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V_{us} from kaon decays

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V_{us}, CKM unitarity, gauge universality

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}} W_\alpha^+ \left(\overline{\mathbf{U}}_L \mathbf{V}_{\mathrm{CKM}} \gamma^\alpha \mathbf{D}_L + \overline{e}_L \gamma^\alpha \nu_{e\,L} + \overline{\mu}_L \gamma^\alpha \nu_{\mu\,L} + \overline{\tau}_L \gamma^\alpha \nu_{\tau\,L} \right) \quad + \quad \mathrm{h.c.}$$

↑ *Single gauge coupling*

↑ *Unitary matrix*

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

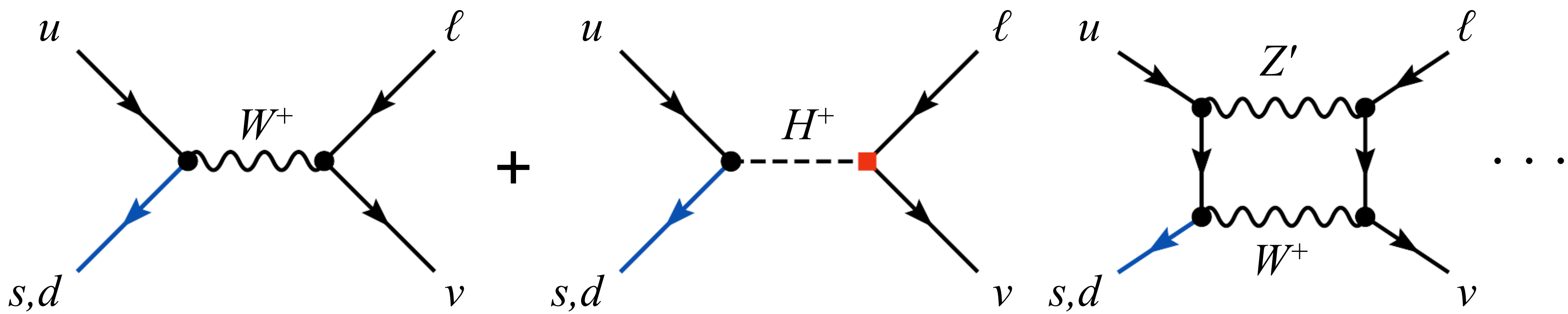
$|V_{ub}|^2 \approx 2\times10^{-5}$

Most precise test of CKM unitarity

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \stackrel{?}{=} \quad G_{\mathrm{CKM}}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2 \right) / M_W^4$$

Physics beyond the Standard Model can break gauge universality:



V_{us} from kaon decays: A modern history

→ 2002 (2004 PDG)	**Old $K_{\ell 3}$ data give 1 – $\lvert V_{ud}\rvert^2$ – $\lvert V_{us}\rvert^2$ = 0.0035(15)** A 2.3σ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-present	**Many new measurements from KTeV, ISTRA+, KLOE, NA48**

- **BRs, lifetimes, form-factor slopes**
- **Much higher statistics** than older measurements
- Importance of **radiative corrections**
- Proper reporting of **correlations** between measurements

2008-beyond	**Much progress on hadronic constants from lattice QCD** **Value of V_{us} used in precision tests of the Standard Model**

Experiment, theory, and evaluation

V_{us} from $K_{\ell 3}$ & $K_{\ell 2}$:
- ~100 measurements of ~10 experimental parameters
- ~20 lattice evaluations of 2 hadronic matrix elements
- Radiative and SU(2)-breaking corrections, ChPT results, etc.

FlaviAnet Kaon WG
2006-2010 (EU 6FP)
http://www.lnf.infn.it/wg/vus

Experimental averages, fits, etc
Selection of results (lattice, ChPT, experiments)
Evaluation, discussion and intepretation
Final report: EPJC 69 (2010) 399
This talk is my attempt at a comprehensive update

Corresponding efforts to synthesize results from lattice QCD (beyond V_{us})

FLAG
http://itpwiki.unibe.ch/flag
Active since 2008 (Europe)

FlaviaNet Lattice Averaging Group
LECs, quark masses, Vus
EPJC 69 (2010) 399

LLVdW
www.latticeaverages.org
Active since 2009 (USA)

Includes hadronic constants for B physics, CKM fits, etc.
PRD 81 (2010) 034503

FLAG-2
Active since May 2012
Europe + USA + Japan
Participation by all major lattice collaborations
Expanded physics scope

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

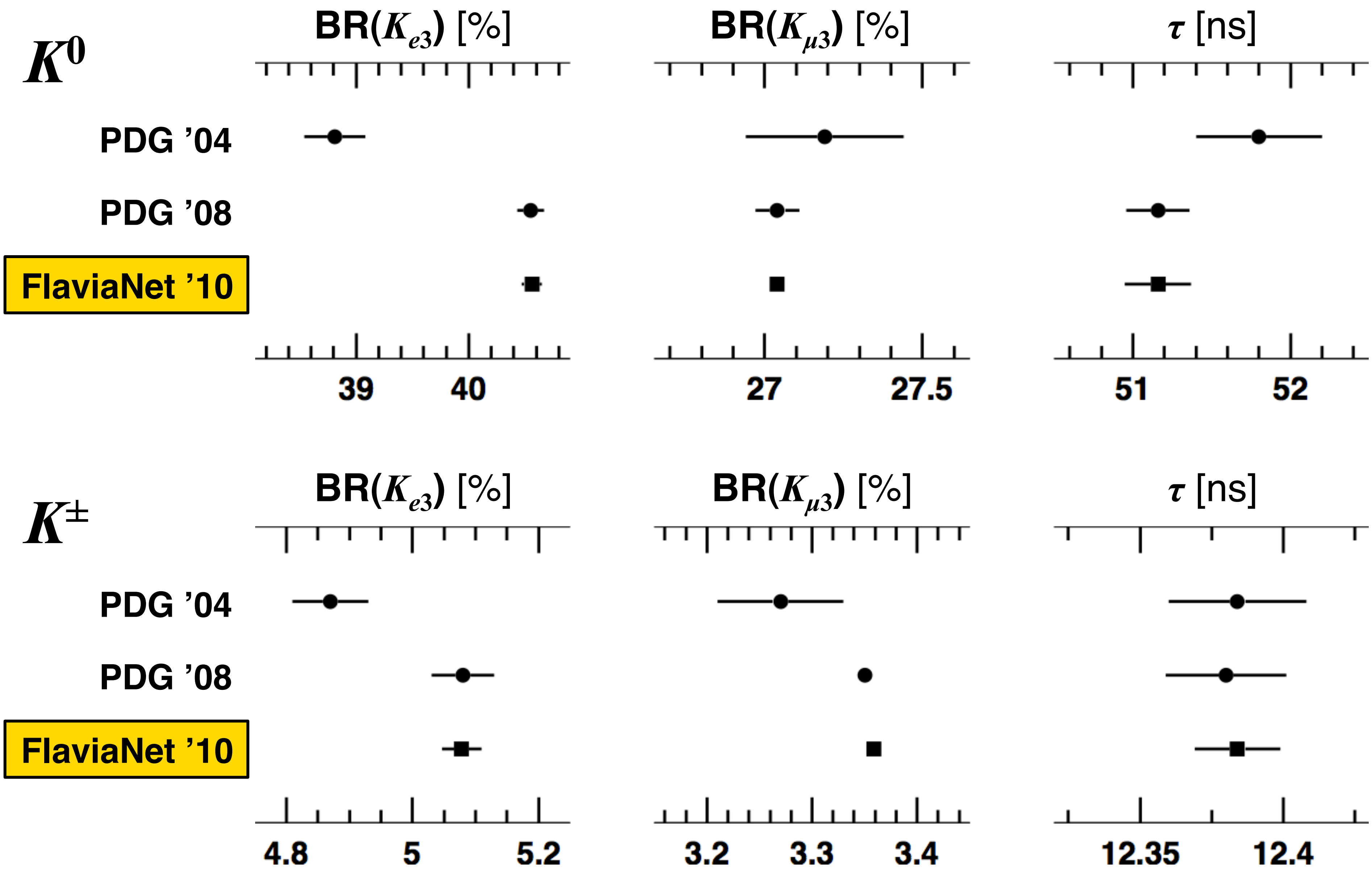
$\Delta_{K\ell}^{\rm EM}$ Form-factor correction for long-distance EM effects

Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	**BR($K^+ \rightarrow \pi^0{}_D e^+ \nu$)/BR($K^+ \rightarrow \pi^0{}_D X^+$)**	**2003**
KTeV	**$\tau(K_S)$**	**2003**
	BR(K_{Le3}), BR($K_{L\mu3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2004**
ISTRA+	**$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$**	**2004**
KLOE	**$\tau(K_L)$**	**2005**
	BR(K_{Le3}), BR($K_{L\mu3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	**2006**
	$\lambda_{+,0}(K_{L\mu3})$	**2007**
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu3}$)	**2008**
NA48	**$\tau(K_S)$**	**2002**
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	**2004**
	BR(K_{Se3}/K_{Le3}), $\lambda_{+,0}(K_{L\mu3})$	**2007**
NA48/2	**BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K+{}_{\mu3}/\pi^+\pi^0$)**	**2007**

Above data set used for official FlaviaNet fits, averages, etc.

Evolution of $K_{\ell 3}$ BR and K lifetime values



Updates: K_L and K_S BRs and lifetimes

FlaviaNet WG

τ_S **= 89.59(6) ps**

KLOE
EPJC 71 (2011)

τ_S **= 89.562(29)(43) ps**

- 2×10^7 $K_S \to \pi^+\pi^-$ decays from 0.4 fb^{-1} '04 data
- Tight track quality cuts & geometric fit
- $\sigma(L_K) \sim$ 0.22-0.27 λ_S (1.3-1.6 mm)
  Measured for 180 bins in (θ_K, ϕ_K)

KTeV
PRD 83 (2011)

τ_S **= 89.589(42)(56) ps**

Re ε'/ε = (21.10 ± 3.43) × 10^{-6}

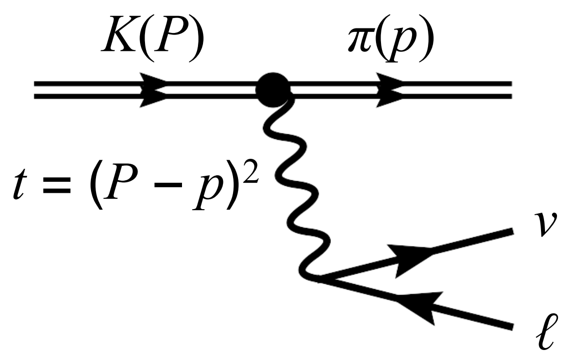
- New analysis of Re ε'/ε with improved Monte Carlo
- From fit to z_{vertex} distribution for regenerator beam without assuming CPT
- Result for Re ε'/ε averaged with NA48 '02 and used to constrain BR($K_L \to \pi^0\pi^0$)/BR($K_L \to \pi^+\pi^-$) in K_L fit

FlaviaNet WG

τ_S **= 89.58(4) ps**

$K_{\ell 3}$ form factors

Hadronic matrix element:



$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \dfrac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element
Parameterize form factors and fit distributions in t (or related variables)

2010 form-factor parameter measurements (K_{e3} and $K_{\mu 3}$)

K_L **KTeV, KLOE, NA48**

K^- **ISTRA+**

NA48/2
2012 preliminary

K^+ and K^- simultaneously
Dedicated min-bias run

2.6 × 10[6] $K^\pm_{\mu 3}$
4.0 × 10[6] $K^\pm_{e3}$

Taylor and pole fits
No dispersive fits (yet)

$K_{\ell 3}$ form-factor parameterizations

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m^2_{\pi^+}} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m^2_{\pi^+}} \right) + \lambda''_{+,0} \left(\frac{t}{m^2_{\pi^+}} \right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Parameterizations incorporating physical constraints

Pole dominance:

$$\tilde{f}_{+,0}(t) = \frac{M^2_{V,S}}{M^2_{V,S} - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m^2_{\pi}} (\Lambda_+ - H(t)) \right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m^2_K - m^2_{\pi}} (\ln C - G(t)) \right]$$

Notes:

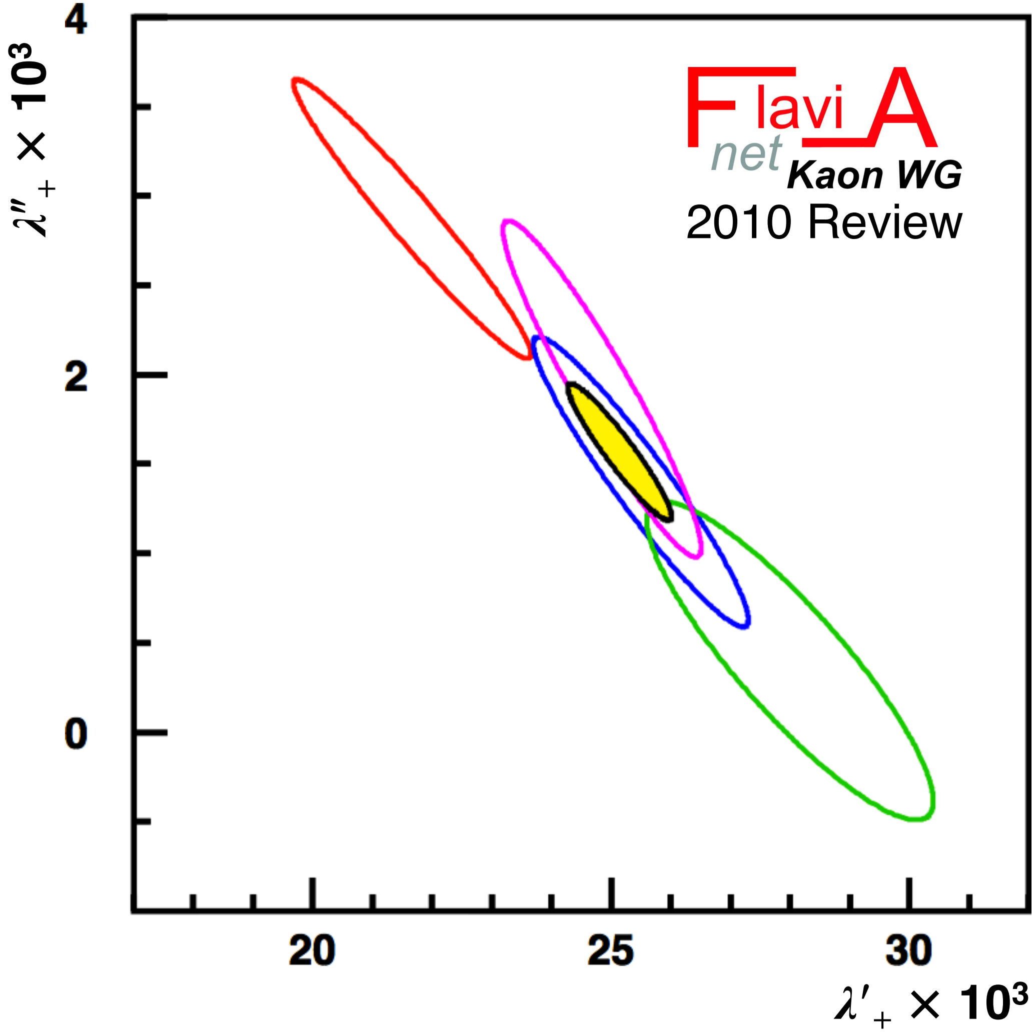
Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Fit to K_{e3} form-factor slopes: 2010

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**



Slope parameters × 10³

$$\lambda'_+ = \mathbf{25.15 \pm 0.87}$$

$$\lambda''_+ = \mathbf{1.57 \pm 0.38}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 5.3/6\ (51\%)$$

Excellent compatibility

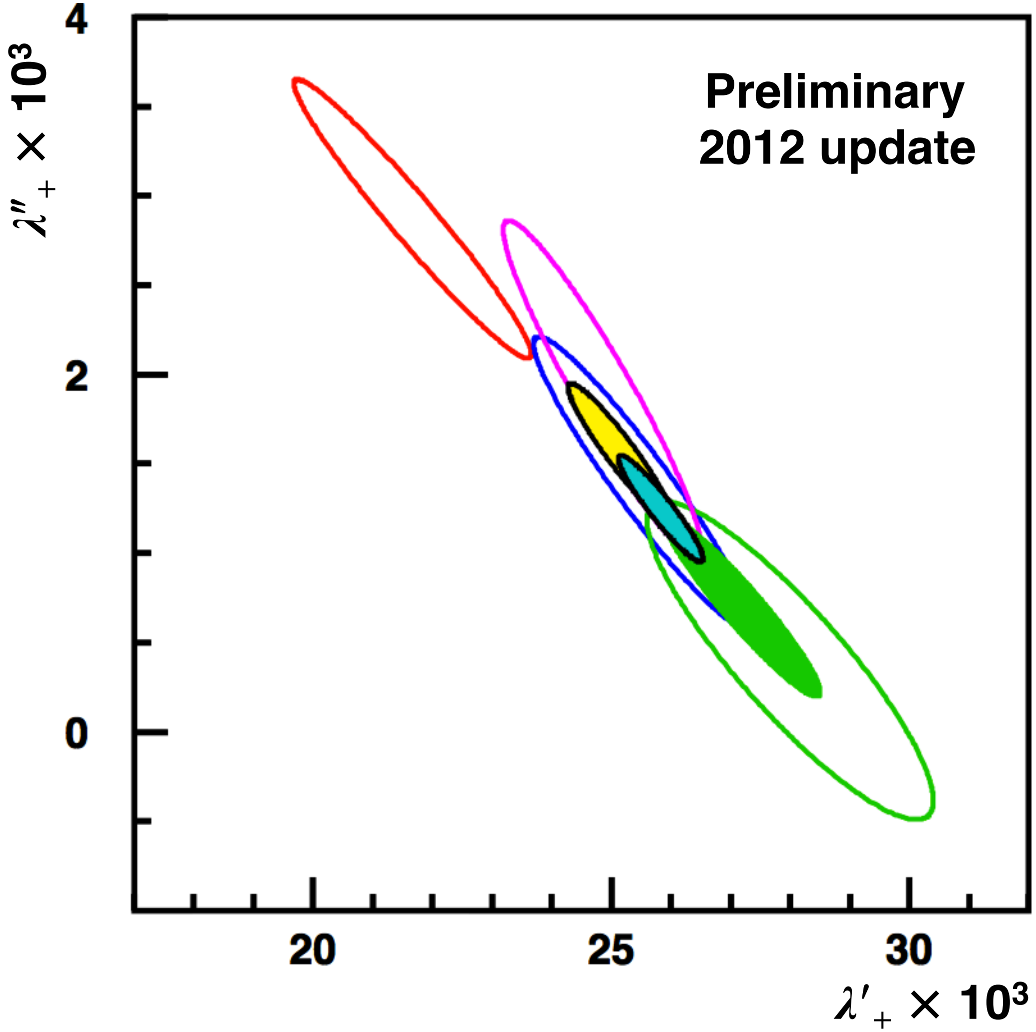
Significance of $\lambda''_+ > 4\sigma$

$I(K^0_{e3})$ **= 0.15463(21)**

$I(K^+_{e3})$ **= 0.15900(22)**

Fit to K_{e3} form-factor slopes: Update

KTeV | KLOE | ISTRA+ | NA48 | NA48/2 '12 prel | 2010 fit | Update



Slope parameters × 10³

$$\lambda'_+ = \mathbf{25.83 \pm 0.71}$$

$$\lambda''_+ = \mathbf{1.25 \pm 0.30}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 7.2/8\ (52\%)$$

Quality of fit unchanged

λ'_+ increased $0.8\sigma_{2010}$

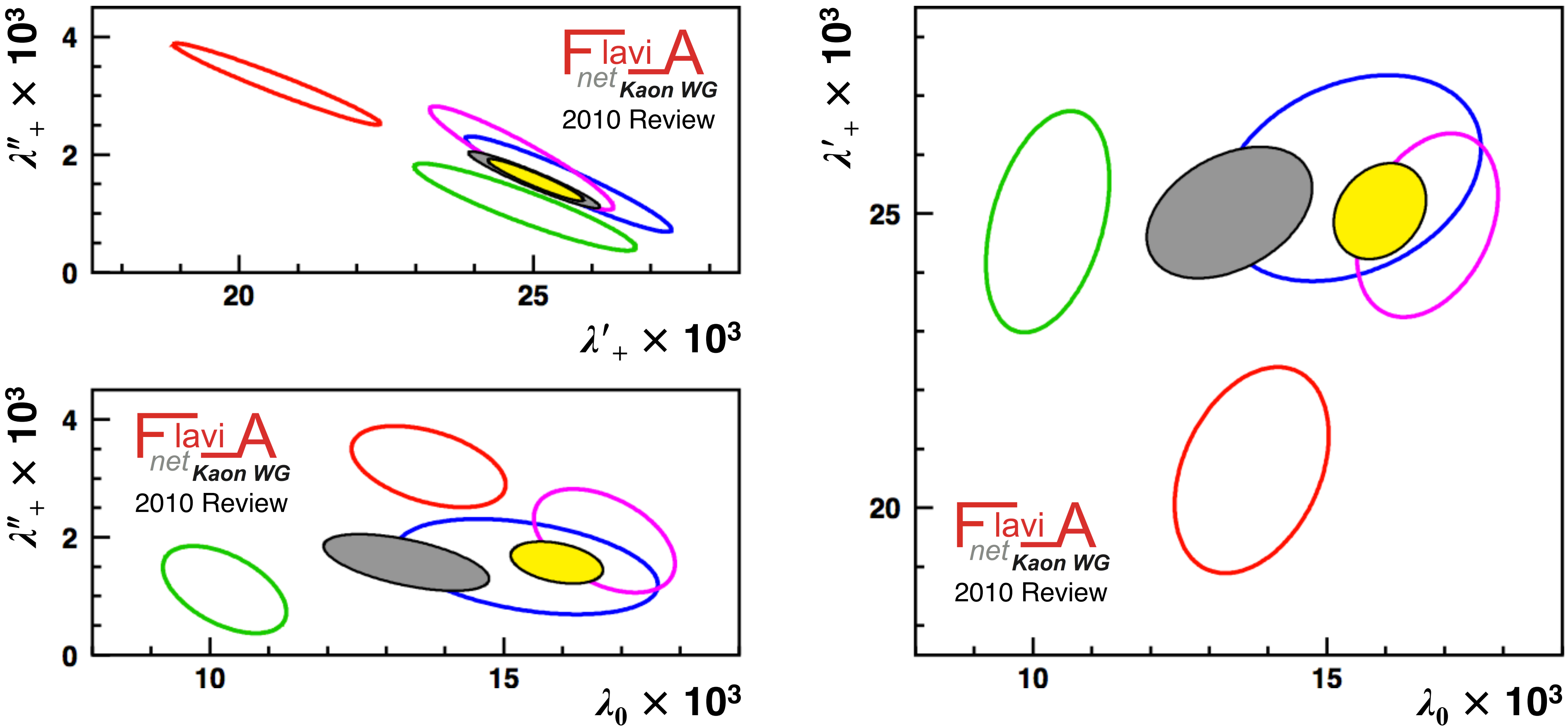
$I(K^0_{e3})$ = 0.15475(18)

$I(K^+_{e3})$ = 0.15912(19)

Increased by +0.08% ($\sim 0.5\sigma_{2010}$)

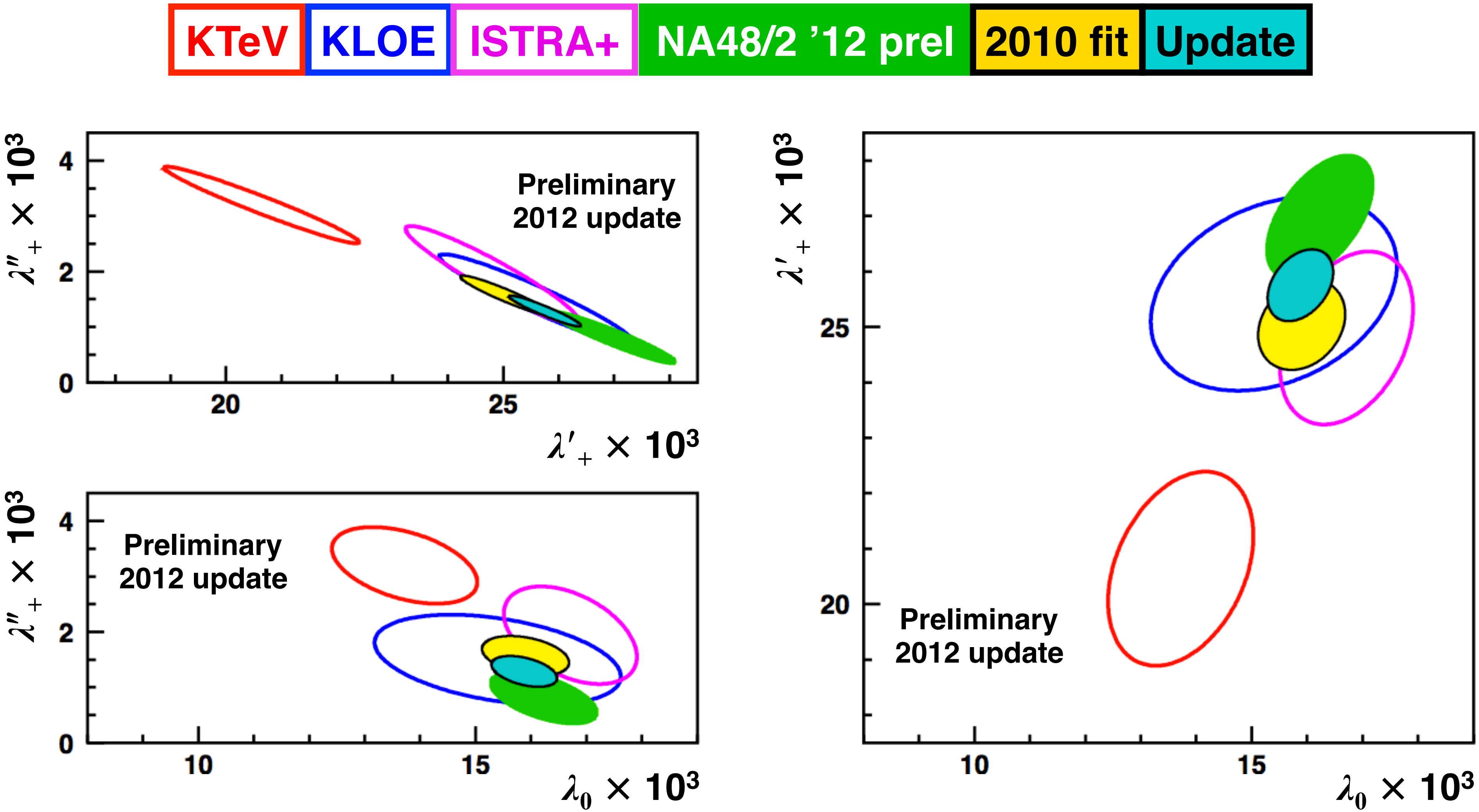
Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: 2010

KTeV	KLOE	ISTRA+	NA48	**2010 fit** (all)	**2010 fit** (no $K_{\mu3}$ NA48)



All data: $\chi^2 = 48/9$ ($P = 3 \times 10^{-7}$)	No NA48 $K_{\mu3}$: $\chi^2 = 12.1/8$ ($P = 14.5\%$)

Fits to $K_{e3} + K_{\mu 3}$ form-factor slopes: Update

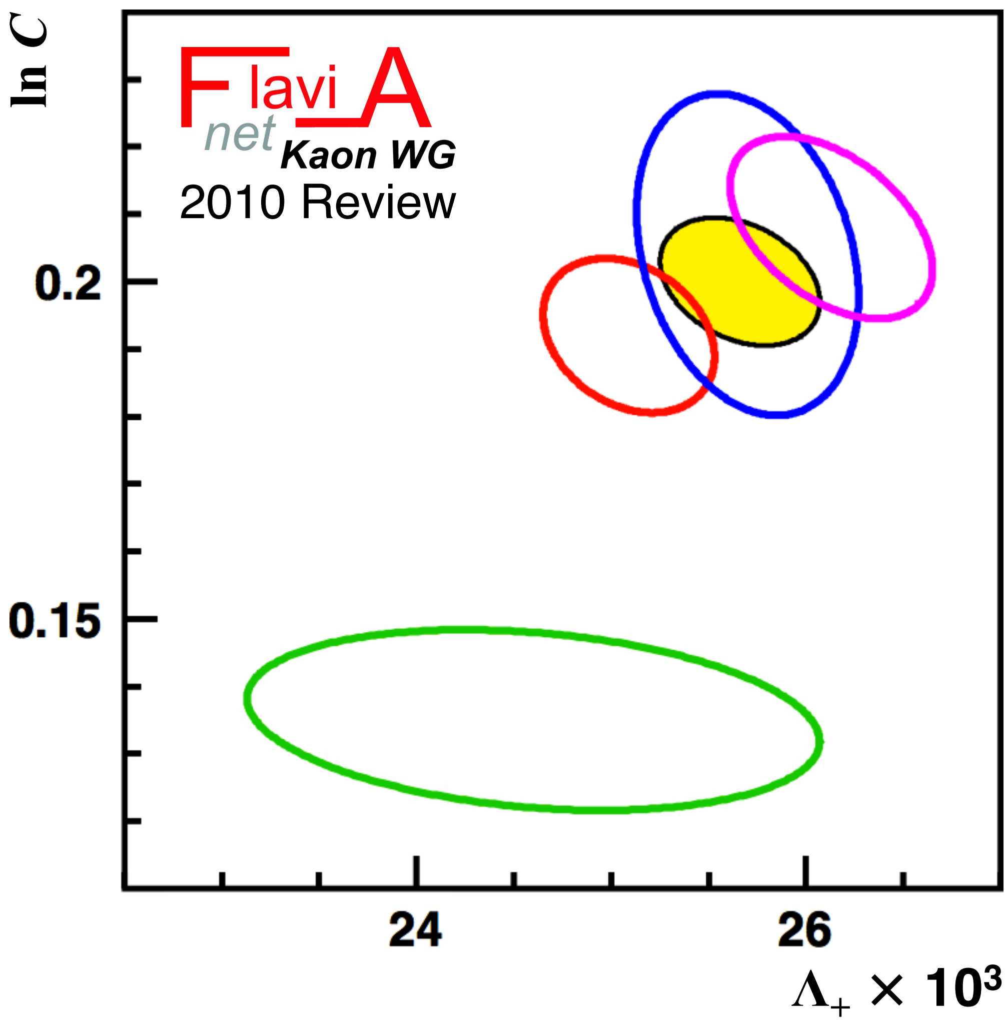


2010: $\chi^2 = 12.1/8$ ($P = 14.5\%$)	Update: $\chi^2 = 14.3/11$ ($P = 22.0\%$)

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{e3} + K_{\mu 3}$ averages from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**

For NA48, only K_{e3} data included in 2010 fit



$\Lambda_+ \times 10^3$	=	**25.61 ± 0.41**
$\ln C$	=	**0.2004(91)**
$\rho(\Lambda_+, \ln C)$	=	−0.328
χ^2/ndf	=	5.6/5 (34%)

Integrals

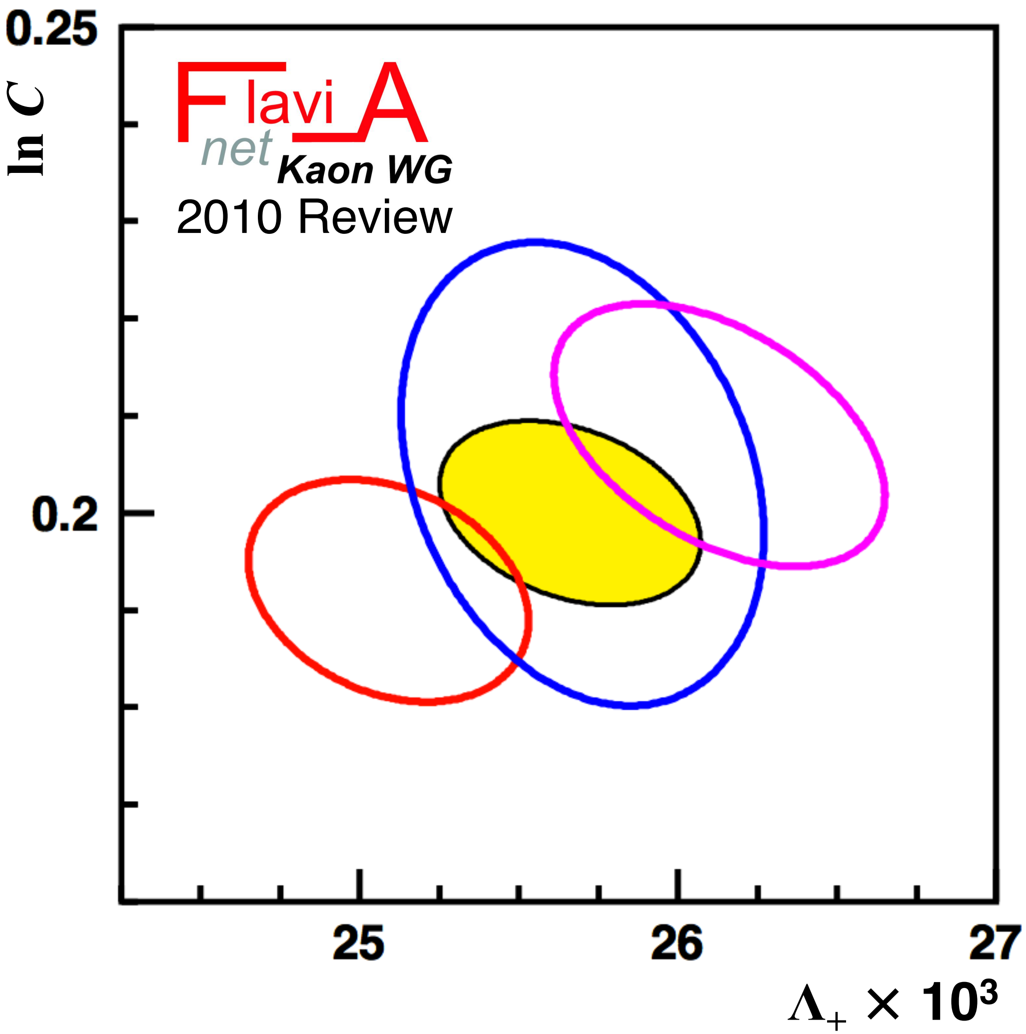
Mode	Quad-lin	Disp
$K^0{}_{e3}$	0.15457(20)	**0.15476(18)**
$K^+{}_{e3}$	0.15894(21)	**0.15922(18)**
$K^0{}_{\mu 3}$	0.10266(20)	**0.10253(16)**
$K^+{}_{\mu 3}$	0.10564(20)	**0.10559(17)**

Maximum change 0.2% if same data used as for quad-lin fits

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{e3} + K_{\mu 3}$ averages from KTeV | KLOE | ISTRA+ | NA48 | 2010 fit

For NA48, only K_{e3} data included in 2010 fit



$$\Lambda_+ \times 10^3 = \mathbf{25.61 \pm 0.41}$$
$$\ln C = \mathbf{0.2004(91)}$$
$$\rho(\Lambda_+, \ln C) = -0.328$$
$$\chi^2/\text{ndf} = 5.6/5\ (34\%)$$

Integrals

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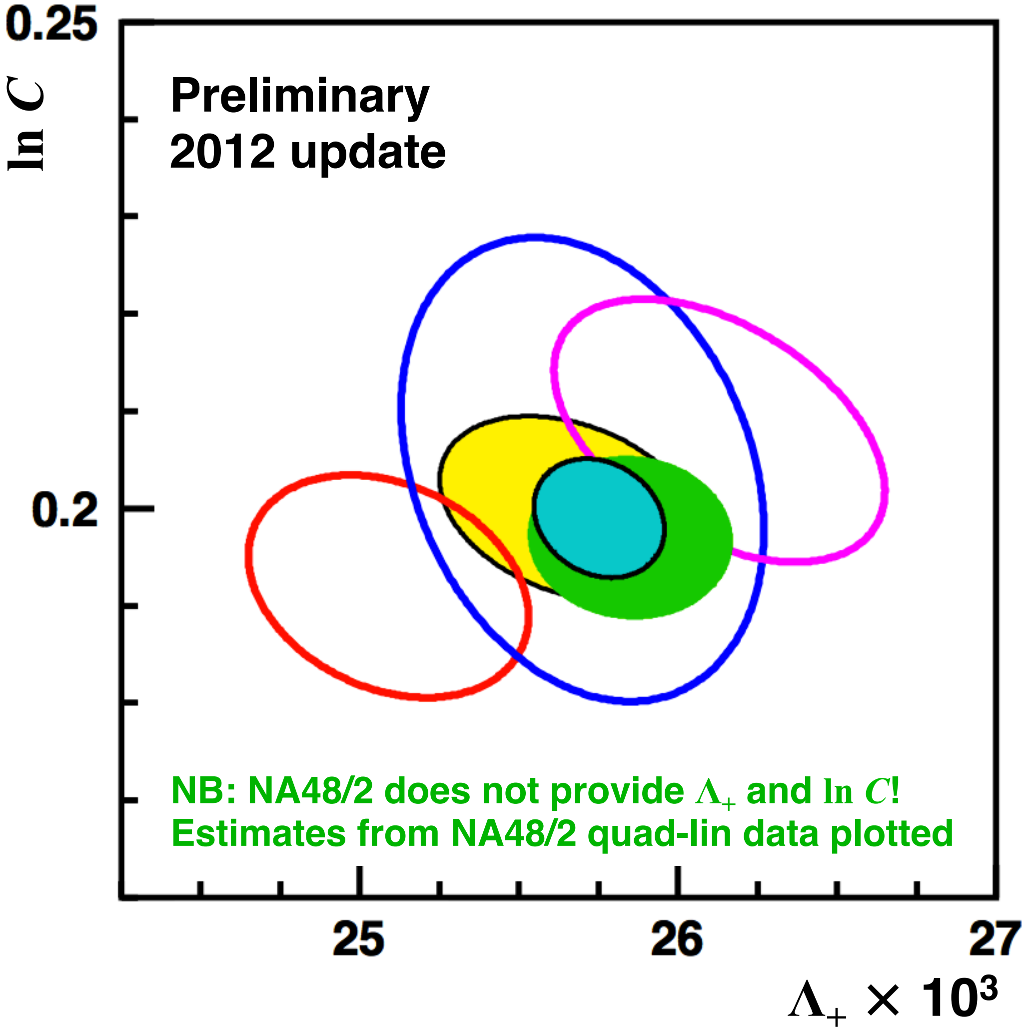
Maximum change 0.2% if same data used as for quad-lin fits

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2 ’12 prel** **2010 fit** **Update**

For NA48, only K_{e3} data included in fits



$\Lambda_+ \times 10^3$	=	**25.75 ± 0.36**
$\ln C$	=	**0.1985(70)**
$\rho(\Lambda_+, \ln C)$	=	−0.202
χ^2/ndf	=	5.9/7 (55%)

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15481(14)**	0.15476(18)
$K^+{}_{e3}$	**0.15927(14)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10253(13)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10558(14)**	0.10559(17)

Only tiny changes in central values

Form factors & the Callan-Treiman relation

Callan-Treiman relation:

$$\tilde{f}_0(t_{\mathrm{CT}}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{\mathrm{CT}}$$

$t_{\mathrm{CT}} = m_K^2 - m_\pi^2$

$\Delta_{\mathrm{CT}} = (-3.5 \pm 0.8) \times 10^{-3}$ ~O(m_u, m_d)

Gasser, Leutwyler ’85

Dispersive representation: $f_0(t_{\mathrm{CT}}) \equiv C$

Test ChPT vs lattice:

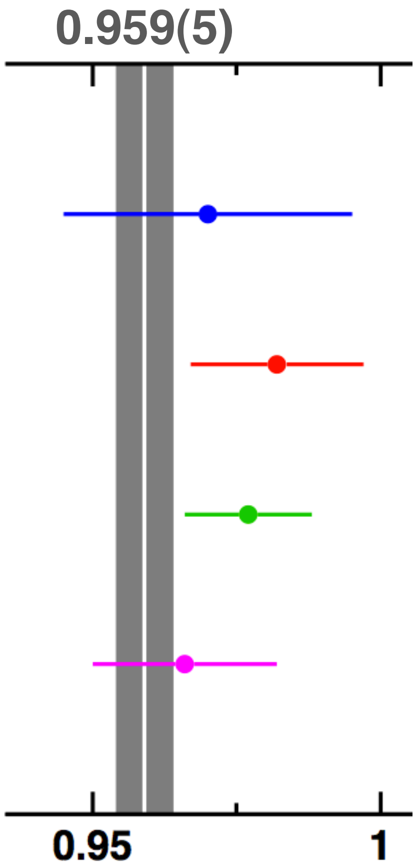
Obtain $f_+(0)$ from ln C

KLOE 0.970(25)

KTeV 0.982(15)

NA48/2 0.977(11)

ISTRA+ 0.966(16)



FLAG averages N_f = 2+1

Use f_K/f_π = **1.193(5)**

Compare $f_+(0)$ = **0.959(5)**

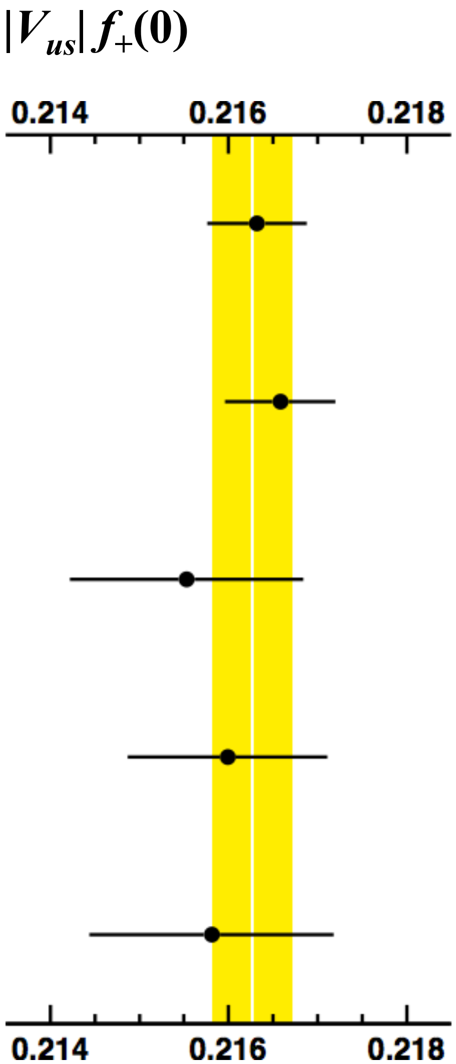
- OR -

Obtain experimental value of

$$R_f \equiv \frac{f_K}{f_\pi}\frac{1}{f_+(0)}$$ No lattice input

	ln C	R_f
2010	0.2004(91)	**1.225(14)**
Update	0.1985(70)	**1.223(12)**
FLAG avg	0.2211(91)	**1.244(8)**

$|V_{us}| f_+(0)$ from world data: 2010

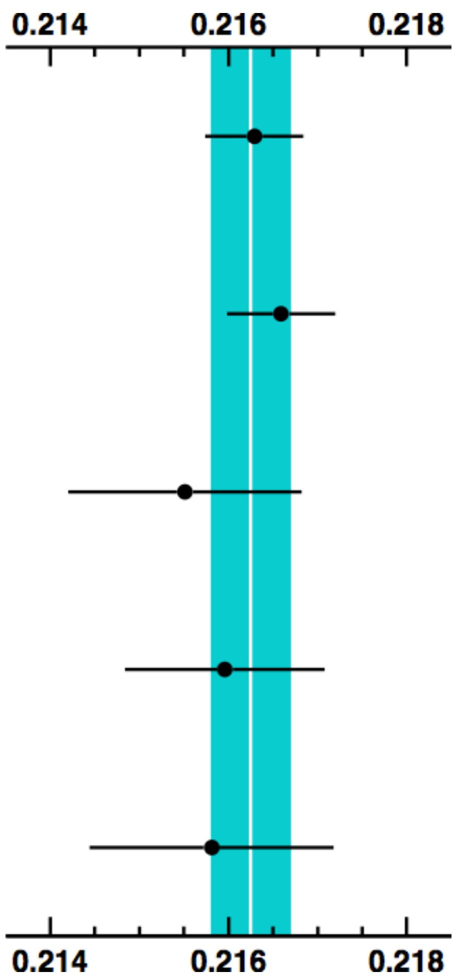


		% err	BR	τ	Δ	Int
			Approx. contrib. to % err from:			
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L \mu 3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^{\pm} e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^{\pm} \mu 3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}| f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

$|V_{us}| f_+(0)$ from world data: Update

$|V_{us}| f_+(0)$



Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	**0.2163(5)**	**0.26**	0.09	0.20	0.11	**0.05**
$K_L \mu 3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	**0.06**
$K_S e3$	**0.2155(13)**	**0.61**	0.60	**0.02**	0.11	**0.05**
$K^{\pm} e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.41	**0.04**
$K^{\pm} \mu 3$	**0.2158(13)**	**0.63**	0.47	0.08	0.41	**0.06**

Average: $|V_{us}| f_+(0)$ = 0.2163(5) χ^2/ndf = 0.84/4 (93%)

Accuracy of $SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\left(\frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}\right)\left[\frac{m_K^2}{m_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right]$$

= **+2.9(4)%** Kastner & Neufeld ’08, used to calculate $|V_{us}|\,f_+(0)$
Quark mass ratio from Ananthanaryan & Moussalam ‘04

Equality of V_{us} values from $K^\pm$ and K^0 data would require **$\Delta^{SU(2)}$ = 2.73(41)%**

Uncertainty on $\Delta^{SU(2)}$ a major contributor to uncertainties for determination of V_{us} for $K^\pm$ decays

Observed value of $\Delta^{SU(2)}$ can be related to quark-mass ratios

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\rm obs}}{(R_{\mu e})_{\rm SM}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\,\rm obs}}{[|V_{us}|\,f_+(0)]^2_{e3,\,\rm obs}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	World data†
K_L	1.054(14)	**1.003(5)**
$K^\pm$	1.014(17)	**0.999(9)**
Avg	1.038(11)	**1.002(5)**

*Assuming current values for form-factor parameters and $\Delta^{\rm EM}$ †K_S not included

As statement on lepton universality

Compare to results from world data:

$\pi \to \ell\nu$ $(r_{\mu e})$ = **1.0042(33)**
Ramsey-Musolf, Su & Tulin '07

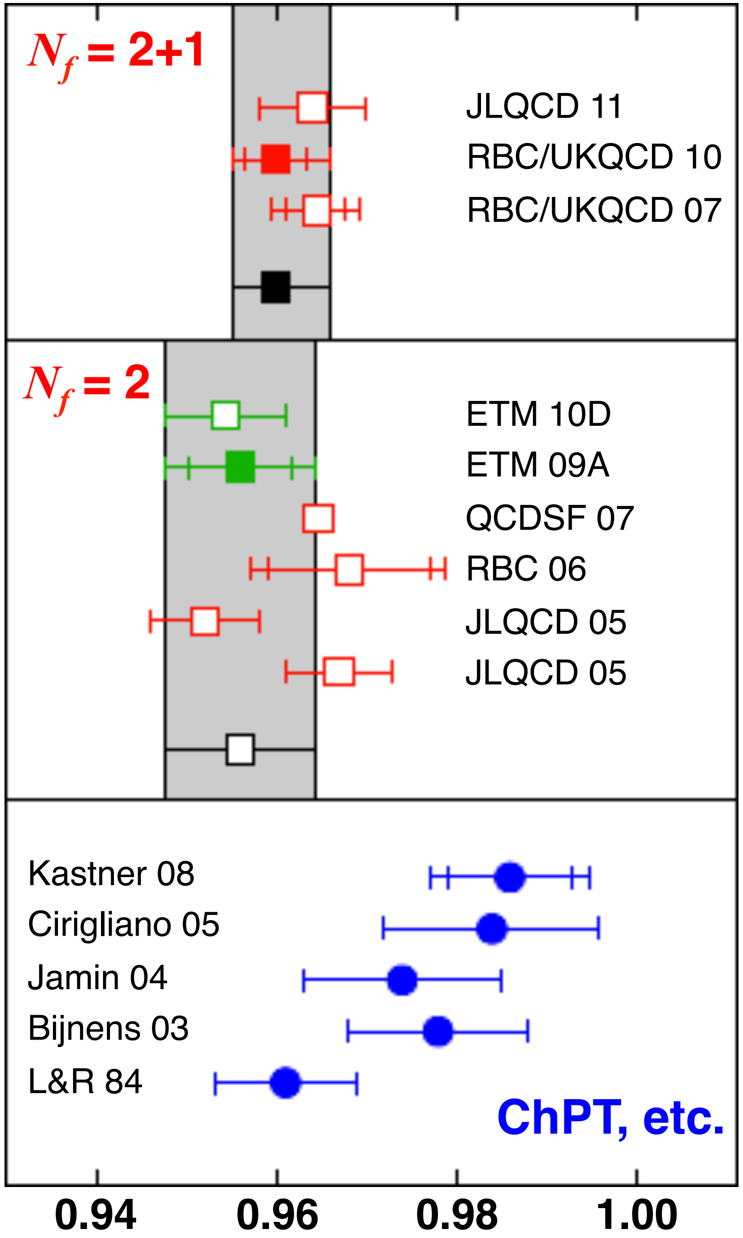
$\tau \to \ell\nu\nu$ $(r_{\mu e})$ = **1.000(4)**
Davier, Hoecker & Zhang '06

As statement on calculation of $\Delta^{\rm EM}$

Cirigliano et al. '08

- Calculation at fixed order e^2p^2
- Fully inclusive for real photons
- Updated LECs for SD terms
- **Confirmed at per-mil level**

Evaluations of $f_+(0)$



ChPT evaluations generally higher than **lattice evaluations**

Preliminary FLAG-2 value for N_f = 2+1
RBC/UKQCD '10

$$f_+(0) = 0.9599(34)_{st}(^{+31}{}_{-47})_{mud}(14)_{FV}$$

FlaviaNet Kaon WG '10
symmetrization of above

$$f_+(0) = 0.959(5)$$

Collaboration	N_f	publ.	$m_{ud} \to 0$	$a \to 0$	FV	$f_+(0)$
JLQCD 11	2+1	C	○	■	★	0.964(6)
RBC/UKQCD 10	2+1	A	○	■	★	$0.9599(34)(^{+31}_{-47})(14)$
RBC/UKQCD 07	2+1	A	○	■	★	0.9644(33)(34)(14)
ETM 10D	2	C	○	★	○	$0.9544(68)_{stat}$
ETM 09A	2	A	○	○	○	0.9560(57)(62)
QCDSF 07	2	C	■	■	★	$0.9647(15)_{stat}$
RBC 06	2	A	■	■	★	0.968(9)(6)
JLQCD 05	2	C	■	■	★	0.967(6), 0.952(6)

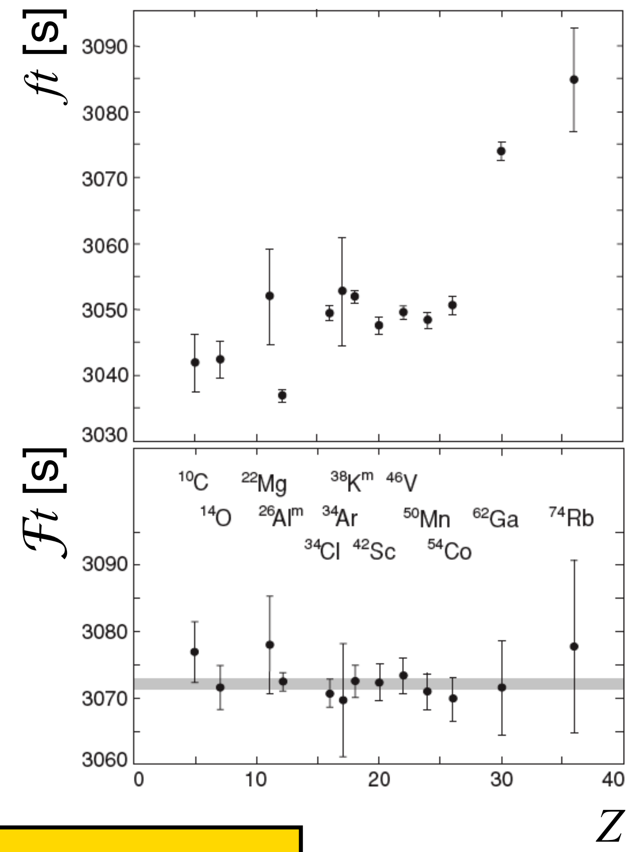
$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \rightarrow 0^+)$

$|V_{us}| f_+(0)$ = 0.2163(5) → $f_+(0)$ = **0.959(5)** → **$|V_{us}|$ = 0.2254(13)**

Hardy & Towner '10

$|V_{ud}|$ = 0.97425(22)

- Survey of 150 measurements of 13 different $0^+ \rightarrow 0^+$ β decays
- 27 new ft measurements including Penning-trap measurements for Q_{EC}
- Some old measurements dropped
- Improved EW radiative corrections [Marciano & Sirlin '06]
- New $SU(2)$-breaking corrections [Towner & Hardy '08]



$\Delta_{CKM} = V_{ud}^2 + V_{us}^2 - 1 = +0.0000(8)$

Exact compatibility with unitarity

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2}\delta_{\rm EM} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

FLAG-2 preliminary average

f_K/f_π = **1.193(5)**

N_f = 2+1

Cancellation of lattice-scale uncertainties

Cirigliano, Neufeld '11

$\delta_{\rm EM}$ = **–0.0069(17)**

Long-distance EM corrections

$\delta_{SU(2)}$ = **–0.0043(5)(11)**

Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

FlaviaNet fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6347(18)

$\tau_{K\pm}$ = 12.384(15) ns

PDG:

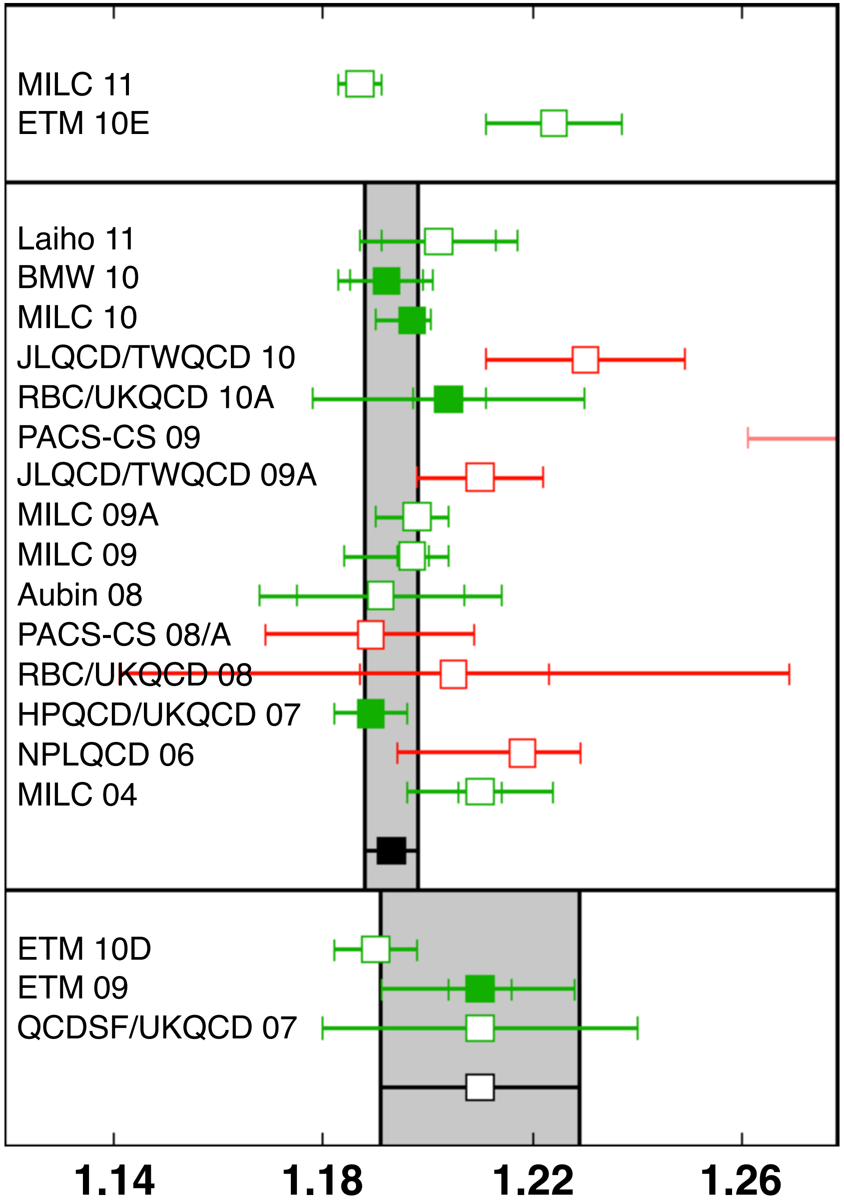
BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

$|V_{us}|/|V_{ud}|$ = 0.2317(11)

New radiative corrections reduce uncertainty by 30%

Lattice evaluations of f_K/f_π



Must be calculated on lattice

Preliminary FLAG-2 value for N_f = 2+1

f_K/f_π **= 1.193(5)**

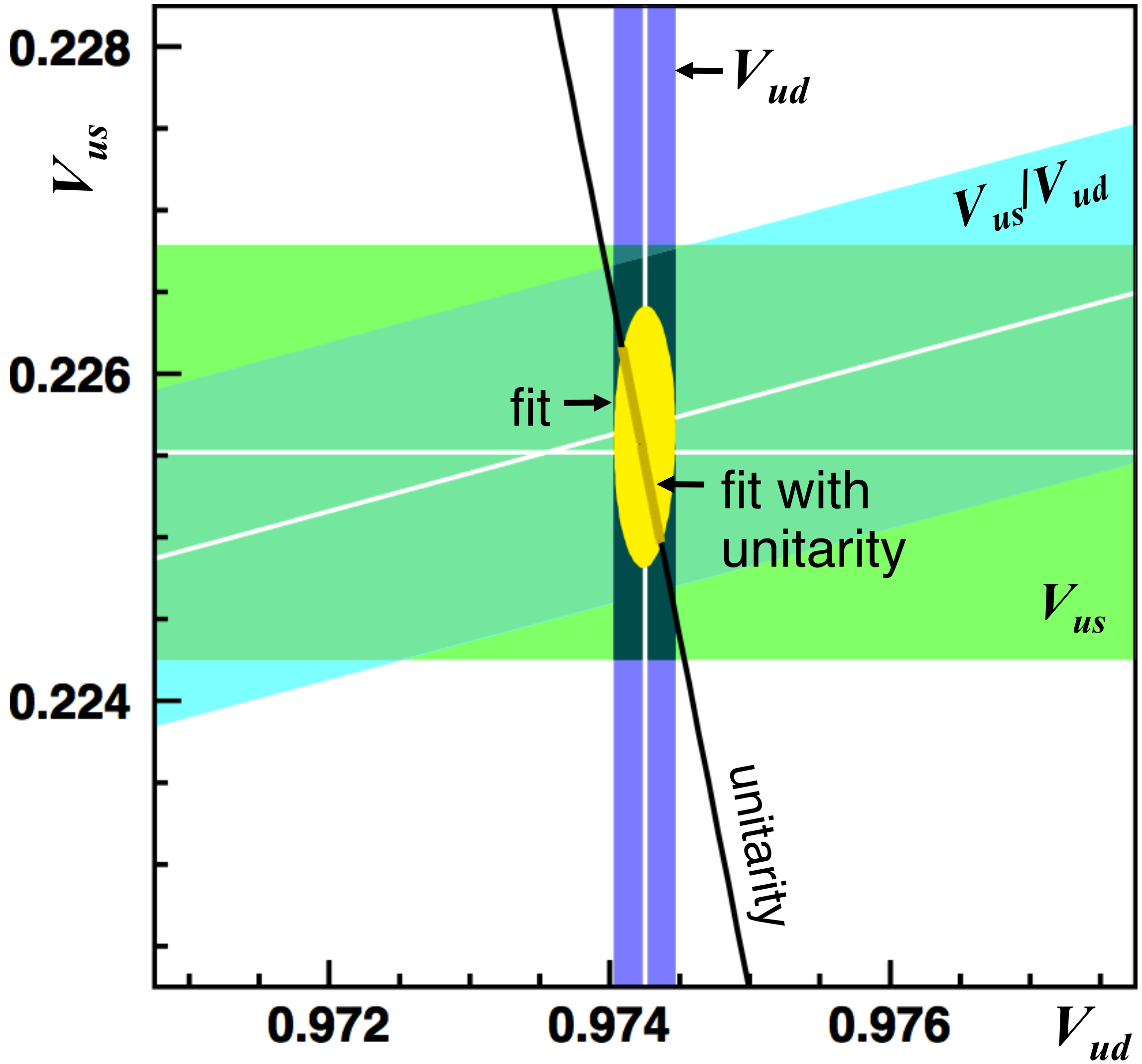
FlaviaNet Kaon WG '10 used f_K/f_π = 1.193(6)

Collaboration	N_f	publ.	m_{ud}	a	FV	f_K / f_π
ETM 10E	2+1+1	C	○	○	○	$1.224(13)_{stat}$
MILC 11	2+1+1	C	○	○	○	$1.1872(42)^{\dagger}_{stat.}$
Laiho 11	2+1	C	○	○	○	$1.202(11)_{stat}(9)_{\chi PT}(2)_{scale}(5)_{m_q}$
MILC 10	2+1	C	○	★	★	$1.197(2)(^{+3}_{-7})$
JLQCD/TWQCD 10	2+1	C	○	■	★	1.230(19)
RBC/UKQCD 10A	2+1	A	○	○	★	1.204(7)(25)
PACS-CS 09	2+1	A	★	■	■	1.333(72)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09A	2+1	C	○	■	■	$1.210(12)_{stat}$
MILC 09A	2+1	C	○	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	A	○	★	★	$1.197(3)(^{+6}_{-13})$
Aubin 08	2+1	C	○	○	○	1.191(16)(17)
PACS-CS 08,	2+1	A	★	■	■	1.189(20)
RBC/UKQCD 08	2+1	A	○	■	★	1.205(18)(62)
HPQCD/UKQCD 07	2+1	A	○	★	○	1.189(2)(7)
NPLQCD 06	2+1	A	○	■	■	$1.218(2)(^{+11}_{-24})$
MILC 04	2+1	A	○	○	○	1.210(4)(13)
ETM 10D	2	C	○	★	○	$1.190(8)_{stat}$
ETM 09	2	A	○	★	○	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	○	○	★	1.21(3)

V_{us} and CKM unitarity: World data

Use $f_+(0)$ = 0.959(5) and f_K/f_π = 1.193(5)
Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$

$|V_{ud}|$ = 0.97425(22)
$|V_{us}|$ = 0.2255(13)
$|V_{us}|/|V_{ud}|$ = 0.2317(11)



Fit results, no constraint

V_{ud} = **0.97425(22)**
V_{us} = **0.2256(8)**
χ^2/ndf = 0.03/1 (86%)
Δ_{CKM} = **+0.0001(6)**

Fit results, unitarity constraint

V_{ud} = **0.97423(14)**
V_{us} = **0.2255(6)**
χ^2/ndf = 0.05/2 (97%)

New physics implications of Δ_{CKM}

Cirigliano, González-Alonso, Jenkins, 2010

Model independent effective-theory approach

Effective Lagrangian for $\mu \sim 1$ GeV with general set of dim-6 operators giving rise to (semi)leptonic transitions

$$\mathcal{L}^{\text{eff}}_{d^j \to u^i \ell \bar{\nu}} = \mathcal{L}^{\text{eff,SM}}_{d^j \to u^i \ell \bar{\nu}} + \frac{v^2}{\Lambda^2} \mathcal{L}^{\text{eff,NP}}_{d^j \to u^i \ell \bar{\nu}}$$

Consider the **flavor-blind** limit (or similar: minimal flavor violation, etc.)
New physics appears as a small difference between G_{CKM} and G_μ

From comparison of operators for $d \to ul\nu$ and $\mu \to e\nu\nu$

$$\Delta_{\text{CKM}} = 4(-\hat{\alpha}^{(3)}_{\varphi\ell} + \hat{\alpha}^{(3)}_{\varphi q} - \hat{\alpha}^{(3)}_{\ell q} + \hat{\alpha}^{(3)}_{\ell\ell}) = \frac{G_{\text{CKM}}}{G_\mu} - 1$$

Strong constraints from precision EW data Weak constraint from LEP-II $e^+e^- \to qq$

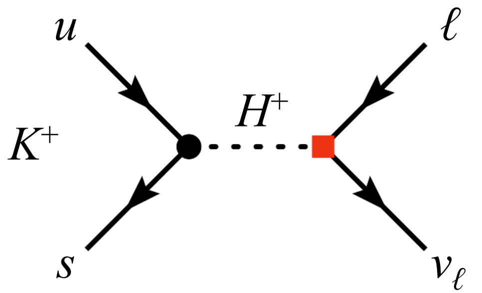
EW fit: Δ_{CKM} = (–4.7 ± 2.9) × 10^{-3}

Exp. V_{ud}, V_{us}: Δ_{CKM} = (–0.1 ± 0.6) × 10^{-3}

→ **$\Lambda^{\text{eff}}_{\text{NP}}$ > 11 TeV (90% CL)**

From 2010 FlaviaNet analysis. Now Δ_{CKM} = (**+0.1** ± 0.6) × 10^{-3}

New physics in $K_{\ell 2}$ decays?



Normally helicity suppressed

Possible contributions from right-handed quark currents
Possible tree-level contribution from H^+ in 2HDM

$$R^2_{\mu 23} \equiv \frac{\Gamma(K^+ \to \ell\nu)}{\Gamma_{SM}(K^+ \to \ell\nu)} = \left[1 - \left(\frac{m^2_{K^+}}{m^2_{H^+}}\right)\frac{\tan^2\beta}{1+\varepsilon_0 \tan\beta}\right]^2$$

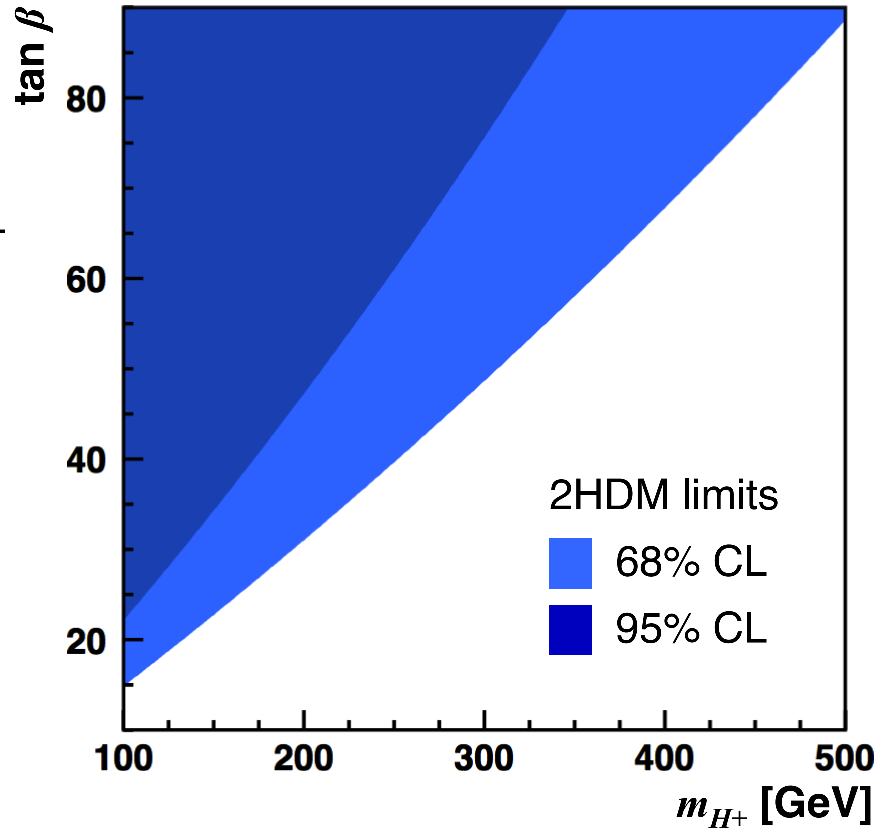
Uncertainty on f_K dominates Γ_{SM}
Evaluate $R_{\mu 23}$ as

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+(0)}\right)^{-1} \left(\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi}\right)_{\mu 2} \frac{|V_{ud}|_{0^+\to 0^+}}{[|V_{us}|f_+(0)]_{\ell 3}}$$

Perform 3-parameter fit with 1 constraint:

$$[V_{us}(K_{\ell 3})]^2 + [V_{ud}(0^+ \to 0^+)]^2 = 1$$

$R_{\mu 23}$ = 1.001(6) χ^2 = 0.003/1 (96%)



Summary and conclusions

V_{ud} = **0.97425(22)** **±0.02%**
V_{us} = **0.2256(8)** **±0.36%**
Δ_{CKM} = **+0.0001(6)**

Plus ça change, plus c'est la même chose!

Hard to improve on 0.23% experimental precision

Most recent progress has been on the theory side

Lattice results for $f_+(0)$ and f_K/f_π to 0.5%

$SU(2)$ corrections to $K_{\ell 2}$, $\pi_{\ell 2}$

Parameters for $|V_{us}|\, f_+(0)$ with large uncertainties:

Experiment: **BR(K_{e3})** and **BR($K_{\mu 3}$)** for K_S and K^+

Theory: $\Delta^{SU(2)}$ for K^+ modes