Global fits of the Unitarity Triangle using the Scan Method

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Fit Methodology

- The scan method is a frequentist-based fitting technique of the CKM matrix that makes explicit the fact that the distribution of theory uncertainties is a priori unknown
- It therefore accounts for theory uncertainties in the QCD parameters f_{Bs}, f_{bs}/f_{bd} , B_{Bs}, B_{bs}/B_{Bd}, and B_K and the CKM matrix elements V_{ub} and V_{cb} by scanning over the range in the theory uncertainty using a fixed grid or MC methods, at each scan point doing a fit based on experimental uncertainties only
- Typically, we combine measurements of Δm_d, Δm_s, ε_K, V_{cb}, V_{ub}, V_{us}, V_{ud}, $\sin 2\beta$, α, and γ in the χ^2
- We constrain the mean values of the QCD parameters η_{cc}, η_{ct}, η_{tt}, and η_b to their predicted values with Gaussian functions in which the width is fixed to the "statistical" uncertainty obtained from the lattice ➔ yields the terms (T_n) in the χ^2
- The ensemble of acceptable fits is used to define a 95% CL range in the unitarity triangle parameters

Fit Methodology

- For each point in the theory parameter space we minimize the function

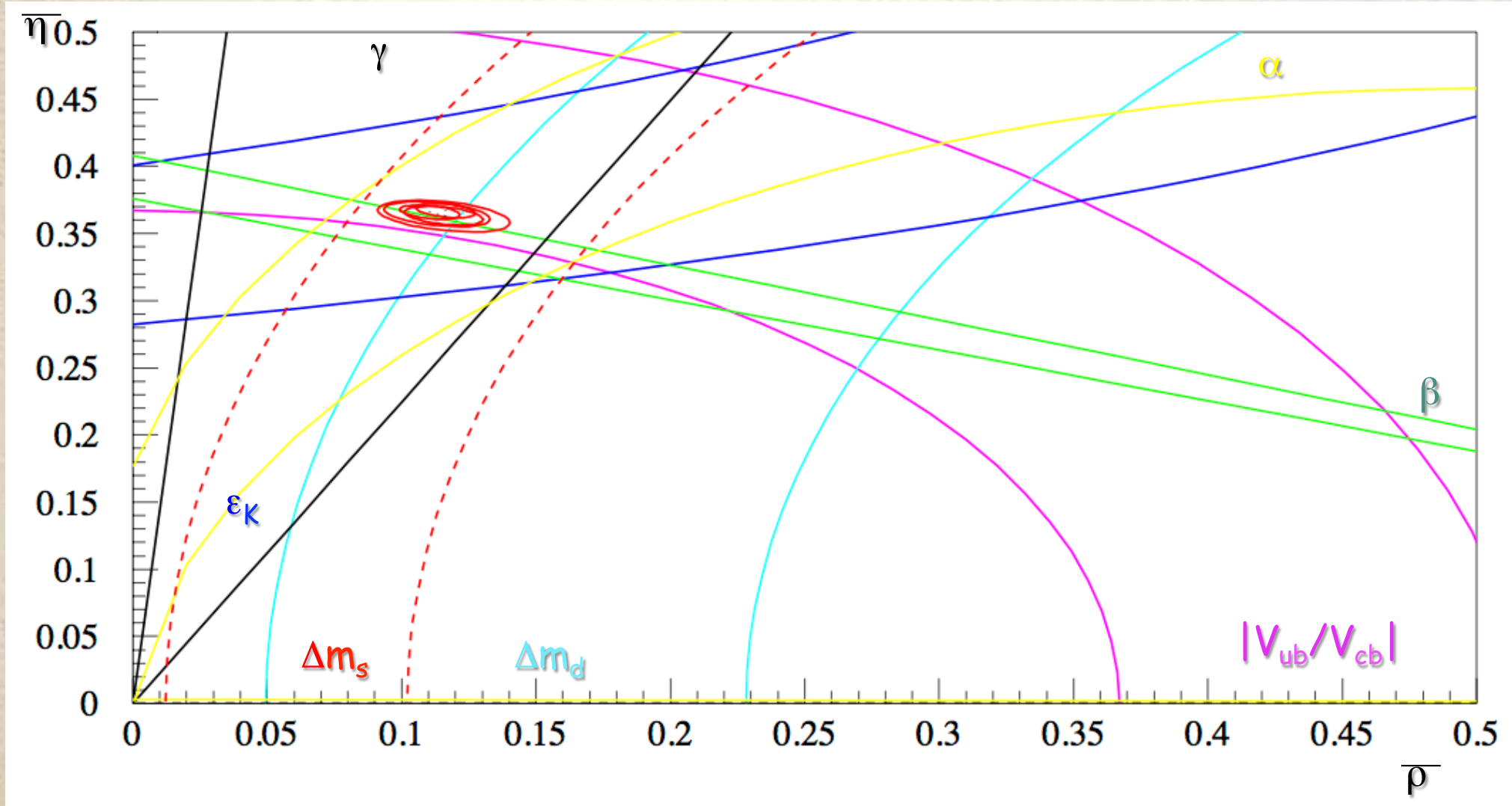
p_i are parameters $(A, \lambda, \ldots)$, t_m are QCD parameters M_k are measurements

$$\chi^2(\bar{\rho},\bar{\eta};p_i,t_i) = \left(\frac{\left\langle \Delta m_{B_{d,s}} \right\rangle - \Delta m_{B_{d,s}}(\bar{\rho},\bar{\eta};p_i,t_m)}{\sigma_{\Delta m_{d,s}}}\right)^2 + \left(\frac{\left\langle \left|V_{cb,ub,ud,us}\right| \right\rangle - V_{cb,ub,ud,us}(\bar{\rho},\bar{\eta};p_i,t_m)}{\sigma_{V_{cb,ub,ud,us}}}\right)^2$$
$$+ \left(\frac{\left\langle \left|\varepsilon_K\right| \right\rangle - \left|\varepsilon_K\right|(A,\bar{\rho},\bar{\eta};p_i,t_m)}{\sigma_{\varepsilon}}\right)^2 + \left(\frac{\left\langle a_{\psi K_s} \right\rangle - \sin 2\beta(\bar{\rho},\bar{\eta},p_i)}{\sigma_{\sin 2\beta}}\right)^2 + \sum_k \left(\frac{\left\langle \mathcal{M}_k \right\rangle - \mathcal{M}_k(p_i)}{\sigma_{\mathcal{M}_k}}\right)^2$$
$$+ \sum_n \left(\frac{\left\langle \mathcal{T}_n \right\rangle - \mathcal{T}_n(p_i,t_m)}{\sigma_{\mathcal{T}_n}}\right)^2 + \left(\frac{\left\langle \alpha \right\rangle - \alpha(\bar{\rho},\bar{\eta},p_i)}{\sigma_{\alpha}}\right)^2 + \left(\frac{\left\langle \gamma \right\rangle - \gamma(\bar{\rho},\bar{\eta};p_i)}{\sigma_{\gamma}}\right)^2$$

- We add measurements for m_t, m_c, B masses, B lifetimes, other CKM matrix elements to account for possible correlations
- The region encompassing all accepted fits in the $\bar{\rho}$-$\bar{\eta}$ plane then defines the range at **95%** CL of parameters compatible with the CKM ansatz without assumption as to the distribution of the theory uncertainties
- The contours of all accepted fits are overlaid
- We also study correlations among the theory parameters

Fit Methodology

- For illustrative purpose we show the overlay a few contours



Inputs for Comparing Different Methods

- We use the legacy book inputs to compare our results with those from CKMfitter and UTFIT

- Note that V_{ub} and V_{cb} has only an experimental uncertainty

  ➔ no scanning over these parameters here

- α and γ are input here by by their central values and uncertainty

68% CL contours

Observable	Value
m_t	174.65±0.11 Gev/c^2
m_c	1.29±0.11 Gev/c^2
m_c [PDG12]	1.275±0.025 Gev/c^2
Δm_d	0.507±0.004 ps^{-1}
Δm_s	17.70±0.08 ps^{-1}
ε_K	(2.228±0.0011)*10^{-3}
V_{cb}	(4.06±0.13)*10^{-2}
V_{ub}	(3.89±0.44)*10^{-3}
V_{us}	0.2254±0.0009
V_{ud}	0.97427±0.00021
sin2β	0.676±0.02
α [°]	90±5
γ [°]	76±10

Inputs for Comparing Different Methods

- We use the lattice QCD parameters given by the lattice group before the ICHEP 2012 conference

- The η parameters are not scanned but are treated with Gaussian uncertainties (we retain the possibility of scanning them as well)

- Parameterize η_{cc} and its uncertainties in terms of m_c and α_s

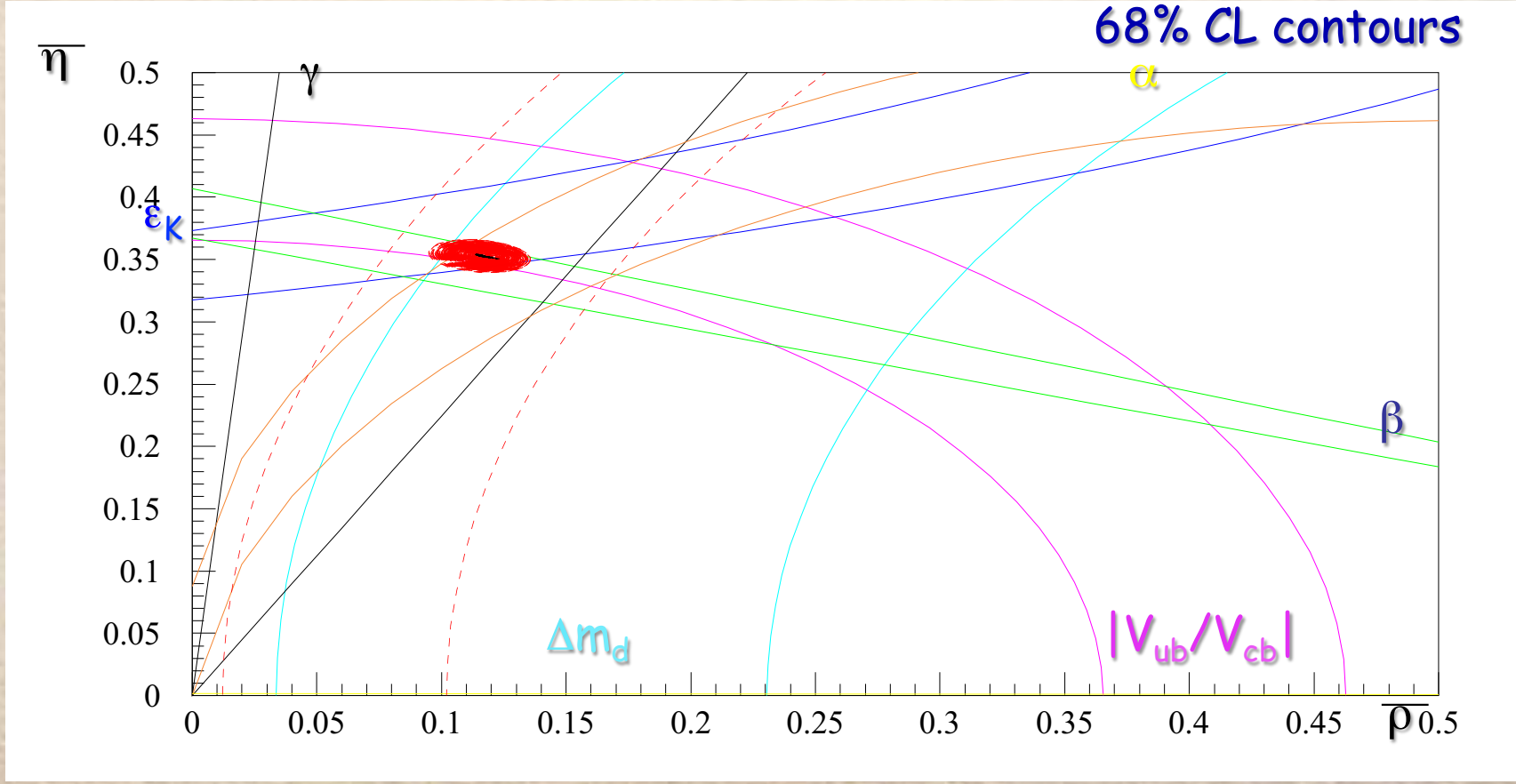
- So here we scan over B_K, f_{Bs} and f_{Bs}/f_{Bd} only

Observable	Value
f_{Bs} [MeV]	250±5.4±11
f_{Bs}/f_{Bd}	1.215±0.012±0.015
B_{Bs}	1.33±0.06
B_{Bs}/B_{Bd}	1.05±0.07
B_K	0.737±0.006±0.020
η_{cc}	1.39±0.35
η_{tc}	0.5765±0.0065
η_{tt}	0.47±0.04
η_b	0.551±0.007

Our Fit Results for 68% Contours

- First, we test the performance wrt CKMfitter and Utfit fitting 19 measurements (V_{ud}, V_{us}, V_{cb}, V_{ub}, ε_K, Δm_d, Δm_s, sin 2β, α, γ, f_{Bs}, B_{Bs}, f_{Bs}/f_{Bd} , B_{Bs}/B_{bd}, B_K, m_t, m_c τ_{Bd}, τ_{Bs}) with 13 parameters (ρ, η, A, λ,f_{Bs}, B_{Bs}, f_{Bs}/f_{Bd} , B_{Bs}/B_{bd}, B_K, m_t, m_c τ_{Bd}, τ_{Bs})

- For fits with $P(\chi^2)>32\%$, we plot 1σ contours in the $\bar{\rho}$-$\bar{\eta}$ plane ➔ plot shows the overlay of all individual contours of accepted fits



Comparison with CKMfitter & UTfit

- The allowed region is the envelope of all accepted contours
- All 3 methods yield similar results at 68% CL
- We redo the fits by separating experimental uncertainties from theory uncertainties in V_{ub} and V_{ub} using the approach in (arXiv: 0806.0530)

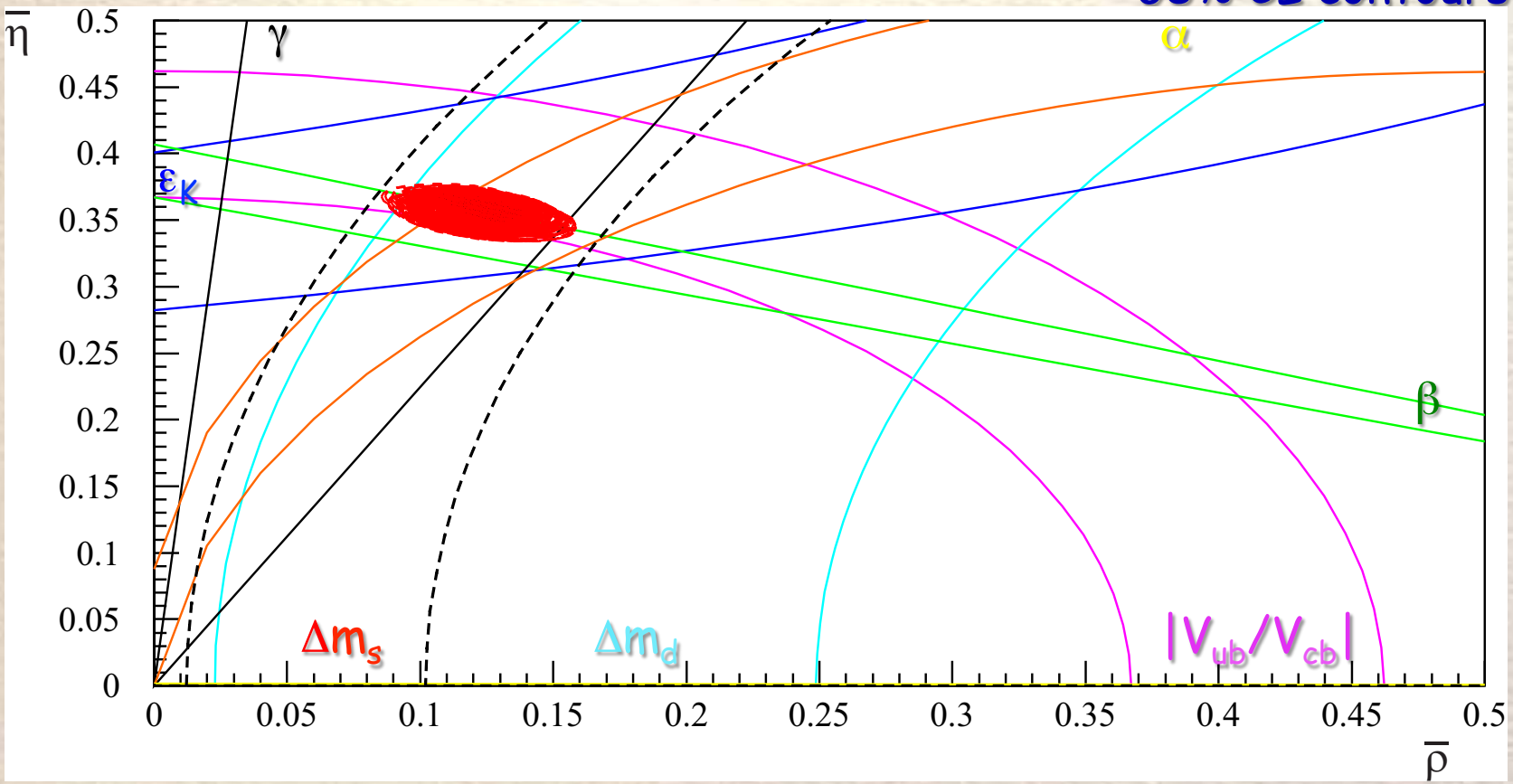
Parameter	Scan method	Scan m. (V_{ub},V_{cb})	CKMfitter	UTfit
$\bar{\rho}$	$0.121^{+0.015}_{-0.025}$	$0.132^{+0.027}_{-0.06}$	0.121±0.02	0.125±0.022
$\bar{\eta}$	$0.351^{+0.015}_{-0.012}$	$0.359^{+0.025}_{-0.025}$	0.349±0.012	0.347±0.014
β [°]	$21.8^{+0.6}_{-0.7}$	$22.5^{+0.4}_{-1.3}$	21.7±1	21.6±0.8
α [°]	$87.2^{+2.3}_{-4.0}$	$87.7^{+9.2}_{-5.3}$	87.5±3.2	87.9±3.4
γ [°]	$71.0^{+4.1}_{-2.2}$	$69.8^{+9.8}_{-4.6}$	70.9±3.2	70.4±3.4

- Central value is that with highest $P(\chi^2)$ uncertainties are maximum & minimum values of the envelope of all contours

V_{cb}	0.0406±0.006±0.011
V_{ub}	0.00389±0.0001±0.0048

Results with Scanning over V_{ub} and V_{cb}

- Now we redo the same fits but we scan over V_{ub} and V_{cb} in addition



- The allowed $\bar{\rho}$-$\bar{\eta}$ region now increases by a factor of ~ 2

New Fit Inputs

- Since ICHEP some inputs have changed

Observable	Value
V_{cb}	$(4.1\pm0.0069\pm0.009)*10^{-2}$
V_{ub}	$(4.14\pm0.31\pm0.39)*10^{-3}$
$\sin2\beta$	0.676 ± 0.02
α [°]	90+2.2-2.1
γ [°]	76.4+4.1-3.8

our inputs see later

Observable	Value
f_{Bs} [MeV]	$227.6\pm2.2\pm4.5$
f_{Bs}/f_{Bd}	$1.201\pm0.012\pm0.012$
B_{Bs}	$1.33\pm0.018\pm0.06$
B_{Bs}/B_{Bd}	$1.05\pm0.025\pm0.07$
B_K	$0.7643\pm0.0034\pm0.00908$

- V_{ub} and V_{cb} are taken from PDG 2012 but we separate theory from experimental uncertainties (provided by R. Kowalewski)
  ➔ Note: V_{ub} and V_{cb} is an average over barely consistent inclusive & exclusive results that include scale factors of 2.6 and 2, respectively

- α is extracted by fitting all measured branching fractions, CP asymmetries, S and C amplitudes for time dependent results

- γ is extracted from GLW, ADS and Dalitz plot analyses in $B\to D^{(*)}K$, $B\to DK^*$, $B\to D^{(*)}\pi$, $B\to D^{(*)}\rho$ (combine γ and $\sin(2\beta+\gamma)$ measurements)

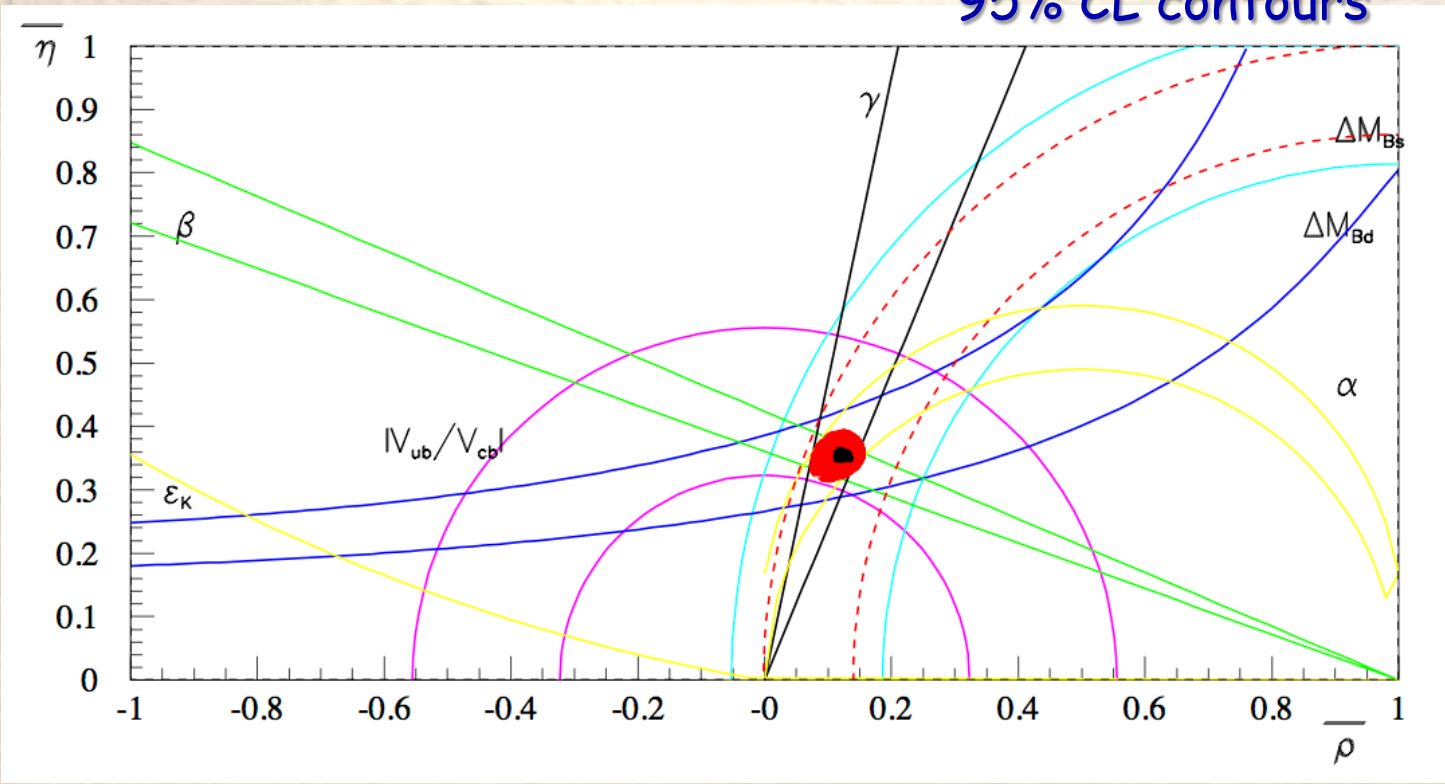
- Use latest lattice QCD parameters

CKM Fits with Updated Inputs

- Fit 23 measurements with updated inputs (no $\mathcal{B}(B \to \tau \nu)$) and and 13 fit parameters

- Plot a 95% CL contour for each accepted fit

95% CL contours



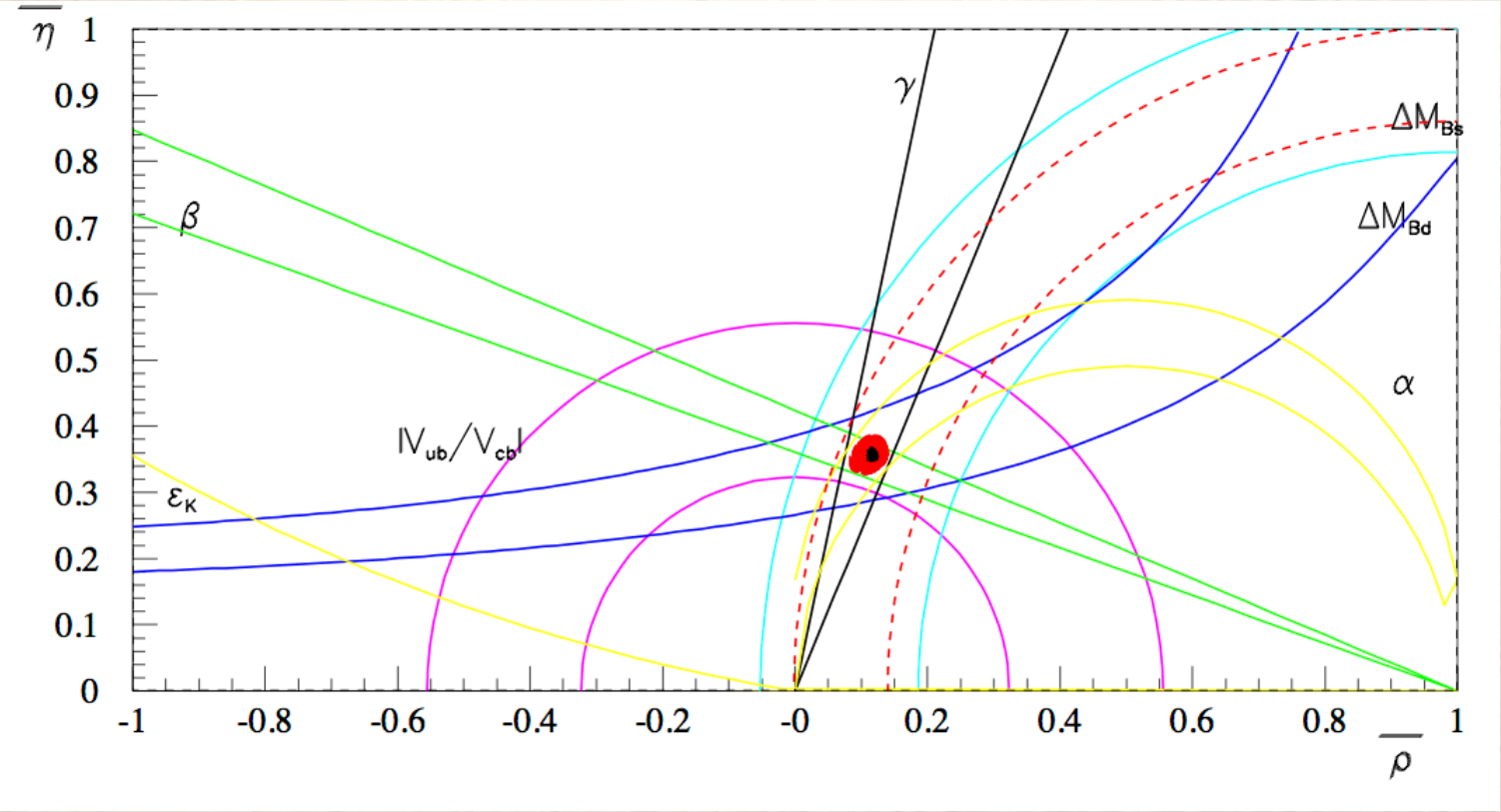
- About 37% of all points in the theory parameter space give successful fits

Global Fits of the CKM matrix

- Same fits but include now $\mathcal{B}(B \to \tau\nu)=(1.66\pm0.33)\times10^{-4}$ (HFAG)
  (24 measurements and 13 fit parameters)

does not include new Belle measurement

95% CL contours



- The number of successful fits is now reduced by another factor of 3

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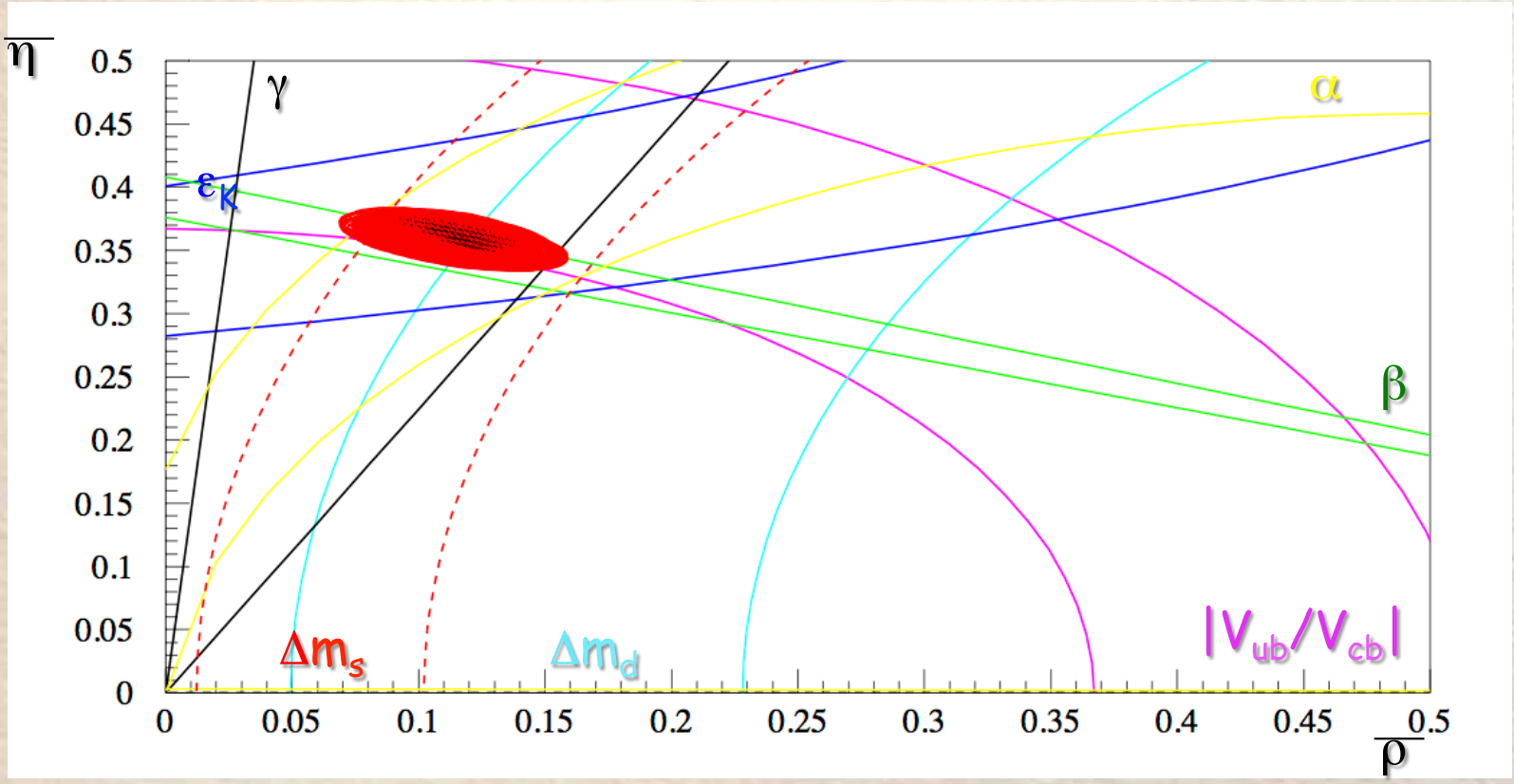

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Results with and without $\mathcal{B}(B\to\tau\nu)$

- $\mathcal{B}(B\to\tau\nu)$ reduces allowed region in the $\bar{\rho}$-$\bar{\eta}$ plane ➔ allowed region is still quite large

- Overall fit probability becomes worst, loose about half of the previously successful fits

Parameter	Scan-fit w/o $B\to\tau\nu$	Scan-fit w $B\to\tau\nu$
$\bar{\rho}$	0.071—0.148	0.082—0.14
$\bar{\eta}$	0.314—0.393	0.327—0.387
β [°]	19.2—24.1	20.0—23.7
α [°]	80.5—91.1	81.7—89.7
γ [°]	67.6—78.1	68.7—76.6

(95% CL regions)

- The 95% CL regions in $\bar{\rho}$-$\bar{\eta}$ are maximum and minimum values of the envelope of all contours, the 95% CL regions on the angles are calculated from the $\bar{\rho}$-$\bar{\eta}$ values

Fits to Extract Angles

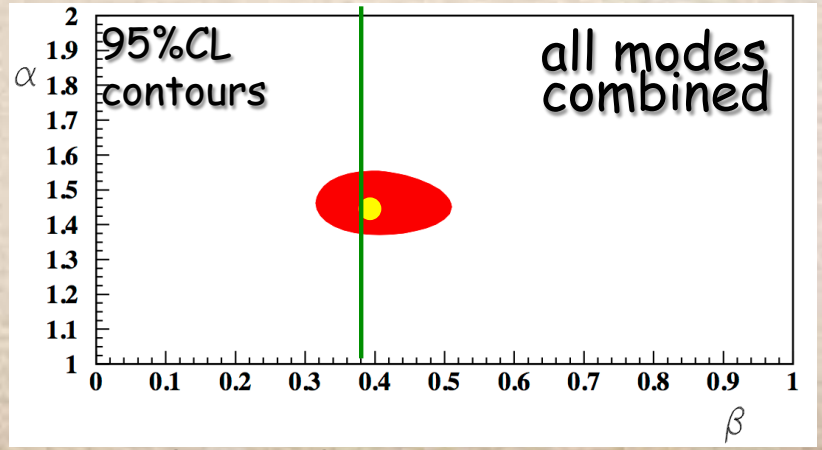
- We fit branching fractions, CP asymmetries, S & C of time-dependent asymmetries of all measured $b\to u\bar{u}d$ and $b\to u\bar{u}s$ modes to determine γ
- Following the Gronau-Rosner approach, the amplitudes are parameterized in terms of tree, color-suppressed tree, penguin, singlet penguin, W-annihilation/W-exchange, electroweak and color-suppressed electroweak diagrams (up to λ^3 beyond leading order)
- For $b\to u\bar{u}s$ modes, SU(3) breaking is included for penguin, tree, color-suppressed tree and singlet penguin amplitudes (suppressed by λ)
- For $B\to PP$ modes, we use 42 measurement to fit 20 parameters
- For $B\to PV$ modes, we use 69 measurement to fit 35 parameters
- For $B\to VV$ modes, we use 52 measurement to fit 34 parameters
- For all modes combined including 17 $B\to a_1 P$ measurements and 13 parameters, we fit a total of **173** measurements with **92** parameters ($P(\chi^2)=10\%$)
- Note that **no** V_{ub}, V_{cb} and sin 2β measurements are used here

Fits to Extract α

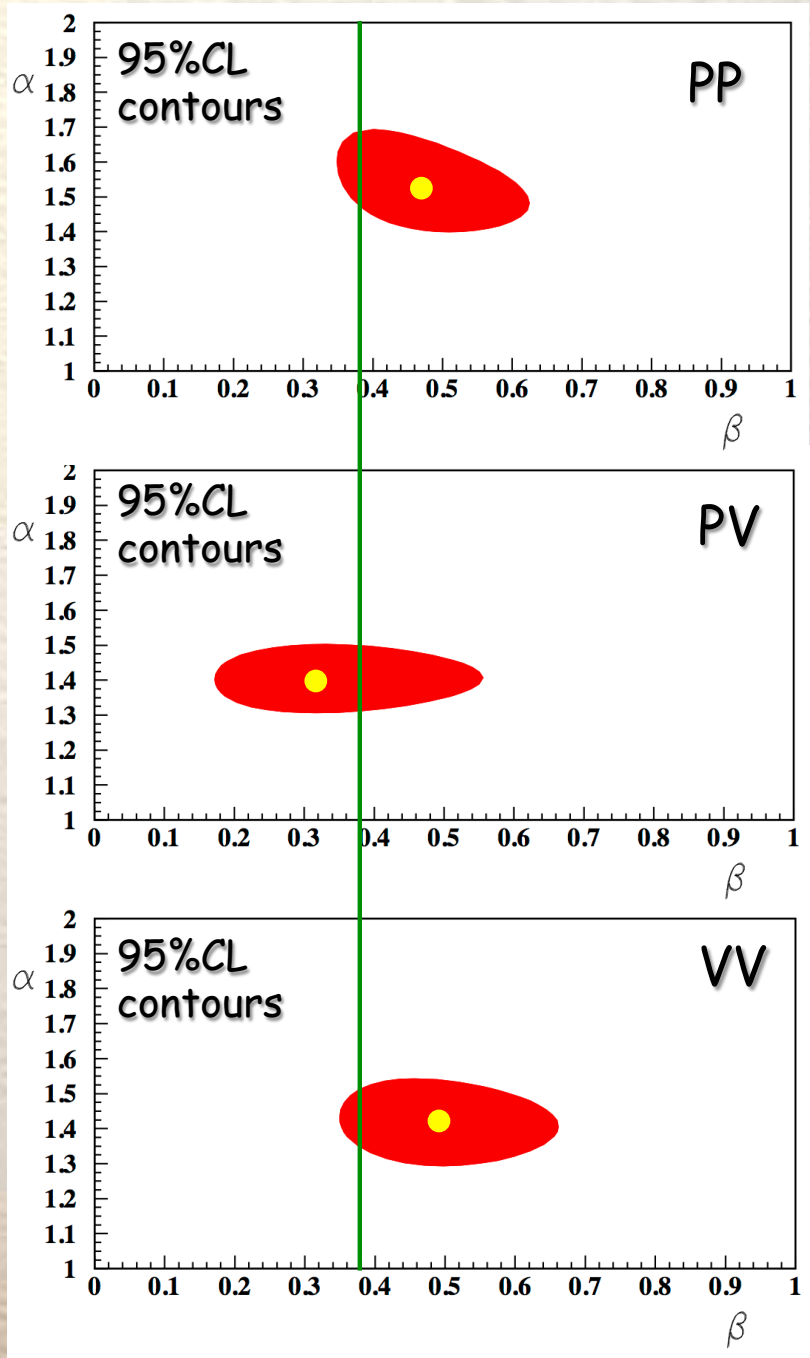
- We extract 95% CL α-β contours for the individual and combined modes

Mode	α [$\pm1\sigma$]	β [$+1\sigma$ -1σ]
$B\to PP$	1.53 ± 0.06	$0.46^{+0.06}_{-0.05}$
$B\to PV$	1.41 ± 0.07	$0.32^{+0.06}_{-0.05}$
$B\to VV$	1.42 ± 0.09	$0.48^{+0.13}_{-0.10}$
combined	$1.478^{+0.038}_{-0.037}$	$0.364^{+0.029}_{-0.028}$

- The combination of all measurements constrains α, while β is less constrained than from direct sin2β measurement from $c\bar{c}s$ modes (β=0.374±0.014)



Green line shows measured β



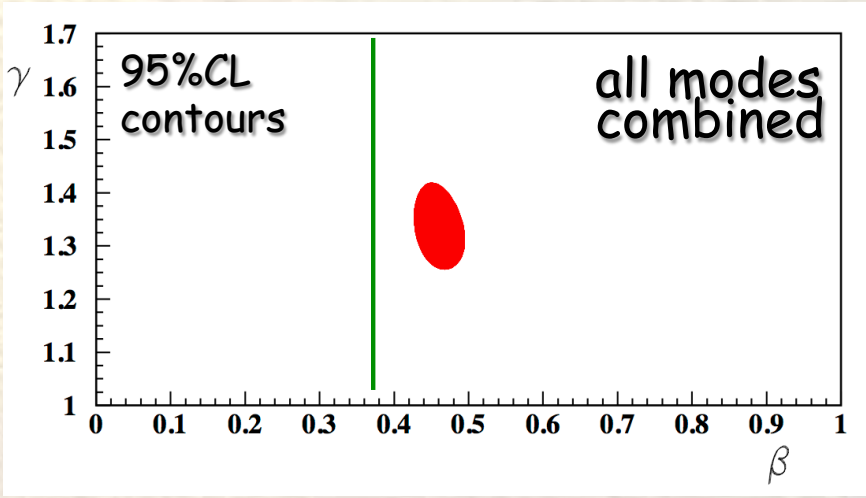
Fits to extract γ

- Use all available GLW, ADS and Dalitz plot measurements in $B\to D^{(*)}K$, $B\to DK^*$, $B\to D^{(*)}\pi$, $B\to DK^*$ and the $\sin(2\beta+\gamma)$ results from $B\to D^{(*)}\pi$ and $B\to D\rho$ time-dependent analyses

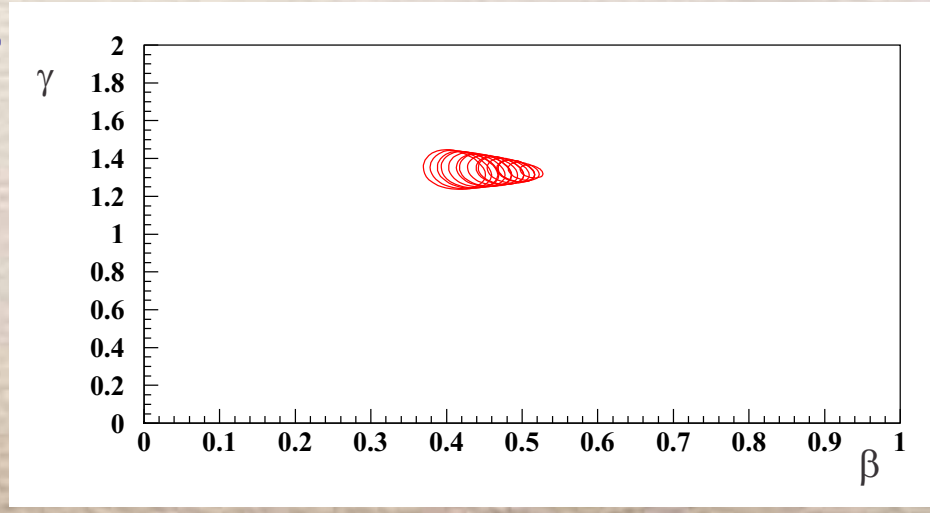
- Parameterize measurements in terms of $r=|A(b\to u)/A(b\to c)|$ separating out CKM factor and strong phase

- Use 56 measurements to extract 19 fit parameters, constrain $V_{us}V^*_{ub}/V_{cs}V^*_{cb}$ and $V_{ud}V^*_{ub}/V_{cd}V^*_{cb}$ to measured ratios (central values) ➔ $P(\chi^2)=6\%$

- β extracted in this approach is higher than that from the $\sin 2\beta$ measurements



Scan over V_{ub} and V_{cb} shows β -V_{ub}/V_{cb} correlation



Mode	γ [$\pm1\sigma$]	β [$\pm1\sigma$]
all	$1.33^{+0.072}_{-0.066}$	0.46±0.029

Conclusions

- For the same inputs the three fitting methods yield similar results, even with a scan over B_K, f_{Bs} and f_{Bs}/f_{Bd}
- Due to the discrepancy between inclusive and exclusive V_{ub} and V_{cb} measurements theory uncertainties are sizable and scanning over them increases the allowed region in the $\bar{\rho}$-$\bar{\eta}$ plane significantly
- Measurements of Δm_d, Δm_s, ε_K, V_{cb}, V_{ub}, other CKM parameters, sin2β, α, and γ are in good agreement with the SM, even the inclusion of $B^{\pm} \to \tau^{\pm} \nu$ measurements reduces the allowed region in the $\bar{\rho}$-$\bar{\eta}$ plane
  ➔ number of successful fits decreases by 2/3
- All measurements in $B \to PP$, $B \to PV$, $B \to VV$ and $B \to a_1 P$ modes are consistent with the Standard Model at the 10% level ➔ yield a precise measurement of α (correlations between α and β are small)
- All GLW, ADS and Dalitz plot measurements in $B \to D^{(*)}K$, $B \to DK^*$, $B \to D^{(*)}\pi$, $B \to DK^*$ and the sin(2β+γ) results from $B \to D^{(*)}\pi$ and $B \to D\rho$ time-dependent analyses are in agreement with the Standard Model ➔ yield a precise measurement of γ

Conclusions

- With this method, making no assumptions about the distribution of theory errors within specified ranges, there is no tension with the Standard Model
- As long as there acceptable fits within the broad region of reasonable values of theory parameters, we find consistency with the SM