Kaon semi-leptonic form factor in lattice QCD

CKM Workshop Cincinnatti

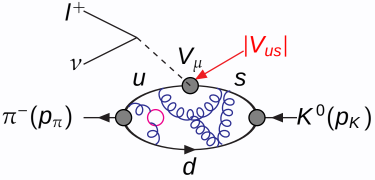
28.09-02.10.2012

Andreas Jüttner
University of Southampton

Motivation

- tests and constraints for the SM $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \delta$
- non-perturbative physics contribution needed in model-independent way
- kaons and pions rather clean in lattice QCD (no real multi-scale issue)
- so far no new results but new simulation data
- new experimental data is challenging us
- new paradigm for data analysis in the future?

$|V_{us}|$ from K_{l3} decay



$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} \, I \, S_{\rm EW}[1+\Delta_{SU(2)}+\Delta_{\rm EM}] \times |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

- I phase space integral (via FF shape from experiment)
- $S_{\rm EW}$ short distance EW corrections
- $\Delta_{SU(2)}$ Iso-spin breaking corrections
- $\Delta_{\rm EM}$ long distance EM corrections

$f_+^{K\pi}(0)|V_{us}| = 0.2163(5) \rightarrow$ sub-1%-precision for $f_+^{K\pi}(0)$ required

FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

$|V_{us}|$ from K_{l3} decay

Form factor in ChiPT: $f_+^{K\pi}(0) = 1 + f_2(m_\pi, m_K, m_\eta) + f_4(m_\pi, m_K, m_\eta, \mathrm{LEC}) + \ldots$

- f_2 known function of the meson masses $f_2^{\mathrm{phys}} = -0.023$ ($K^0 \to \pi^- l\nu$)
  (Gasser, Leutwyler, Nucl.Phys.B250,1985.)
  (f_2 depends on the expansion paramter f: $f_2 \propto \frac{1}{f}(H_{\pi K} + H_{\eta K})$)
  cf. study in RBC/UKQCD Eur.Phys.J. C69 (2010) 159-167
- f_4 NNLO contribution not available in closed from
  (Post, Schilcher Eur.Phys.J., 2002, Bijnens, Talavera Nucl.Phys.B, 2003, Jamin, Oller, Pich JHEP 2004)
  Bijnen's Fortran code *see e.g. Bernard, Passemar JHEP 1004 (2010) 001*

Since f_2 is precisely known it suffices to determine

$$\Delta f = f_+^{K\pi}(0) - (1 + f_2)$$

$\longrightarrow$ need $< 20\%$ precision for Δf

Lattice QCD

What is . . . ?

	QCD
N_c	3
N_f, fundamental	1+1+1+1+1+1
$SU(2)$ iso-spin brk.	$\checkmark$
m_π	135MeV
V	∞
a	0

Lattice QCD

What is . . . ?

	QCD	Lattice QCD
N_c	3	3
N_f, fundamental	1+1+1+1+1+1	0, 2, 2+1, 2+1+1
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m_π	135MeV	$\lesssim m_\pi^{\rm sim}$
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all approximations need to be removed or studied in detail

Definition of the form factor

Vector current (method V)

$$\langle\pi(p')|V_\mu|K(p)\rangle = f_+^{K\pi}(q^2)(p+p')_\mu + f_-^{K\pi}(q^2)(p-p')_\mu,\; f_0^{K\pi}(0) = f_+^{K\pi}(0)$$

Ward identity (method S)

$$\langle\pi(p')|S|K(p)\rangle|_{q^2=0} = f_0^{K\pi}(0)\frac{m_K^2-m_\pi^2}{m_s-m_q}$$

- both computations completely well defined
- symmetry constraint in method V supresses cut-off effects but not for method S
- basic technique: define ratios of correlation functions with good signal/noise properties *Hashimoto et al., Phys.Rev. D61 (1999) 014502*

K_{l3}-decay - the computation

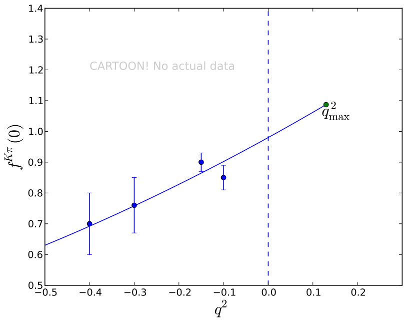
1a) compute $f_0^{K\pi}(q^2)$

meson momenta in a finite box:

$$\vec{p} = \vec{n}\frac{2\pi}{L},\; n_i \in \pm\{0,1,2,\ldots\}$$

interpolate to $q^2 = 0$

phenomenological ansatz
(e.g. pole)



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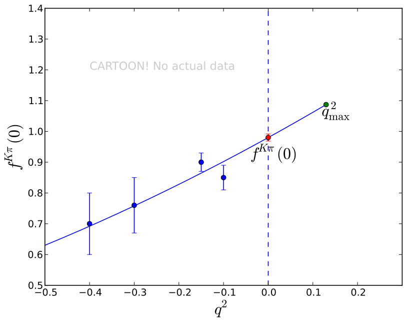
1b) better: twisted bcs → arbitrary momenta → simulation directly at $q^2 = 0$
UKQCD JHEP05 (2007) 016



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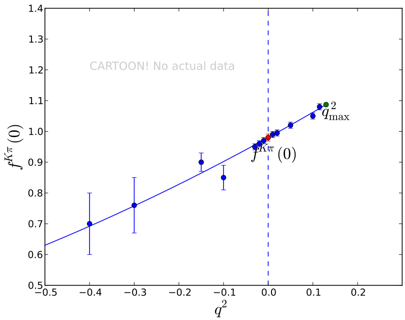

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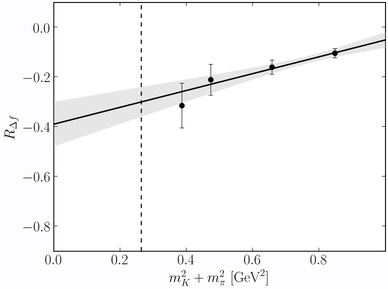
2) extrapolate to physical quark mass
Gasser&Leutwyler, Nucl.Phys.B, 1985

$$\Delta f = f_+^{K\pi}(0, m_\pi, m_K) - (1 + f_2(m_\pi, m_K))$$

chiral extrapolations about to become unnecessary $\rightarrow$ simulation with physical quark mass



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3) study cut-off and finite volume effects

Players

Collaboration	published	N_f	fermion formulation	lightest pion mass	technique	q^2-dependence	a-dependence
MILC		2+1+1	stag.	$135_{valence}$	S	twbc	✓
MILC		2+1	stag.	$270_{valence}$	S	twbc	✓
JLQCD		2+1	ovl.	290	V	twbc+interpol.	
RBC/UKQCD	✓	2+1	DWF	170	V	twbc	✓
ETM	✓	2	TM	260	V	twbc+interpol	✓

fermion formulations:

- stag. = staggered
- ovl. = overlap
- DWF = Domain Wall Fermions
- TM = twisted mass

technique:

- V = direct computation of vector current
- S = computation via Ward identity

q^2-dependence:

- twbc = twisted boundary conditions
- interpol. = interpolation in momentum trasnfer to $q^2 = 0$

Error budget

typical result *RBC+UKQCD Phys. Rev. Lett. 2008*:

$$f_+^{K\pi}(0) = 0.9644(33)^{\rm stat}(34)^{q^2,\chi}(14)^{a}$$

$$|V_{us}^{K_{l3}}| = 0.2242 \quad \begin{matrix} (5)^{\Gamma} & (8)^{\rm stat} & (8)^{q^2,\chi} & (3)^{a} \\ (0.2\%)^{\Gamma} & (0.3\%)^{\rm stat} & (0.4\%)^{q^2,\chi} & (0.1)^{a}\% \end{matrix}$$

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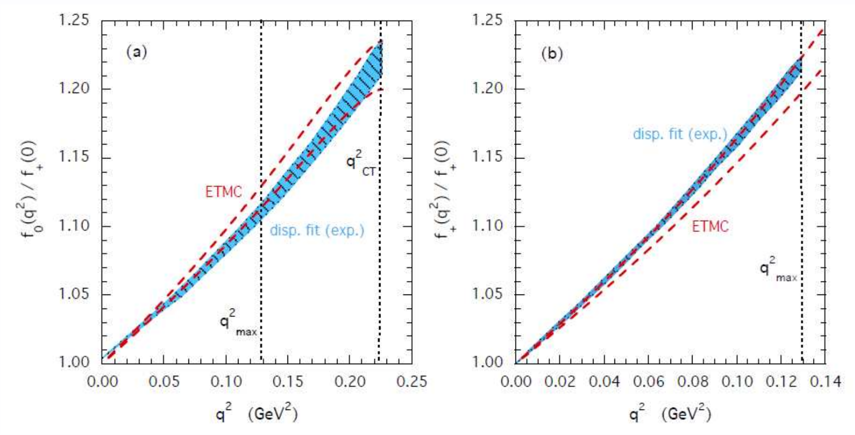
- stat: in Monte Carlo simulation naively scales $\propto 1/\sqrt{N}$, but also new techniques can help
- δq^2: is absent in simulations using twisted boundary conditions
- $\delta\chi$: the simulated pion mass is now very close or already at the physical point and this uncertainty will soon disappear
- δL: I think this needs more detailed studies
- δa: simulating various lattice spacings allows for reducing cut-off effects but anyway symmetry-suppressed in method V
- Γ: $KLOE-2$ says they will push down the uncertainty on Γ

Update on ETM effort *arXiv:1012.3573*

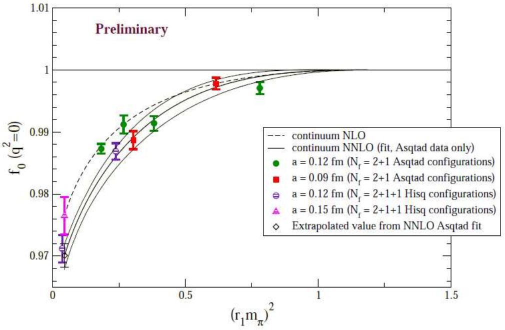


- preliminary results
- detailed study of q^2-dependence
- pole-ansatz for q^2 plus SU(2) χPT NLO-inspired model with NNLO-terms and Callan-Treiman constraint with f_K/f_π
- dashed lines ETM result, blue line dispersive fit to exp. data by

Bernard, Oertel, Passemar Phys. Lett. B638 (2006) 480; Phys. Rev. D80 (2009) 034034

New MILC effort *Gamiz Lattice 2012*

Summary of MILC results with $N_f = 2+1$ and $N_f = 2+1+1$ staggered fermions



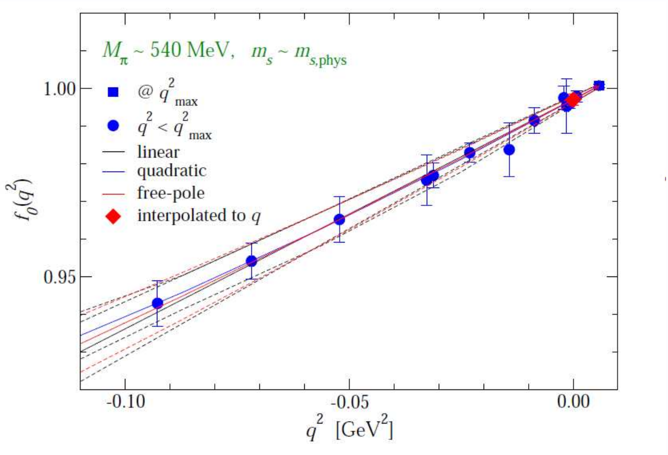
$N_f = 2+1$ error budget

source	%
stat+extrapol	0.2-0.3
chiral extrapol	0.1
cut-off	0.15-0.2
m_s mistuning	0.2
FSE	?
total	0.35-0.50

- preliminary results
- compared to other collabs cut-off effects large, but they have many ensembles and can try to control it
- for $N_f = 2+1$ staggered χPT at NLO supplemented with continuum NNLO expressions
- $N_f = 2+1$ $\quad m_{\pi,\mathrm{min}} \approx 270\mathrm{MeV}$
  $N_f = 2+1+1$ $\quad m_{\pi,\mathrm{min}} \approx m_{\pi,\mathrm{phys}}$
- aiming at 0.35-0.50% final uncertainty for $N_f = 2+1$

JLQCD effort *Kaneko Lattice 2012*

Summary of JLQCD results with $N_f = 2 + 1$ overlap fermions



- preliminary results
- $m_{\pi,\mathrm{min}} = 260$MeV, NLO+model analysis
- preliminary analaysis uncertainty $\approx 0.8\%$ for $N_f = 2 + 1$

JLQCD effort *Kaneko Lattice 2012*

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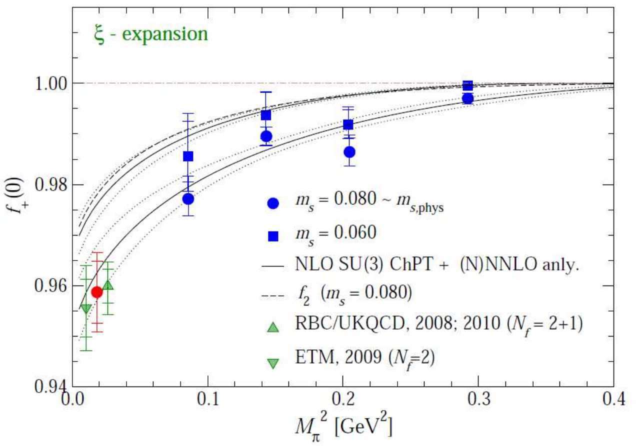

- large q^2-range
- including many points around $q^2 = 0$, twisted boundary conditions
- allows for charge radius determination

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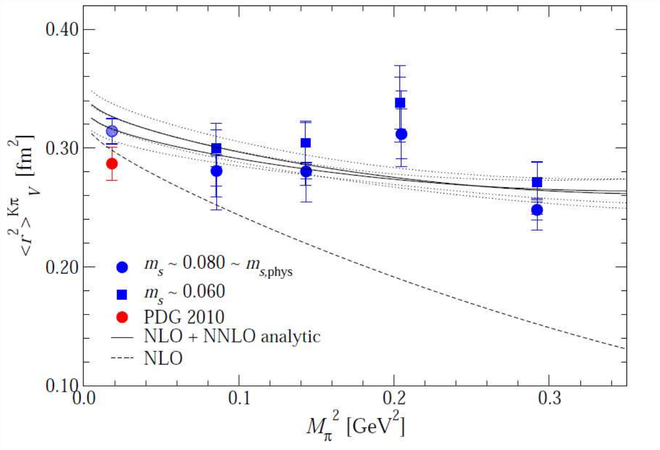

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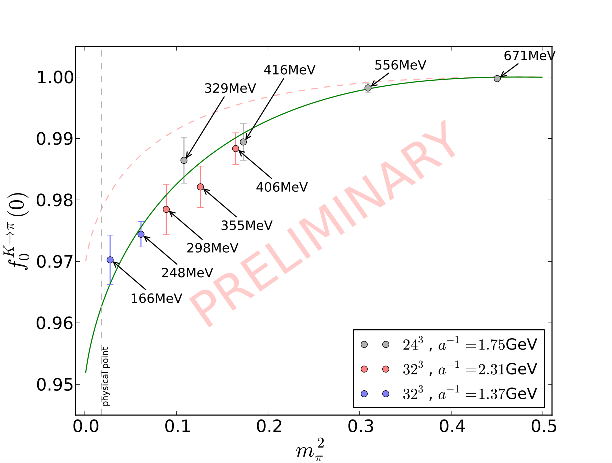
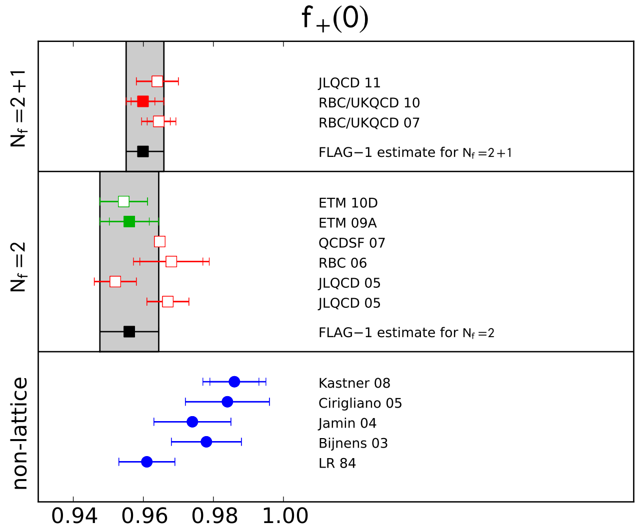
Update on RBC/UKQCD effort *Sivalingam Lattice 2012*



- preliminary results
- $N_f = 2 + 1$ $m_{\pi,\text{min}} \approx 166\text{MeV}$ NLO+model analysis
- twisted bcs

Where are we



$f_+(0)$
$N_f = 2+1$
JLQCD 11
RBC/UKQCD 10
RBC/UKQCD 07
FLAG−1 estimate for $N_f = 2+1$
$N_f = 2$
ETM 10D
ETM 09A
QCDSF 07
RBC 06
JLQCD 05
JLQCD 05
FLAG−1 estimate for $N_f = 2$
non-lattice
Kastner 08
Cirigliano 05
Jamin 04
Bijnens 03
LR 84
0.94
0.96
0.98
1.00

FLAG (advertisement)

Eur.Phys.J. C71 (2011)

- Flavour Lattice Averaging Group (**FLAG**) (previously FlaviaNet Lattice Averaging Group): allow also an outsider to judge the quality and ’state-of-the-art’-fulnes of lattice results relevant to flavor physics

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  - chiral extrapolation
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  - finite volume errors
  - renormalisation
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- FLAG-2: FLAG-1 and `latticeaverages.org` join:
  - around 30 members from
    - US, EU, Japan
    - Alpha, BMW, ETMC, RBC/UQCD, CLS, Fermilab, HPQCD, JLQCD, MILC, PACS-CS, SWME
  - $m_{u,d}$, m_s, $f_+^{K\pi}(0)$, f_K/f_π, B_K, NLO LEC's + α_s, $B_{(s)}$ and $D_{(s)}$ leptonic and semi-leptonic decay and mixing
  - in preparation

Analyses (together with f_K/f_π)

Is the CKM matrix unitary?

- experimental results

$$|V_{us}f_+^{K\pi}(0)| = 0.2163(5) \qquad f_K/f_\pi|V_{us}/V_{ud}| = 0.2758(5)$$

FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

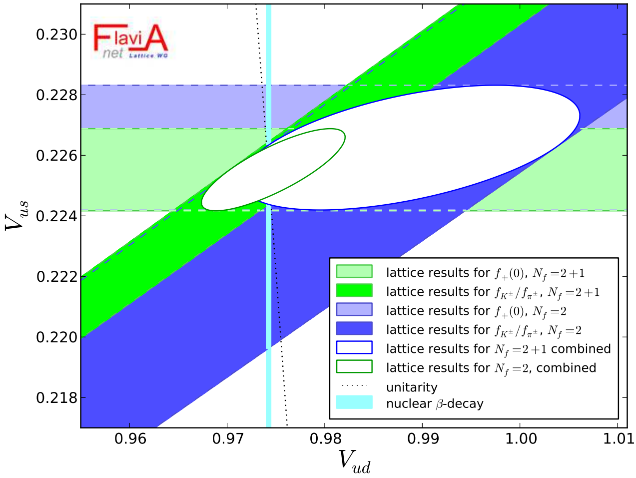
- SM test
  - $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
  - f_K/f_π and $|V_{ud}|$ from experiment
  - $f_+^{K\pi}(0)$ and f_K/f_π from lattice

FLAG Eur.Phys.J. C71 (2011) 1695:

	$f_+^{K\pi}(0)$& $\|V_{ud}\|_{\rm exp}$	f_K/f_π & $\|V_{ud}\|_{\rm exp}$	combined$_{\rm no\|V_{ud}\|exp}$
$N_f = 2+1$	1.0000(7)	0.9999(6)	1.002(15)
$N_f = 2$	1.0004(10)	0.9985(16)	1.037(36)

$N_f = 2+1$: $f_+^{K\pi}(0)$ by RBC/UKQCD 10, f_K/f_π FLAG-average over RBC/UKQCD 10A, HPQCD/UKQCD 07, BMW 10, MILC 10
$N_f = 2$: $f_+^{K\pi}(0)$ by ETM 09A, f_K/f_π ETM 09

Analyses



FlaviA net Lattice WG
0.230
0.228
0.226
0.224
0.222
0.220
0.218
V_{us}
0.96
0.97
0.98
0.99
1.00
1.01
V_{ud}
lattice results for $f_+(0)$, $N_f = 2+1$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1$
lattice results for $f_+(0)$, $N_f = 2$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2$
lattice results for $N_f = 2+1$ combined
lattice results for $N_f = 2$, combined
unitarity
nuclear β-decay

Analyses (together with f_K/f_π)

Use SM-unitarity

- experimental results

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FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- ignore $|V_{ub}| = 4 \cdot 10^{-3}$, then 3 equations, 4 unknowns
- provide either
  - $f_+^{K\pi}(0) \to$ predict $|V_{ud}|$, $|V_{us}|$, f_K/f_π
  - $f_K/f_\pi \to$ predict $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$
- analysis within the SM (relying on the assumption of first-row unitarity) *FLAG Eur.Phys.J. C71 (2011) 1695*:

	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$	$f_+^{K\pi}(0)$	f_K/f_π
$N_f = 2+1$	0.2253(9)	0.97428(21)	0.9599(38)	1.1927(50)
$N_f = 2$	0.2251(18)	0.97433(42)	0.9604(75)	1.194(10)

$N_f = 2+1$: $f_+^{K\pi}(0)$ by RBC/UKQCD 10, f_K/f_π FLAG-average over RBC/UKQCD 10A, HPQCD/UKQCD 07, BMW 10, MILC 10
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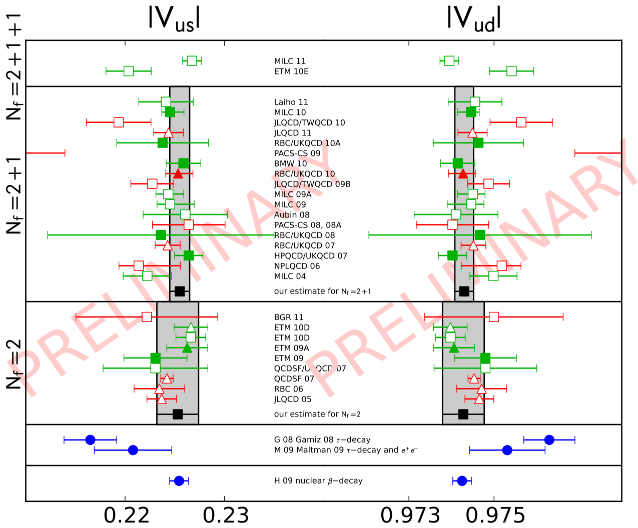
Analyses (together with f_K/f_π)



Outlook

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\mathrm{EW}}[1 + \Delta_{SU(2)} + \Delta_{\mathrm{EM}}] \times |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

- $\Delta_{SU(2)}$ and Δ_{EM} are now also being studied in lattice QCD (+QED) → talk by Nazario Tantalo
  - we will be able to provide predictions for individual iso-spin channels
  - lattice QCD+QED includes QED effects

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  - we will be able to provide predictions for individual iso-spin channels
  - lattice QCD+QED includes QED effects
- dominant systematics removed:
  - q^2-interpolation (twisted boundary conditions)
  - chiral extrapolation (simulations at the physical point)
- QCD-part in most simulations statistics limited which is good for us
- ETM, JLQCD MILC and RBC/UKQCD have worked a lot to improve upon previous results, will hopefully soon be published

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757