b and c mass determination

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HPQCD collaboration

CKM12,
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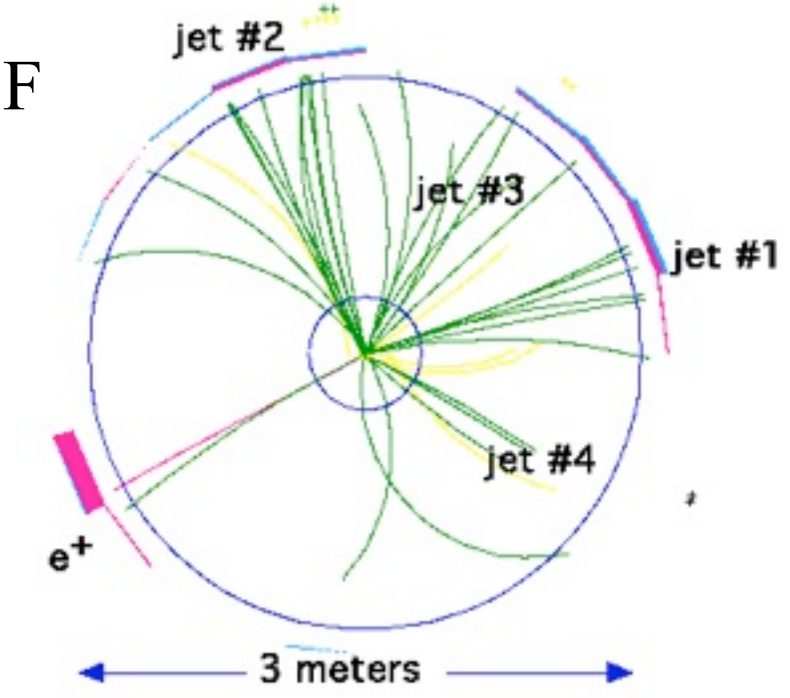
Quark masses are fundamental parameters of the SM but cannot be directly determined from experiment.

CDF

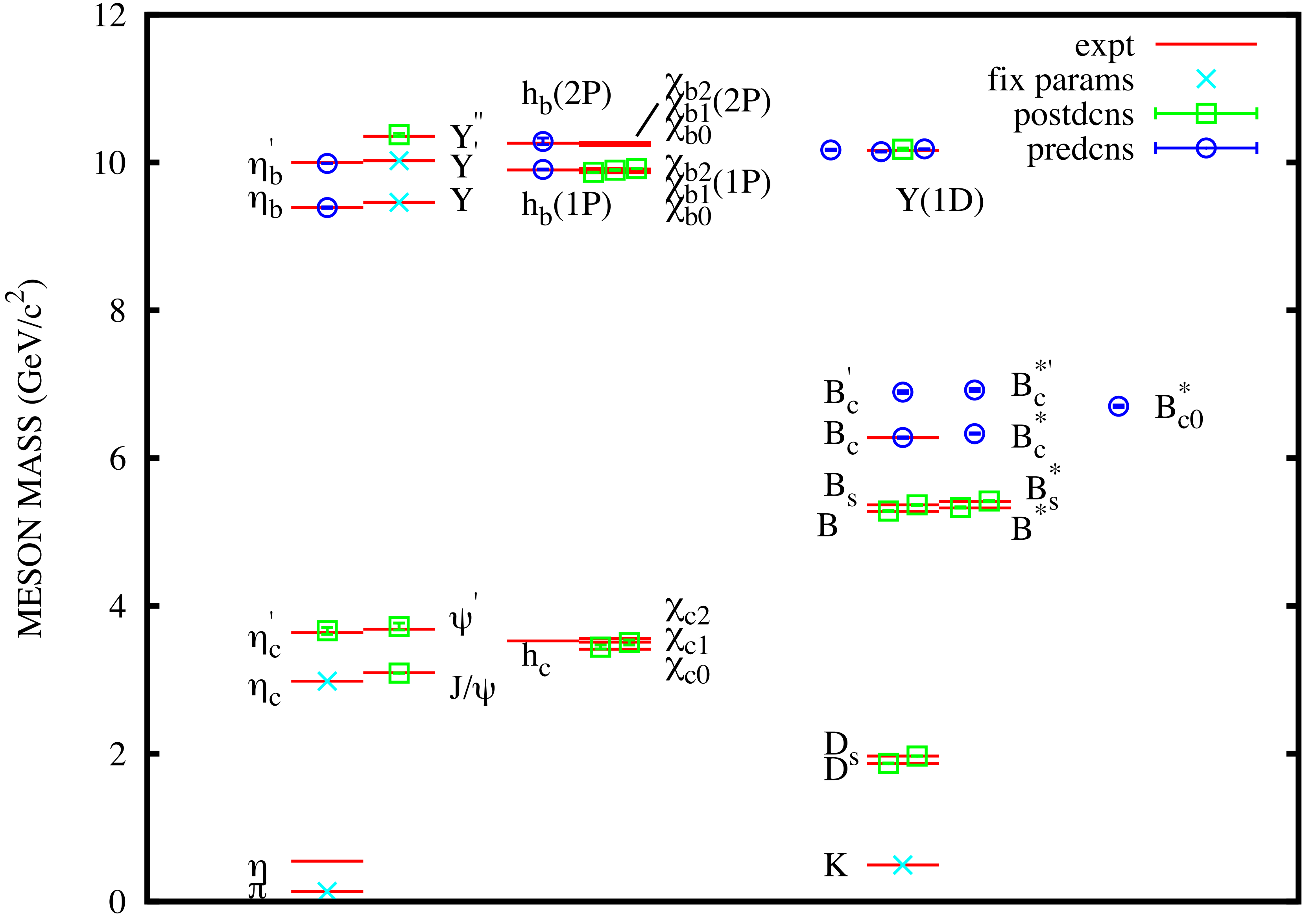


Well-defined masses are scheme and scale-dependent.
Convention to use $\overline{MS}$

Masses are then input to theoretical expressions for SM cross-sections e.g. $H \to b\overline{b}$

Comparison of accurate masses from multiple approaches is a strong test of QCD. m_b and m_c can be accurately determined from continuum methods and lattice QCD.

Lattice QCD works directly with the QCD Lagrangian. Can tune bare mass parameters very accurately using experimentally very well-determined hadron masses.



R. Dowdall et al, HPQCD, 1207.5149

Conversion of lattice quark masses to $\overline{MS}$ scheme

- Direct methods: Determine $m_{q,latt}$ in lattice QCD.

$$m_{\overline{MS}}(\mu) = Z(\mu a) m_{latt}$$

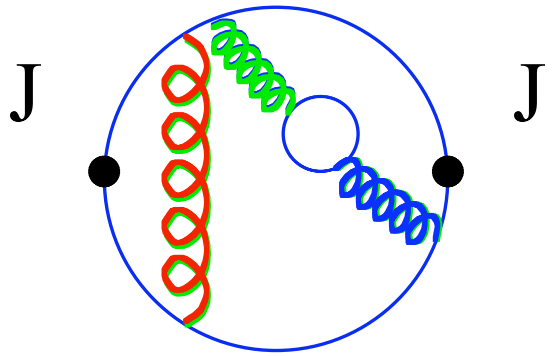
Calculate Z in lattice QCD pert. th. or use ‘nonpert’ lattice matching.

Error dominated by that of Z and continuum extrapolation.

Note: Z cancels in mass ratios.

- Indirect methods: (after tuning m_{latt}) match a uv-finite quantity calculated in lattice QCD to continuum pert. th. in terms of $\overline{MS}$ quark mass

e.g. Current-current correlator method for heavy quarks



`HPQCD + Chetyrkin et al, 0805.2999`

Issues with handling ‘heavy’ quarks on the lattice:

$$L_q = \overline{\psi}(\not{D} + m)\psi \to \overline{\psi}(\gamma\cdot\Delta + ma)\psi$$

Δ is a finite difference on the lattice - leads to discretisation errors. What sets the scale for these?

For light hadrons the scale is Λ_{QCD}

For heavy hadrons the scale can be m_Q

$$\underset{\uparrow}{E} = E_{a=0}(1 \underset{\uparrow}{+} A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

hadron energy assuming O(m_Qa) improved

$m_c a \approx 0.4, m_b a \approx 2$ for $a \approx 0.1\mathrm{f}m$

➡ can use improved light quark action for c on fine lattices. Less clear for b - non rel. actions have $(\Lambda a)^n$ errors

➡ best approach to c and b not necessarily same

Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quark method:

- meson has $E(\vec{p}=0) = M$
- PCAC relation (if enough chiral symmetry) gives $Z_A = 1$
- same action as for u, d, s, so cancellation in ratios

Relativistic approaches in use (for mass determination) :

- Highly improved staggered quarks (HISQ) HPQCD

  $\alpha_s(am)^2, (am)^4$ + small taste-changing

- Twisted mass ETM

  $(am)^2$

- clover/smeared clover Wupp-Reg

  $\approx (am)^2 \quad Z$

Use various lattice QCD gluon configs inc. u/d, u/d/s and NOW u/d/s/c sea quarks.

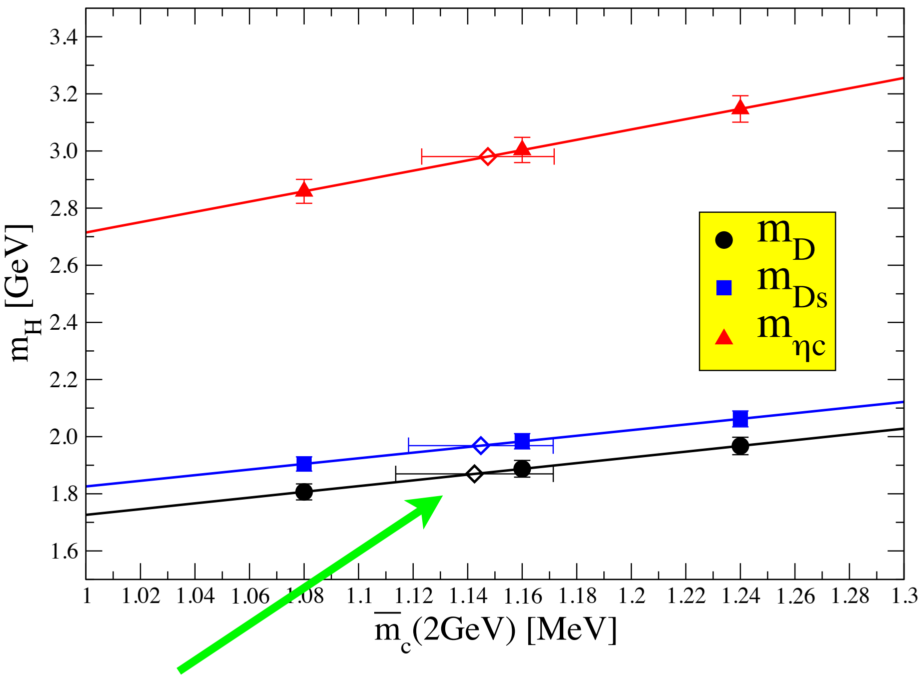
Direct determination of m_c

Blossier et al, ETM,1010.3659

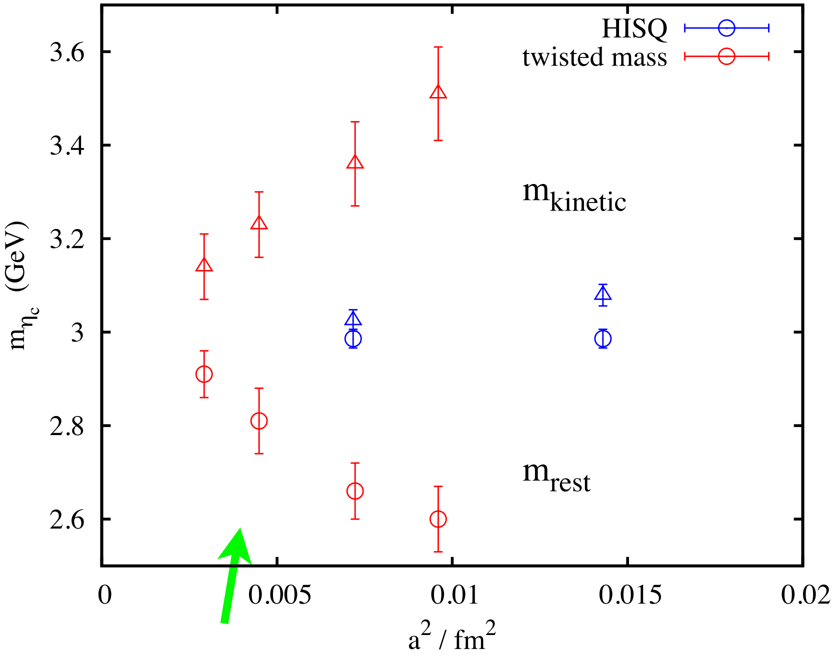
Fix lattice m_c from meson mass, checking D, D_s, η_c

$$m_{\overline{MS}}(\mu) = Z(\mu a) m_{latt}$$

Z from RI-MOM method - fix to MOM nonpert. on lattice and then match to $\overline{MS}$ through α_s^3 - error 2%



final m_c



test disc. errors from meson dispn relation

Becirevic+Sanfilippo, 1206.1445

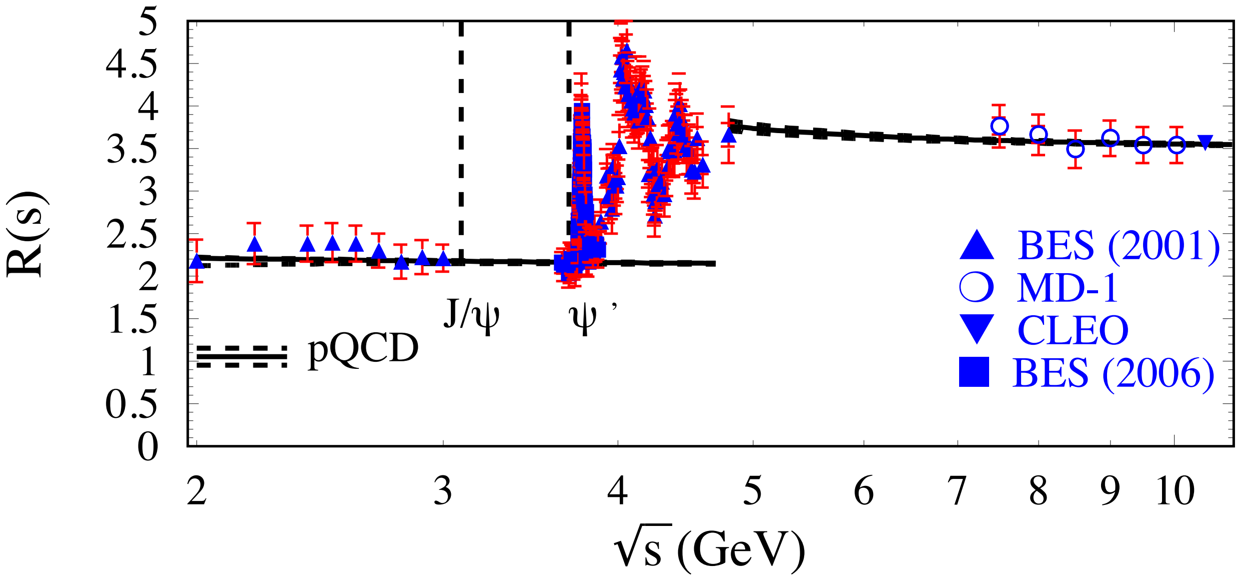
Donald et al, HPQCD, 1208.2855

Current-current correlator method for m_c

Continuum: extract charm piece of:

`e.g. Kuhn et al, hep-ph/0702103`

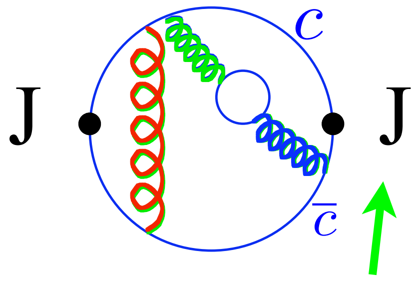
$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/(3s)}$$ from experiment,



relate to

$$\left(\frac{d}{dq^2}\right)^n \Pi_c(q^2)|_{q^2=0}$$

$$\Pi_c(q^2) = \frac{3}{16\pi^2} e_c^2 \sum_{n\geq 0} C_n \left(\frac{q^2}{4(\overline{m}_c(\mu))^2}\right)^n$$



=V here

with C_n a power series in $\alpha_s(\mu)$, known through α_s^3 for some n

Current-current correlator method for lattice m$_c$

HPQCD + Chetyrkin et al, 0805.2999

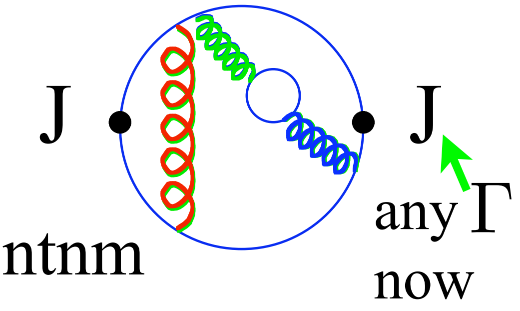
- Fix m$_q$ to m_c in correlators by getting m_{η_c} correct.
- Time moments of correlators are equiv. to contnm quantities used. Simplify by ratio to tree level ('free') .

$$G(t) = a^6 \sum_{\vec{x}} (am_c)^2 < 0|j_5(\vec{x},t)j_5(0,0)|0 >$$

$$G_n = \sum_t (t/a)^n G(t)$$

$$R_{n,latt} = G_4/G_4^{(0)} \qquad n = 4$$

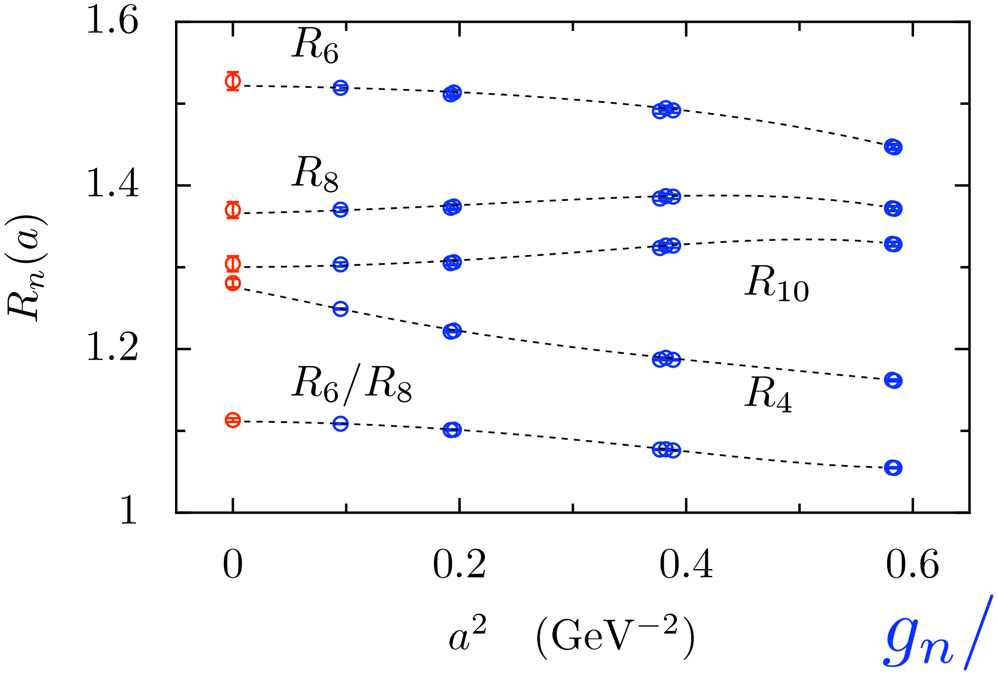
$$= \frac{am_{\eta_c}}{2am_c}(G_n/G_n^{(0)})^{1/(n-4)} \qquad n = 6, 8, 10 \ldots$$



contnm

$$\left(\frac{d}{dq^2}\right)^n \Pi_c(q^2)|_{q^2=0}$$

- extrapolate to a=0 (and physical sea quark masses).

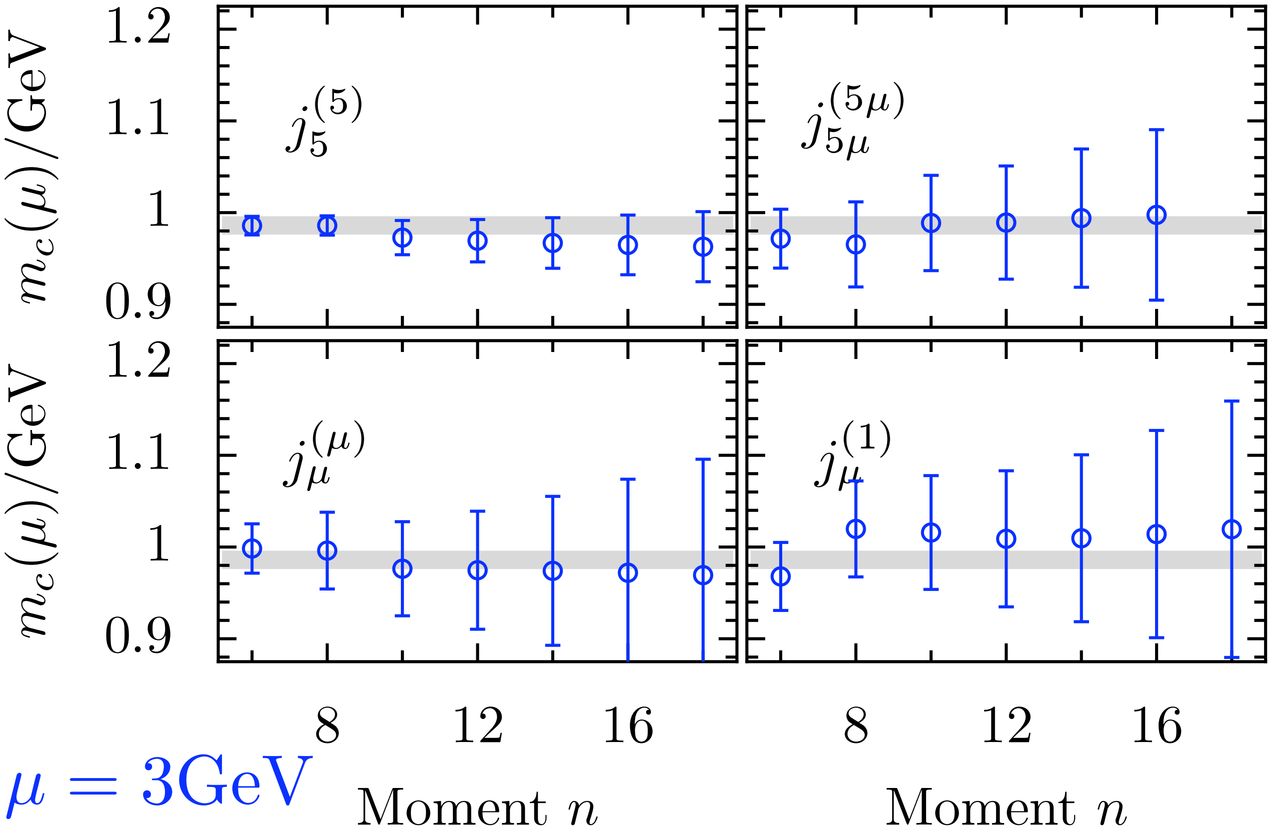


$$R_{n,cont} = g_4/g_4^0 \quad n = 4$$

$$= \frac{m_{\eta_c}}{2\overline{m}_c(\mu)} g_n/g_n^0$$

$$n = 6, 8, 10 \ldots$$

$$g_n/g_n^0 = 1 + \sum_i c_i(\mu/\overline{m}(\mu))\alpha_{\overline{MS}}(\mu)^i$$



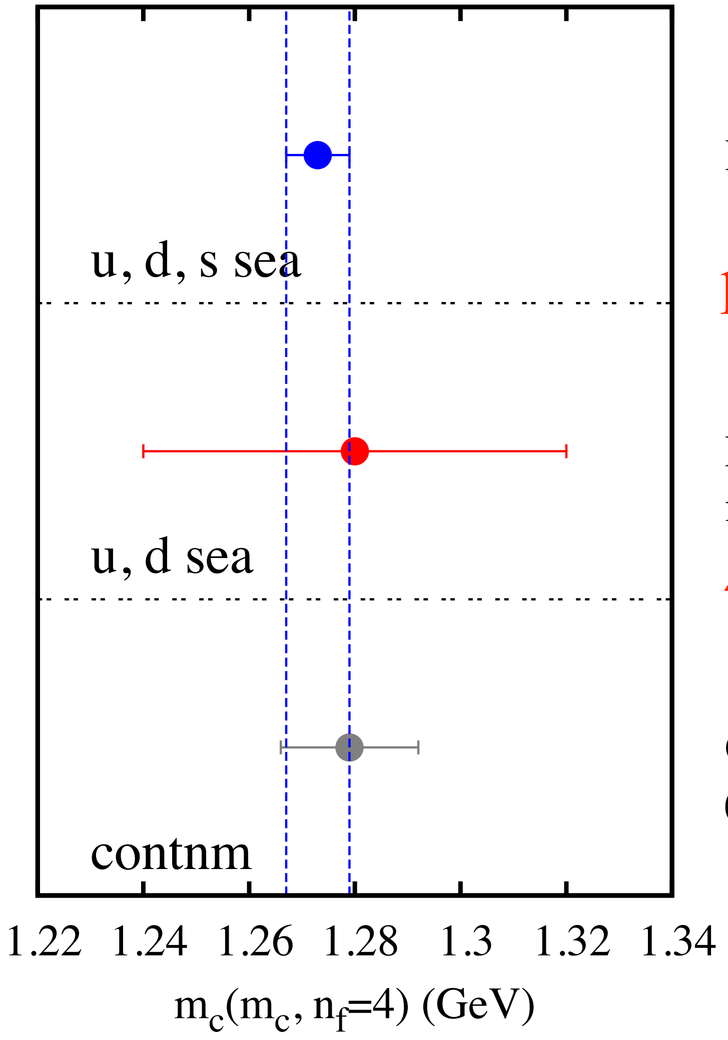
$\mu = 3\text{GeV}$

extract m_c from ratio to m_{η_c} Different j agree, but pseudoscalar best. Dependence on $m_{u,d}$ tiny.

Can also determine α_s

Results for m_c

dominant error

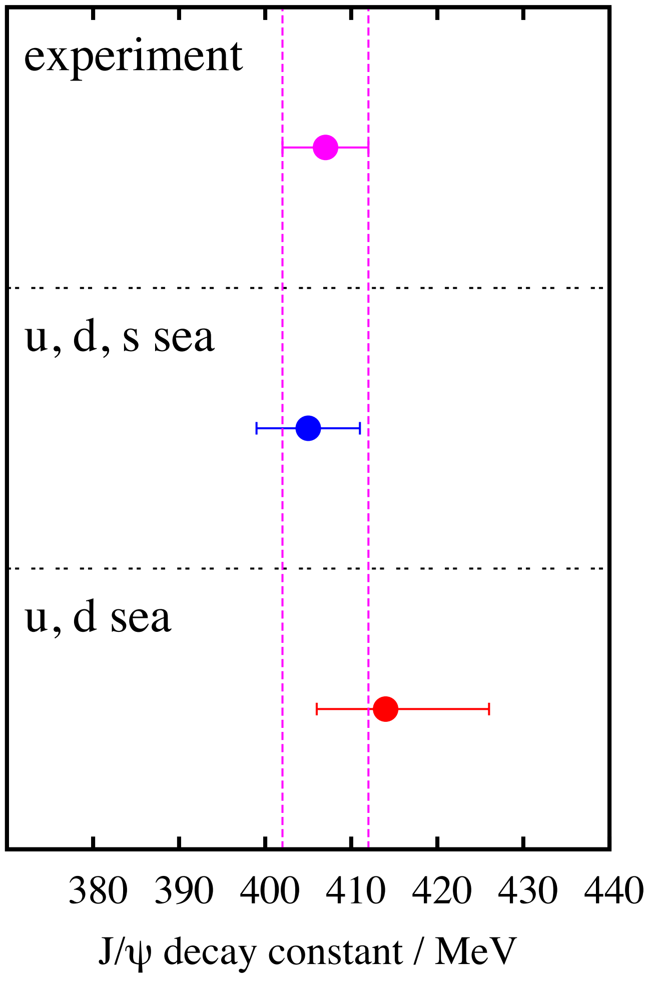


HPQCD HISQ
1004.4285
pert.th.

ETMC 1010.3659
$n_f=2$
Z, a extrap.

Chetyrkin et al
0907.2110
expt. R

Lattice QCD: Obtain other charm physics at the same time ..



Particle Data Grou
average

HISQ
this paper

`Donald et al,`
`HPQCD,`
`1208.2855`

Twisted mass
1206.1445

1% errors possible

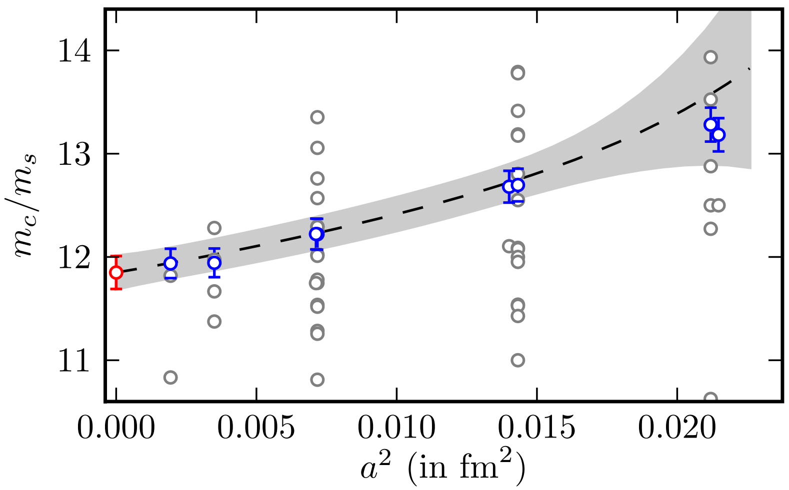
In progress: ETM results from current-current correlators1111.5252

m_c/m_s

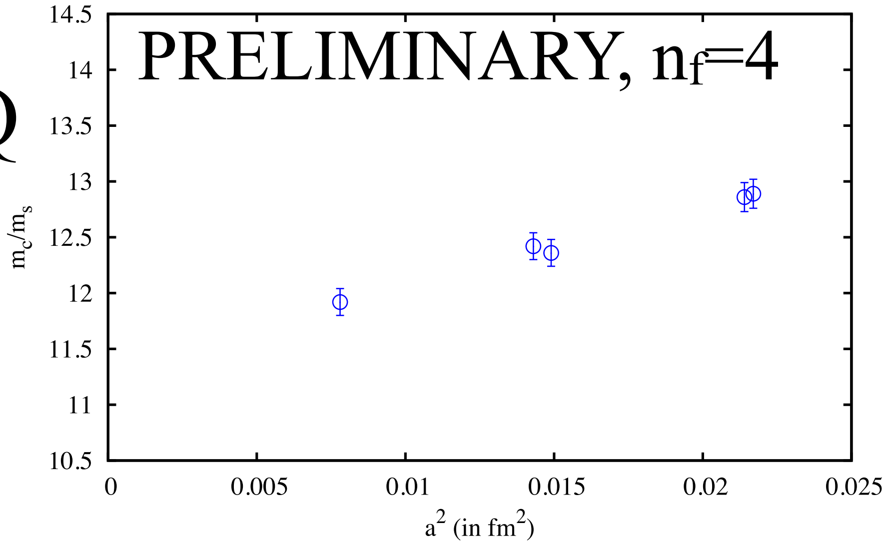
Mass ratio can be obtained directly from lattice QCD if same quark formalism is used for both quarks. Ratio is at same scale and for same n_f.

$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

Not possible with any other method ...



HISQ

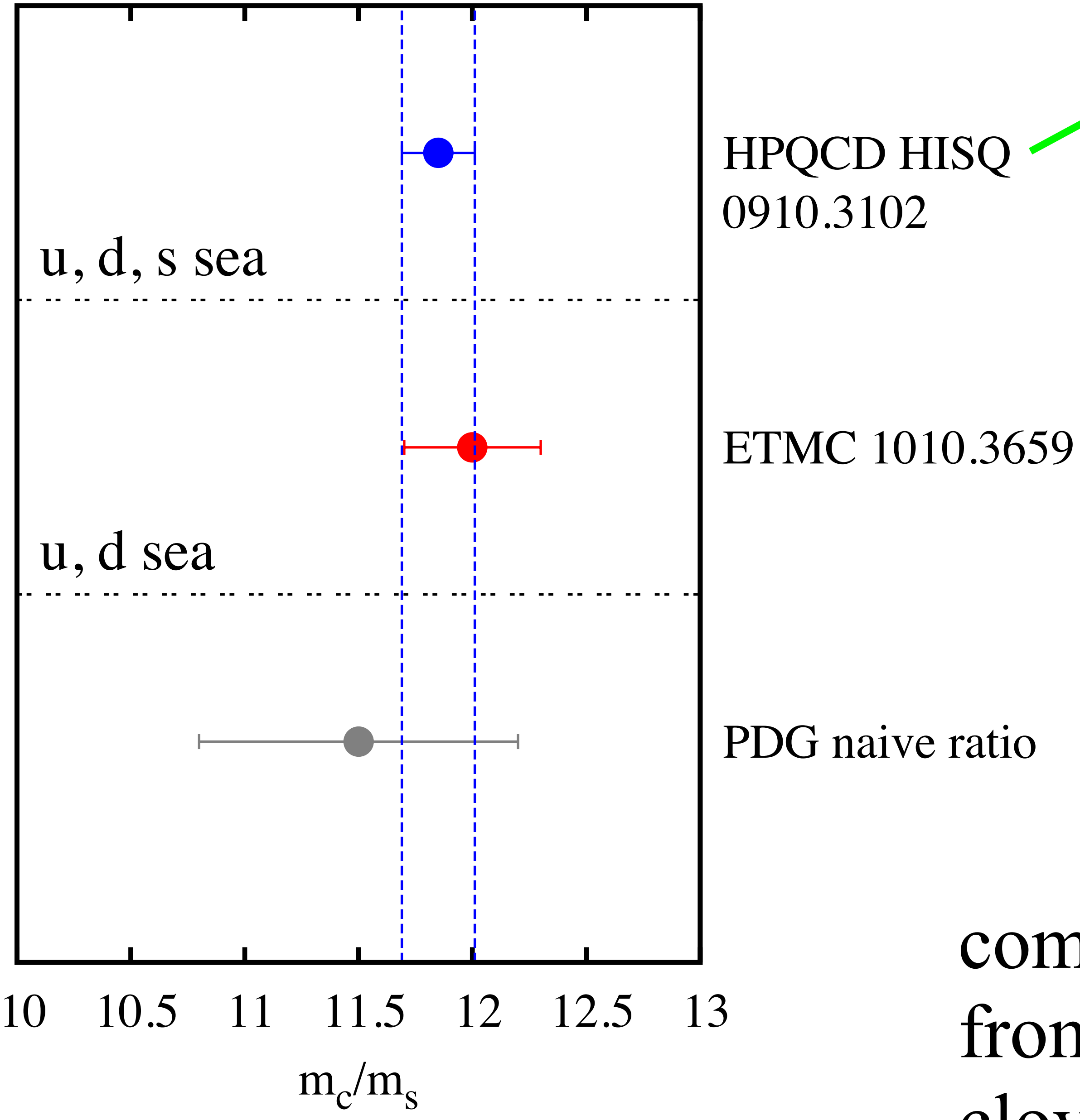


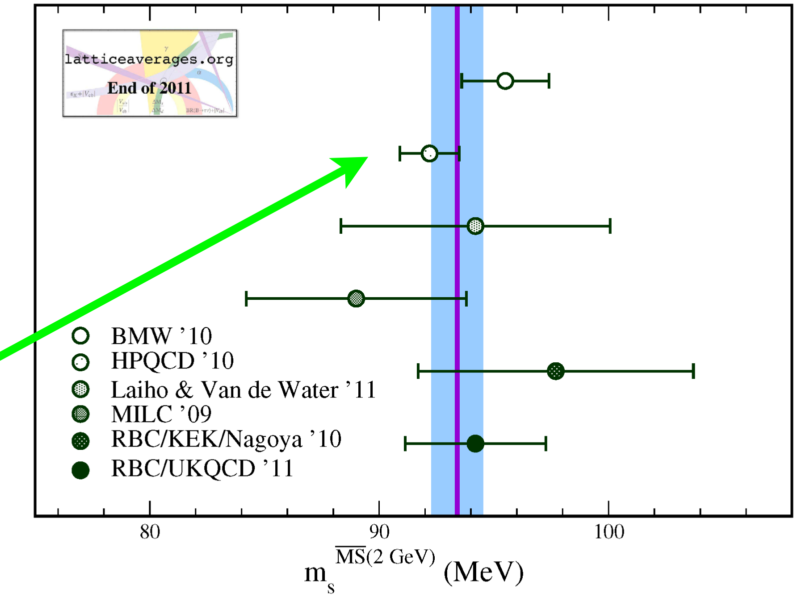
$$\frac{m_c}{m_s} = 11.85(16) \quad n_f = 3$$

`R. Dowdall et al,HPQCD, 1207.5149`

`C. Davies et al,HPQCD, 0910.3102`

m_c/m_s comparison





Allows us to leverage accurate m_s from accurate m_c

coming soon : results from ratio using smeared clover C. McNeile et al, QWG11

Bottom quarks in lattice QCD - heavy or light?

Several options have been used for m_b:

- Relativistic methods extrapolated to b

HISQ, TM

`HPQCD 1004.4285,`
`ETM 1107.1441`

- Nonrelativistic method at b:
  NRQCD - disc. nonrel. expansion of L_q, now radiatively improved through $\alpha_s v_b^4$

`HPQCD, 1105.5309,`
`1110.6887`

- HQET methods. Most advanced inc. 1/M corrections and step-scaling to tune coefficients nonperturbatively

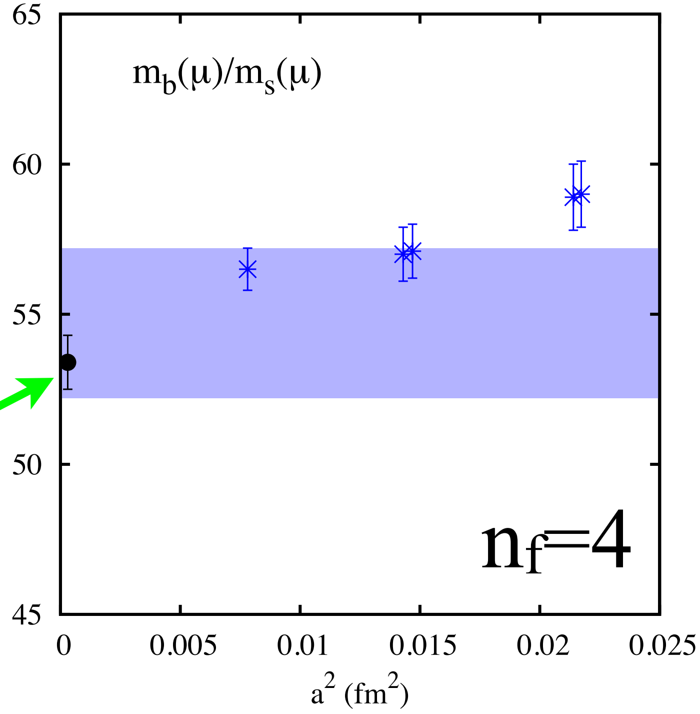
`Alpha, 1203.6516`

Direct methods for m_b

NRQCD - one-loop determination of Z_m so far `HPQCD, 1110.6887`

Check m_b/m_s from HISQ-HISQ

`HPQCD, 1004.4285; 0910.3102`



Alternative - use the binding energy

For nonrelativistic actions there is a calculable energy offset, E_0, so that:

$$n_Q\overline{m} = Z_{m,\overline{MS}}\left[M_{meson,expt} - (E_{latt} - n_Q E_0)\right]$$

$n_Q=2$, heavy-heavy;
$n_Q=1$, heavy-light

NRQCD: two-loop determination of E_0 underway `C. Monahan et al, HPQCD`

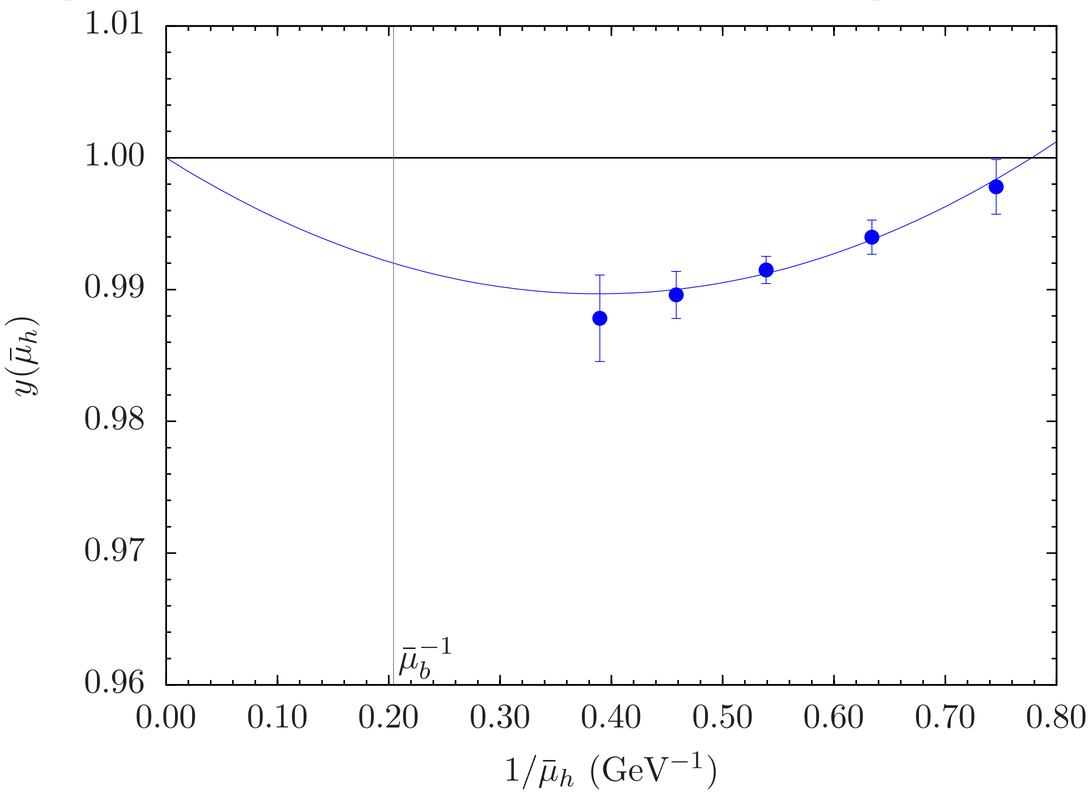
HQET: determine E0 using nonpert. stepscaling. Heavy-light only. `Alpha, 1203.6516`

Ratio method

ETM, 1107.1441

Use relativistic method (twisted mass here) and extrapolate ratios of heavy-light meson mass to quark pole mass using:

$$lim_{m\to\infty}\left(\frac{M_{hl}}{m_{pole}}\right) = constant$$



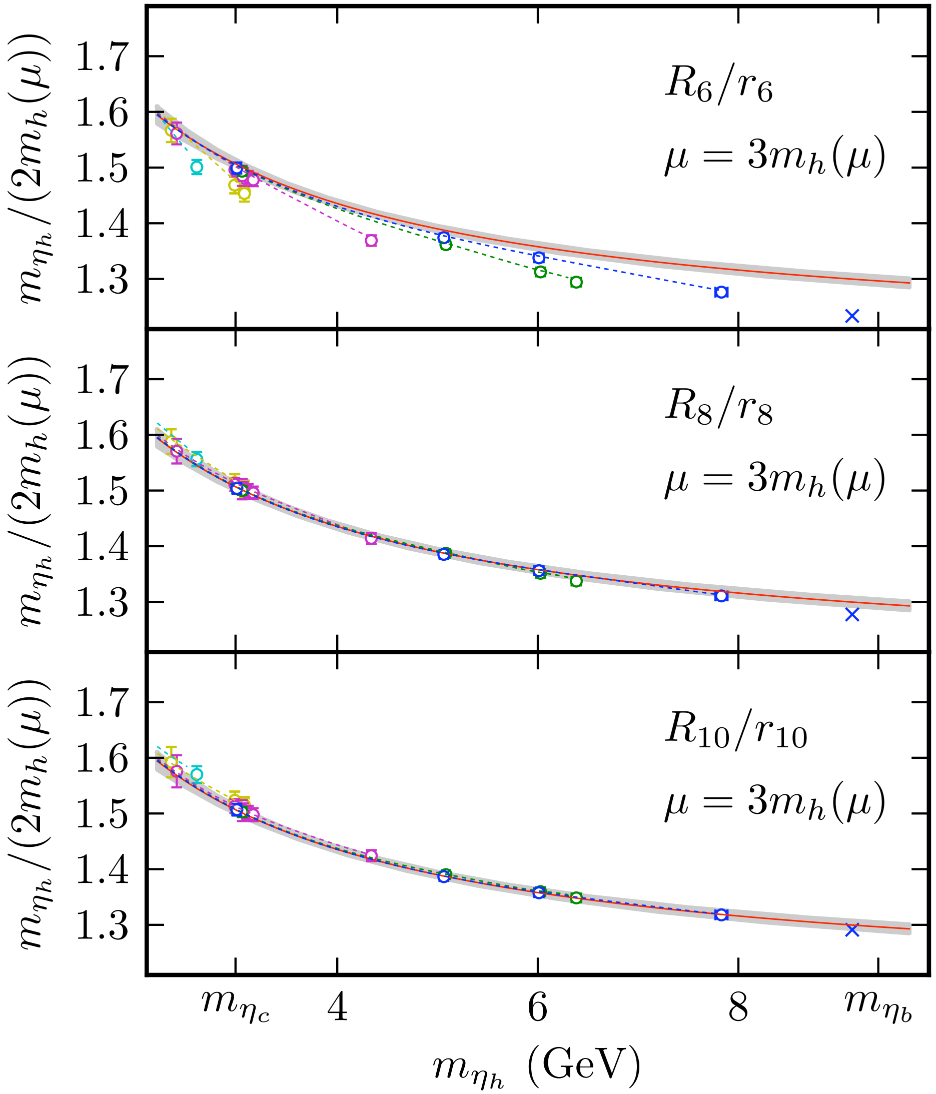
Use HQET to interpolate to b from c and known static limit and reconstruct m_b.

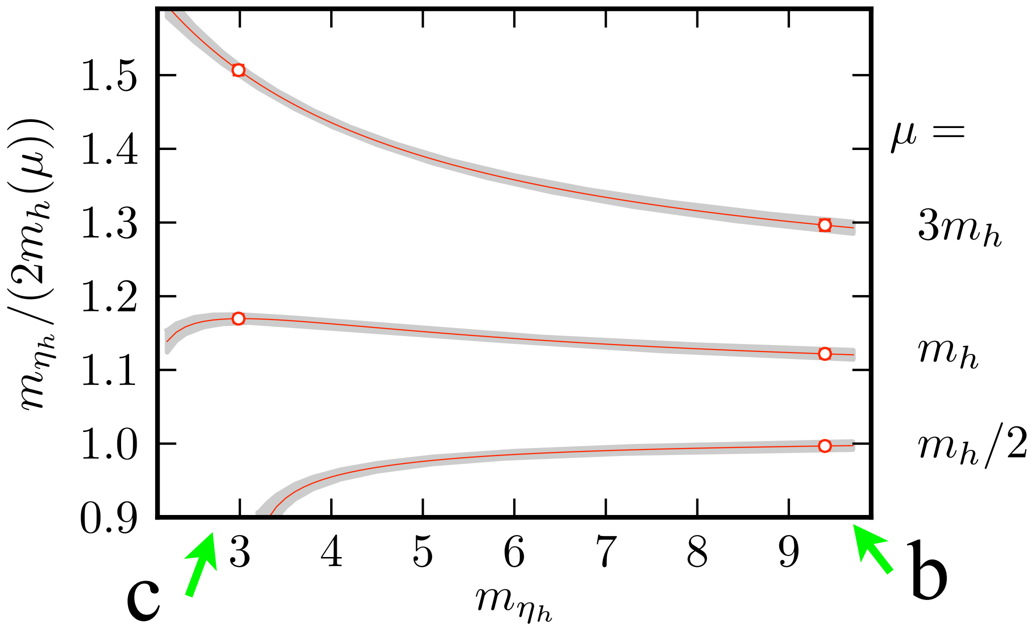
Errors 3% at present from interpoln and fixing scale.

Current-current correlator method -HISQ

`HPQCD,`
`1004.4285`

- Repeat calcln for $m_q \geq m_c$ inc. ultrafine lattices



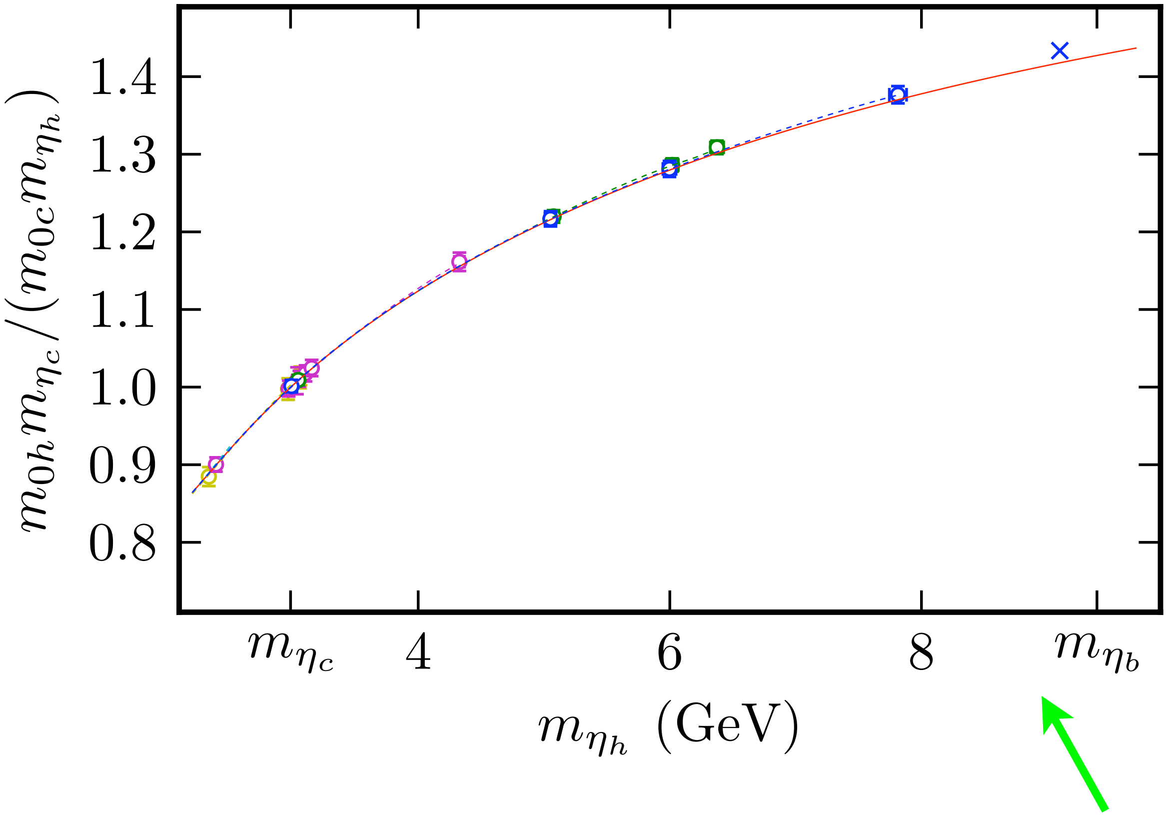


Can determine m_h/m_{η_h} for heavy quarks - extrapolate (slightly) to b.

$$\overline{m}_b^{n_f=5}(\overline{m}_b) = 4.164(23)\mathrm{GeV}$$

Agrees well with contnm results using $R_{e^+e^-}$

m_b/m_c from lattice QCD



$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0}$$

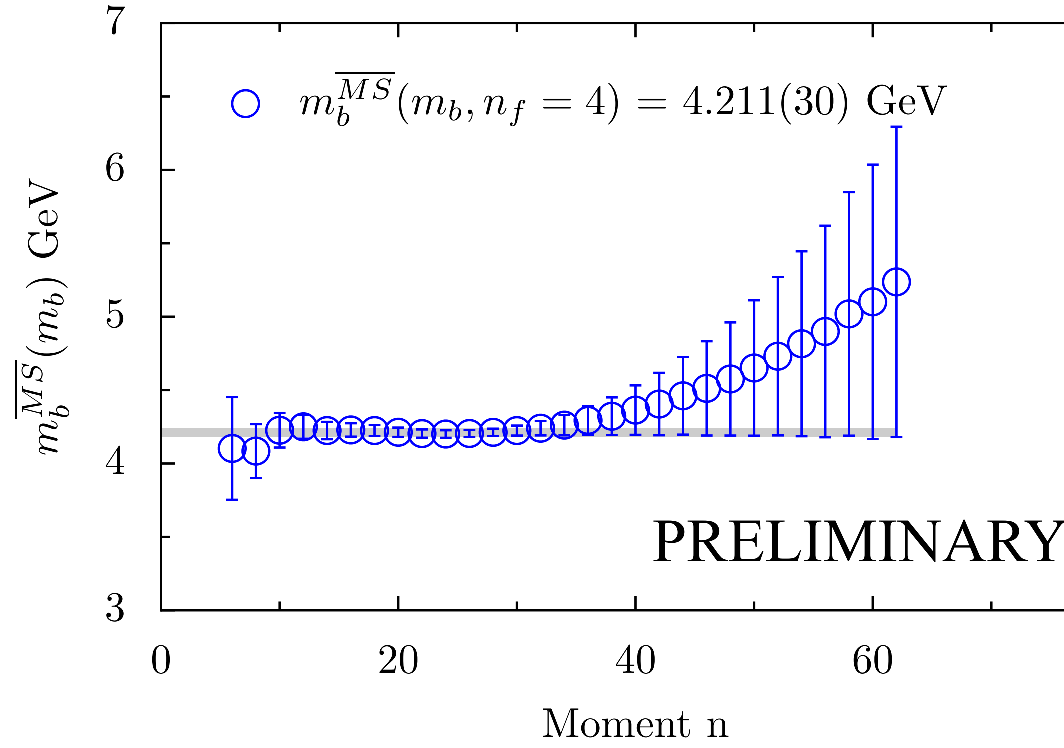
$$= \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

completely nonperturbative determination of ratio gives:

$$\frac{m_b}{m_c} = 4.49(4)$$

Agrees with that from current-current correlator method - test of pert. th.

Current-current correlator method for NRQCD



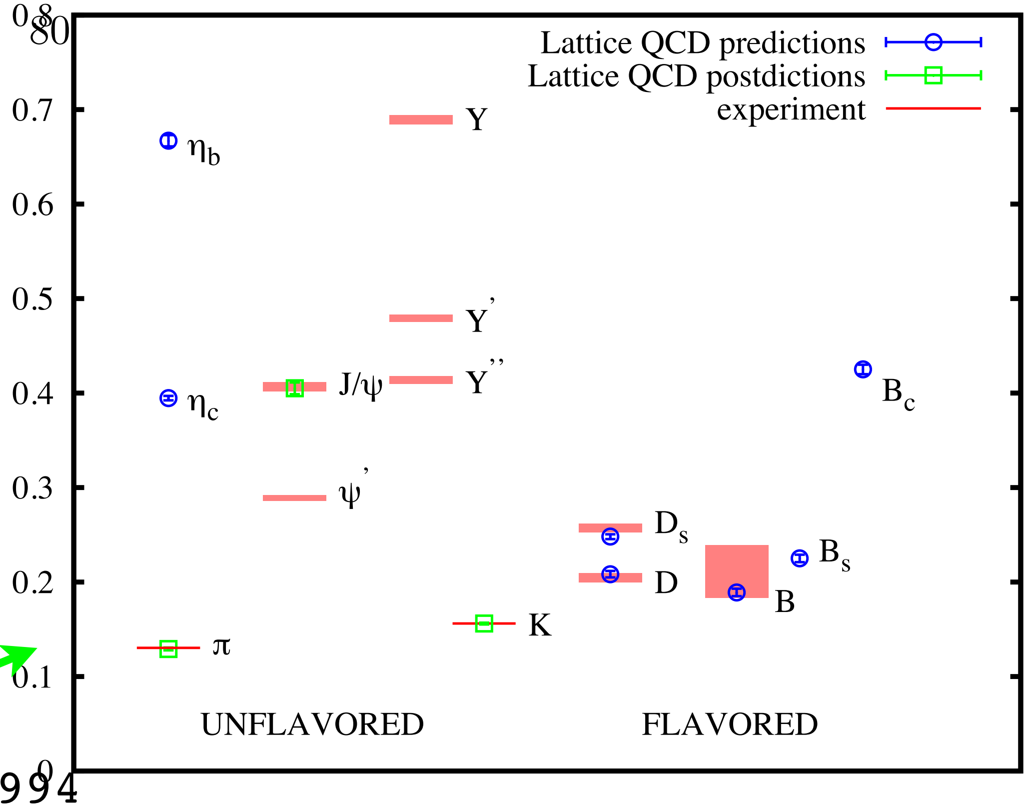
radiatively improved NRQCD on 2+1+1 gluon configs.

in progress ...

R. Dowdall et al, HPQCD

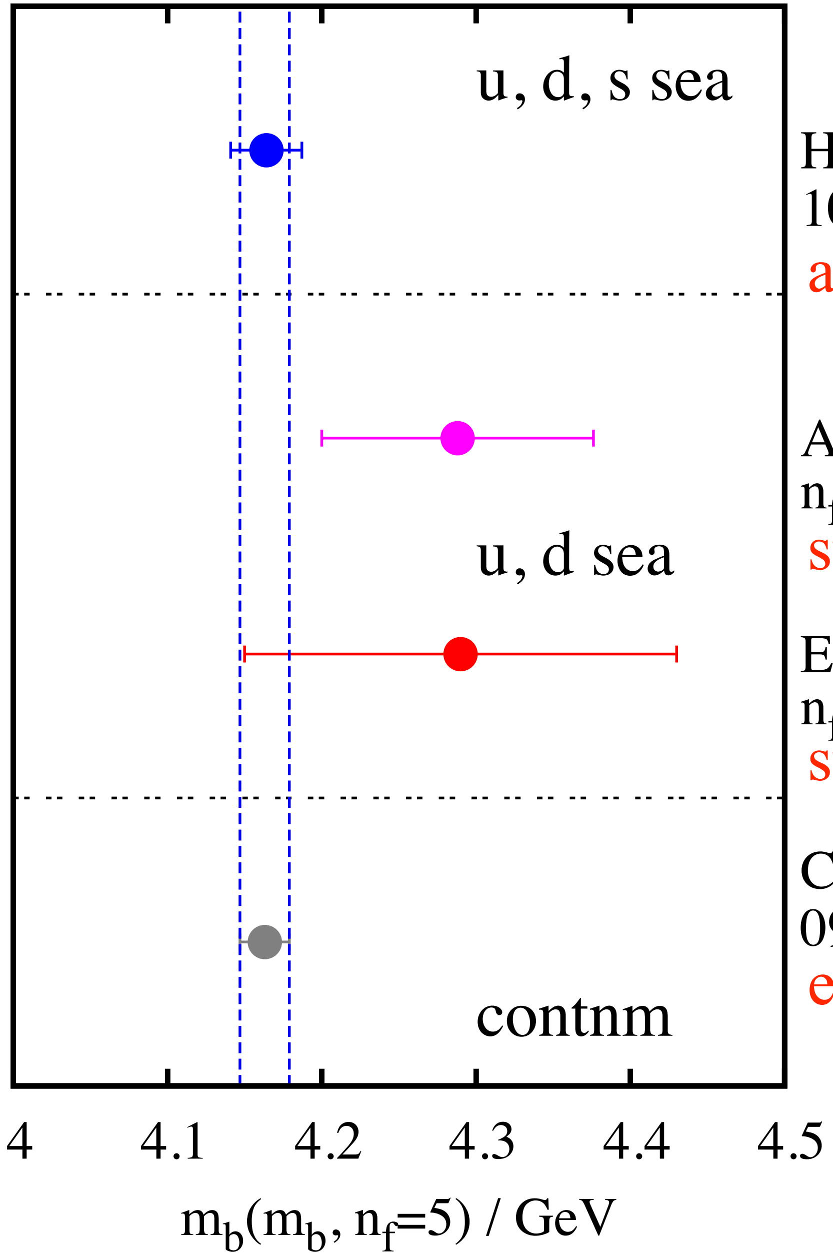
Using vector current which needs renormln factor so not as accurate as HISQ result.

Will give Υ decay constant at same time to fill in “spectrum” HPQCD, 1207.0994



Results for m_b

dominant error



HPQCD HISQ
1004.4285
a extrapolation

ALPHA Trento12
$n_f=2$
statistical

ETMC 1107.1441
$n_f=2$
statistical/extrapolation

Chetyrkin et al
0907.2110
expt R

0.5% errors possible

New lattice results to come shortly ....

Conclusions

$\overline{m_c}(\overline{m_c})$ is determined to 1% and
$\overline{m_b}(\overline{m_b})$ to 0.5% from continuum and lattice methods.

Will be hard to improve $\overline{m_c}$ further.
$\overline{m_b}$ can be improved from lattice QCD e.g using relativistic methods on finer lattices

Lattice QCD methods have advantages:

- lots of checks from meson masses and decay constants
- ratios of masses determined accurately

Lots of new lattice QCD determinations in progress using a variety of formalisms. Watch this space ...

Error budget for HISQ current-current method

1004.4285

	$m_c(3)$	$m_b(10)$	m_b/m_c	$\alpha_{\overline{\rm MS}}(M_Z)$
a^2 extrapolation	0.2%	0.6%	0.5%	0.2%
Perturbation theory	0.5	0.1	0.5	0.4
Statistical errors	0.1	0.3	0.3	0.2
m_h extrapolation	0.1	0.1	0.2	0.0
Errors in r_1	0.2	0.1	0.1	0.1
Errors in r_1/a	0.1	0.3	0.2	0.1
Errors in m_{η_c}, m_{η_b}	0.2	0.1	0.2	0.0
α_0 prior	0.1	0.1	0.1	0.1
Gluon condensate	0.0	0.0	0.0	0.2
Total	0.6%	0.7%	0.8%	0.6%