Lattice progress and future prospects for f_K/f_π

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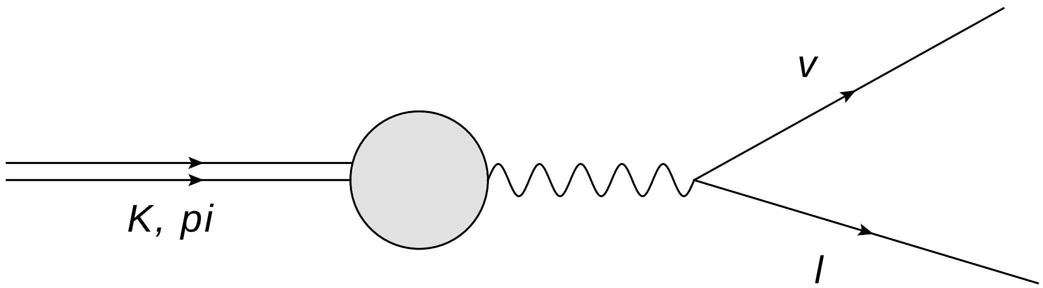
Leptonic decays



$$\frac{\Gamma(K\to \ell\overline{\nu}_\ell)}{\Gamma(\pi\to \ell\overline{\nu}_\ell)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-\frac{m_\ell^2}{m_K^2}\right)^2}{m_\pi\left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2}\left[1+\frac{\alpha}{\pi}(C_K - C_\pi)\right] \qquad (1)$$

$$\Gamma = (\text{known factor})\,(\text{CKM factor})(\text{QCD factor}) \qquad (2)$$

Lattice Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

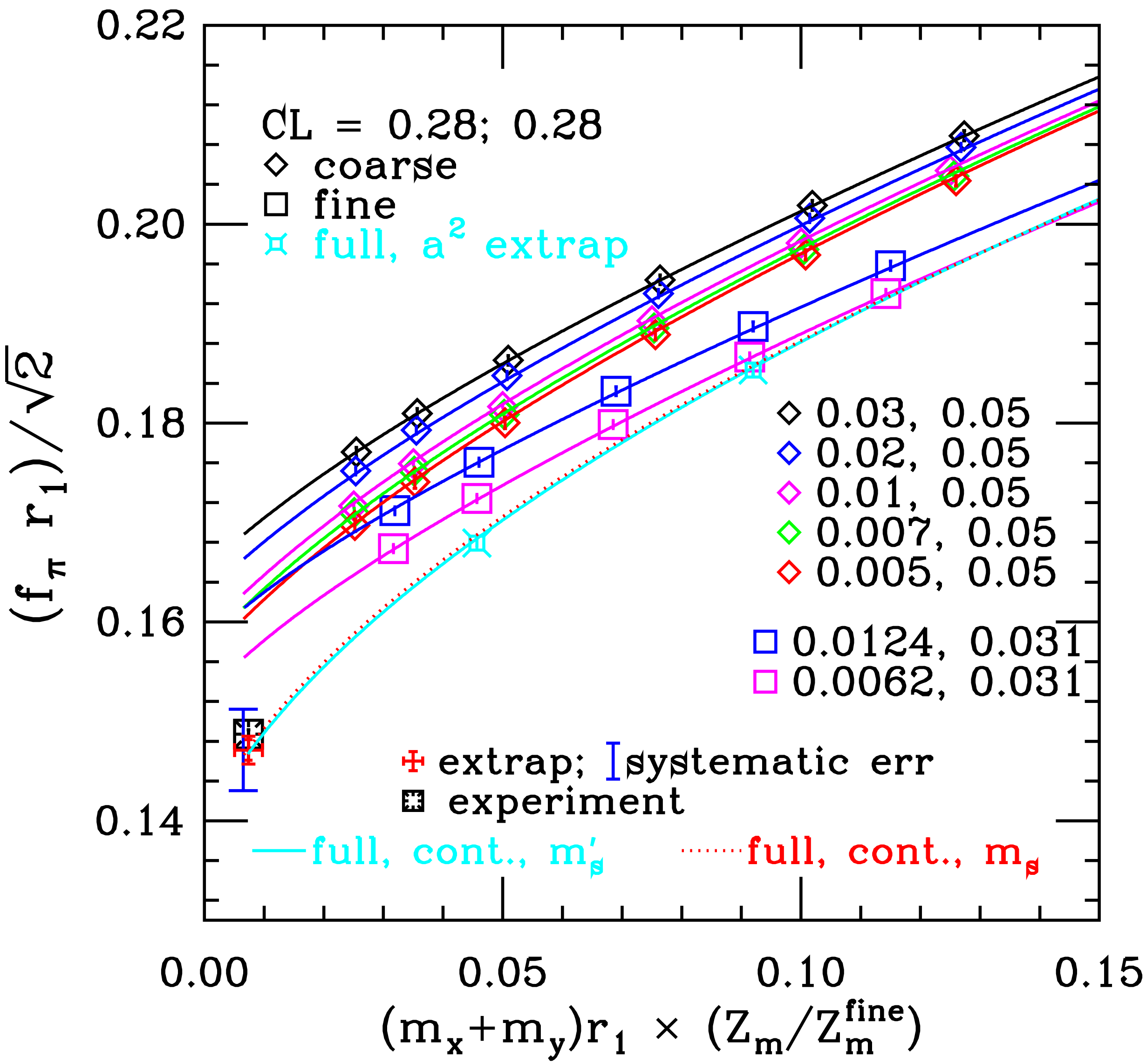
3.) Finite volume effects

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

6.) Quenching. Uncontrolled!

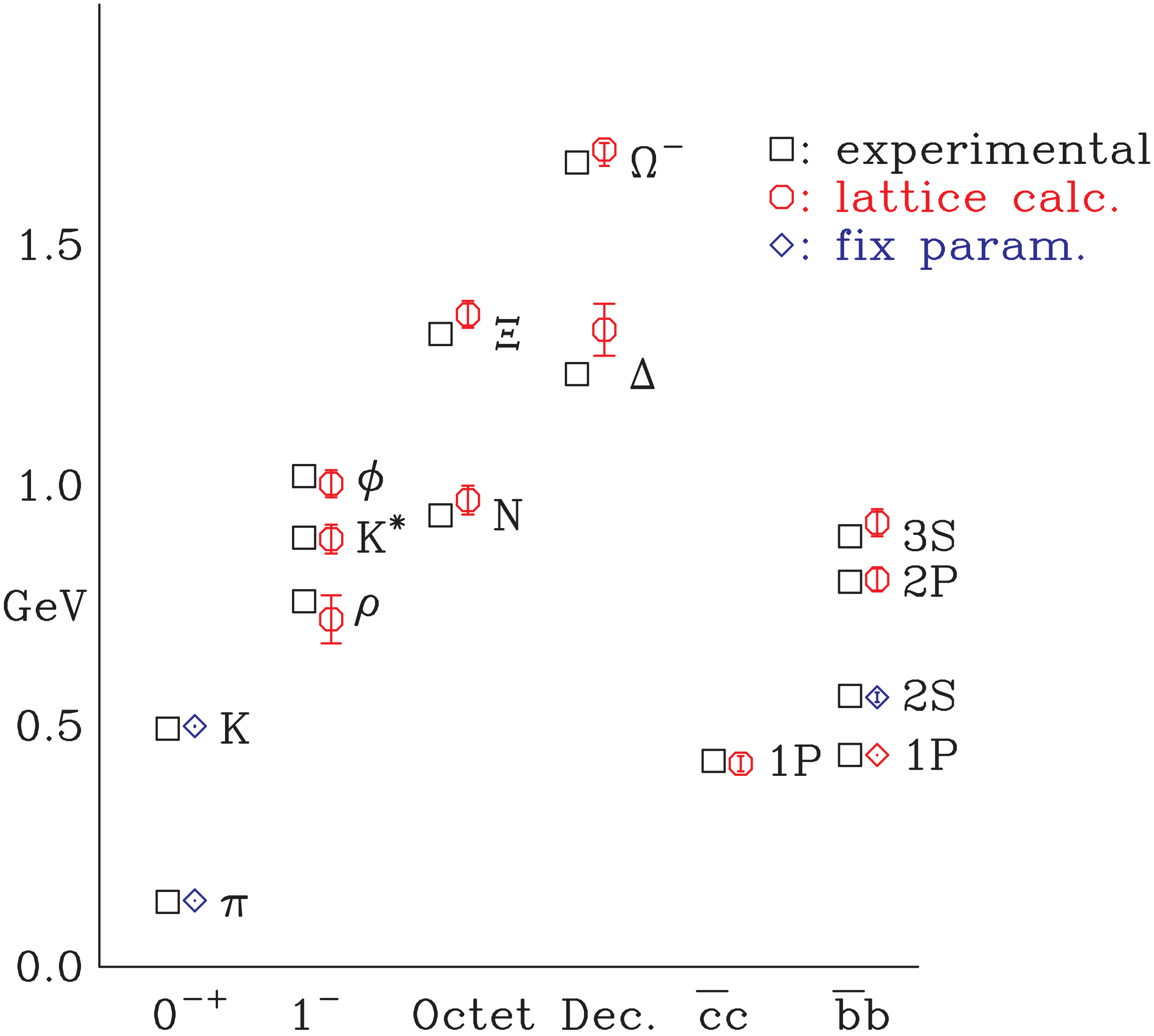
Example Chiral Extrapolation



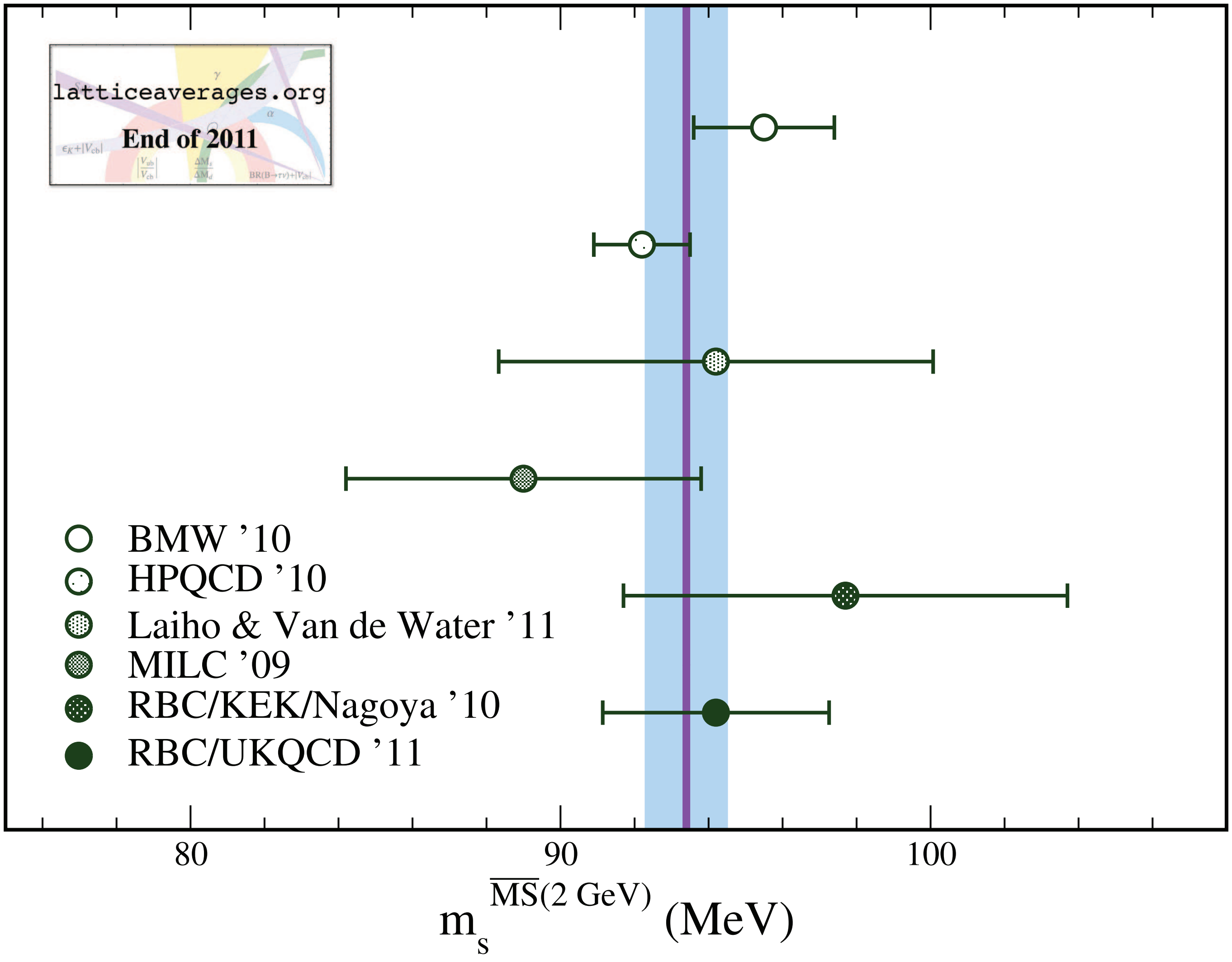
(MILC, hep-lat/0407028)

Light Hadron spectrum

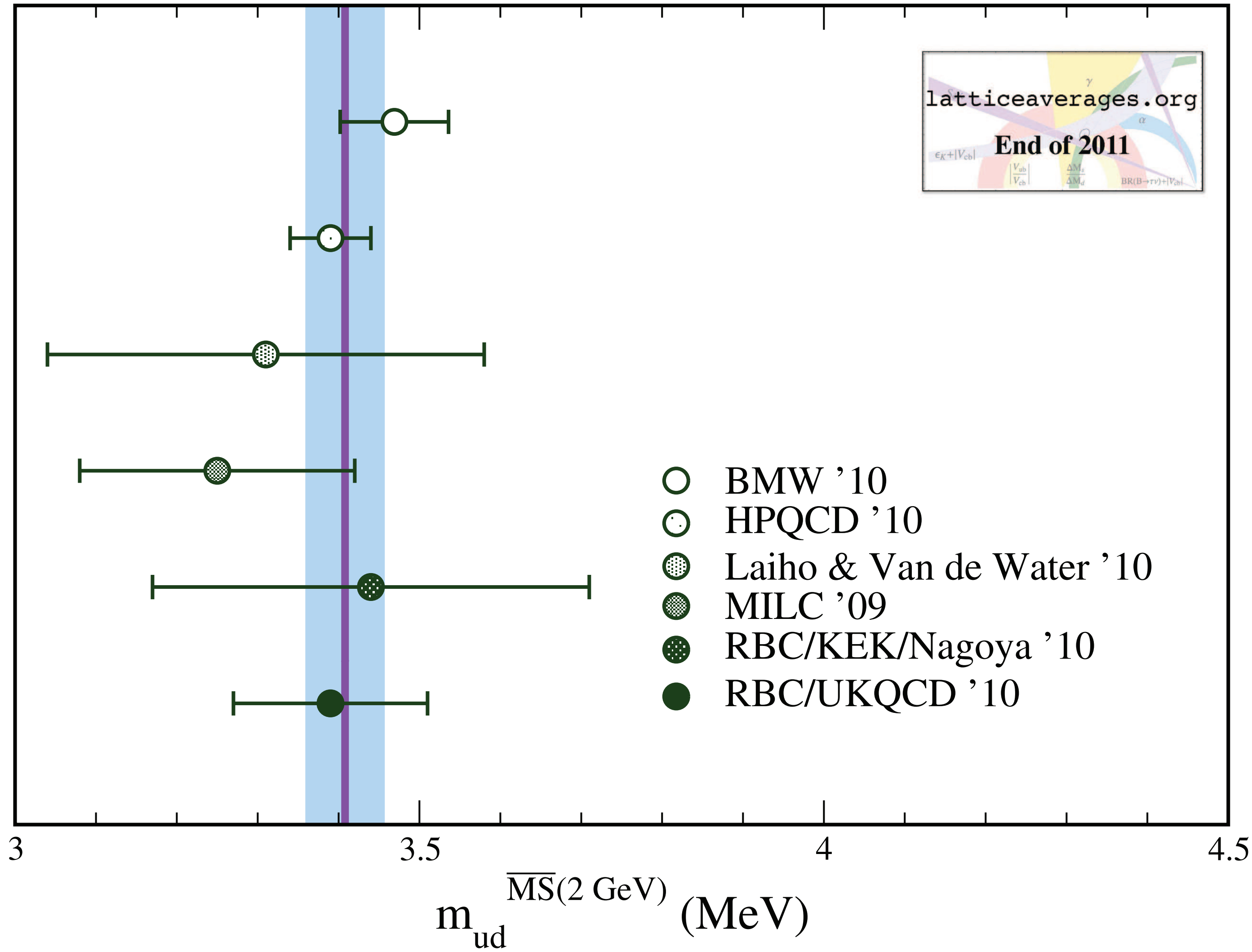
MILC and HPQCD, Rev.Mod.Phys. 82:1349 (2010)



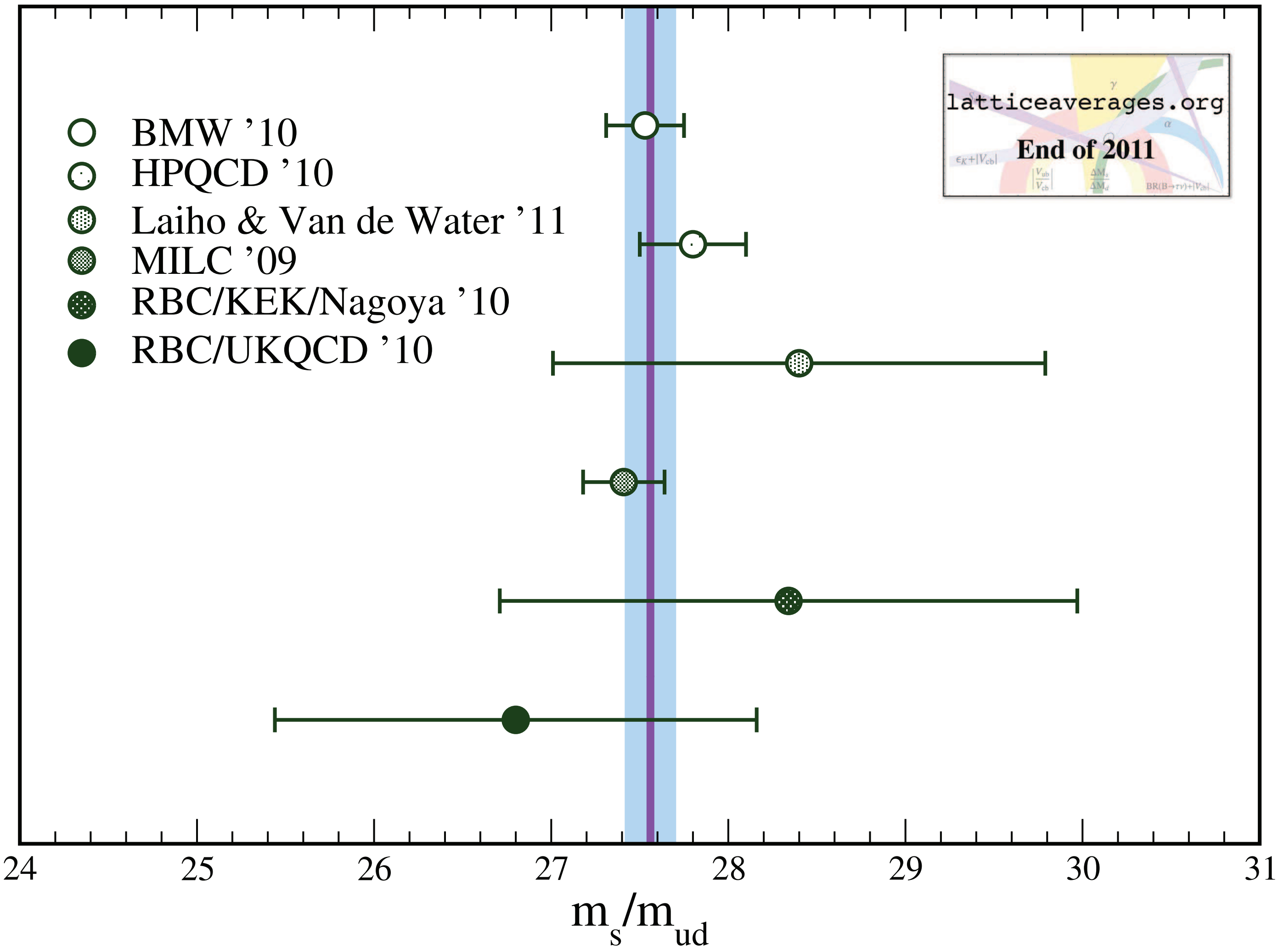
Strange quark mass



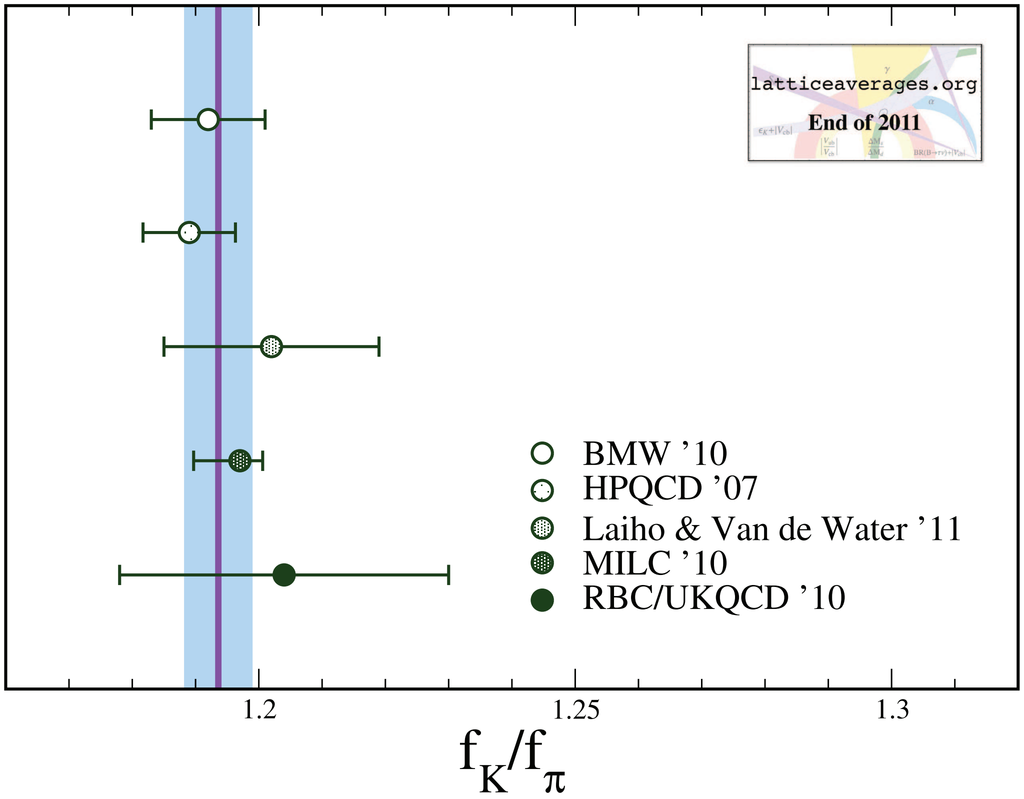
Light-quark mass



Quark-mass ratio

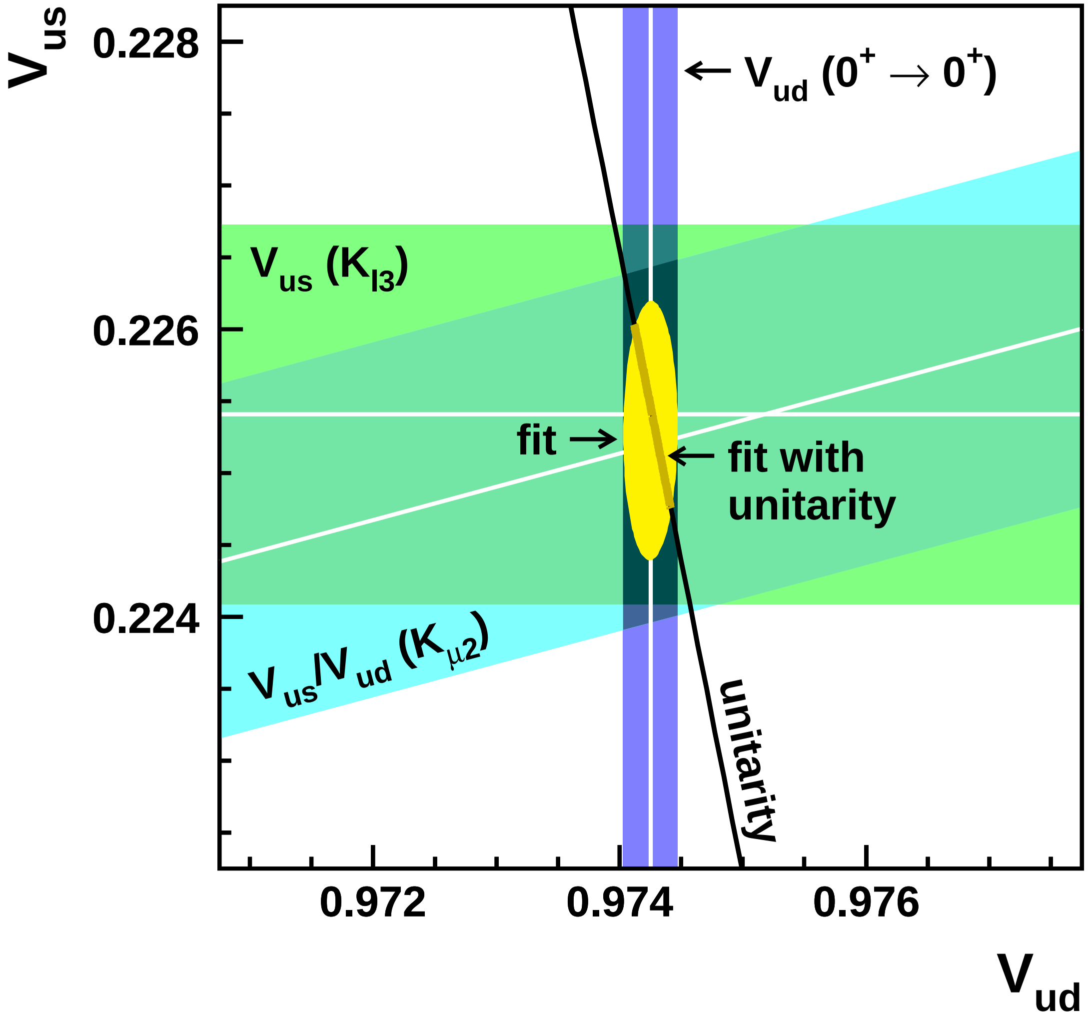


f_K/f_π



BMW, MILC, and HPQCD dominate the 2+1 flavor world average
$f_K/f_\pi = 1.1936(53)$.

First row unitarity constraint



Plot from Flavianet.

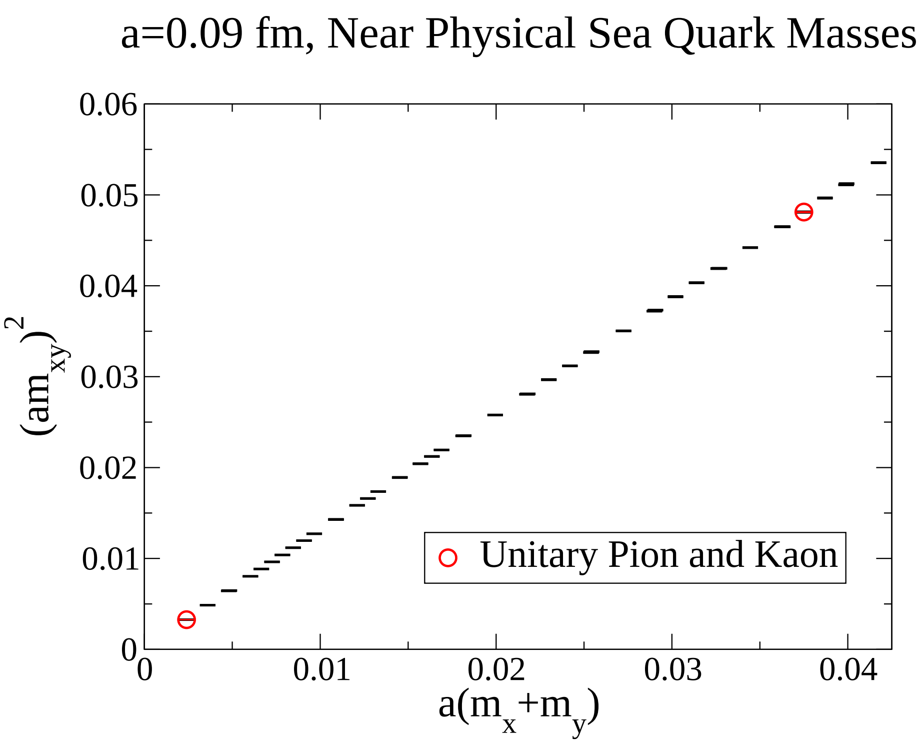
New MILC calculation

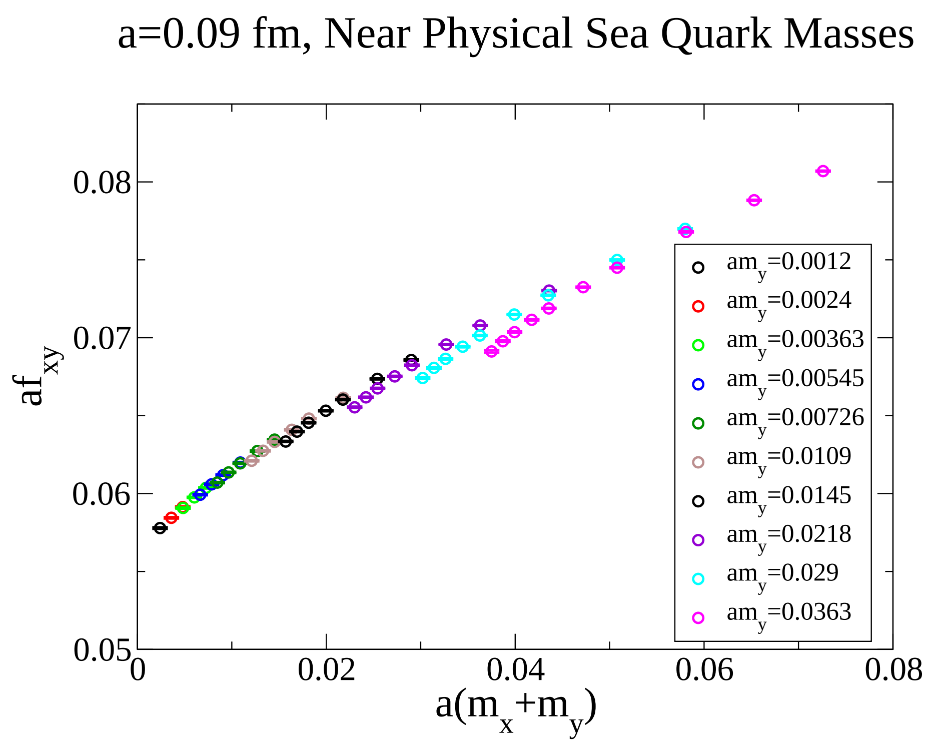
Other lattice groups are working on f_K/f_π, but I will focus on the work of MILC as representative of current efforts.

The new MILC calculation is significantly improved over what was done previously:

- Highly Improved Staggered Quark (HISQ) action, so smaller discretization effects
- 2+1+1 flavors, so charm quark is included in the sea
- Multiple volumes to check explicitly finite size effects
- Physical light quark masses

Interpolating in quark masses





$$(am_{xy})^2 = A_1 + B_1 a(m_x + m_y), \qquad (3)$$

$$af_{xy} = A_2 + B_2 a(m_x + m_y), \qquad (m_x, m_y) \text{ near } (m_l, m_l)$$

$$af_{xy} = A_3 + B_3 am_x + C_3 am_y, \qquad (m_x, m_y) \text{ near } (m_l, ms) \qquad (4)$$

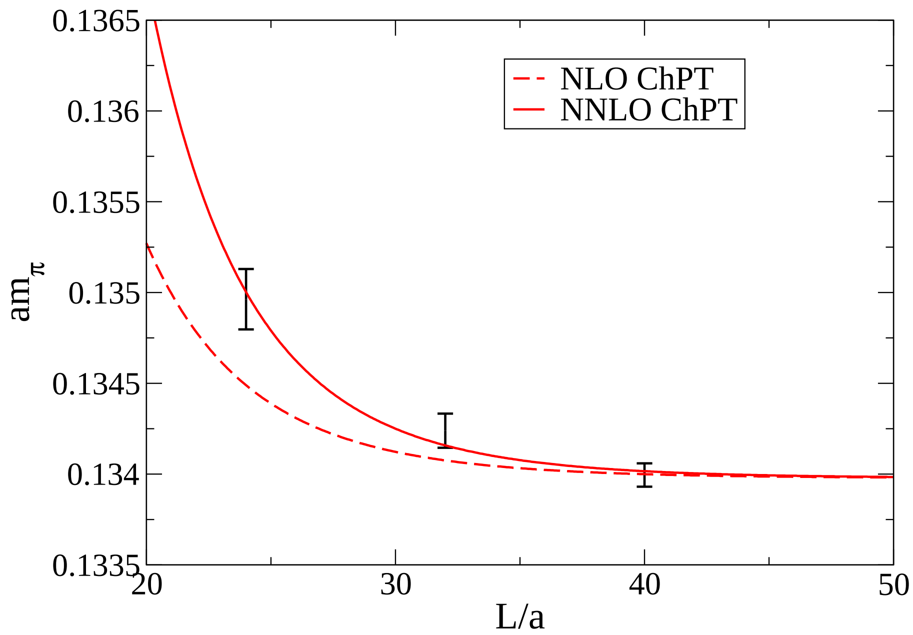
Fixing the quark masses

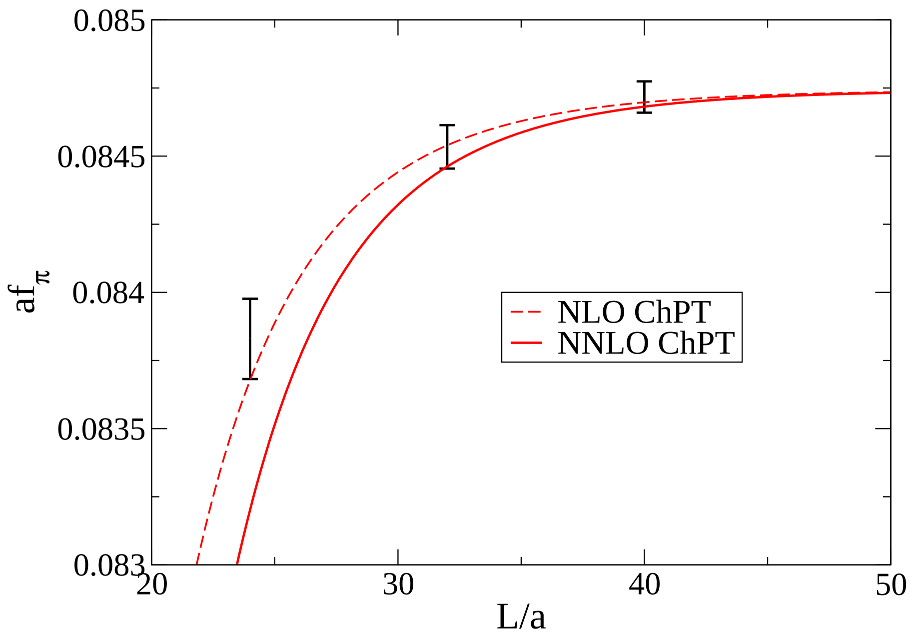
Do linear interpolations of lattice values of f_{xx}/m_{xx} and m_{xy}/m_{xx} and use physical (f_π/m_π) to obtain m_l and m_K/m_π to obtain m_s.

Linearly interpolate f_{xx} and f_{xy} to $m_x = m_l$ and $m_y = m_s$ to obtain physical f_K/f_π.

This avoids a direct determination of the lattice scale.

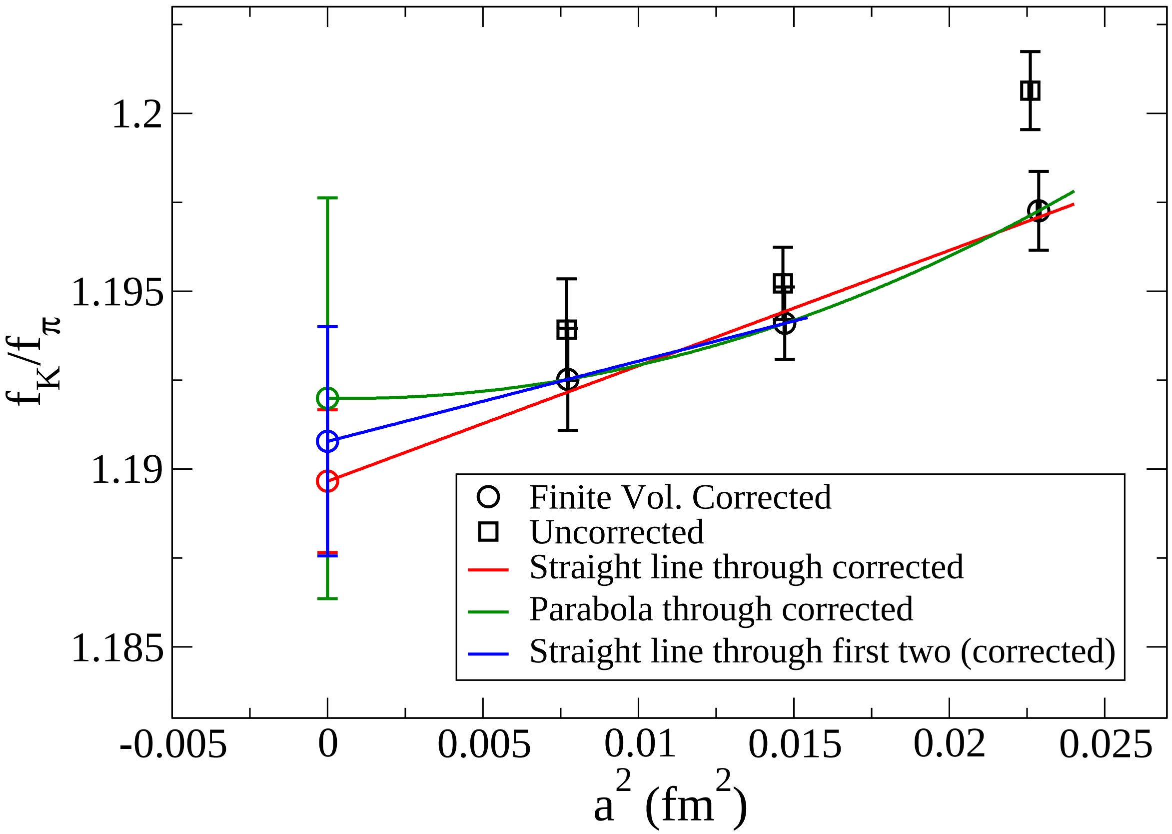
Finite volume effects





For the final results we restrict ourselves to $m_\pi L > 4$ and pions near the physical point, where 2-loop chiral perturbation theory is a good estimate of finite-size effects.

Continuum Extrapolation



If we take the central value from the linear extrapolation and a continuum extrapolation systematic error from the difference between linear and quadratic fits, we find $f_K/f_\pi = 1.1897(20)_{\rm stat}(23)_{\rm extrap}$. Other errors are likely small, but are not yet quoted.

Longer term

Current error on world average is 0.44%.

New MILC result will have error $\sim 0.25\%$.

Better statistics and a finer lattice spacing should help reduce this even further.

Around 0.1% we need to worry about emission of a soft photon and $f_{\pi,K}$ becomes a form factor, so usual improvements that come with doing the same calculation with bigger computers and better lattice/actions and algorithms will hit a wall without more thought.