Theory News on $B_{s(d)} \to \mu^+\mu^-$ Decays

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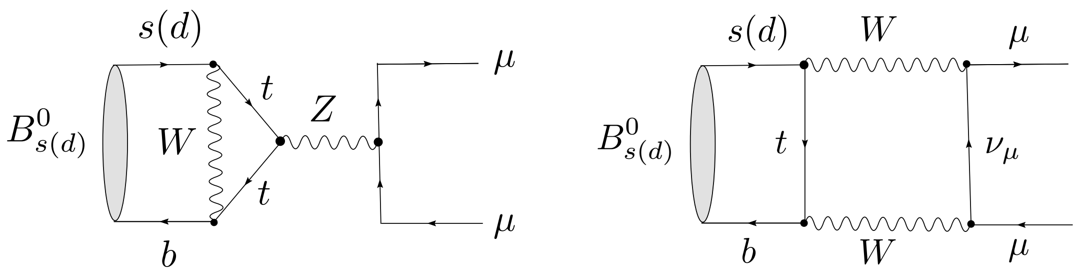
WG III @ CKM 2012, Cincinnati, 28 September – 2 October 2012

- Setting the Stage
- Recent Development: $\Delta\Gamma_s \neq 0 \to$ affects B_s BRs in a subtle way ...
- Impact on $B_s \to \mu^+\mu^-$ (?): $\Rightarrow$ BR $\oplus$ new window for New Physics
- Conclusions

Setting the Stage

General Features of $B^0_{s(d)} \to \mu^+\mu^-$ Decays

- Only loop contributions in the Standard Model (SM):



$\Rightarrow$ $\boxed{\text{strongly suppressed \& sensitive to New Physics (NP)}}$

- Hadronic sector: only $B_{s(d)}$-decay constant $f_{B_{s(d)}}$ enters: [$\to$ talk by E. Gamiz]

$\Rightarrow$ $\boxed{B^0_{s(d)} \to \mu^+\mu^- \text{ belong to the cleanest rare } B \text{ decays}}$

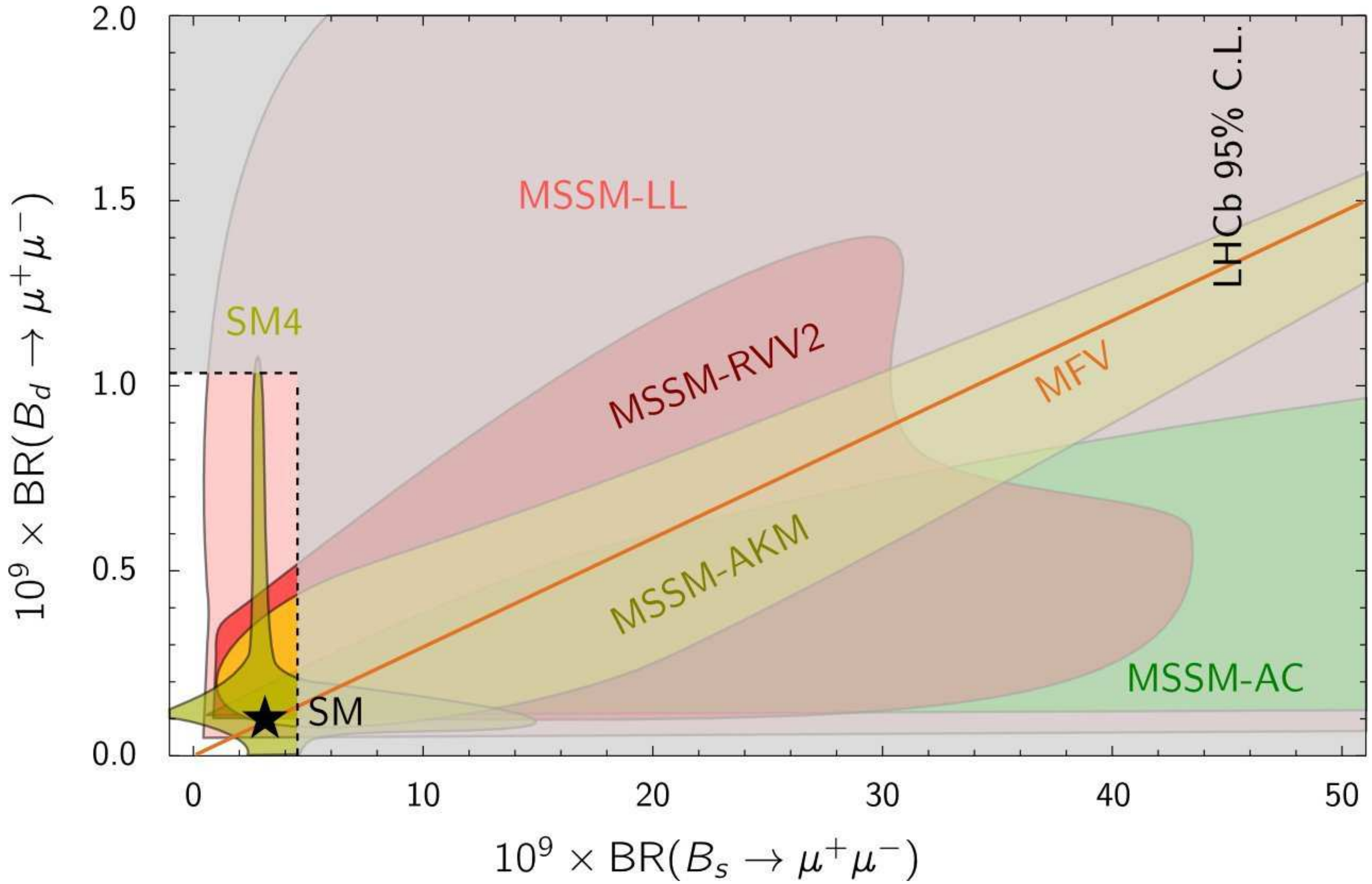
- SM predictions: $\mathrm{BR}(B_s \to \mu^+\mu^-) = (3.23 \pm 0.27) \times 10^{-9}$
  $\mathrm{BR}(B_d \to \mu^+\mu^-) = (1.07 \pm 0.10) \times 10^{-10}$

[Buras, Girrbach, Guadagnoli & Isidori (2012); address also soft photon corrections]

$\boxed{\text{NP may – in principle – enhance BRs significantly...}}$

[Babu & Kolda, Dedes *et al.*, Foster *et al.*, Carena *et al.*, Isidori & Paradisi, ...]

- Situation in different supersymmetric flavour models, showing also the impact of recent LHCb upper bounds on $\mathrm{BR}(B_{s,d} \to \mu^+\mu^-)$:



[D. Straub (2010); A.J. Buras & J. Girrbach (2012) $\oplus$ talk by C. Bobeth]

Experimental Upper Bounds (95% C.L.):

[Review: J. Albrecht (2012) $\oplus$ talks by F. Archilli, B. Gaur & K. Pitts]

- Tevatron: $\to$ *"legacy"* ...
  - DØ (2010): BR$(B_s \to \mu^+\mu^-) < 51 \times 10^{-9}$
  - CDF (2011): BR$(B_{s(d)} \to \mu^+\mu^-) < 31\,(46)\ \times 10^{-9}$

- Large Hardon Collider: $\to$ *future* ...
  - ATLAS (2012): BR$(B_s \to \mu^+\mu^-) < 22 \times 10^{-9}$
  - CMS (2012): BR$(B_{s(d)} \to \mu^+\mu^-) < 7.7\,(1.8)\ \times 10^{-9}$
  - LHCb (2012): BR$(B_{s(d)} \to \mu^+\mu^-) < 4.5\,(1.0)\ \times 10^{-9}$

  $\Rightarrow$ *LHC combination:* BR$(B_{s(d)} \to \mu^+\mu^-) < 4.2 \times 10^{-9}\ (8.1 \times 10^{-10})$

  [BR$(B_{s(d)} \to \mu^+\mu^-)_{\rm SM} = (3.23 \pm 0.27) \times 10^{-9}\ ((1.07 \pm 0.10) \times 10^{-10})$]

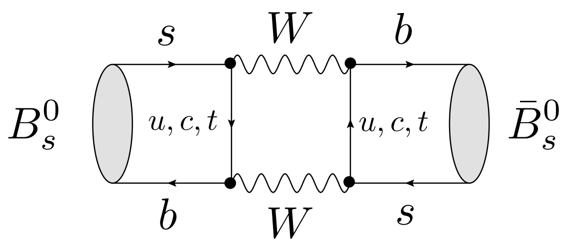
- Note: *the* limiting factor for the BR$(B_s \to \mu^+\mu^-)$ measurement – and all B_s branching ratios – is the ratio of f_s/f_d fragmentation functions.

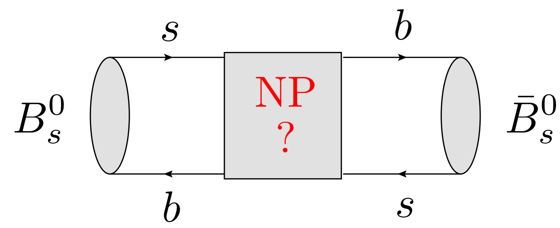
[Details: R.F., Serra & Tuning (2010); Fermilab Lattice & MILC Collaborations (2012)]

$$\boxed{\text{Recent Development:}}$$

- $\diamond$ concerning a – seemingly – unrelated topic:

B_s^0–$\bar{B}_s^0$ Mixing & $\Delta\Gamma_s$





- Quantum mechanics: $\Rightarrow \quad |B_s(t)\rangle = a(t)|B_s^0\rangle + b(t)|\bar{B}_s^0\rangle$
  - Mass eigenstates: $\Delta M_s \equiv M_{\rm H}^{(s)} - M_{\rm L}^{(s)}, \quad \Delta\Gamma_s \equiv \Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}$
  - Time-dependent decay rates: $\Gamma(B_s^0(t) \to f), \quad \Gamma(\bar{B}_s^0(t) \to f)$
- Key feature of the B_s-meson system: $\boxed{\Delta\Gamma_s \neq 0}$
  - Expected theoretically since decades [Recent review: A. Lenz (2012)].
  - Recently established by LHCb at the $6\,\sigma$ level:

$$y_s \equiv \frac{\Delta\Gamma_s}{2\,\Gamma_s} \equiv \frac{\Gamma_{\rm L}^{(s)} - \Gamma_{\rm H}^{(s)}}{2\,\Gamma_s} = 0.088 \pm 0.014$$

$$\tau_{B_s}^{-1} \equiv \Gamma_s \equiv \frac{\Gamma_{\rm L}^{(s)} + \Gamma_{\rm H}^{(s)}}{2} = (0.6580 \pm 0.0085)\,\text{ps}^{-1}$$

B_s Branching Ratios:

- $\Delta\Gamma_s \neq 0 \Rightarrow$ *special care* has to be taken when dealing with the concept of a branching ratio ...

- How to *convert* measured "experimental" B_s branching ratios into "theoretical" B_s branching ratios?

[De Bruyn, R.F., Knegjens, Koppenburg, Merk and Tuning
Phys. Rev. **D 86** (2012) 014027 [arXiv:1204.1735 [hep-ph]]]

Experiment vs. Theory

- Untagged B_s decay rate: $\rightarrow$ *sum of two exponentials:*

$$\langle\Gamma(B_s(t)\to f)\rangle \equiv \Gamma(B^0_s(t)\to f)+\Gamma(\bar{B}^0_s(t)\to f) = R^f_{\rm H}e^{-\Gamma^{(s)}_{\rm H}t}+R^f_{\rm L}e^{-\Gamma^{(s)}_{\rm L}t}$$

$$= \left(R^f_{\rm H}+R^f_{\rm L}\right)e^{-\Gamma_s t}\left[\cosh\left(\frac{y_s\,t}{\tau_{B_s}}\right)+\mathcal{A}^f_{\Delta\Gamma}\sinh\left(\frac{y_s\,t}{\tau_{B_s}}\right)\right]$$

- "Experimental" branching ratio: [I. Dunietz, R.F. & U. Nierste (2001)]

$$\mathrm{BR}\,(B_s\to f)_{\rm exp}\equiv\frac{1}{2}\int_0^\infty\langle\Gamma(B_s(t)\to f)\rangle\,dt$$

$$=\frac{1}{2}\left[\frac{R^f_{\rm H}}{\Gamma^{(s)}_{\rm H}}+\frac{R^f_{\rm L}}{\Gamma^{(s)}_{\rm L}}\right]=\frac{\tau_{B_s}}{2}\left(R^f_{\rm H}+R^f_{\rm L}\right)\left[\frac{1+\mathcal{A}^f_{\Delta\Gamma}\,y_s}{1-y_s^2}\right] \qquad (6)$$

- "Theoretical" branching ratio: [R.F. (1999); S. Faller, R.F. & T. Mannel (2008); ...]

$$\mathrm{BR}\,(B_s\to f)_{\rm theo}\equiv\frac{\tau_{B_s}}{2}\langle\Gamma(B^0_s(t)\to f)\rangle\Big|_{t=0}=\frac{\tau_{B_s}}{2}\left(R^f_{\rm H}+R^f_{\rm L}\right) \qquad (8)$$

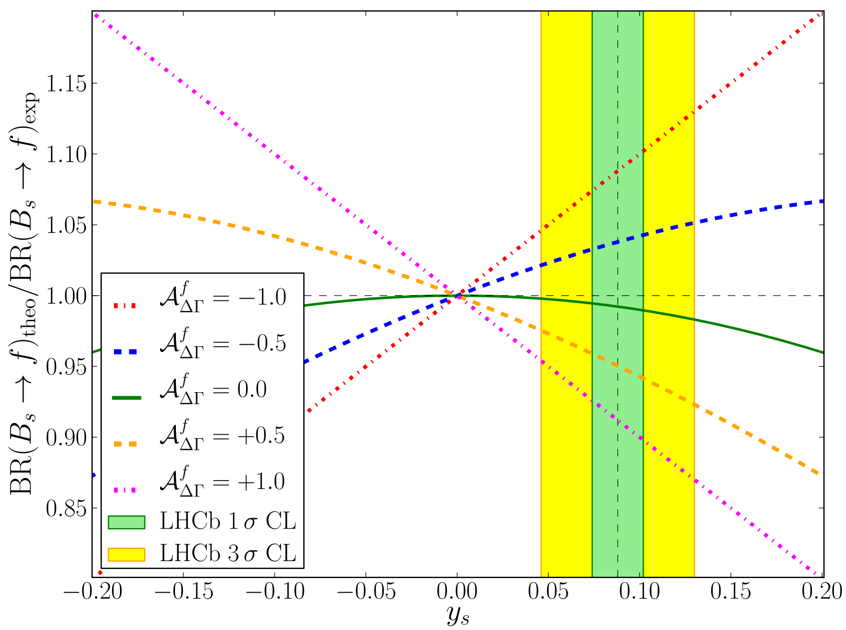
  - By considering $t=0$, the effect of B^0_s–$\bar{B}^0_s$ mixing is "switched off".
  - The advantage of this definition is that it allows a straightforward comparison with the BRs of B^0_d or B^+_u mesons by means of $SU(3)_{\rm F}$.

Conversion of B_s Decay Branching Ratios

- Relation between $\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}}$ and the measured $\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}}$:

$$\mathrm{BR}\,(B_s \to f)_{\mathrm{theo}} = \left[\frac{1-y_s^2}{1+\mathcal{A}_{\Delta\Gamma}^{f}\,y_s}\right]\mathrm{BR}\,(B_s \to f)_{\mathrm{exp}} \qquad (9)$$

- While $y_s = 0.088 \pm 0.014$ has been measured, $\mathcal{A}_{\Delta\Gamma}^{f}$ depends on the considered decay and generally involves non-perturbative parameters:



$\Rightarrow$ $\boxed{\text{differences can be as large as } \mathcal{O}(10\%) \text{ for the current value of } y_s}$

- Compilation of theoretical estimates for specific B_s decays:

$B_s \to f$	$\mathrm{BR}(B_s \to f)_{\rm exp}$	$\mathcal{A}_{\Delta\Gamma}^f(\mathrm{SM})$	$\mathrm{BR}\,(B_s \to f)_{\rm theo}\,/\mathrm{BR}\,(B_s \to f)_{\rm exp}$ From Eq. (9)	From Eq. (11)
$J/\psi f_0(980)$	$(1.29^{+0.40}_{-0.28}) \times 10^{-4}$ [18]	0.9984 ± 0.0021 [14]	0.912 ± 0.014	0.890 ± 0.082 [6]
$J/\psi K_{\rm S}$	$(3.5 \pm 0.8) \times 10^{-5}$ [7]	0.84 ± 0.17 [15]	0.924 ± 0.018	N/A
$D_s^-\pi^+$	$(3.01 \pm 0.34) \times 10^{-3}$ [9]	0 (exact)	0.992 ± 0.003	N/A
K^+K^-	$(3.5 \pm 0.7) \times 10^{-5}$ [18]	-0.972 ± 0.012 [13]	1.085 ± 0.014	1.042 ± 0.033 [19]
$D_s^+D_s^-$	$(1.04^{+0.29}_{-0.26}) \times 10^{-2}$ [18]	-0.995 ± 0.013 [16]	1.088 ± 0.014	N/A

TABLE I: Factors for converting $\mathrm{BR}\,(B_s \to f)_{\rm exp}$ (see (6)) into $\mathrm{BR}\,(B_s \to f)_{\rm theo}$ (see (8)) by means of Eq. (9) with theoretical estimates for $\mathcal{A}_{\Delta\Gamma}^f$. Whenever effective lifetime information is available, the corrections are also calculated using Eq. (11).

How can we avoid theoretical input? $\to$

- Effective B_s decay lifetimes:

$$\tau_f \equiv \frac{\int_0^\infty t\,\langle\Gamma(B_s(t)\to f)\rangle\,dt}{\int_0^\infty \langle\Gamma(B_s(t)\to f)\rangle\,dt} = \frac{\tau_{B_s}}{1-y_s^2}\left[\frac{1+2\,\mathcal{A}_{\Delta\Gamma}^f y_s + y_s^2}{1+\mathcal{A}_{\Delta\Gamma}^f y_s}\right]$$

$$\Rightarrow \boxed{\mathrm{BR}\,(B_s \to f)_{\rm theo} = \left[2-\left(1-y_s^2\right)\tau_f/\tau_{B_s}\right]\mathrm{BR}\,(B_s \to f)_{\rm exp}} \qquad (11)$$

$\to$ advocate the use of this relation for Particle Listings (PDG, HFAG)

$B_s \to VV$ Decays

- Another application is given by B_s decays into two vector mesons:
  - Examples: $B_s \to J/\psi\phi$, $B_s \to K^{*0}\bar{K}^{*0}$, $B_s \to D_s^{*+}D_s^{*-}$, ...
- Angular analysis of the vector-meson decay products has to be performed to disentangle the CP-even $(0, \parallel)$ and CP-odd $(\perp)$ states (labelled by k):

$$f^{\rm exp}_{VV,k} = \frac{\mathrm{BR}^{VV,k}_{\rm exp}}{\mathrm{BR}^{VV}_{\rm exp}}, \quad \mathrm{BR}^{VV}_{\rm exp} \equiv \sum_k \mathrm{BR}^{VV,k}_{\rm exp} \quad \Rightarrow \quad \sum_k f^{\rm exp}_{VV,k} = 1.$$

- Conversion of the "experimental" into the "theoretical" branching ratios:
  - Using *theory info* about $\mathcal{A}^{VV,k}_{\Delta\Gamma} = -\eta_k\sqrt{1-C^2_{VV,k}}\cos(\phi_s+\Delta\phi_{VV,k})$:

$$\mathrm{BR}^{VV}_{\rm theo} = (1-y_s^2)\left[\sum_{k=0,\parallel,\perp} \frac{f^{\rm exp}_{VV,k}}{1+y_s\mathcal{A}^{VV,k}_{\Delta\Gamma}}\right]\mathrm{BR}^{VV}_{\rm exp}$$

  - Using *effective lifetime measurements:*

$$\mathrm{BR}^{VV}_{\rm theo} = \mathrm{BR}^{VV}_{\rm exp} \sum_{k=0,\parallel,\perp}\left[2-(1-y_s^2)\frac{\tau_k^{VV}}{\tau_{B_s}}\right] f^{\rm exp}_{VV,k}$$

[See also LHCb, arXiv:1111.4183; S. Descotes-Genon, J. Matias & J. Virto (2011)]

$\boxed{\text{Key } B_s \text{ Decay: } B_s \to \mu^+\mu^-}$

- Upper bounds on the branching ratio are becoming stronger and stronger, thereby approaching the SM prediction ...
- What is the impact of $\Delta\Gamma_s \neq 0$ on these analyses?

$\to$ *opens actually a new window for New Physics*

[De Bruyn, R.F., Knegjens, Koppenburg, Merk, Pellegrino and Tuning
Phys. Rev. Lett. **109** (2012) 041801 [arXiv:1204.1737 [hep-ph]]]

The General $B_s \to \mu^+\mu^-$ Amplitudes

- Low-energy effective Hamiltonian for $\bar{B}^0_s \to \mu^+\mu^-$: $\boxed{\text{SM} \oplus \text{NP}}$

$$\mathcal{H}_{\text{eff}} = -\frac{G_{\text{F}}}{\sqrt{2}\pi} V_{ts}^* V_{tb} \alpha \left[C_{10} O_{10} + C_S O_S + C_P O_P + C'_{10} O'_{10} + C'_S O'_S + C'_P O'_P \right]$$

[G_{F}: Fermi's constant, $V_{qq'}$: CKM matrix elements, α: QED fine structure constant]

- Four-fermion operators, with $P_{L,R} \equiv (1 \mp \gamma_5)/2$ and b-quark mass m_b:

$$\begin{array}{rclrcl} O_{10} & = & (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5\ell), & O'_{10} & = & (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma_5\ell) \\ O_S & = & m_b(\bar{s}P_R b)(\bar{\ell}\ell), & O'_S & = & m_b(\bar{s}P_L b)(\bar{\ell}\ell) \\ O_P & = & m_b(\bar{s}P_R b)(\bar{\ell}\gamma_5\ell), & O'_P & = & m_b(\bar{s}P_L b)(\bar{\ell}\gamma_5\ell) \end{array}$$

[Only operators with non-vanishing $\bar{B}^0_s \to \mu^+\mu^-$ matrix elements are included]

- The Wilson coefficients C_i, C'_i encode the short-distance physics:
  - SM case: only $C_{10} \neq 0$, and is given by the *real* coefficient C_{10}^{SM}.
  - *Outstanding feature of* $\bar{B}^0_s \to \mu^+\mu^-$: sensitivity to (pseudo-)scalar lepton densities $\to O_{(P)S}$, $O'_{(P)S}$; WCs are still largely unconstrained.

[W. Altmannshofer, P. Paradisi & D. Straub (2011) $\to$ model-independent NP analysis]

$\rightarrow$ *convenient to go to the rest frame of the decaying $\bar{B}^0_s$ meson:*

- Distinguish between the $\mu^+_{\rm L}\mu^-_{\rm L}$ and $\mu^+_{\rm R}\mu^-_{\rm R}$ helicity configurations:

$$|(\mu^+_{\rm L}\mu^-_{\rm L})_{\rm CP}\rangle \equiv (\mathcal{CP})|\mu^+_{\rm L}\mu^-_{\rm L}\rangle = e^{i\phi_{\rm CP}(\mu\mu)}|\mu^+_{\rm R}\mu^-_{\rm R}\rangle$$

[$e^{i\phi_{\rm CP}(\mu\mu)}$ is a convention-dependent phase factor $\rightarrow$ cancels in observables]

- General expression for the decay amplitude [$\eta_{\rm L}=+1$, $\eta_{\rm R}=-1$]:

$$A(\bar{B}^0_s\to\mu^+_\lambda\mu^-_\lambda) = \langle\mu^-_\lambda\mu^+_\lambda|\mathcal{H}_{\rm eff}|\bar{B}^0_s\rangle = -\frac{G_{\rm F}}{\sqrt{2}\pi}V^*_{ts}V_{tb}\alpha$$

$$\times f_{B_s}M_{B_s}m_\mu C_{10}^{\rm SM}e^{i\phi_{\rm CP}(\mu\mu)(1-\eta_\lambda)/2}\left[\eta_\lambda P+S\right]$$

- Combination of Wilson coefficient functions [CP-violating phases $\varphi_{P,S}$]:

$$P\equiv|P|e^{i\varphi_P}\equiv\frac{C_{10}-C'_{10}}{C_{10}^{\rm SM}}+\frac{M^2_{B_s}}{2m_\mu}\left(\frac{m_b}{m_b+m_s}\right)\left(\frac{C_P-C'_P}{C_{10}^{\rm SM}}\right)\xrightarrow{\rm SM}1$$

$$S\equiv|S|e^{i\varphi_S}\equiv\sqrt{1-4\frac{m^2_\mu}{M^2_{B_s}}}\frac{M^2_{B_s}}{2m_\mu}\left(\frac{m_b}{m_b+m_s}\right)\left(\frac{C_S-C'_S}{C_{10}^{\rm SM}}\right)\xrightarrow{\rm SM}0$$

[f_{B_s}: B_s decay constant, M_{B_s}: B_s mass, m_μ: muon mass, m_s: strange-quark mass]

The $B_s \to \mu^+\mu^-$ Observables

- Key quantity for calculating the CP asymmetries and the untagged rate:

$$\xi_\lambda \equiv -e^{-i\phi_s}\left[e^{i\phi_{\rm CP}(B_s)}\frac{A(\bar{B}^0_s \to \mu^+_\lambda\mu^-_\lambda)}{A(B^0_s \to \mu^+_\lambda\mu^-_\lambda)}\right]$$

$$\Rightarrow A(B^0_s \to \mu^+_\lambda\mu^-_\lambda) = \langle \mu^-_\lambda\mu^+_\lambda|\mathcal{H}^\dagger_{\rm eff}|B^0_s\rangle \text{ is also needed ...}$$

- Using $(\mathcal{CP})^\dagger(\mathcal{CP}) = \hat{1}$ and $(\mathcal{CP})|B^0_s\rangle = e^{i\phi_{\rm CP}(B_s)}|\bar{B}^0_s\rangle$ yields:

$$A(B^0_s \to \mu^+_\lambda\mu^-_\lambda) = -\frac{G_{\rm F}}{\sqrt{2}\pi}V_{ts}V^*_{tb}\alpha f_{B_s}M_{B_s}m_\mu C^{\rm SM}_{10}$$

$$\times\, e^{i[\phi_{\rm CP}(B_s)+\phi_{\rm CP}(\mu\mu)(1-\eta_\lambda)/2]}\left[-\eta_\lambda P^* + S^*\right]$$

- The convention-dependent phases cancel in ξ_λ $[\eta_{\rm L} = +1,\ \eta_{\rm R} = -1]$:

$$\xi_\lambda = -\left[\frac{+\eta_\lambda P + S}{-\eta_\lambda P^* + S^*}\right] \quad \Rightarrow \quad \boxed{\xi_{\rm L}\xi^*_{\rm R} = \xi_{\rm R}\xi^*_{\rm L} = 1}$$

CP Asymmetries:

- Time-dependent rate asymmetry: $\rightarrow$ *requires tagging of* B_s^0 *and* $\bar{B}_s^0$*:*

$$\frac{\Gamma(B_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)-\Gamma(\bar{B}_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)}{\Gamma(B_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)+\Gamma(\bar{B}_s^0(t)\to\mu_\lambda^+\mu_\lambda^-)}=\frac{C_\lambda\cos(\Delta M_s t)+S_\lambda\sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B_s})+\mathcal{A}_{\Delta\Gamma}^\lambda\sinh(y_s t/\tau_{B_s})}$$

- Individual observables: $\rightarrow$ *theoretically clean (no dependence on* f_{B_s}*):*

$$C_\lambda\equiv\frac{1-|\xi_\lambda|^2}{1+|\xi_\lambda|^2}=-\eta_\lambda\left[\frac{2|PS|\cos(\varphi_P-\varphi_S)}{|P|^2+|S|^2}\right]\quad\stackrel{\rm SM}{\longrightarrow}\quad 0$$

$$S_\lambda\equiv\frac{2\,\mathsf{Im}\,\xi_\lambda}{1+|\xi_\lambda|^2}=\frac{|P|^2\sin2\varphi_P-|S|^2\sin2\varphi_S}{|P|^2+|S|^2}\quad\stackrel{\rm SM}{\longrightarrow}\quad 0$$

$$\mathcal{A}_{\Delta\Gamma}^\lambda\equiv\frac{2\,\mathsf{Re}\,\xi_\lambda}{1+|\xi_\lambda|^2}=\frac{|P|^2\cos2\varphi_P-|S|^2\cos2\varphi_S}{|P|^2+|S|^2}\quad\stackrel{\rm SM}{\longrightarrow}\quad 1$$

- Note: $\mathcal{S}_{\rm CP}\equiv S_\lambda$, $\mathcal{A}_{\Delta\Gamma}\equiv\mathcal{A}_{\Delta\Gamma}^\lambda$ are *independent* of the muon helicity λ.

- Difficult to measure the muon helicity: $\Rightarrow$ *consider the following rates:*

$$\Gamma(\overset{(-)}{B}{}^0_s(t) \to \mu^+\mu^-) \equiv \sum_{\lambda=\mathrm{L,R}} \Gamma(\overset{(-)}{B}{}^0_s(t) \to \mu^+_\lambda\mu^-_\lambda)$$

- Corresponding CP-violating rate asymmetry: $\rightarrow$ $C_\lambda \propto \eta_\lambda$ *terms cancel:*

$$\frac{\Gamma(B^0_s(t) \to \mu^+\mu^-) - \Gamma(\bar{B}^0_s(t) \to \mu^+\mu^-)}{\Gamma(B^0_s(t) \to \mu^+\mu^-) + \Gamma(\bar{B}^0_s(t) \to \mu^+\mu^-)} = \frac{\mathcal{S}_{\mathrm{CP}} \sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B_s}) + \mathcal{A}_{\Delta\Gamma} \sinh(y_s t/\tau_{B_s})}$$

- Practical comments:
  - It would be most interesting to measure this CP asymmetry since a non-zero value immediately signaled CP-violating NP phases.
    [See, e.g., Buras & Girrbach ('12) for Minimal $U(2)^3$ models [Barbieri *et al.*])]
  - Unfortunately, this is challenging in view of the tiny branching ratio and as B^0_s, $\bar{B}^0_s$ *tagging* and time information are required.

[Previous studies of CP asymmetries of $B^0_{s,d} \to \ell^+\ell^-$ (assuming $\Delta\Gamma_s = 0$):
Huang and Liao (2002); Dedes and Pilaftsis (2002), Chankowski *et al.* (2005)]

Untagged Rate and Branching Ratio:

- The first measurement concerns the "experimental" branching ratio:

$$\mathrm{BR}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{exp}} \equiv \frac{1}{2}\int_0^\infty \langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle\, dt$$

$\to$ *time-integrated untagged rate*, involving

$$\langle \Gamma(B_s(t) \to \mu^+\mu^-)\rangle \equiv \Gamma(B^0_s(t) \to \mu^+\mu^-) + \Gamma(\bar{B}^0_s(t) \to \mu^+\mu^-)$$

$$\propto e^{-t/\tau_{B_s}}\left[\cosh(y_s t/\tau_{B_s}) + \mathcal{A}_{\Delta\Gamma}\sinh(y_s t/\tau_{B_s})\right]$$

- Conversion into the "theoretical" branching ratio: $\to$ *NP searches:*

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = \left[\frac{1 - y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}\, y_s}\right]\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{exp}}$$

- $\mathcal{A}_{\Delta\Gamma}$ depends on NP and is hence unknown: $\in [-1, +1] \Rightarrow$ *two options:*
  - Add extra error: $\Delta\mathrm{BR}(B_s \to \mu^+\mu^-)|_{y_s} = \pm y_s \mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{exp}}$.
  - $\mathcal{A}^{\mathrm{SM}}_{\Delta\Gamma} = 1$ gives *new SM reference value* [rescale $\mathrm{BR}_{\mathrm{SM}}$ by $1/(1-y_s)$]:

$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}}|_{y_s} = (3.54 \pm 0.30) \times 10^{-9}.$$

Effective $B_s \to \mu^+\mu^-$ Lifetime:

$\diamond$ *Collecting more and more data* $\oplus$ *include decay time information* $\Rightarrow$

- Access to the effective $B_s \to \mu^+\mu^-$ lifetime:

$$\tau_{\mu^+\mu^-} \equiv \frac{\int_0^\infty t\,\langle\Gamma(B_s(t)\to\mu^+\mu^-)\rangle\,dt}{\int_0^\infty \langle\Gamma(B_s(t)\to\mu^+\mu^-)\rangle\,dt}$$

- $\mathcal{A}_{\Delta\Gamma}$ can then be extracted: $\mathcal{A}_{\Delta\Gamma} = \frac{1}{y_s}\left[\frac{(1-y_s^2)\tau_{\mu^+\mu^-} - (1+y_s^2)\tau_{B_s}}{2\tau_{B_s} - (1-y_s^2)\tau_{\mu^+\mu^-}}\right]$

- Finally, extraction of the "theoretical" BR: $\to$ *clean expression:*

$$\mathrm{BR}\left(B_s\to\mu^+\mu^-\right) = \underbrace{\left[2-\left(1-y_s^2\right)\frac{\tau_{\mu^+\mu^-}}{\tau_{B_s}}\right]\mathrm{BR}\left(B_s\to\mu^+\mu^-\right)_{\mathrm{exp}}}_{\to\ \text{only measurable quantities}}$$

  – It is *crucial* that $\mathcal{A}_{\Delta\Gamma}$ does *not* depend on the muon helicity.

  – *Important new measurement for the high-luminosity LHC upgrade:*
  $\Rightarrow$ precision of 5% or better appears feasible for $\tau_{\mu^+\mu^-}$ ...

Constraints on New Physics

- Information from the $B_s \to \mu^+\mu^-$ branching ratio:

$$R \equiv \frac{\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm exp}}{\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm SM}} = \left[\frac{1+\mathcal{A}_{\Delta\Gamma} y_s}{1-y_s^2}\right]\left(|P|^2 + |S|^2\right)$$

$$= \left[\frac{1+y_s \cos 2\varphi_P}{1-y_s^2}\right]|P|^2 + \left[\frac{1-y_s \cos 2\varphi_S}{1-y_s^2}\right]|S|^2 \stackrel{\rm LHC}{<} 1.3$$

  – Unknown CP-violating phases φ_P, $\varphi_S \Rightarrow |P|, |S| \leq \sqrt{(1+y_s)R} < 1.2$

  – R does not allow a separation of the P and S contributions:

    $\Rightarrow$ *large NP could be present, even if the BR is close to the SM value.*

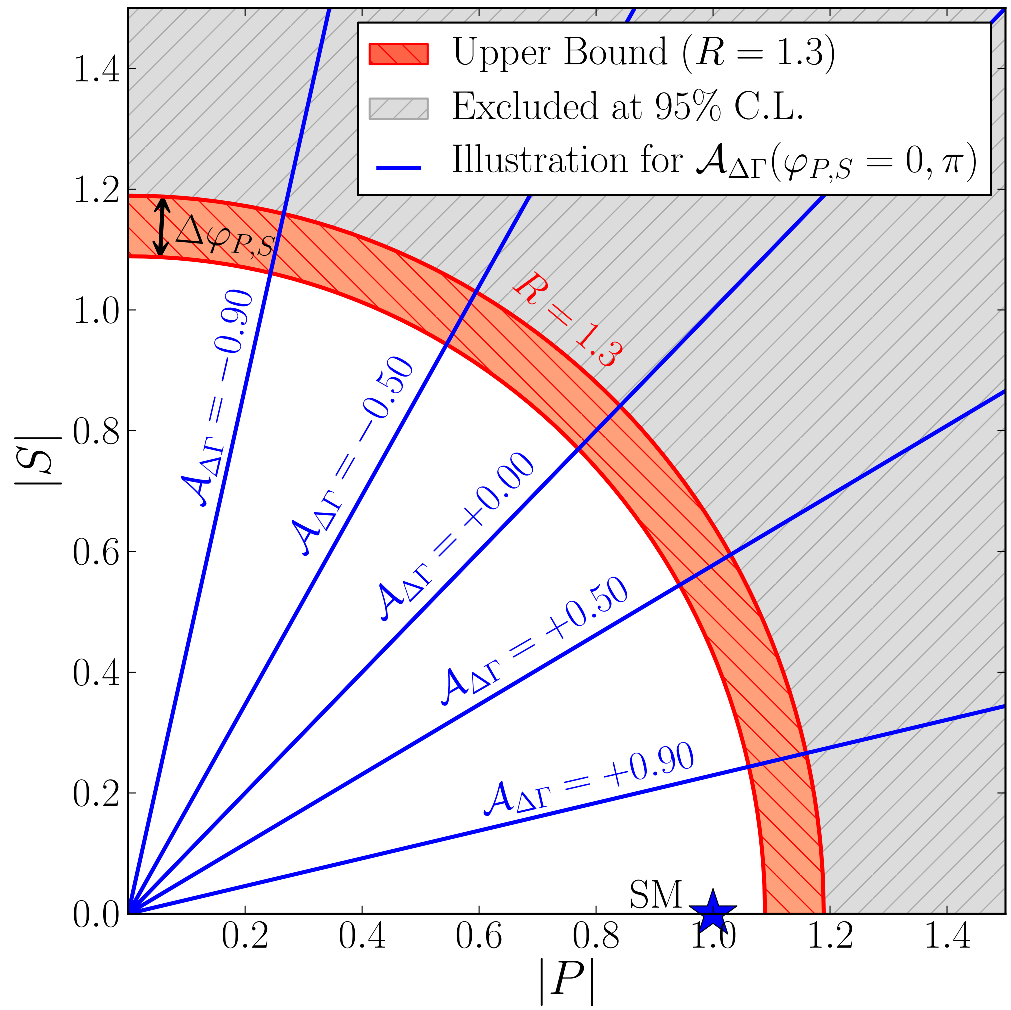
- Further information from the measurement of $\tau_{\mu^+\mu^-}$ yielding $\mathcal{A}_{\Delta\Gamma}$:

$$|S| = |P|\sqrt{\frac{\cos 2\varphi_P - \mathcal{A}_{\Delta\Gamma}}{\cos 2\varphi_S + \mathcal{A}_{\Delta\Gamma}}}$$

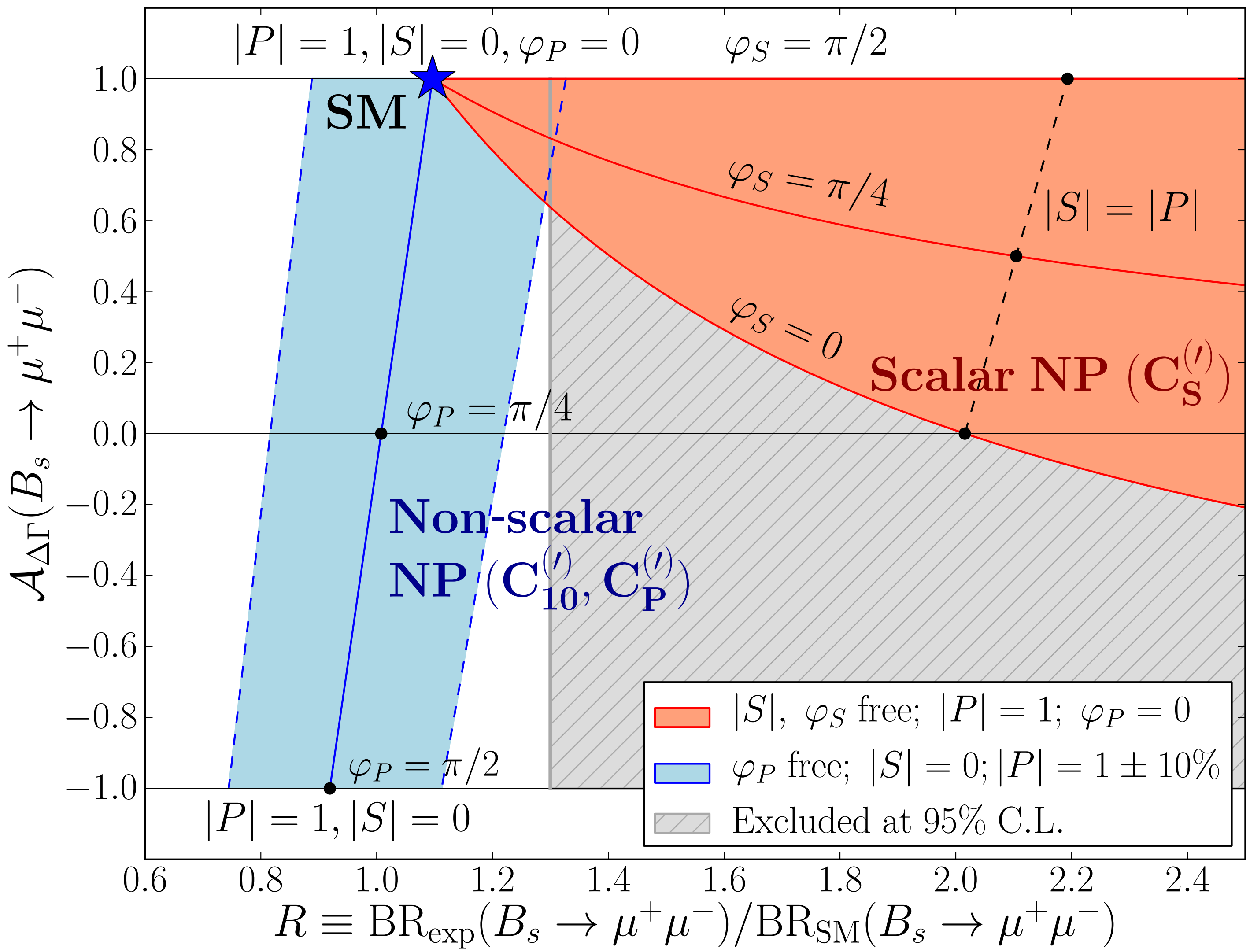
$\Rightarrow$ $\boxed{\text{offers a new window for New Physics in } B_s \to \mu^+\mu^-}$

How does the situation in NP parameter space look like?

- Current constraints in the $|P|$–$|S|$ plane and illustration of those following from a future measurement of the $B_s \to \mu^+\mu^-$ lifetime yielding $\mathcal{A}_{\Delta\Gamma}$:



- Illustration of the allowed regions in the R–$\mathcal{A}_{\Delta\Gamma}$ plane for scenarios with scalar or non-scalar NP contributions:



- Authors have started to include the effect of $\Delta\Gamma_s$ in analyses of the constraints on NP that are implied by BR$(B_s \to \mu^+\mu^-)_{\rm exp}$:

O. Buchmueller, R. Cavanaugh, M. Citron, A. De Roeck, M. J. Dolan, J. R. Ellis, H. Flächer and S. Heinemeyer *et al.*, "The CMSSM and NUHM1 in Light of 7 TeV LHC, $B_s \to \mu^+\mu^-$ and XENON100 Data," arXiv:1207.7315 [hep-ph]

T. Hurth and F. Mahmoudi, "The Minimal Flavour Violation benchmark in view of the latest LHCb data," arXiv:1207.0688 [hep-ph]

A. J. Buras and J. Girrbach, "On the Correlations between Flavour Observables in Minimal $U(2)^3$ Models," arXiv:1206.3878 [hep-ph]

W. Altmannshofer and D. M. Straub, "Cornering New Physics in $b \to s$ Transitions," arXiv:1206.0273 [hep-ph]

D. Becirevic, N. Kosnik, F. Mescia and E. Schneider, "Complementarity of the constraints on New Physics from $B_s \to \mu^+\mu^-$ and from $B \to K\ell^+\ell^-$ decays," arXiv:1205.5811 [hep-ph]

F. Mahmoudi, S. Neshatpour and J. Orloff, "Supersymmetric constraints from $B_s \to \mu^+\mu^-$ and $B \to K^*\mu^+\mu^-$ observables," arXiv:1205.1845 [hep-ph]

T. Li, D. V. Nanopoulos, W. Wang, X. -C. Wang and Z. -H. Xiong, "Rare B decays in the flip SU(5) Model," JHEP **1207** (2012) 190 arXiv:1204.5326 [hep-ph]

...

Conclusions

Exciting Times for Leptonic Rare B Decays

- BR($B_d \to \mu^+\mu^-$): experimental upper bound $\sim 8 \times$ BR$(B_d \to \mu^+\mu^-)_{\rm SM}$.

- BR($B_s \to \mu^+\mu^-$): experimental upper bound ...

... is moving closer and closer to the SM prediction:

$\Rightarrow$ will we see a signal soon?

... or will we go below the SM expectation?

- Recent news on a – seemingly – unrelated topic:

LHCb has established $\Delta\Gamma_s \neq 0$ at the $6\,\sigma$ level $\Rightarrow$

  – Care has to be taken when dealing with B_s decay branching ratios.
  – "Experimental" vs. "theoretical" branching ratios ...

$\Rightarrow$ what is the impact of $\Delta\Gamma_s$ on NP searches with $B_s \to \mu^+\mu^-$?

$\rightarrow$ the muon helicity of $B_s \to \mu^+\mu^-$ has *not* to be measured:

- The theoretical $B_s \to \mu^+\mu^-$ SM branching ratio has to be rescaled by $1/(1-y_s)$ for the comparison with the experimental branching ratio:

$\Rightarrow$ *new SM reference:* $\boxed{\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm SM}|_{y_s} = (3.54 \pm 0.30) \times 10^{-9}}$

- $B_s \to \mu^+\mu^-$ is a sensitive probe for physics beyond the SM:
  - y_s can be *included* in the constrains for NP from $\mathrm{BR}(B_s \to \mu^+\mu^-)_{\rm exp}$.

- The effective lifetime $\tau_{\mu^+\mu^-}$ offers a new observable (yielding $\mathcal{A}_{\Delta\Gamma}$):
  - Allows the extraction of the "theoretical" $B_s \to \mu^+\mu^-$ branching ratio.
  - New *theoretically clean* observable to search for NP: $\mathcal{A}_{\Delta\Gamma}^{\rm SM} = +1$
    - $*$ In contrast to the BR no dependence on the B_s-decay constant f_{B_s}.
    - $*$ May reveal NP effects even if the BR is close to the SM prediction: still largely unconstrained (pseudo-)scalar operators $O_{(P)S}$, $O'_{(P)S}$.

$\Rightarrow$ $\boxed{\textit{exciting study the LHC upgrade physics programme!}}$