Obtaining V_{ub} exclusively

a theoretical perspective

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Why exclusive $|V_{ub}|$?

and why calculate form factors with LCSR?

- Uncertainty on $|V_{ub}|^{\text{incl}} \sim 10\%$ ($< 2\%$ on $|V_{cb}|^{\text{incl}}$) due to large $b \to cl\nu$ background
- Competitive $|V_{ub}|^{\text{excl}}$ from $B \to \pi l \nu$, depends on $f_+(q^2)$ (as $m_l \to 0$) from Lattice QCD[1] ($q^2 \gtrsim 15\,\mathrm{GeV}^2$) or QCD sum rules on the light-cone (LCSR) ($q^2 \lesssim 6 - 7\,\mathrm{GeV}^2$)
- Also possible via other B decays, e.g. $B \to \rho l \nu$ and even via Λ_b decays, i.e. $\Lambda_b \to p l \nu$
- For Λ_b case, for $m_l \to 0$ there are four form factors, $f_{1,2}(q^2)$ and $g_{1,2}(q^2)$ which have recently been calculated in LCSR

[1] See next talk by Ruth van de Water

What is LCSR?

taking the example of f_+ for $B \to \pi$

On one hand....

In physical region, correlator dominated by B pole:

$$\Pi_\mu = i\, m_b \int d^D x e^{-i\, p_B \cdot x} \langle \pi(p)|\, T\{\bar{u}(0)\gamma_\mu b(0)\bar{b}(x) i\gamma_5 d(x)\}|0\rangle,$$
$$= (p_B + p)_\mu \Pi_+(p_B^2, q^2) + (p_B - p)_\mu \Pi_-(p_B^2, q^2).$$

into

$B \to \pi$ transition ($f_+(q^2)$)

$$\langle \pi(p)|\bar{u}\gamma_\mu b|B(p_B)\rangle = (p_B + p)_\mu f_+(q^2) + (p_B - p)_\mu f_-(q^2)$$

B meson decay (f_B)

$$m_b \langle 0|\bar{d} i\gamma_5 b|B\rangle = m_B^2 f_B$$

$$\Pi_+(p_B^2, q^2) = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \int_{s > m_B^2} ds \frac{\rho_{\rm had}}{s - p_B^2},$$

($\rho_{\rm had}$ is spectral density of the higher-mass hadronic states)

What is LCSR?

on the other hand..

In Euclidean region ($p_B^2 - m_B^2$ is large and negative): light-cone expand about $x^2 = 0$[2]

$$\Pi_+(p_B^2, q^2) = \sum_n \int du\, \mathcal{T}_+{}^{(n)}(u, p_B^2, q^2, \mu^2)\phi^{(n)}(u, \mu^2) = \int ds \frac{\rho_{\rm LC}}{s - p_B^2},$$

$\mathcal{T}_+^{(n)}(u, \mu^2)$: perturbatively calculable hard kernels
$\phi^{(n)}(u, \mu^2)$: non-perturbative LCDAs at twist n e.g. n=2, $\phi(u, \mu^2)$:

$$\langle \pi(p)|\bar{u}(0)\gamma_\mu\gamma_5\, d(x)|0\rangle = -if_\pi p_\mu \int_0^1 du\, e^{i\bar{u}p\cdot x}\phi(u, \mu^2) + \ldots,$$

where $\phi(u, \mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)$

[2] Factorisation theorem not proven to all orders, verified at given order by cancellation of IR and soft divergences

What is LCSR?

....which leads to the sum rule

Above the continuum threshold s_0, a continuum of states contribute and approximation of quark-hadron duality is thought to be reasonable, such that

$$\rho_{\rm had} = \rho_{\rm LC}\,\Theta(s - s_0).$$

Subtracting from both sides, and Borel transforming (M^2=Borel parameter):

Sum rule for $f_+(q^2)$

$$f_+(q^2) = \frac{1}{f_B m_B^2}\int_{m_b^2}^{s_0} ds\,\rho_{\rm LC}\,e^{-(s-m_B^2)/M^2},$$

State of $f_+(q^2)$ in 2012

15 years since twist-2 NLO corrections calculated

- 1997: NLO twist-2 corrections were calculated[3]
- 2000: LO corrections up to twist-4 were calculated[4]
- 2004: NLO twist-3 corrections[5]
- 2008: $\overline{\mathrm{MS}}$ m_b is used in place of the pole mass [6]

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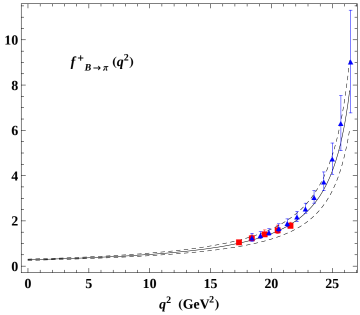
[3] A. Khodjamirian et al, Phys. Lett. B **410** (1997) 275 [arXiv:hep-ph/9706303]; E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417** (1998) 154 [arXiv:hep-ph/9709243]

[4] A. Khodjamirian et al, Phys. Rev. D **62** (2000) 114002 [arXiv:hep-ph/0001297]

[5] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232]

[6] G. Duplancic et al, J. Phys. Conf. Ser. **110** (2008) 052026

State of the art LCSR 2011



- Analysis using 2008 NLO calculation[a] ($\overline{\mathrm{MS}}$ scheme for m_b)
- Use a_2, a_4 from F_π, LCSR+new JLab
- $\Delta\zeta(0, 12\mathrm{GeV}^2) = 4.59^{+1.00}_{-0.85}\,\mathrm{ps}^{-1}$

$$\Delta\zeta(0, q^2_{\max}) = \frac{1}{|V_{ub}|^2}\int_0^{q^2_{\max}} dq^2\,\frac{d\Gamma}{dq^2}(\Lambda_b \to p l \nu)$$

[a] A. Khodjamirian, T. .Mannel, N. Offen, Y. -M. Wang, Phys. Rev. **D83** (2011) 094031, 1103.2655 [hep-ph]

[b] Similar results for BGL/BCL (z-type) or BK/BZ (simple pole), dominated by B^*

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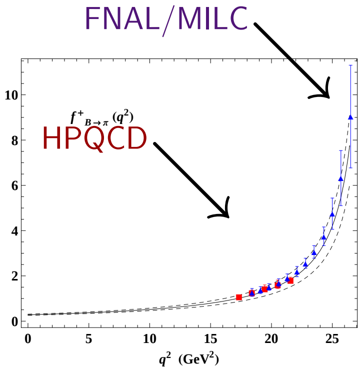

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- Extrapolate by fitting to BCL q^2 parameterisation[b], compare to Lattice

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Why two-loop corrections in LCSR?

A. Bharucha, JHEP **1205** (2012) 092 [arXiv:1203.1359 [hep-ph]].

- Improved determination of $|V_{ub}|$!
- Test argument that radiative corrections to $f_+ f_B$ and f_B should cancel when both calculated in sum rules, in view of the sizeable two-loop contribution to f_B in QCDSR

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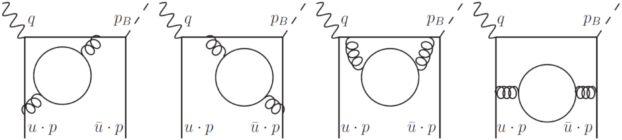
$\Rightarrow$ Calculate subset of two-loop radiative corrections for twist-2 contribution to $f_+(0)$ proportional to β_0 (good approximation to the complete next-to-next-to-leading order (NNLO) result.)

NLO twist two diagrams + fermion loop insertion

$$\Pi_\mu^{(2)} = \mathcal{N} \int_0^1 du\, \phi(u,\mu^2) \int \frac{d^D k}{(2\pi)^D} \frac{\Gamma(\epsilon)\Gamma(2-\epsilon)^2}{\Gamma(4-2\epsilon)} \left(\frac{-k^2}{4\pi\mu^2}\right)^{-\epsilon} \frac{1}{k^2} \left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}\right) F_\mu^{\mathrm{T}}$$

Higher order expansion in ϵ

$$\frac{-ig^{\alpha\beta}}{k^2} \to \frac{-i}{k^2}\left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}\right)$$



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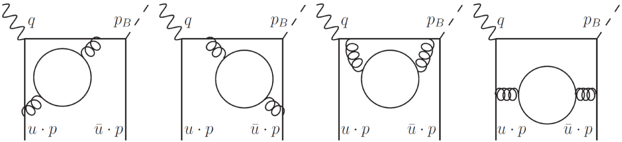
Higher order expansion in ϵ

Propagator changes

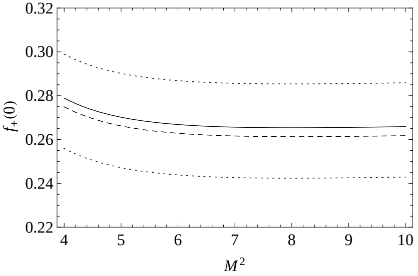
$$\frac{-ig^{\alpha\beta}}{k^2} \to \frac{-i}{k^2}\left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}\right)$$



$q^2 = 0$, but still 3 scales

Use HypExp, FeynCalc and Polylog Identities

Results



- $f_+(0)$ $(0.262^{+0.020}_{-0.023})$ at $\mathcal{O}(\alpha_s^2\beta_0)$ (solid) with uncertainties$\lesssim$ 9% (dotted), compared to $\mathcal{O}(\alpha_s)$ result (dashed), as a function of Borel parameter M^2
- Despite $\sim$ 9% $\mathcal{O}(\alpha_s^2\beta_0)$ corrections to f_B, change in $f_+(0)$, only $\sim$ 2%

This enforces the stability of LCSR w.r.t. higher order corrections, confirmation f_B from sum rules not Lattice QCD

Recent update from Babar analysis[7],

$|V_{ub}| = (3.34 \pm 0.10 \pm 0.05 +^{+0.29}_{-0.26})10^{-3}$, same analysis quotes

$|V_{ub}| = (3.46 \pm 0.06 \pm 0.08^{+0.37}_{-0.32})10^{-3}$ using $\Delta\zeta(0, 12\,\mathrm{GeV}^2)$

[7] J. P. Lees *et al.* [BABAR Collaboration], arXiv:1208.1253 [hep-ex].

Form factors for $\Lambda_b \longrightarrow p$ decays

A new route to exclusive $|V_{ub}|$?

- Heavy-light baryon interpolating current, $\eta = \epsilon^{ijk}(u_i C\Gamma_b d_j)\tilde{\Gamma}_b c_k$, choice of Γ_b and $\tilde{\Gamma}_b$ debated since 80s
- Λ_b^* with $J^P = 1/2^-$, mass (Just measured at LHCb!) 5.91 GeV, $m_{\Lambda_b} = 5.62$ GeV.
- 3 point sum rules+HQET[8]. Large continuum dependence, treatment of -ve parity baryons not clear, large 4 quark resonance contribution
- LCSR+HQET, Λ_b DA[9] (Use CZ current for p, couples to Δ), and in full QCD with the p DA[10], using a mixing parameter β in Γ_b and $\tilde{\Gamma}_b$ (introduces new systematic error), and including the Λ_b^* in continuum
- Recently, Λ_b^* contribution separated in SR [11], results less dependent on current, use both $\Gamma_b = \gamma_5(\gamma_5\gamma_\lambda)$ and $\tilde{\Gamma}_b = 1(\gamma_\lambda)$.

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[8] e.g. R. S. Marques de Carvalho, F. S. Navarra, M. Nielsen, E. Ferreira and H. G. Dosch, Phys. Rev. D **60** (1999) 034009 [hep-ph/9903326]

[9] e.g. Y. -M. Wang, Y. -L. Shen and C. -D. Lu, Phys. Rev. D **80** (2009) 074012 [arXiv:0907.4008 [hep-ph]]

[10] K. Azizi, M. Bayar, Y. Sarac and H. Sundu, Phys. Rev. D **80** (2009) 096007. [arXiv:0908.1758 [hep-ph]]

[11] A. Khodjamirian, C. .Klein, T. .Mannel and Y. -M. Wang, JHEP **1109** (2011) 106 [arXiv:1108.2971 [hep-ph]].

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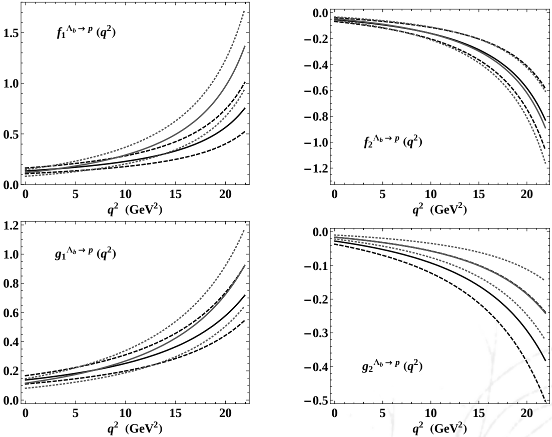

Axial vector current: $\Delta\zeta(0, 11\,\mathrm{GeV}^2) = 5.5^{+2.5}_{-2.0}\mathrm{ps}^{-1}$

Pseudoscalar current: $\Delta\zeta(0, 11\,\mathrm{GeV}^2) = 5.6^{+3.2}_{-2.9}\mathrm{ps}^{-1}$

Not yet competitive with $B \longrightarrow \pi$ but offers independent determination

Summary

and Outlook

Updates from LCSR on f_+ in 2011 and 2012:

- Updated 2008 analysis, e.g. $a_{2,4}$ from F_π, gives $|V_{ub}| = (3.50^{+0.38}_{-0.33}|_{th.} \pm 0.11|_{exp.})10^{-3}$
- At $\mathcal{O}(\alpha_s^2\beta_0)$, despite $\sim 9\%$ increase in f_B from QCDSR, LCSR prediction for $f_+(0)$ increases by $\sim 2\%$ to $f_+(0) = 0.262^{+0.020}_{-0.023}$, $|V_{ub}| = (3.34 \pm 0.10 \pm 0.05 +^{0.29}_{-0.26})10^{-3}$

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Exciting progress on form factors for $\Lambda_b \to p$:

- Removing contribution of negative parity baryons, form factors have reduced dependence on choice of Γ_b and $\tilde{\Gamma}_b$
- $\Delta\zeta$ is calculated for extraction of V_{ub} when experimental data available.

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Things for the future:

- Use $\mathcal{O}(\alpha_s^2\beta_0)$ $f_+(0)$ and Lattice in BCL type analysis to determine $|V_{ub}|$
- Remaining twist-2 NNLO corrections to $f_+(q^2)$
- Progress in $\Lambda_b \to p$ form factors: improvements on p DA and gluon radiative corrections

Thanks for listening![8]

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Renormalisation and Convolution

- We first perform the gluon self-energy renormalisation and mass renormalisation in the $\overline{\mathrm{MS}}$ scheme,

$$\Pi_\mu^{(2),\mathrm{ren.}} = \Pi_\mu^{(2)} - \mathbf{Z}_{\mathbf{3}\mathrm{YM}}^{(\mathbf{1})}\Pi_\mu^{(1)} - \mathbf{Z}_{\mathbf{m}}^{(\mathbf{2})}\Pi_\mu^{(0)} - \Pi_\mu^{(2),T_{\mathrm{IR}}},$$

- $\phi(u,\mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2)C_n^{3/2}(2u-1)$, UV structure of asymptotic DA ($a_n = 0$ for $n > 1$) can be factorised into $\mathbf{Z}_\phi(u,v)$,

$$V(u,v) = -\frac{1}{Z_\phi(u,v)}\left(\mu^2\frac{\partial}{\partial\mu^2}Z_\phi(u,v)\right),\ \mu^2\frac{d}{d\mu^2}\phi(u,\mu^2) = \int_0^1 dv\, V(u,v)\phi(v,\mu^2)$$

- $Z_\phi^{(2)}(u,v)$, i.e. the $\mathcal{O}(\alpha_s^2 N_f)$ contribution to $Z_\phi(u,v)$, can then be reconstructed from the evolution kernel,

$$Z_\phi(u,v) = \delta(u,v) + \frac{\alpha_s}{4\pi}\,\frac{1}{\epsilon}\,2V_0(u,v) + \left(\frac{\alpha_s}{4\pi}\right)^2\frac{1}{\epsilon^2}N_f C_F\left(\frac{1}{2}V_0(u,v) + \epsilon\, V_N(u,v)\right) + \ldots$$

- $\Pi_\mu^{(2),\phi_{UV}} = -\Pi_\mu^{(2),T_{\mathrm{IR}}}$

Which value of f_B?

Sum rule:

$$f_+(0) = \left(\int_{m_b^2}^{s_0} ds\, \rho_{\Pi_+}(s,0) e^{(m_B^2 - s)/M^2} + \mathrm{T}4_c\, e^{m_B^2/M^2} \right) / (m_B^2 f_B)$$

[9] M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005 [arXiv:hep-ph/0108135].

[10] K. G. Chetyrkin and M. Steinhauser, Eur. Phys. J. C **21** (2001) 319 [arXiv:hep-ph/0108017].

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- Require mathematica package Rvs.m from authors[10] as only semi-analytical results available

[9] M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005 [arXiv:hep-ph/0108135].

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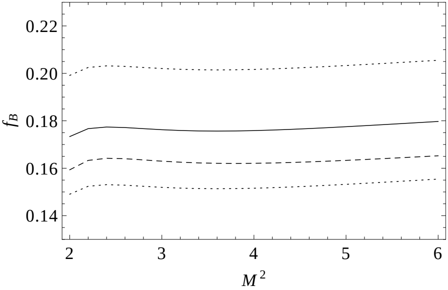
Sum rule for f_B

Careful choice of s_0 and M^2 required, to satisfy the following:



- Sum rule depends weakly on s_0 and M^2, but have a clear extremum as fn. of them
- Sum rule for m_B fulfilled to less than 0.1%
- Continuum contribution is under control
- Contributions of higher orders and twists should be suppressed

Choosing a suitable a_2

Progress from the Lattice:

- Non-perturbative LCDA require calculation of moments from sum rules or lattice
- UKQCD and RBC collaborations predict $a_2(2\,\mathrm{GeV})$, using $N_f = 2+1$ domain-wall fermions[11].
- Combine $a_2(\mu)$ with exp. $\gamma\gamma^*\pi$ form factor, obtain $a_4(\mu)$[12].

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For consistency use LCSR result:

- $a_{2,4}(1\,\mathrm{GeV})$ from fitting F_π from LCSR to the exp. data.
- $a_2(1\,\mathrm{GeV}) = 0.17 \pm 0.08$ and $a_4(1\,\mathrm{GeV}) = 0.06 \pm 0.10$[13], consistent with existing LCSR and Lattice QCD predictions.

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Numerics

and uncertainties

- The LCSR approach requires a careful choice of numerical values for the continuum limit s_0 and the Borel parameter M^2 vary s_0 by $\pm 0.5\,\mathrm{GeV}^2$ and M_2 by $\pm 1.2\,\mathrm{GeV}^2$
- we adopt $a_{2,4}(1\,\mathrm{GeV})$ from fitting the LCSR result for the pion electro-magnetic form factor to the experimental data
- Parameters describing twist-3 and 4 DA's, namely η_3, ω_3, η_4 and ω_4, calculated in QCD sum rules, error $\sim 50\%$
- Condensates varied $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3\,\mathrm{GeV}^3$, $\langle \bar{q}\sigma g G q \rangle = (0.8 \pm 0.2)\,\langle \bar{q}q \rangle$
- Use pole mass, $m_b = 4.8 \pm 0.1$ GeV (also calculate with $\overline{\mathrm{MS}}$ and compare
- μ typical virtuality of the b quark, $\sqrt{m_B^2 - m_b^2}$, vary in range $\mu/2$ to 2μ.

The $\overline{MS}$ scheme[14]?

- $\overline{MS}$ scheme said to be natural for the calculation of scattering amplitudes involving a virtual b quark at large space-like momentum scales $\sim m_b$
- We also calculate our result using the $\overline{MS}$ mass for the b quark
- Find $f_+(0) = 0.251^{+0.019}_{-0.030}$

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[14] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic and N. Offen, J. Phys. Conf. Ser. **110** (2008) 052026, A. Khodjamirian, T. Mannel, N. Offen and Y. M. Wang, Phys. Rev. D **83** (2011) 094031 [arXiv:1103.2655 [hep-ph]].

The $\overline{MS}$ scheme[14]?

- $\overline{MS}$ scheme said to be natural for the calculation of scattering amplitudes involving a virtual b quark at large space-like momentum scales $\sim m_b$
- We also calculate our result using the $\overline{MS}$ mass for the b quark
- Find $f_+(0) = 0.251^{+0.019}_{-0.030}$

Comparison at NLO:

b-quark mass	$\overline{\mathbf{MS}}$	pole
$f^+_{B\pi}(0)$	0.263	0.258
tw2 LO	50.5%	39.7%
tw2 NLO	7.4%	17.2 %
tw3 LO	46.7%	41.5 %
tw3 NLO	-4.4%	2.4 %
tw4 LO	-0.2%	-0.9%

[14] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic and N. Offen, J. Phys. Conf. Ser. **110** (2008) 052026, A. Khodjamirian, T. Mannel, N. Offen and Y. M. Wang, Phys. Rev. D **83** (2011) 094031 [arXiv:1103.2655 [hep-ph]].

Parameter	Value	Ref.	Parameter	Value	Ref.
m_π	139.6 MeV	PDG	f_π	130.4 MeV	PDG
m_B	5.28 GeV	PDG	$\alpha_s(M_Z)$	0.118	PDG
η_3	0.015	Ball, 1998	ω_3	-3	Ball, 1998
η_4	10	Ball, 1998	ω_4	0.2	Ball, 1998
$\langle \bar{q}q \rangle$	$(-0.246^{+0.027}_{-0.019})^3\ \mathrm{GeV}^3$	Duplancic, 2008	$\langle \bar{q}\sigma gGq \rangle$	$0.8\ \langle \bar{q}q \rangle$	Ioffe, 2005

Table: Summary of values of parameters used in the numerical analysis. Note the quark condensate is given at the scale 1 GeV.

BCL type analysis

Use parameterisation: $f(t) = \frac{1}{1-q^2/m_R^2} \sum_k \tilde{\alpha}_k \, z^k(t, t_0)$

$$\chi^2_{th} = \sum_{j,k=1}^{8} [f_j^{in} - f_+(q_j^2)] C_{jk}^{-1} [f_k^{in} - f_+(q_k^2)] + (f_+(0) - f_{\mathrm{LCSR}})^2/(\delta f_{\mathrm{LCSR}})^2$$

$$\chi^2_{exp} = \sum_{j,k=1}^{22} [\mathcal{B}_j^{in} - \mathcal{B}_j(f_+)] C_{\mathcal{B}\,jk}^{-1} [\mathcal{B}_k^{in} - \mathcal{B}_k(f_+)] \,, \qquad \chi^2 = \chi^2_{th} + \chi^2_{exp}$$

- f_j^{in} is FF on Lattice at q_j^2
- $\mathcal{B}_j^{in}$ are exp. partial branching fractions
- $\mathcal{B}_j(f_+)$ are integrated over the bins $[q_j^2, q_{j+1}^2]$, with given parametrization for $f_+(q^2)$.

Fitting to exp and theory one can extract V_{ub}