Lattice Calculations of $B \to K/K^* l^+ l^-$ form factors

Ran Zhou

Indiana University

CKM 2012,
Cincinnati, OH

09/30/2012

Lattice Calculations of $B \to K/K^* l^+ l^-$ form factors

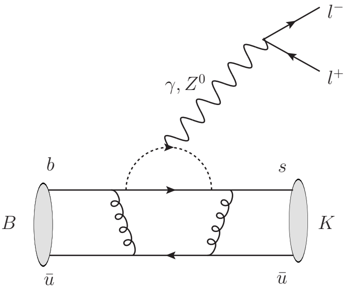
Outline:

- Introduction
- Lattice QCD calculations of $B \to K l^+ l^-$ form factors
- Lattice QCD calculations of $B \to K^* l^+ l^-$ form factors
- Conclusions

Many apologies for the important topics (papers) which are not covered (cited) by this talk.

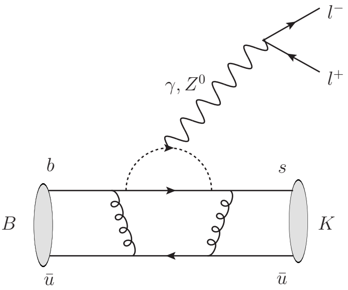
Motivations and theoretical background

$B \to K/K^* l^+ l^-$ semileptonic decay occurs through Penguin diagram ($b \to sll$).



- Standard Model (SM) contributes via FCNC (suppressed)
- Suitable process to detect physics BSM
- Studied by many experiment groups (BABAR, Belle, CDF, LHCb, SuperB, etc.)

Motivations and theoretical background



$$\text{Theoretical predictions} = \text{Known Const.} \times f(V_{nm}) \times \langle K/K^*|\hat{O}|B\rangle$$

- Form factors enter from hadron matrix elements
- The errors in form factors are the main source of the uncertainties in theoretical predictions.
- Lattice QCD can help in calculating $\langle K/K^*|\hat{O}|B\rangle$ non-perturbatively and precisely.

Form factors in $B \to Kll$ semileptonic decays

- Two matrix elements are needed in $B \to Kll$ work:

$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle$, $\langle B(p)|\bar{s}\sigma^{\mu\nu} b|K(k)\rangle$

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = f_+(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu$$

$$= \sqrt{2m_B}\left[f_\parallel \frac{p^\mu}{m_B} + f_\perp p_\perp^\mu \right]$$

$$\begin{cases} f_\parallel(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \dfrac{1}{p_i} \end{cases}$$

$$\begin{cases} f_0(E_K) = \dfrac{2m_B}{m_B^2 - m_K^2}\left[(m_B - E_K) f_\parallel(E_K) + (E_K^2 - m_K^2) f_\perp(E_K)\right] \\ f_+(E_K) = \dfrac{1}{\sqrt{2m_B}}\left[f_\parallel(E_K) + (m_B - E_K) f_\perp(E_K)\right] \end{cases}$$

Form factors in $B \to Kll$ semileptonic decays

Semileptonic $B \to K$ transition from tensor current:

$$q_\nu \langle K(k)|\bar{s}\sigma^{\mu\nu} b|B(p)\rangle = \frac{i f_T}{m_B + m_K}\left[q^2(p^\mu + k^\mu) - (m_B^2 - m_K^2)q^\mu\right]$$

Solve for f_T:

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle K(k)|ib\sigma^{0i}s|B(p)\rangle}{\sqrt{2m_B}k^i}$$

z-expansion on $B \to Kll$ form factors

Lattice QCD measures form factors at small E_K (low q^2). The z-expansion is used to extrapolate form factors along q^2 model-independently .

- z-expansion maps q^2 to z by:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_B \pm m_K)^2$$

- Choose $t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$ such that $z << 1$
- Expand form factors as a function of z.

$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k,$$

where $B(z) = z(q^2, m_R^2)$ is used to count pole structure and $\phi(z)$ is selected such that $\sum_{k=0}^{\infty} a_k^2 \leq 1$

Form factors in $B \to K^* ll$ decay:

Form factors in $B \to K^*$ semileptonic decay are from:

- $\langle K^*(k,\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle$: $V(q^2)$
- $\langle K^*(k,\lambda)|\bar{s}\gamma^\mu \gamma_5 b|B(p)\rangle$: $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$
- $\langle K^*(k)|\bar{s}\sigma_{\mu\nu} b|B(p)\rangle$: $T_1(q^2)$
- $\langle K^*(k)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle$: $T_2(q^2)$, $T_3(q^2)$

Two difficulties in lattice $B \to K^* ll$ form factors calculations:

- The K^* is unstable ($K^* \to K\pi$).
- No χPT available for K^*.

Studies of $B \to K/K^*ll$ form factors from lattice QCD

Quenched lattice QCD:

- A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)
- D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
- L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
- A. Abada et al. Phys. Lett. B 365, 275 (1996)
- T. Bhattacharya et al. Nucl. Phys. Proc. Suppl. 42, 935 (1995)
- K. C. Bowler et al. Phys. Rev. Lett. 72, 1398 (1994)
- C. W. Bernard et al. Phys. Rev. Lett. 72, 1402 (1994)

Quenched lattice studies of $B \to Kll$ form factors

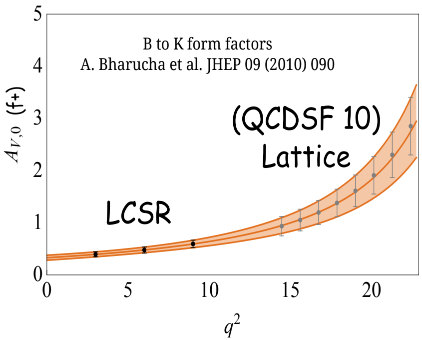


Figure: $B \to Kll$ form factors from LCSR + quenched lattice result.

- Most of lattice QCD results focus on large q^2 only.
- Errors from previous quenched calculations are large. (greater than 15%.)

Quenched lattice studies of $B \to K^* ll$ form factors

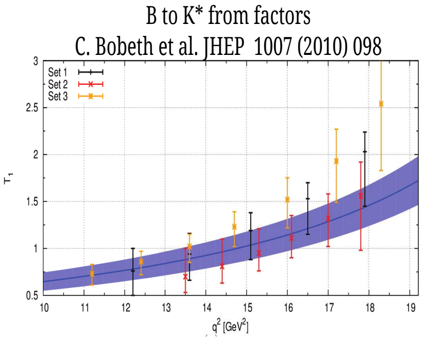


Figure: $B \to K^* ll$ form factors from LCSR + quenched lattice result.

- Errors from previous lattice calculations are large. (greater than 20%.)

Studies of $B \to K/K^* ll$ form factors from lattice QCD

Quenched lattice QCD:

- A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)
- D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
- L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
- A. Abada et al. Phys. Lett. B 365, 275 (1996)
- *etc.*

Recent studies on dynamical Nf=2+1 flavors ensembles:

- FNAL/MILC: ($B \to Kll$) hep-lat/1111.0981
- HPQCD: ($B \to Kll$)
- Cambridge/W&M/Edinburgh group: ($B \to K/K^* ll$) hep-ph/1101.2726
- D. Becirevic et al., hep-ph/1209.0969

Studies of $B \to K/K^*ll$ form factors from lattice QCD

Recent Lattice QCD calculations have these improvements:

- More measurements: smaller statistical error
- Improved actions: discretization error at the higher order of a, am, etc.
- $N_f = 2 + 1$ dynamical configurations: sea quark effects are included.
- Multiple lattice spacings: good for continuum extrapolation
- Close to physical light quark masses: smaller chiral extrapolation error
- Lattice ensembles on physical light quarks masses become available and used by FNAL/MILC, HPQCD and RBC/UKQCD.

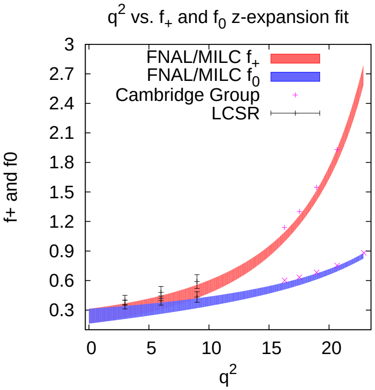
Lattice QCD calculations has been improved systematically towards physical regime in the last decades.

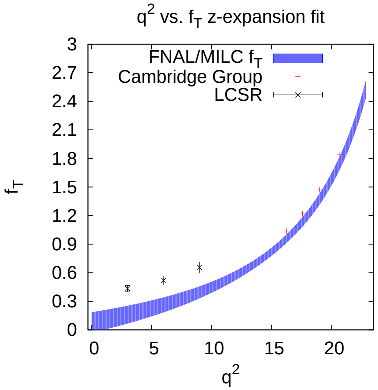
Current lattice studies of $B \to K/K^* ll$ form factors

	FNAL/MILC	HPQCD	Cam./W/Edinb.
	$B \to K$	$B \to K$	$B \to K/K^*$
sea quark	2+1f Asqtad	2+1f Asqtad	2+1f Asqtad
valance s	Asqtad	HISQ	Asqtad
valance b	Fermilab b	NRQCD	NRQCD
usd ens.	4c+5f+2sf+1uf	3c+2f	2c+1f
analysis	SχPT+z-exp.	modified z-exp.	modified z-exp.

- Three lattice groups work on the same form factors with different methods: good for the consistency check.
- The form factors will be on the whole q^2 range and can be compared with LCSR.

Preliminary $B \to Kll$ form factors from FNAL/MILC





- Precise form factors (total error $< 5\%$) at large q^2.
- The form factors on the whole q^2 are available from lattice QCD.
- Consistent with the Cambridge group's preliminary results and LCSR.
- Final results on form factors will be available soon.

Preliminary $B \to Kll$ form factors from HPQCD

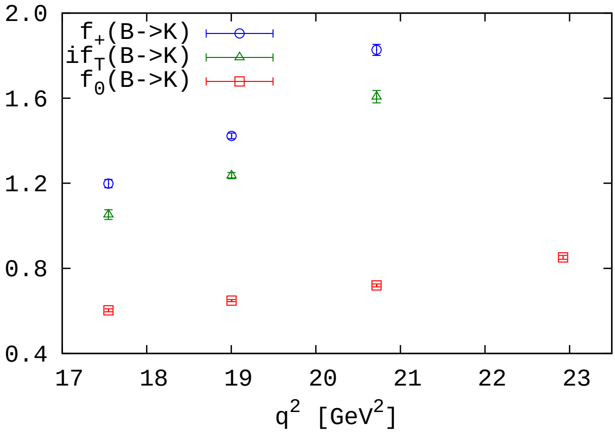


Figure: Preliminary lattice $B \to Kll$ from factors on coarse(a=0.12fm) m_l/m_s=0.2 ensemble. (Thanks C. Bouchard)

$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.

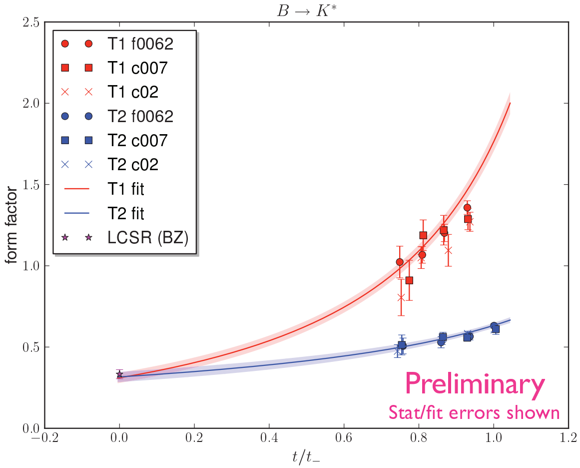


Figure: Preliminary results on $B \to K^* ll$ T_1 and T_2 vs. $q^2/q^2_{\max}$ (by M. Wingate at lattice 2012)

$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.

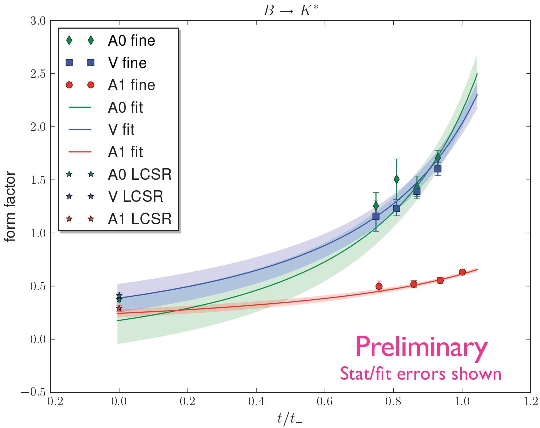


Figure: Preliminary results on $B \to K^* ll$ V, A_0, and A_1 vs. q^2/q^2_{max}. (by M. Wingate at lattice 2012)

Others topics in flavors physics

Other related B semileptonic decays:

- FNAL/MILC: $B \to \pi$, $B_s \to \pi$, $B_s \to K$.
- HPQCD: $B \to \pi$, $B_s \to K$, $B_s \to \phi$.
- RBC/UKQCD: $B \to \pi$

Useful links:

- Lattice QCD annual conference: latest lattice results+two reviews on lattice flavor physics every year.
- Lattice Averages website (www.latticeaverages.org)
- CKM conference (2012, 2010, etc.)

Conclusions

- Lattice QCD is an important tool to calculate form factors in the $B \to K/K^* ll$.
- $B \to Kll$ form factors ($< 5\%$ error at large q^2) from FNAL/MILC will be available soon. The results from groups be available in the future.

We expect to see more contributions from lattice QCD to flavor physics results in the future.