Extracting Weak Phases cleanly from Charmless 3-Body B Decays

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Talk based on work done in collaboration with M. Imbeault, N. Rey-Le Lorier and B. Bhattacharya.

The standard way to obtain clean information about CKM phases is through the measurement of indirect CPV in $B/\bar{B} \to f$. This requires that f be a CP eigenstate.

Conventional wisdom: one cannot obtain such clean information from 3-body decays because final states such as $K_S\pi^+\pi^-$ are not CP eigenstates – the value of its CP depends on whether the relative $\pi^+\pi^-$ angular momentum is even (CP $+$) or odd (CP $-$).

Exceptions:

1. If the final state contains truly identical particles – e.g. $K_S\pi^0\pi^0$ – it is a CP eigenstate. (Here the relative $\pi^0\pi^0$ angular momentum is even $\Longrightarrow$ CP $+$.)
2. $B_d^0 \to K^+K^-K_S$: Belle used an isospin analysis to differentiate CP $+$ and CP $-$. They found that it is dominantly CP $+$. K. Abe *et al.* [Belle Collaboration], hep-ex/0208030; Y. Grossman, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D **68**, 015004 (2003).

Even for these exceptions, cannot measure indirect CPV and get clean weak-phase information. Problem: this only works if decay is dominated by amplitudes with a single weak phase. But in general these decays receive significant contributions from amplitudes with a different weak phase. Need a way of dealing with this "pollution."

Recently it was shown that all of these difficulties can be overcome.

M. Imbeault, N. Rey-Le Lorier, D. L., Phys. Rev. D **84**, 034040 (2011), 034041 (2011);

N. Rey-Le Lorier, D. L., Phys. Rev. D **85**, 016010 (2012).

Dalitz Plots

Consider $B \to P_1 P_2 P_3$, in which each P_i has momenta p_i. Can construct the three Mandelstam variables:

$$s_{12} \equiv (p_1 + p_2)^2 \quad , \quad s_{13} \equiv (p_1 + p_3)^2 \quad , \quad s_{23} \equiv (p_2 + p_3)^2 \ .$$

These are not independent:

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2 \ .$$

Dalitz plot: given in terms of two Mandelstam variables, say s_{12} and s_{13}. Great advantage: can extract the full amplitude of the decay. Write

$$\mathcal{M}(B \to P_1 P_2 P_3) = \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}) \ .$$

Sum is over all decay modes (resonant and non-resonant). c_j and θ_j are the magnitude and phase of the j contribution, relative to one of the channels. The distributions F_j describe the dynamics of the individual decay amplitudes, and take different forms for the various contributions.

Key point: in the experimental Dalitz-plot analyses, explicit expressions for the F_j are assumed (e.g. Breit-Wigner). Then a maximum likelihood fit over the entire Dalitz plot gives the best values of the c_j and θ_j.

$\Longrightarrow$ the decay amplitude $\mathcal{M}(s_{12}, s_{13})$ is known.

Can now fix the CP of the final state. E.g. suppose the final state has CP $+$ when the amplitude is symmetric under $P_2 \leftrightarrow P_3$ (as is the case for the final state $K_S\pi^+\pi^-$). We can find this amplitude from the above:

$$\mathcal{M}_{sym} = \frac{1}{\sqrt{2}} \left[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) \right] \ .$$

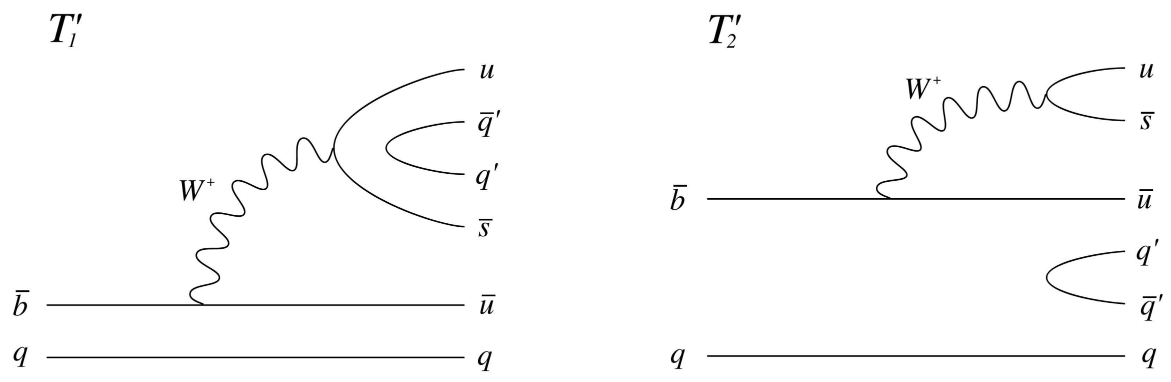
Using this, it is possible to compute the $B \to P_1 P_2 P_3$ observables. E.g. the indirect (mixing-induced) CP asymmetry is given by

$$S = \mathrm{Im} \left[e^{-2i\phi_M} \frac{\bar{\mathcal{M}}_{sym}}{\mathcal{M}_{sym}} \right] \ .$$

Note: all observables are momentum dependent – they take different values at each point in the Dalitz plot.

Diagrams

In order to remove the pollution due to additional decay amplitudes, one first expresses the full amplitude in terms of diagrams. In 3-body diagrams we add the subscript “1” (“2”) if the popped quark pair is between two non-spectator final-state quarks (two final-state quarks including the spectator). All amplitudes are expressed in terms of color-allowed and color-suppressed tree, gluonic penguin, and EWP diagrams; annihilation/exchange-type diagrams are neglected.



The above figure shows the T_1' and T_2' diagrams contributing to $B \to K\pi\pi$ (as this is a $\bar{b} \to \bar{s}$ transition, the diagrams are written with primes). The other diagrams (C_1', C_2', P_1', P_2', P'_{EW1}, P'_{EW2}, P'^{C}_{EW1}, P'^{C}_{EW2}) are obtained similarly from the 2-body diagrams.

Note: unlike the 2-body diagrams, the 3-body diagrams are momentum dependent. This must be taken into account whenever the diagrams are used.

Now, in $B \to K\pi$ decays there are relations between the EWP and tree diagrams under flavor SU(3) symmetry. Recently it was shown that similar EWP-tree relations hold for $B \to K\pi\pi$ decays [M. Imbeault, N. Rey-Le Lorier, D. L., Phys. Rev. D **84**, 034041 (2011)]. Taking $c_1/c_2 = c_9/c_{10}$ (which holds to about 5%), these take the simple form

$$P'_{EW1} = \kappa T'_1 \;, \qquad P'_{EW2} = \kappa T'_2 \;,$$
$$P'^{C}_{EW1} = \kappa C'_1 \;, \qquad P'^{C}_{EW2} = \kappa C'_2 \;,$$

where

$$\kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2} \;,$$

with $\lambda_p^{(s)} = V^*_{pb} V_{ps}$.

$\exists$ important caveat. Under SU(3), the final state in $B \to K\pi\pi$ involves three identical particles, so that the six permutations of these particles (the group S_3) must be taken into account. But the EWP-tree relations hold only for the totally symmetric state. Thus, the analysis must be carried out for this state. The fully symmetric state can be found from the Dalitz plot. Instead of the amplitude which is symmetric only under $P_2 \leftrightarrow P_3$, we define

$$\begin{aligned}\mathcal{M}_{fully\ sym} &= \frac{1}{\sqrt{6}}\left[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) + \mathcal{M}(s_{12}, s_{23})\right. \\ &\qquad \left. + \mathcal{M}(s_{23}, s_{12}) + \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23})\right] .\end{aligned}$$

Once the full decay amplitudes are expressed in terms of diagrams, one can perform an analysis like that done with 2-body decays – one can combine the amplitudes for different decays in order to isolate and extract a CKM phase. I now give an example of such an analysis. (Note: SU(3) is assumed.)

$B \to K\pi\pi$

$B^+/B_d^0 \to K\pi\pi$: there are 6 decays. Decays with two π^0's are excluded as being too difficult experimentally. Also, $B^+ \to K^0\pi^+\pi^0$ is not independent – its amplitude is proportional to that of $B_d^0 \to K^+\pi^0\pi^-$. There are therefore only three $B \to K\pi\pi$ decays to consider.

The $B \to K\pi\pi$ amplitudes in which the $\pi\pi$ pair is symmetrized are:

$$
\begin{aligned}
2A(B_d^0 \to K^+\pi^0\pi^-)_{sym} &= T_1' e^{i\gamma} + C_2' e^{i\gamma} - \kappa\,(T_2' + C_1')\ , \\
\sqrt{2}A(B_d^0 \to K^0\pi^+\pi^-)_{sym} &= -T_1' e^{i\gamma} - C_1' e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} + \tilde{P}'_{tc} \\
&\quad + \kappa\left(\frac{1}{3}T_1' + \frac{2}{3}C_1' - \frac{1}{3}C_2'\right)\ , \\
\sqrt{2}A(B^+ \to K^+\pi^+\pi^-)_{sym} &= -T_2' e^{i\gamma} - C_1' e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} + \tilde{P}'_{tc} \\
&\quad + \kappa\left(\frac{1}{3}T_1' - \frac{1}{3}C_1' + \frac{2}{3}C_2'\right)\ .
\end{aligned}
$$

$B \to KK\bar{K}$

There are four $B \to KK\bar{K}$ decays in which the final KK pair is in a symmetric isospin state. However, $B^+ \to K^+K^+K^-$ and $B^+ \to K^+K^0\bar{K}^0$ are not independent – their amplitudes are proportional to those of $B_d^0 \to K^+K^0K^-$ and $B_d^0 \to K^0K^0\bar{K}^0$, respectively. These are

$$
\begin{aligned}
\sqrt{2}A(B_d^0 \to K^+K^0K^-)_{sym} &= -T_2' e^{i\gamma} - C_1' e^{i\gamma} - \tilde{P}_{uc}' e^{i\gamma} + \tilde{P}_{tc}' \\
&\quad + \kappa\left(\frac{1}{3}T_1' - \frac{1}{3}C_1' + \frac{2}{3}C_2'\right) , \\
A(B_d^0 \to K^0K^0\bar{K}^0)_{sym} &= \tilde{P}_{uc}' e^{i\gamma} - \tilde{P}_{tc}' \\
&\quad + \kappa\left(\frac{2}{3}T_1' + \frac{1}{3}C_1' + \frac{1}{3}C_2'\right) .
\end{aligned}
$$

Note: since SU(3) is assumed, $B \to KK\bar{K}$ diagrams in which the popped quark pair is $s\bar{s}$ are equivalent to $B \to K\pi\pi$ diagrams with a popped $u\bar{u}$ or $d\bar{d}$.

Note also: $A(B^+ \to K^+\pi^+\pi^-)_{sym} = A(B_d^0 \to K^+K^0K^-)_{sym}$.

Can combine diagrams into “effective diagrams” T'_a, T'_b, P'_a, P'_b, C'_a, giving

$$\begin{aligned}
2A(B_d^0 \to K^+\pi^0\pi^-)_{sym} &= T'_a e^{i\gamma} + T'_b e^{i\gamma} - C'_a - \kappa T'_b ~, \\
\sqrt{2}A(B_d^0 \to K^0\pi^+\pi^-)_{sym} &= -T'_a e^{i\gamma} - P'_a e^{i\gamma} + P'_b ~, \\
\sqrt{2}A(B_d^0 \to K^+K^0K^-)_{sym} &= -P'_a e^{i\gamma} + P'_b - C'_a ~, \\
A(B_d^0 \to K^0K^0\bar{K}^0)_{sym} &= P'_a e^{i\gamma} - T'_b e^{i\gamma} - \frac{1}{\kappa} C'_a e^{i\gamma} \\
&\quad - P'_b + \kappa T'_a + \kappa T'_b + C'_a ~.
\end{aligned}$$

$\exists$ 5 effective diagrams describing the four amplitudes $\Longrightarrow$ 10 theoretical parameters: 5 magnitudes of diagrams, 4 relative phases, and γ. But $\exists$ 11 (momentum-dependent) experimental observables: the decay rates and direct asymmetries for $B_d^0 \to K^+\pi^0\pi^-$, $B_d^0 \to K^0\pi^+\pi^-$, $B_d^0 \to K^+K^0K^-$ and $B_d^0 \to K^0K^0\bar{K}^0$, and the indirect asymmetries of $B_d^0 \to K^0\pi^+\pi^-$, $B_d^0 \to K^+K^0K^-$ and $B_d^0 \to K^0K^0\bar{K}^0$. With more observables than theoretical parameters, γ can be extracted from a fit.

Note: this analysis applies to every point in the Dalitz plot $\Longrightarrow$ get multiple measurements of γ. These can be averaged, reducing the error.

This is a broad overview of the $K\pi\pi$/$KK\bar{K}$ method of measuring γ. However, yesterday (Sept. 30, 4:30, WG IV), B. Bhattacharya presented some details of the analysis.

Work in progress: only 14 points in the Dalitz plots used for preliminary analysis, SU(3) breaking not taken into account, not all sources of error included, have to deal with discrete ambiguities (multiple overlapping solutions).

With these caveats, initial result is

$$\gamma = \left(81^{+4}_{-5}\ (\mathrm{avg.}) \pm 4\ (\mathrm{std.\ dev.})\right)^\circ\ .$$

This is consistent with independent direct measurements of γ. The PDG gives $\gamma = \left(66^{+11}_{-10}\right)^\circ$.

Conclusions

The value found for γ is preliminary, and there are still important issues to address (complete error analysis, discrete ambiguities, etc.). However, the result is extremely encouraging. It does indeed appear that one can cleanly extract weak-phase information from 3-body B decays, contrary to what was previously thought.

I strongly encourage the experimentalists to incorporate this method into their measurements of the 3-body $K\pi\pi$/$KK\bar{K}$ Dalitz plots. [E.g. see discussions of $B \to KK\bar{K}$ in talks by E. Ben-Haim (Saturday, Sept. 29, 2:50, WG IV) and E. Pucchio (Sunday, Sept. 30, 9:15, WG V).] There is no doubt that many of the outstanding question marks could be better treated with a complete analysis of the experimental data, and the error on γ might be reduced even further.

Given that 3-body decays can be used for weak-phase measurements, it is important to come up with other methods for extracting phase information. B. Bhattacharya and I have examined ways of measuring β_s using 3-body B_s^0 decays, and other techniques for measuring weak phases are surely possible.