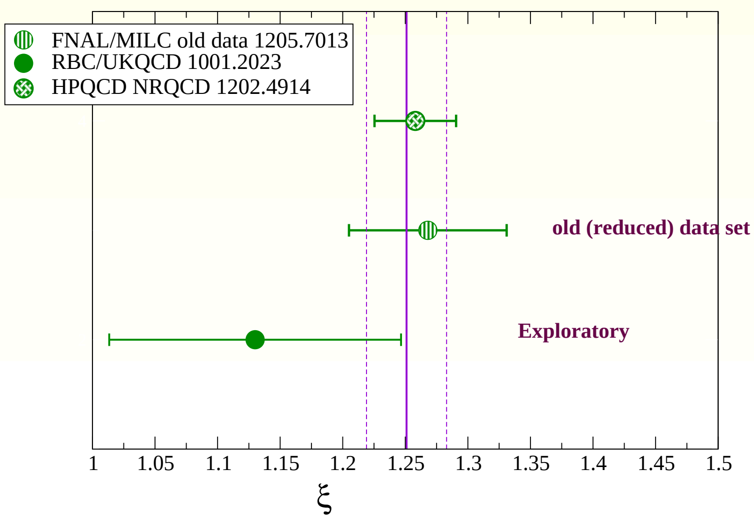

Results for $\xi = \dfrac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$

$$\xi^{\mathbf{lat}} = 1.251 \pm 0.032$$

3. B^0 mixing: SM parameters

Bag parameters

HPQCD 2009

$$\hat{B}_{B_s} = 1.33(6) \qquad \hat{B}_{B_d} = 1.26(11) \qquad B_{B_s}/B_{B_d} = 1.05(7)$$

Also, from FNAL/MILC old data: $\frac{B_{B_s}}{B_{B_d}} = 1.06(11)$

* These two calculations are not optimized to get bag parameters, except for $\hat{B}_{B_s}$ (no correlations between matrix elements and decay constants, not direct extrapolations ...)

$\rightarrow$ significant improvement is possible.

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Bag parameters: **ETMC**, Lattice 2012 $N_f = 2$

$$\left.\frac{B_{B_s}}{B_{B_d}}\right|_{\rm LO\,matching} = 1.023(18)(?)$$ **Preliminary**

3. B^0 **mixing: BSM**

SM predictions + BSM contributions = experiment

$\rightarrow$ constraints on BSM building **Dobrescu and Krnjaic**, 1104.2893; **Altmannshofer and Carena**, 1110.0843; **Buras and Girrbach**, 1201.1302 ...

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FNAL/MILC, Lattice 2011 **Preliminary**

	B_d^0		B_s^0	
[GeV^2]	BBGLN	BJU	BBGLN	BJU
$f_{B_q}^2 B_{B_q}^{(1)}$	0.0411(75)		0.0559(68)	
$f_{B_q}^2 B_{B_q}^{(2)}$	0.0574(92)	0.0538(87)	0.086(11)	0.080(10)
$f_{B_q}^2 B_{B_q}^{(3)}$	0.058(11)	0.058(11)	0.084(13)	0.084(13)
$f_{B_q}^2 B_{B_q}^{(4)}$	0.093(10)		0.135(15)	
$f_{B_q}^2 B_{B_q}^{(5)}$	0.127(15)		0.178(20)	

* Errors in $f_{B_q}\sqrt{B_{B_q}}$: 6-9%

* Errors for all $f_{B_q^i}\sqrt{B_{B_q}^i}$: 5-9%

Errors should decrease when including smaller a and more ensembles

* $\langle Q_1\rangle, \langle Q_3\rangle, \langle R_0\rangle$ will allow new prediction for $\Delta\Gamma_s$.

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Target: Matrix elements, bag parameters, ξ, $\frac{\langle Q_3\rangle}{\langle Q_1\rangle}$, $\frac{\langle R_0\rangle}{\langle Q_1\rangle}$,

$$\langle R_0\rangle \equiv \langle Q_2\rangle + \langle Q_3\rangle + 1/2\langle Q_1\rangle, \cdots$$

4. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Bag parameters $B_{B_{s,d}}$ describing $B-$**meson** mixing in the SM can be can be used for theoretical prediction of $\mathcal{B}r(B \to \mu^+\mu^-)$
Buras, hep-ph/0303060.

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Need to include the effects of a non-vanishing $\Delta\Gamma_s$ to compare with experiment **K. de Bruyn et al.**, 1204.1737

$$\mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \to \mathcal{B}r(B_q \to \mu^+\mu^-)_{y_s} \equiv \mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \times \frac{1}{1-y_s}$$

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* Using:

** **HPQCD (Gámiz et al.)**, 0902.1815: $\hat{B}_s = 1.33(6)$, $\hat{B}_d = 1.26(11)$

** **PDG2012** $\tau_{B_s} = 1.497(15)\mathrm{ps}$ and $\tau_{B_d} = 1.519(7)\mathrm{ps}$

** **LHCb** measurement: $\Delta\Gamma_s = 0.116(19)\mathrm{ps}^{-1}$

4. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

$$\mathcal{B}r(B_s \to \mu^+\mu^-)_{y_s} = (3.65 \pm 0.20) \times 10^{-9}$$

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Improved $f_{B_{s,d}}^{lattice}$ makes direct theoretical calculation competitive.

Buras and Girrbach,1204.5064

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* Using the same inputs as in **Buras and Girrbach**,1204.5064, except for
    ** **PDG2012** $\tau_{B_s} = 1.497(15)\text{ps}$ and $\tau_{B_d} = 1.519(7)\text{ps}$
    ** **LLV averages**: $f_B = 190.6(4.7)\text{MeV}$ and $f_{B_s} = 227.6(5.0)\text{MeV}$.
* And again including the correction $1/(1 - y_s)$

$$\mathcal{B}r(B_s \to \mu^+\mu^-)_{y_s} = (3.64 \pm 0.23) \times 10^{-9}$$
$$\mathcal{B}r(B_d \to \mu^+\mu^-) = (1.07 \pm 0.10) \times 10^{-10}$$

Most stringest experimental bounds **LHCb, 1203.4493**:

$$\mathcal{B}r(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9} \qquad \mathcal{B}r(B_d \to \mu^+\mu^-) < 8.1 \times 10^{-10}$$

5. Future prospects and goals

Expected improvements in lattice calculations of B-meson decay constants and mixing parameters.

	current errors (%)	estimated errors in 2 years (%)
$f_B\sqrt{B_B}$	6-9	4-5
ξ	3-4	1.5-2
$B_{B_{s,d}}$	5-9	3?
f_B, f_{B_s}	2-2.5	≤ 2

* Increase of the number of calculations:

HPQCD, FNAL/MILC old data, **RBC/UKQCD** exploratory

$\Longrightarrow$ **HPQCD, FNAL/MILC** new data, **ETMC, RBC/UKQCD**

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+ Calculations at the physical quark masses

$\rightarrow$ will allow high precision tests of the SM.

5. Future prospects and goals

In the next year:

* First unquenched calculation in the continuum limit of matrix elements needed for $\Delta\Gamma_q$ at leading order in $1/m_b$: errors $\leq 10\%$

* First unquenched calculation of B mixing matrix elements contributing BSM: errors $\leq 10\%$

* Unquenched calculation of matrix elements contributing to $D^0 - \bar{D}^0$ mixing in the SM and beyond.

** Work in progress by **ETMC, FNAL/MILC**. Some preliminary results in **N. Carrasco**, Lattice 2012 with $N_f = 2$.

Preliminary	$B_D^{\overline{MS}}(2\ \mathrm{GeV})$
B_1	0.77(03)(?)
B_2	0.73(04)(?)
B_3	1.37(11)(?)
B_4	0.96(06)(?)
B_5	1.22(13)(?)