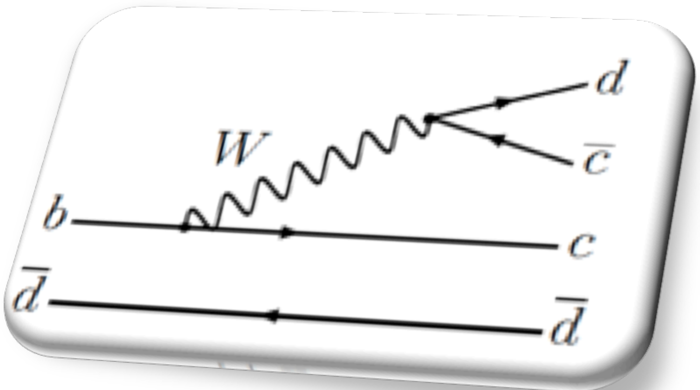
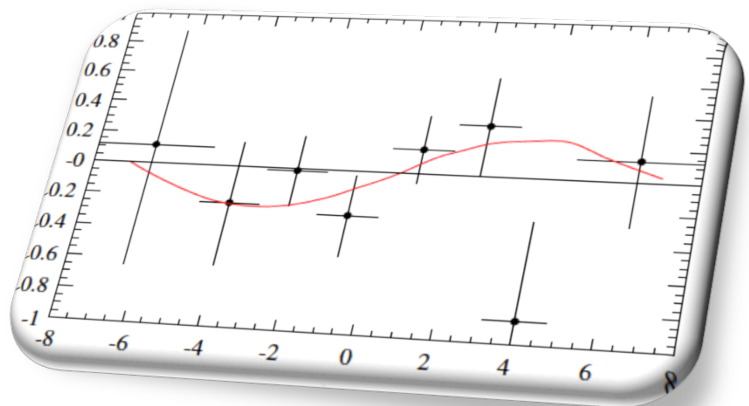
7th International Workshop on the CKM Unitarity Triangle
September 28 - October 2 2012, Cincinnati

Results on the CKM angle β from *BABAR*





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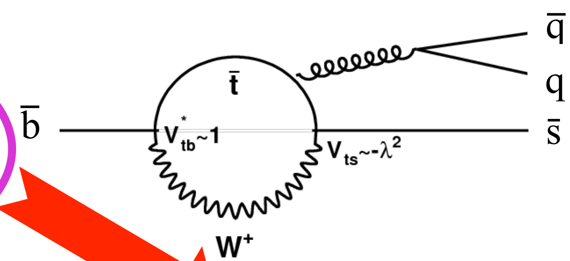
On behalf of the ***BABAR*** collaboration

Overview

General introduction

β from $b \to q\bar{q}s$

- Time dependent $B^0 \to K_S K_S K_S$
- Dalitz plot analysis of $B^0 \to K^+K^-K_S$

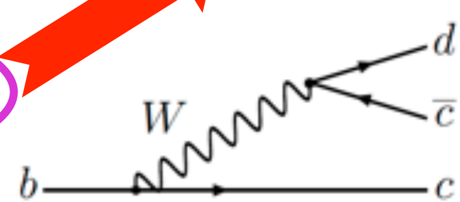


Comparison to β from $b \to c\bar{c}s$ (e.g. $B^0 \to J/\psi\ K_S$)

Other contributions from penguin processes may result in differences

β from $b \to c\bar{c}d$

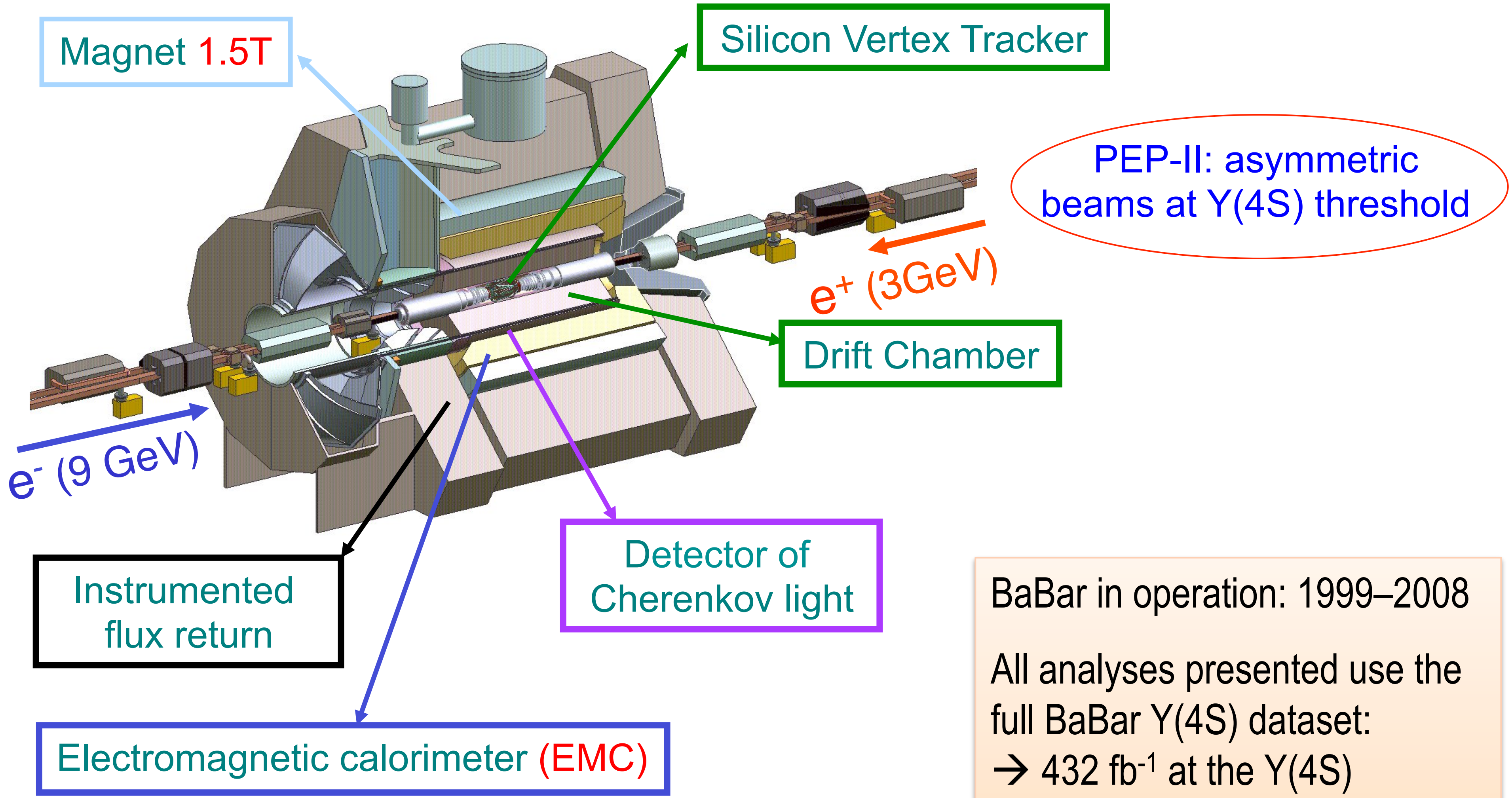
- Partially reconstructed $B^0 \to D^{*+}D^{*-}$



General introduction

- The BaBar detector and dataset
- Common analysis techniques
- Time dependent analysis and flavor tagging

The BaBar detector and dataset

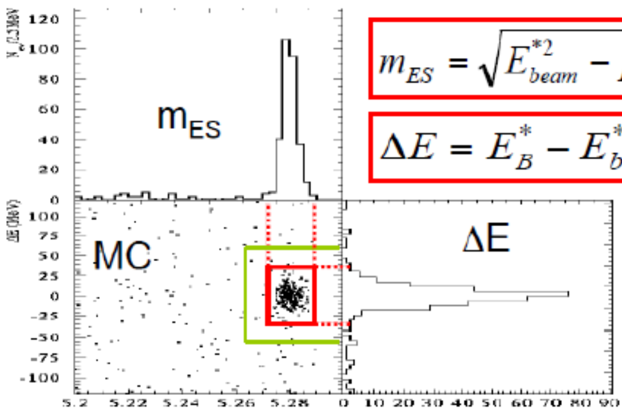


BaBar in operation: 1999–2008

All analyses presented use the full BaBar Y(4S) dataset:
→ 432 fb^{-1} at the Y(4S)
→ 467M $B\bar{B}$ pairs

Common analysis techniques

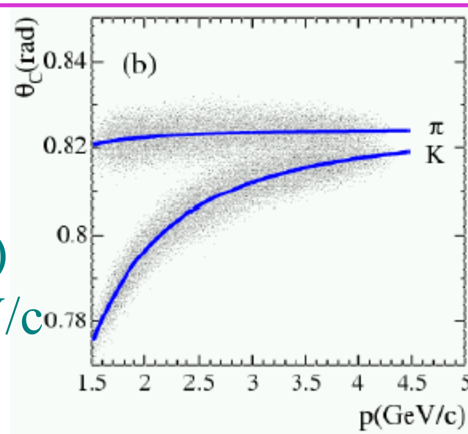
Kinematics of fully reconstructed B



$$m_{ES} = \sqrt{E^{*2}_{beam} - p^{*2}_B}$$

$$\Delta E = E^*_B - E^*_{beam}$$

Good charged particle ID (in particular K/π) up to few GeV/c



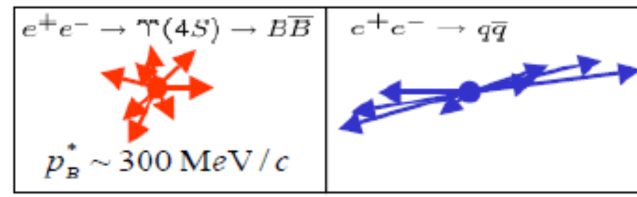
Background characterization:

→ Mainly continuum: $e^+e^- \to q\bar{q}$ (q = u,d,s,c).

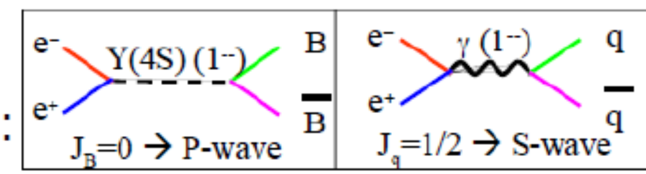
Suppression by multi-variable classifiers based on event-shape variables:

Fisher discriminant, Neural Networks (NN)…

Topology:



Angular distribution:

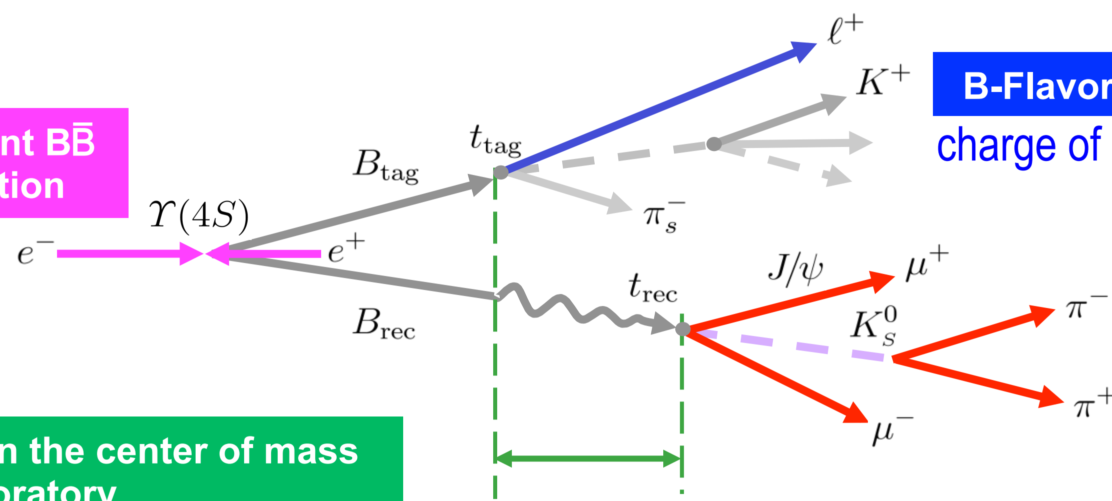


→ Background from B decays: classified by kinematic and topological properties

Variables are often combined to a likelihood function, used in a maximum likelihood fit for signal/background separation and to measure parameters of interest

Time dependent measurements, flavor tagging

Coherent $B\bar{B}$ production



B-Flavor tagging

charge of leptons, K, π

Exclusive B meson reconstruction

$B\bar{B}$ Almost at rest in the center of mass
Boosted in the laboratory

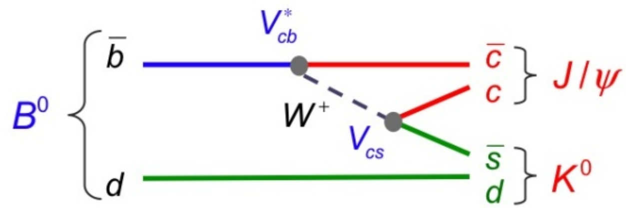
$\beta\gamma \sim 0.56$
$\Delta t \sim \tau_B \approx 1.5\text{ps} \Leftrightarrow \Delta z \sim 250\mu\text{m}$

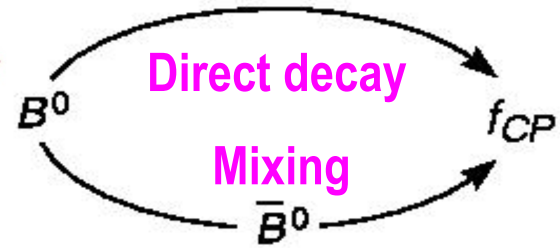
$$\Delta t \equiv t_{rec} - t_{tag} \approx \Delta z / \beta\gamma c$$

CKM angle β

Time dependent CP asymmetry in $b \to c\bar{c}s$

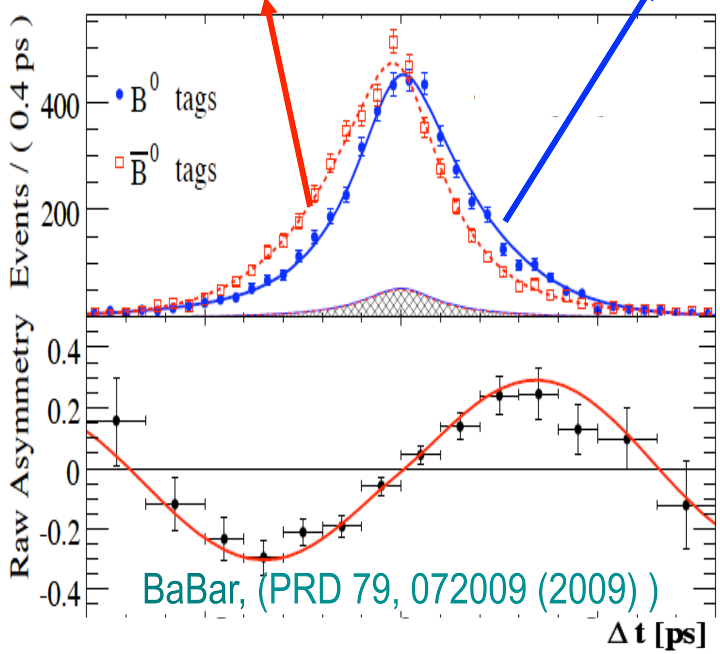
- With the "Golden Mode" ($B^0 \to J/\psi\ K^0_S$): "Golden" because there is ~only one decay amplitude





$$A_{CP}(\Delta t) = \frac{\Gamma[\bar{B}^0 \to J/\psi K_S](\Delta t) - \Gamma[B^0 \to J/\psi K_S](\Delta t)}{\Gamma[\bar{B}^0 \to J/\psi K_S](\Delta t) + \Gamma[B^0 \to J/\psi K_S](\Delta t)} = S\sin(\Delta m_d t) - C\cos(\Delta m_d t)$$

indirect (S) — direct (C)



BaBar, (PRD 79, 072009 (2009))

$$C_f = 0$$

$$S_f = -\eta_{CP} \sin 2\beta$$

$\Rightarrow$ Extraction of $\sin 2\beta$ from A_{cp}

β from $b \to q\bar{q}s$

- Time dependent CP asymmetry in $B^0 \to K_S K_S K_S$
  arXiv:1111.3636 [hep-ex], Phys.Rev.D85:054023 (2012)
  (in the same paper: amplitude analysis)

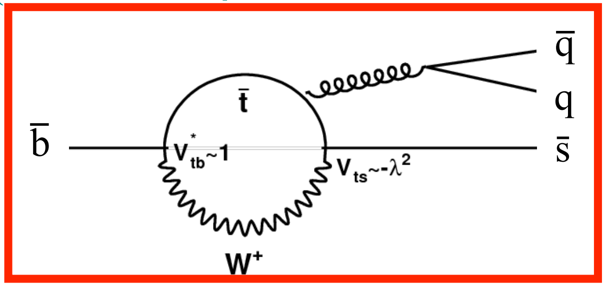
- CP violation in amplitude analysis of $B^0 \to K^+ K^- K_S$
  arXiv:1201.5897 [hep-ex], Phys.Rev.D85:112010 (2012)
  (in the same paper: $B^+ \to K^+ K^- K^+$ and $B^+ \to K_S K_S K^+$)

sin2β from b→s penguins

- Within the standard model (SM):

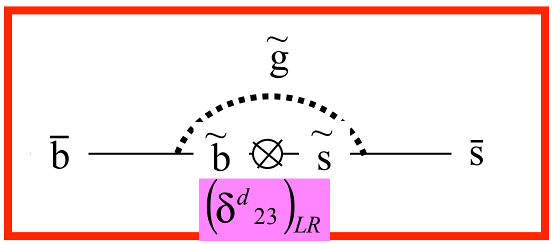
$$S_{c\bar{c}s} = S_{q\bar{q}s} + \Delta S_{SM} = -\eta_{CP} \sin 2\beta$$
$$C_{c\bar{c}s} \approx C_{q\bar{q}s} = 0$$

(dominant phase is the same as in b→c$\bar{c}$s)



Standard Model contribution

- New physics in the loop may cause deviation in the values of S and C.

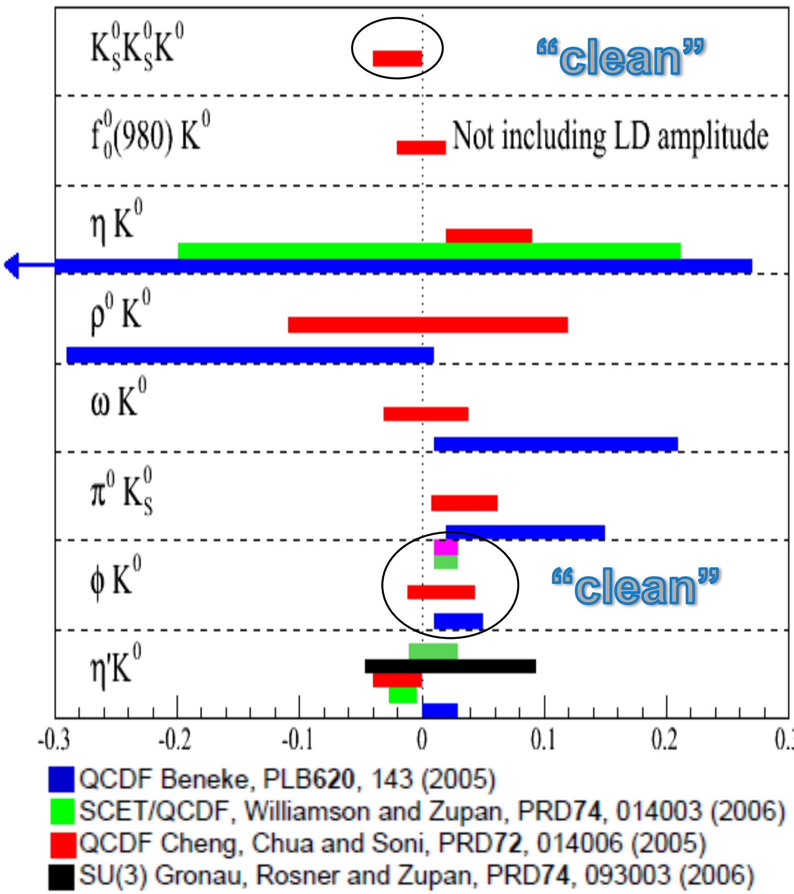


New Physics contribution

- Definitions:

$$\Delta S = S_{c\bar{c}s} - S_{q\bar{q}s}$$
$$\sin 2\beta_{eff} = -\eta_{CP} S_{q\bar{q}s}$$

Theoretical prediction for ΔS_{SM}



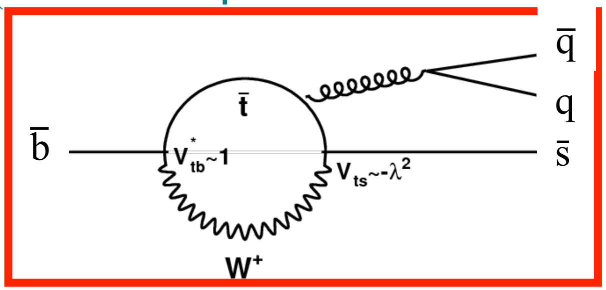
Precise predictions allow for powerful tests of the SM

sin2β from b→s penguins

- Within the standard model (SM):

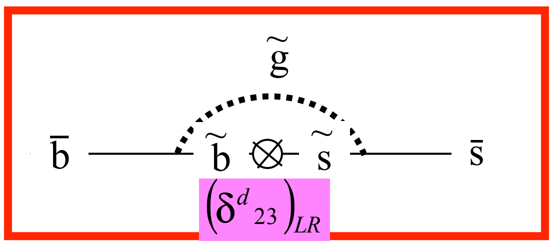
$$S_{c\bar{c}s} = S_{q\bar{q}s} + \Delta S_{SM} = -\eta_{CP}\sin 2\beta$$
$$C_{c\bar{c}s} \approx C_{q\bar{q}s} = 0$$

(dominant phase is the same as in b→$c\bar{c}s$)



Standard Model contribution

- New physics in the loop may cause deviation in the values of S and C.

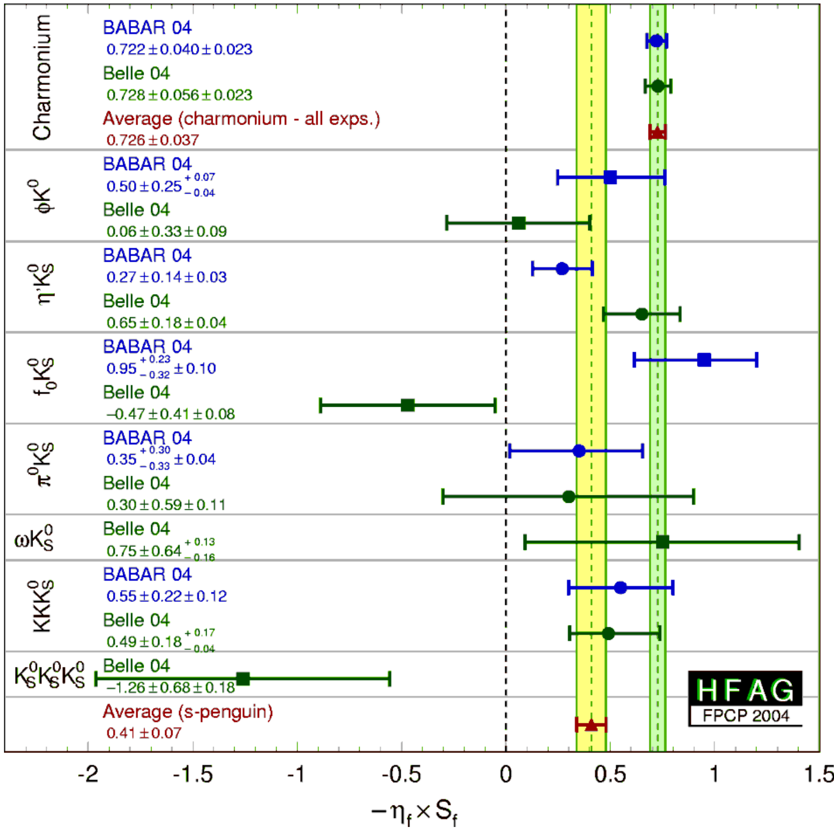


New Physics contribution

- Definitions:

$$\Delta S = S_{c\bar{c}s} - S_{q\bar{q}s}$$
$$\sin 2\beta_{\text{eff}} = -\eta_{CP} S_{q\bar{q}s}$$

In 2004:



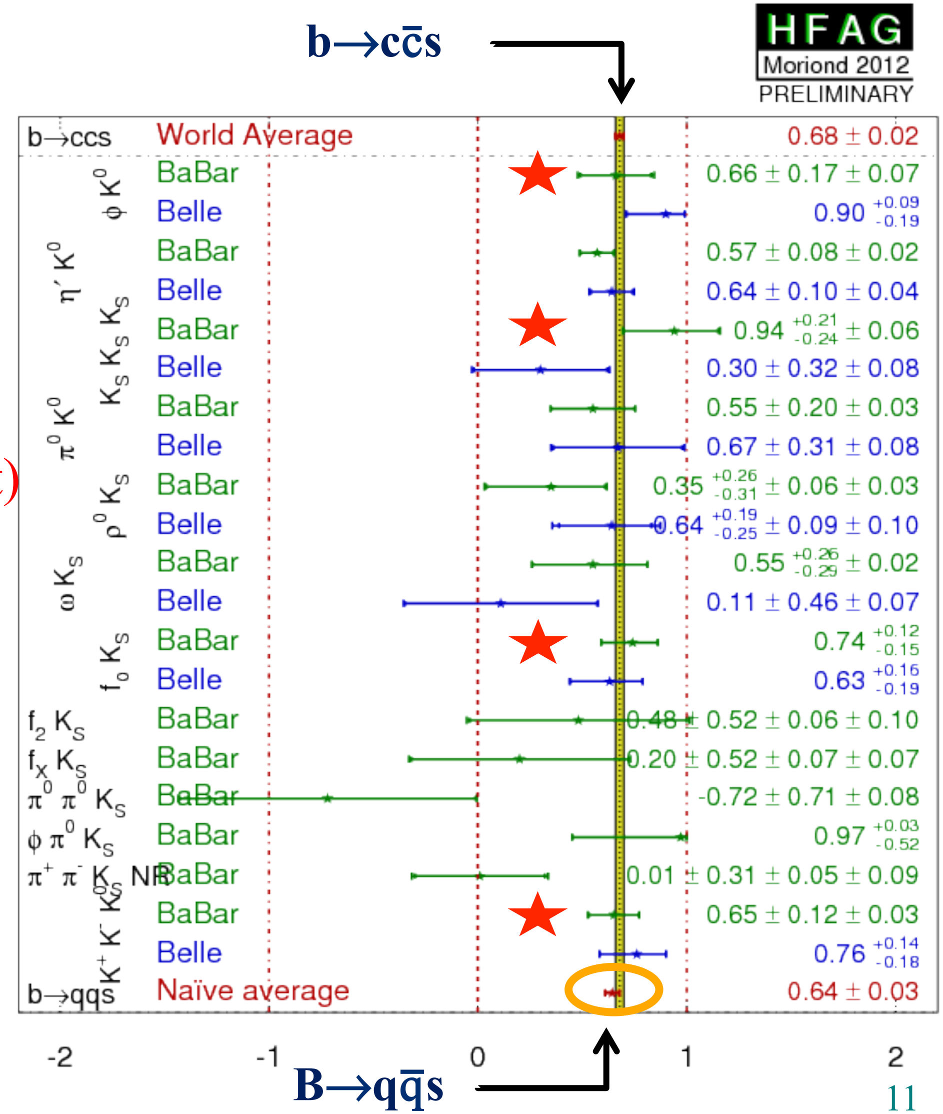
Tensions between sin2β from b→$c\bar{c}s$ and b→$q\bar{q}s$ (**ΔS < 0**)

sin2β from b→s penguins

The situation today is quite different

- **Fresh sin2β world averages (HFAG):**
  - **$b \to c\bar{c}s$: 0.68 ± 0.02**
  - **$b \to q\bar{q}s$: 0.64 ± 0.04** (naïve!)
- Improvements:
  - hints of trends/deviations in previous measurements clarified by B factories
  - several results from (Time Dependent) Dalitz Plot analyses
- Still… **minor** tension persists

Results presented here marked with ★



$B^0 \to K_S K_S K_S$ overview and motivations

- Small theoretical uncertainty $\Rightarrow$ Comparison with $b \to c\bar{c}s$ is more meaningful
- Low background level (difficult to "imitate" 3 K^0_S)

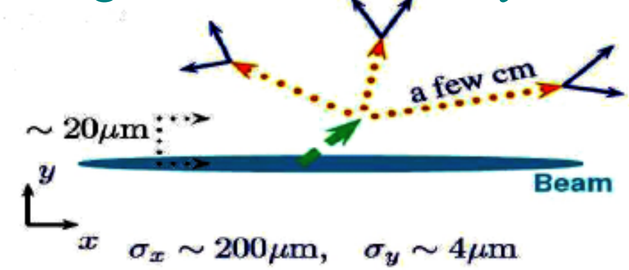
Inclusive time dependent analysis to extract CP asymmetries S and C

$B^0 \to 3K^0_S(\pi^+\pi^-)$

$B^0 \to 2K^0_S(\pi^+\pi^-)\, K^0_S(\pi^0\pi^0)$

CP=+1 eigenstate $\Rightarrow$ possible

K_S is long lived $\Leftrightarrow$ no tracks emerge from the B decay vertex

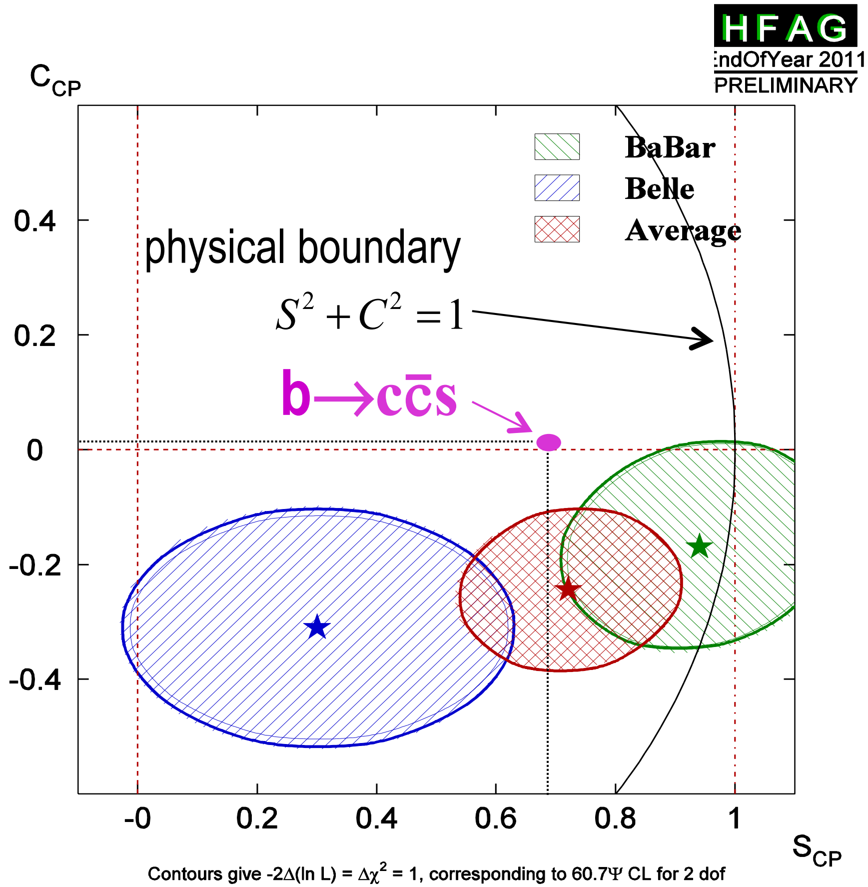


- Maximum likelihood fit, using: m_{ES}, ΔE, Neural network and **Δt**

$$\mathcal{P}^i_{\rm sig}(\Delta t, \sigma_{\Delta t}; q_{\rm tag}, c) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \Big\{ 1 + q_{\rm tag}\frac{\Delta D_c}{2} + q_{\rm tag}\langle D\rangle_c \Big[\mathcal{S}\sin(\Delta m_d \Delta t) - \mathcal{C}\cos(\Delta m_d \Delta t)\Big]\Big\} \otimes \mathcal{R}_{\rm sig}(\Delta t, \sigma_{\Delta t}),$$

Results

$B^0 \to K_S K_S K_S$



$$S = -0.94 \pm {}^{0.24}_{0.21}\ (\text{stat.}) \pm 0.06\ (\text{syst.})$$

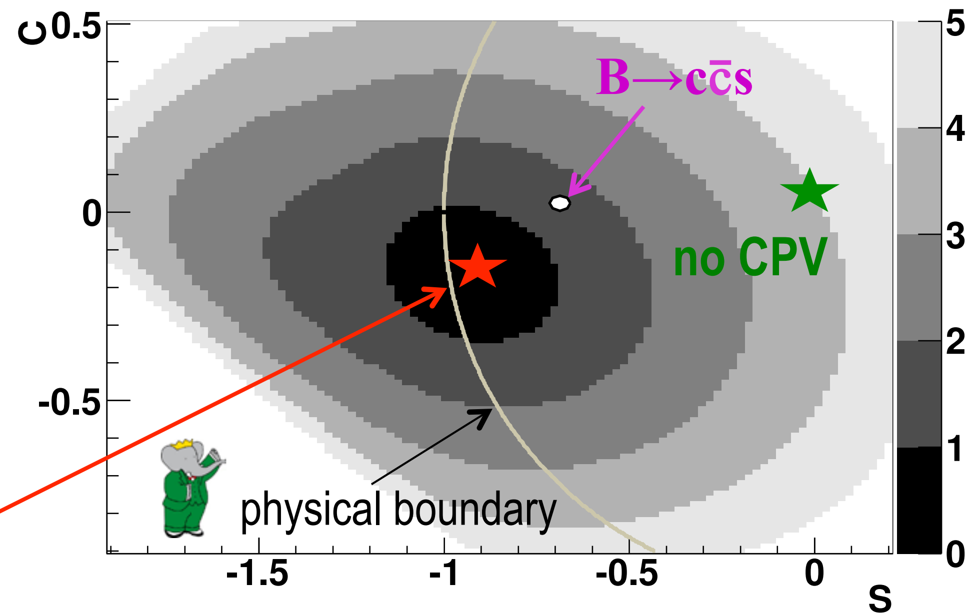
$$C = -0.17 \pm 0.18\ (\text{stat.}) \pm 0.04\ (\text{syst.})$$

Signal yields:

$201 \pm {}^{16}_{15}$ $B^0 \to 3K^0_S(\pi^+\pi^-)$ (Purity = 40%)

$62 \pm {}^{13}_{12}$ $B^0 \to 2K^0_S(\pi^+\pi^-)\ K^0_S(\pi^0\pi^0)$

Confidence level contours (C, S)



First evidence of CPV (at 3.8σ)

$B^0 \to K^+K^-K_S$ overview and motivations

- Time dependent analysis to measure the effective β
  - → includes ϕK_S (small theoretical uncertainty)
  - → not a CP eigenstate! CP content depends on the intermediate state
- Dalitz-plot (DP) analysis to separate intermediate (resonant) CP eigenstates
- DP structure of $B^+ \to K^+K^-K^+$ and $B^+ \to K_SK_SK^+$ used for $B^0 \to K^+K^-K_S$
  - → Large sample (5269±84 signal events, purity = 43%)
  - → 2 K_S in the final state: helpful to study the nature of broad $f_X(1500)$

  (For details on $B^+ \to K^+K^-K^+$ and $B^+ \to K_SK_SK^+$: Eugenia Puccio, WG V, Sunday morning)
- Direct access to phases: no trigonometric ambiguities (next slide…)
- Reconstruction of both $K_S \to \pi^+\pi^-$ and $K_S \to \pi^0\pi^0$

Dalitz plot and the isobar model

$B^0 \to K^+K^-K_S$

- Each intermediate resonance in P → 1 2 3 appears as a structure in the DP according to its mass, width and spin
- Parameterization of intermediate state amplitudes:

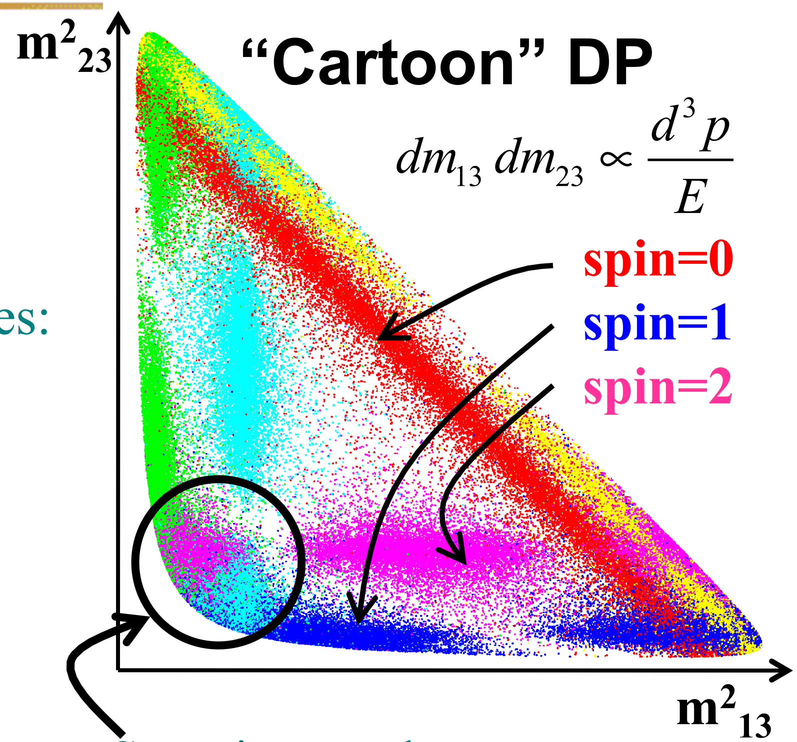
$A \sim \Sigma\, c_i\, \mathbf{F}(m^2_{13}, m^2_{23})$ B decays

$\bar{A} \sim \Sigma\, \bar{c}_i\, \mathbf{F}(m^2_{13}, m^2_{23})$ $\bar{B}$ decays

complex e.g. Breit-Wigner

$$\frac{d\Gamma}{ds_{12}ds_{23}d\Delta t} = \frac{1}{(2\pi)^3}\frac{1}{32m^3_{B^0}}\frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \times[|\mathcal{A}|^2+|\bar{\mathcal{A}}|^2 - Q(1-2w)(|\mathcal{A}|^2-|\bar{\mathcal{A}}|^2) \times\cos\Delta m_d\Delta t + Q(1-2w) \times 2\,\mathrm{Im}[e^{-2i\beta}\bar{\mathcal{A}}\mathcal{A}^*]\sin\Delta m_d\Delta t],$$

Directly extracted parameters: isobar amplitudes c_i
Other parameters (S, C, A_{CP}, phases, Branching Fractions) are computed from them



Superimposed resonant contributions
→ Interference
→ access to phases with no ambiguity such as $\sin 2\beta_{\text{eff}} = \sin(180° - 2\beta_{\text{eff}})$

Determining the signal model

$B^0 \to K^+K^-K_S$

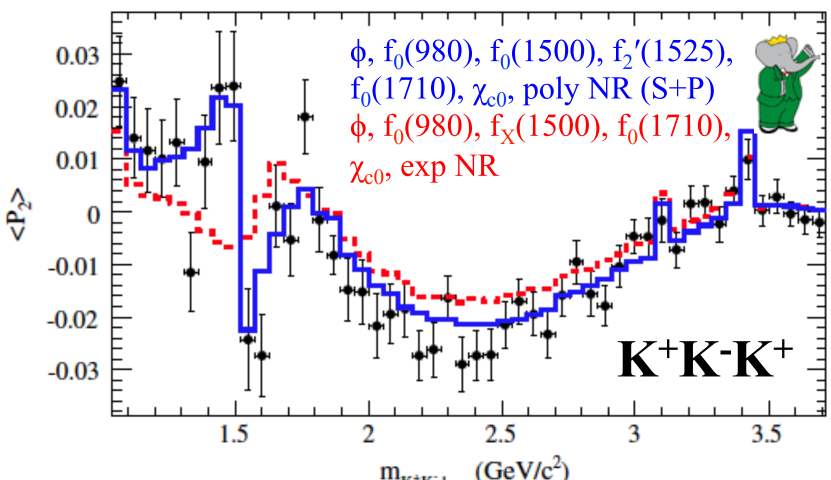
- Prior to fitting CPV parameters, the nominal DP models are established
  - → CPV parameters set to the SM ones
  - → Legendre polynomial moments vs invariant masses, used to compare data and fit

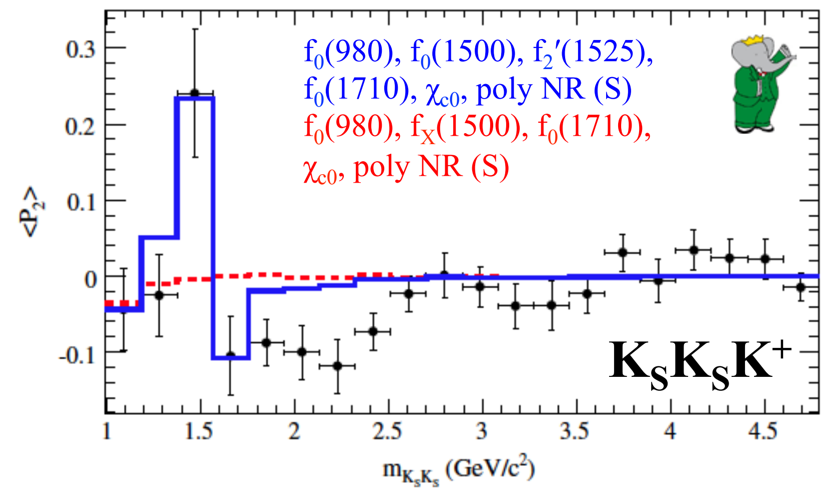
$$\langle P_\ell(\cos\theta_3)\rangle \equiv \int_{-1}^{1} d\Gamma P_\ell(\cos\theta_3) d\cos\theta_3$$

- **$K^+K^-K^+$**: $\phi(1020)$, $f_0(980)$, $f_0(1500)$, $f_2'(1525)$, $f_0(1710)$, χ_{c0}, poly. NR
- **$K_SK_SK^+$**: , $f_0(980)$, $f_0(1500)$, $f_2'(1525)$, $f_0(1710)$, χ_{c0}, poly. NR

} Best fits

In the 3 modes: no need for the $f_X(1500)$
good description with $f_0(1500)$, $f_2'(1525)$, $f_0(1710)$)





Results

$B^0 \to K^+K^-K_S$

N_{sig} = 1579±46 (Purity = 18%)

BF = $(25.4\pm0.9\pm0.8)\times10^{-6}$ [$\chi_{c0}K$ excluded]

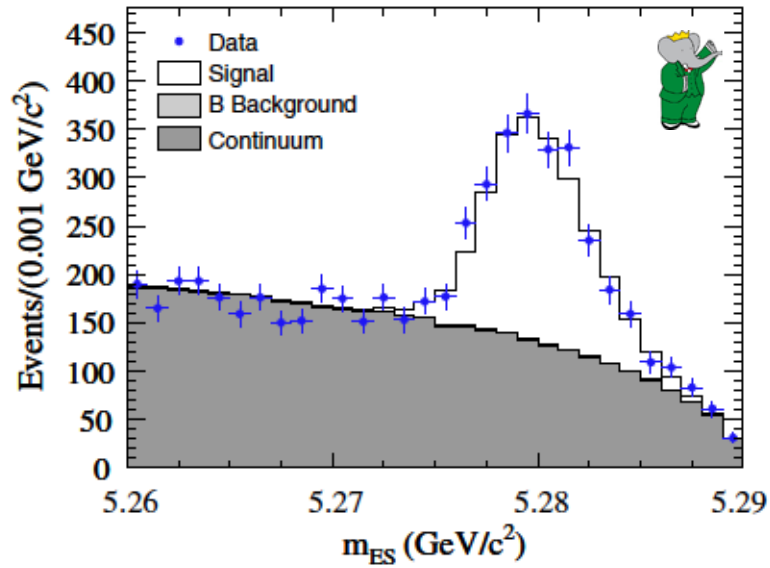
$\beta_{eff}(\phi K_S) = (21\pm6\pm2)°$ ← most precise

$\beta_{eff}(f_0 K_S) = (18\pm6\pm4)°$

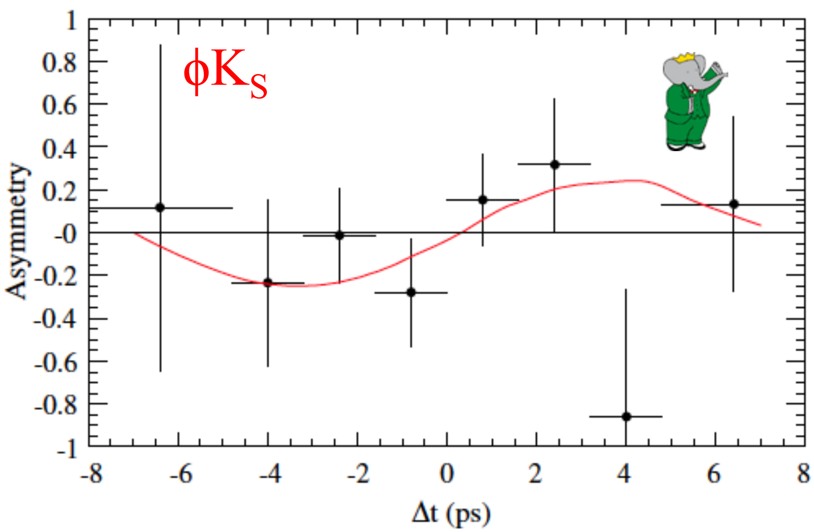
β_{eff}(non-ϕ, non-f_0) = $(20.3\pm4.3\pm1.2)°$

Agree with SM

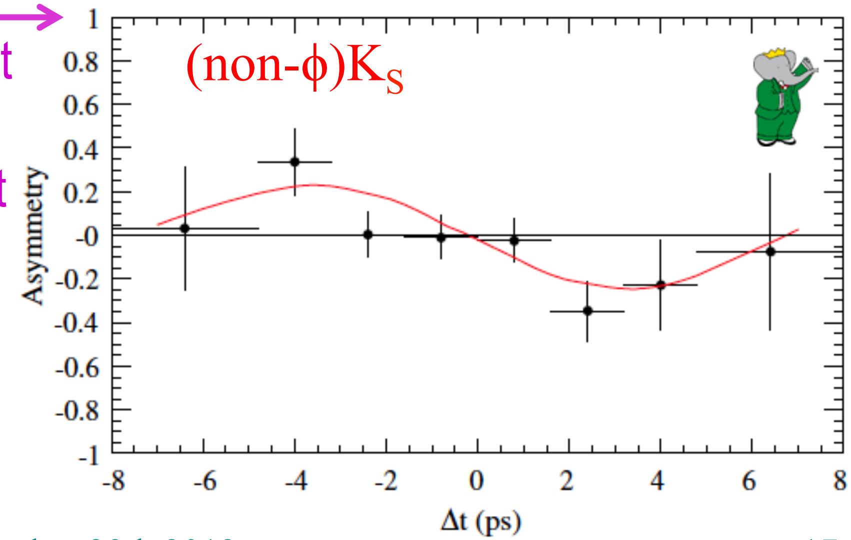
90°-β_{eff} excluded at 4.8σ (ambiguity in $J/\psi K_S$)



Four local minima within 9 units of -2lnL (mostly degenerate in β)



different CP content



β from $b \to c\bar{c}d$

- Time dependent CP asymmetry of partially reconstructed $B^0 \to D^{*+}D^{*-}$ decays

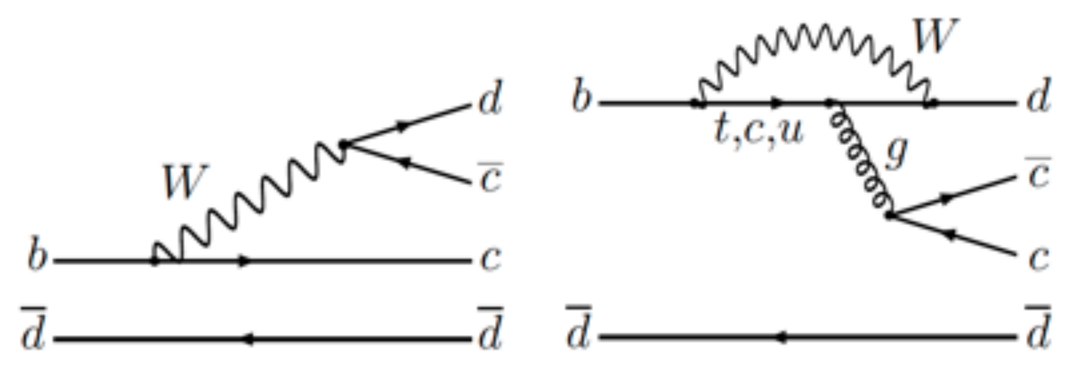
  arXiv:1208.1282 [hep-ex]

  (Submitted to Phys.Rev.D)

$\sin 2\beta$ from $B^0 \to D^{*+}D^{*-}$ decays

$B^0 \to D^{*+}D^{*-}$

- In such $b \to c\bar{c}d$ transitions, TD asymmetry is a measure of $S_\eta \cong \eta \sin(2\beta)$, provided that contribution from penguins can be neglected.



- VV final state → admixture of CP=+1 and CP=-1 amplitudes.

- Theoretical calculations based on factorization and heavy quark symmetry: in the SM penguin contributions lead to corrections of ~few % to the determination of $\sin 2\beta$ from the TD CPV asymmetry

  Z. Z. Xing, Phys. Lett. **B443**, 365 (1998).
  Z. Z. Xing, Phys. Rev. **D61**, 014010 (1999).

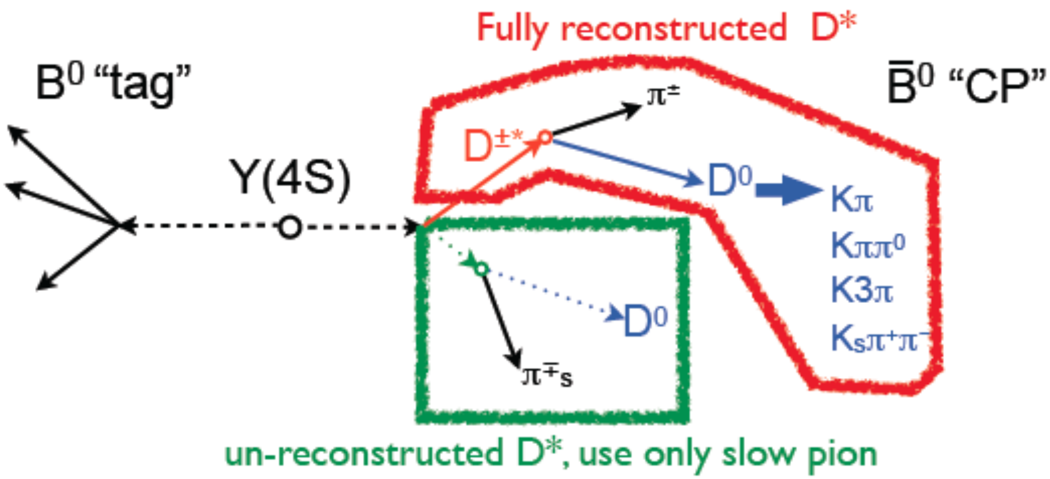
- Large deviation in S_η with respect to that measured in $b \to c\bar{c}s$ transitions could be an indication of physics beyond the SM

  M. Gronau, J. L. Rosner and D. Pirjol, Phys. Rev. D **78**, 033011 (2008).
  Y. Grossman and M. P. Worah, Phys. Lett. **B395**, 241 (1997).
  R. Zwicky, Phys. Rev. D **77**, 036004 (2008).

Analysis strategy
partial reconstruction

$B^0 \to D^{*+}D^{*-}$

- Angular analysis needed to separate CP eigenstates
  (possible with fully reconstructed events).
- BaBar and Belle full reconstruction analyses measured the CP even component CP parameters S_+ and C_+, and the fraction $R_\perp$ of CP-odd amplitude:
  $$R_\perp=0.158\pm0.028\pm0.006$$
- In a partial reconstruction analysis, we measure average S and C parameters which are related to C_+ and S_+ by the relations $C=C_+$ and $S=S_+(1-2R_\perp)$



→ One fully reconstruct one D*
→ Match reconstructed D* with a slow pion of opposite sign
→ Powerful discriminating variable: recoiling D^0 mass m_{rec}
→ Another useful extracted information: missing D^0 direction

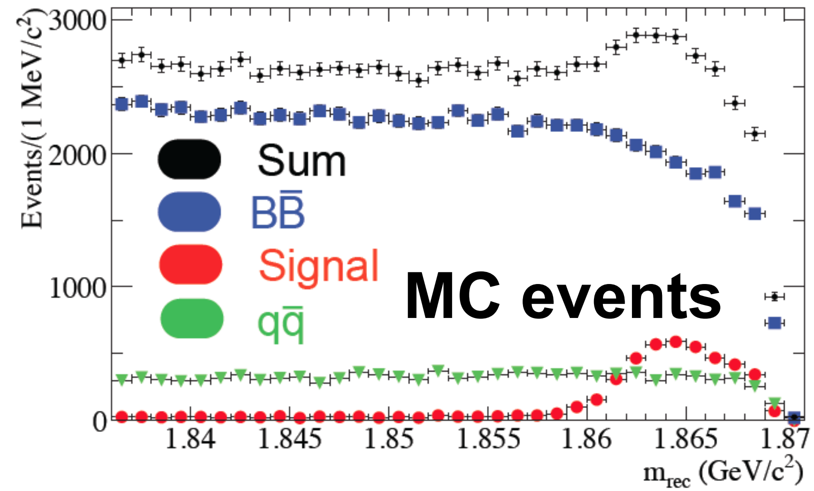
- Pros: gain in statistics (with an almost independent sample)
  Cons: Higher background, larger systematic uncertainty

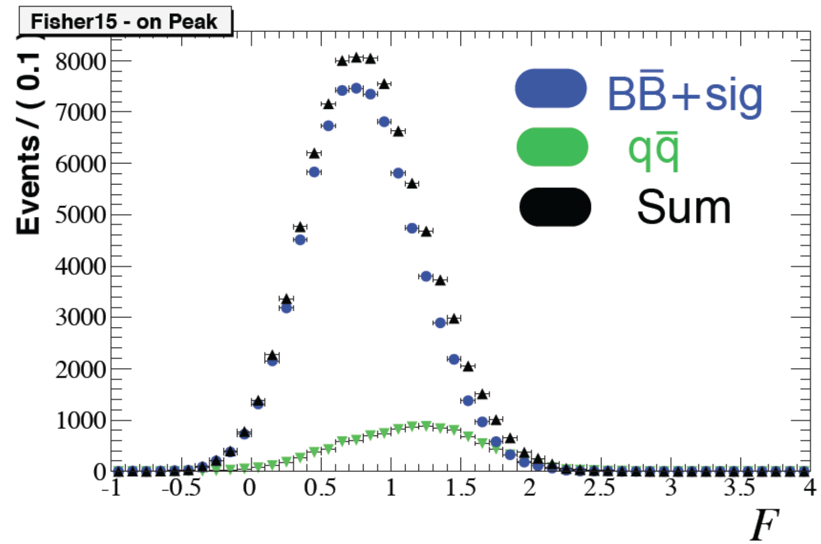
Analysis strategy

Variables and maximum likelihood fit

$B^0 \rightarrow D^{*+}D^{*-}$

- Recoil mass m_{rec} :
  → Signal peaks at D^0 mass
- Other fit variables: Fisher discriminant, Δt
- Additional dilution due to tagging tracks from unreconstructed D^0.
- Separate fits to two categories: Lepton or K
- Three stages in fit:
  - Kinematic fit (variable shapes, signal fraction)
  - Determining the mistag probabilities; determining the additional dilution (unreconstructed D^0) from data.
  - Time dependent fit, to extract the CP parameters S and C



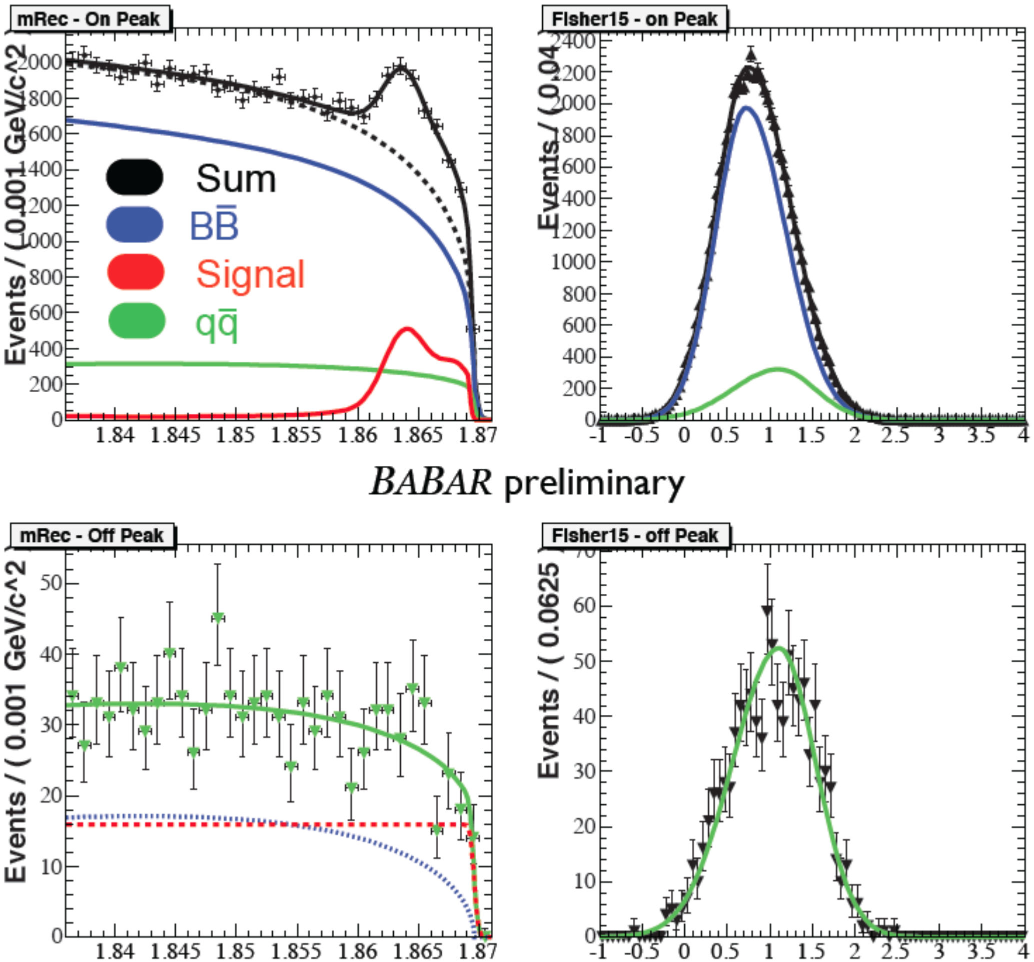


Results

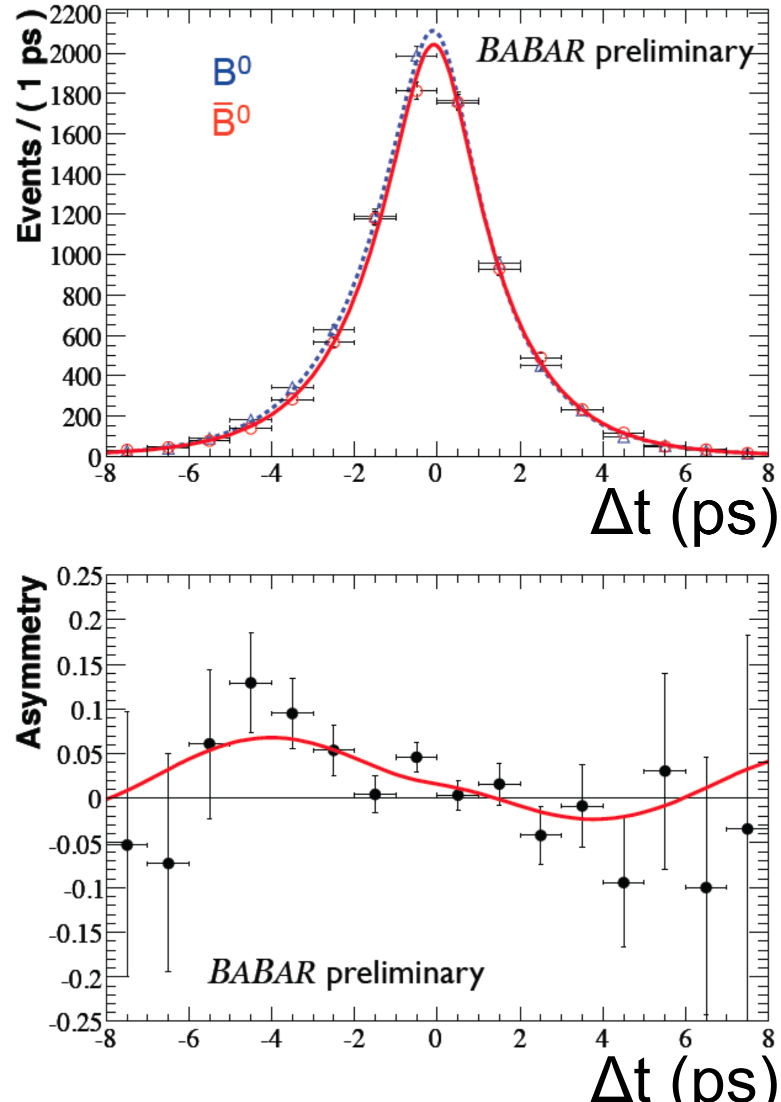
Kaon tags

$B^0 \to D^{*+}D^{*-}$

Kinematic fit



Time dependent fit

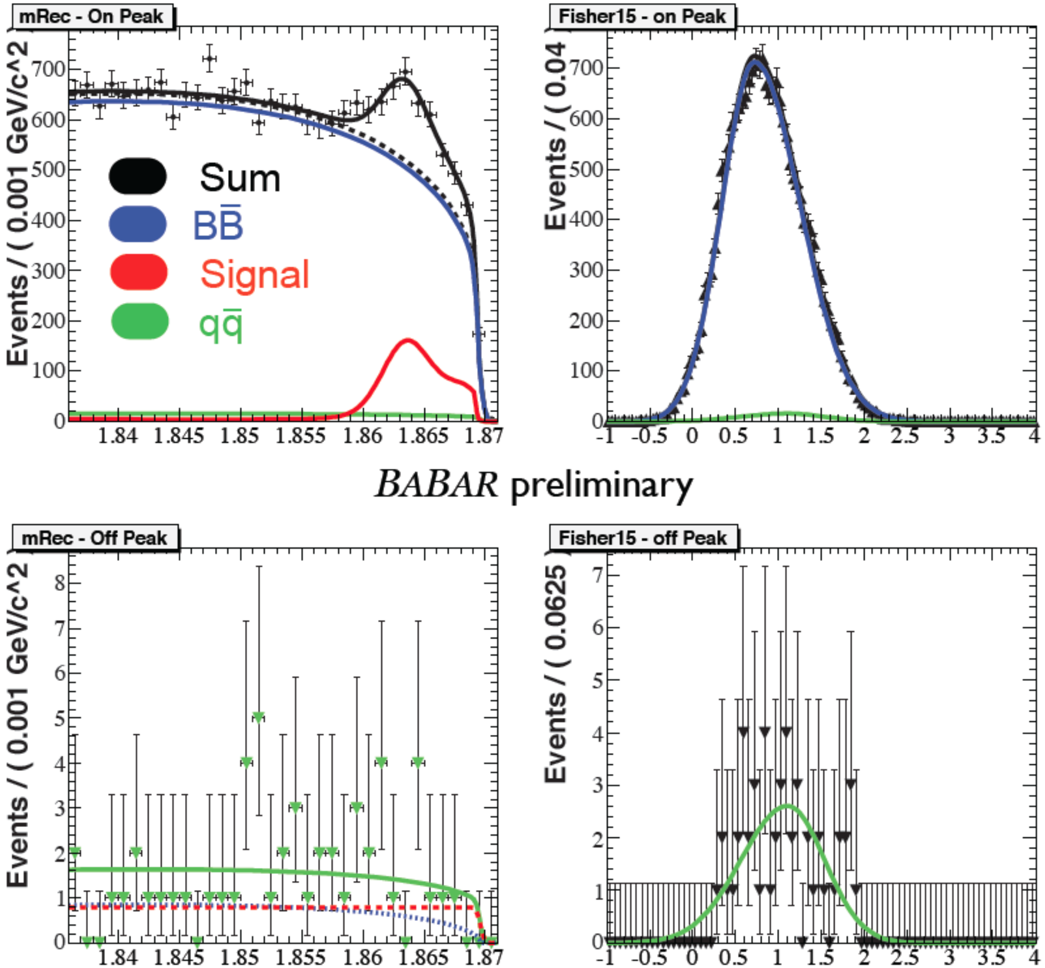


$$C = +0.12 \pm 0.11$$
$$S = -0.42 \pm 0.16$$

Results
Lepton tags

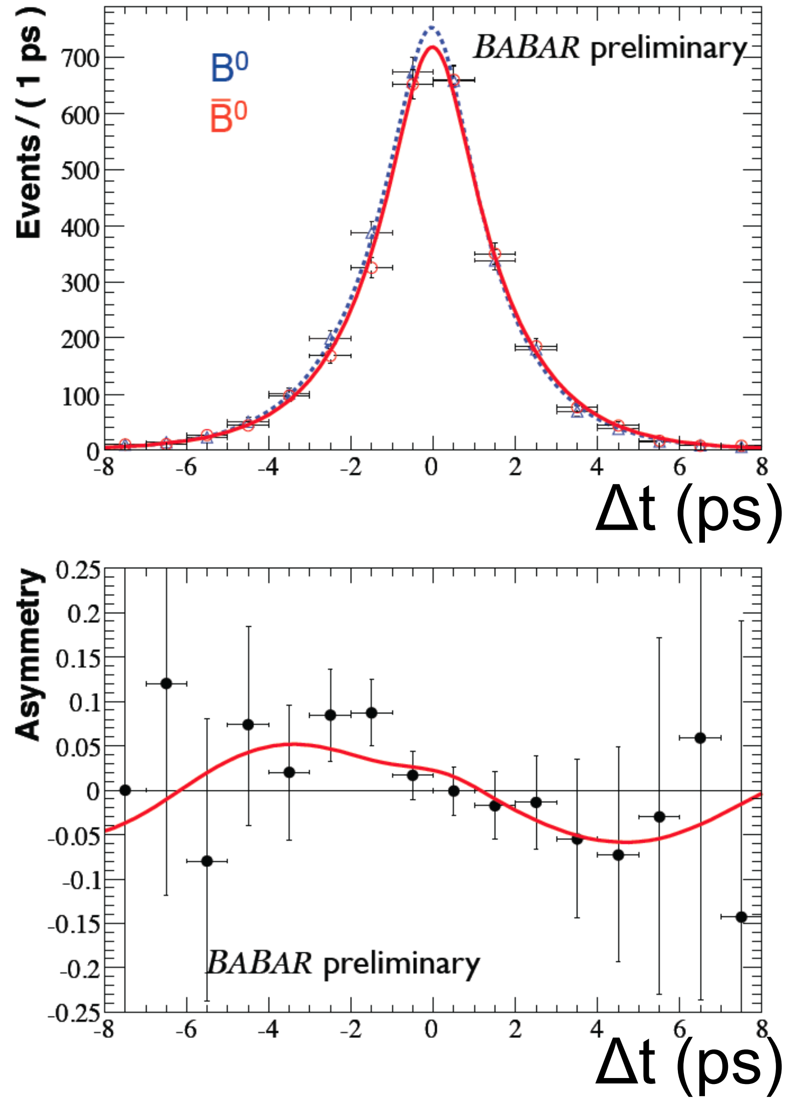
$B^0 \to D^{*+}D^{*-}$

Kinematic fit



$$C = +0.20 \pm 0.15$$
$$S = -0.21 \pm 0.20$$

Time dependent fit



Results

Combination of kaon and lepton tags

$B^0 \rightarrow D^{*+}D^{*-}$

Inclusive combination (preliminary):

$$C = +0.15 \pm 0.09 \pm 0.04$$
$$S = -0.34 \pm 0.12 \pm 0.05$$

Assuming negligible penguin contributions:

$S_+ = -S_-$; $C = C_+$

$S = S_+ (1 - 2R_\perp)$,

then using[1] $R_\perp = 0.158 \pm 0.02$

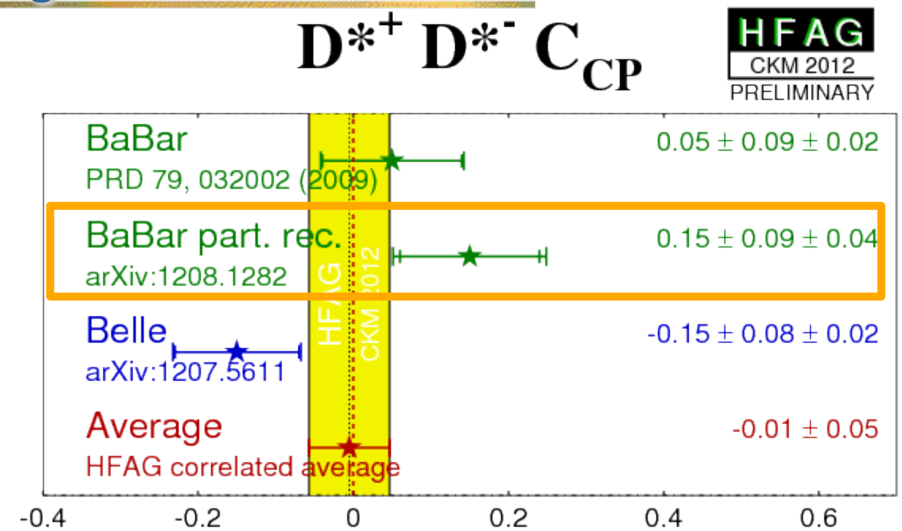
$$C_+ = +0.15 \pm 0.09 \pm 0.04$$
$$S_+ = -0.49 \pm 0.18 \pm 0.07 \pm 0.04$$

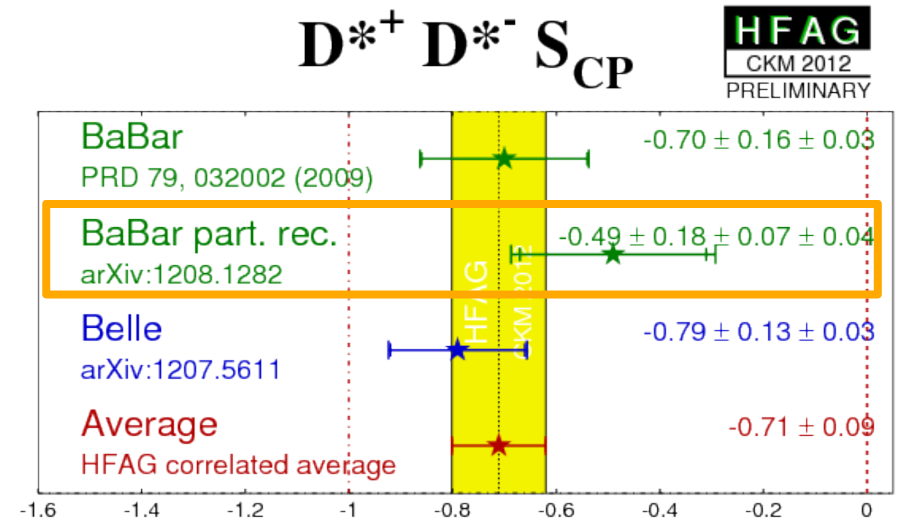
stat. syst. $R_\perp$

preliminary

The result is compatible with the latest (fully reconstructed) from BaBar[1] and Belle[2], and in agreement with SM predictions

Improves previous BaBar errors by ~20%

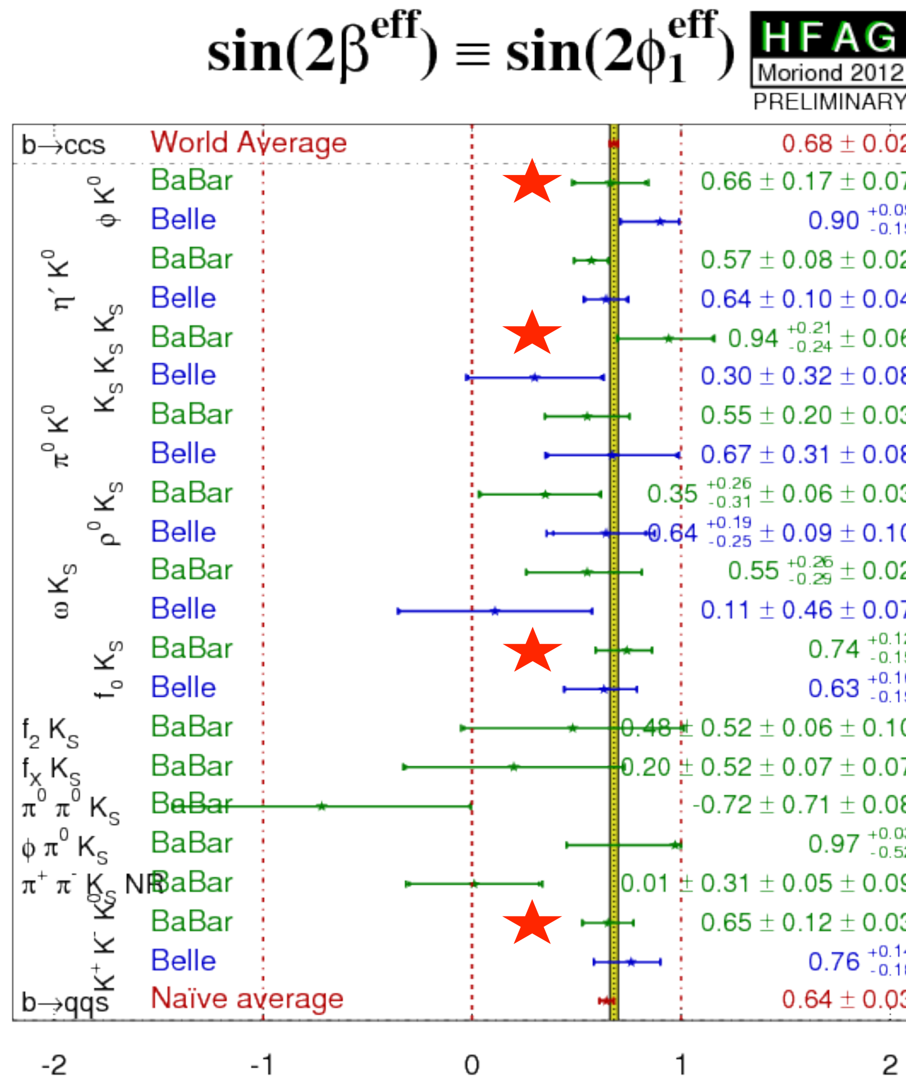




[1] B. Aubert et al. (BABAR collaboration), Phys. Rev. D79, 032002 (2009) ;
[2] B. Vervink et al. (Belle collaboration), Phys. Rev. D80, 111104 (2009).; Belle Collab. EPS 2011 Preliminary

Summary and Conclusions

- BaBar continues to produce physics results, adding more information and using more sophisticated analysis techniques to improve the precision of measurements in hadronic B decays
- All measurements of β presented here, in $b\to q\bar{q}s$ and $b\to c\bar{c}d$ processes, agree with β in $b\to c\bar{c}s$ processes (standard model prediction)
- Larger samples are needed to push further the comparisons with $b\to c\bar{c}s$, and tell whether or not there could be indications for new physics…

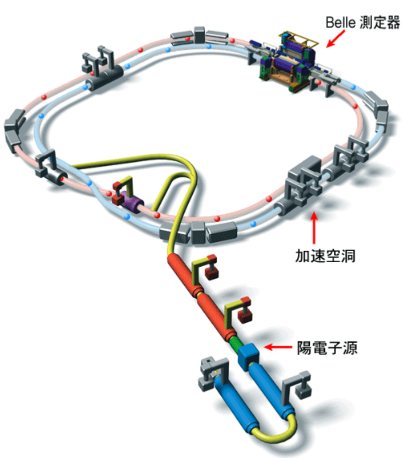


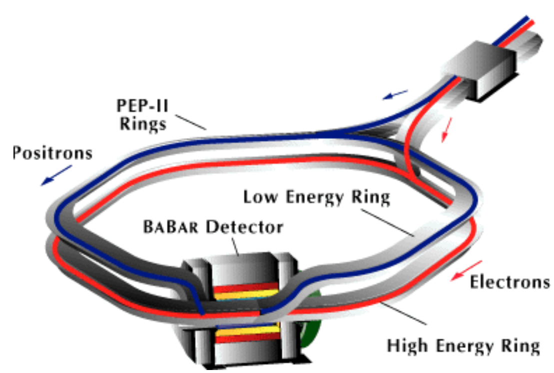
★ contribution from analyses shown here

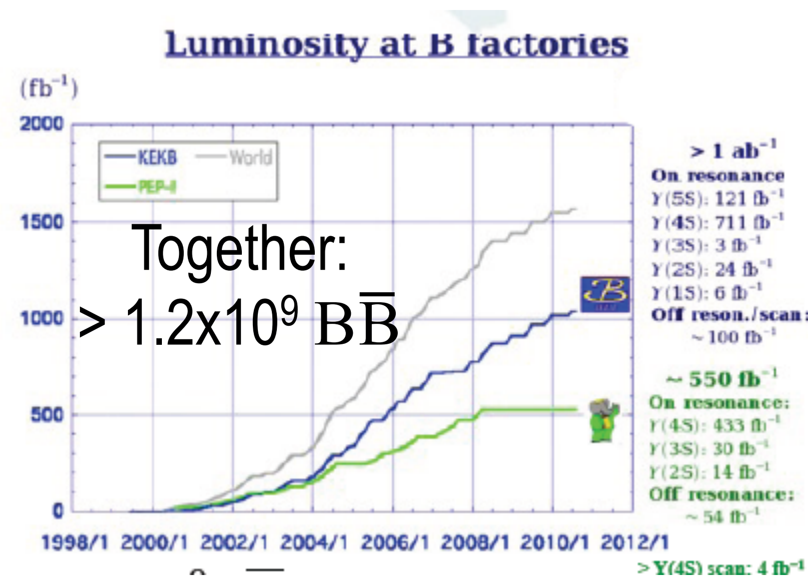
Backup

More on B-Factories

- Data taking periods over for the B-Factories
  In April 2008 for BABAR
  In June 2010 for Belle
- Outstanding luminosity records
  BABAR: 433 fb^{-1} @ $\Upsilon(4S)$ + ~54 fb^{-1} 40 MeV below
  Belle: 711 fb^{-1} ~100 fb^{-1}







Together:
> 1.2×10^{9} $B\overline{B}$

TD analysis - Backgrounds and Yields

$B^0 \rightarrow K^0_S K^0_S K^0_S$

TABLE VI: Summary of B-background modes included in the fit model of the time-dependent analysis. The expected number of events takes into account the branching fractions ($\mathcal{B}$) and efficiencies. In case there is no measurement, the branching fraction of an isospin-related channel is used. All the fixed yields are varied by ±100% for systematic uncertainties.

Submode	Background mode	Varied	$\mathcal{B}$ [$\times 10^{-6}$]	Number of events
$B^0 \rightarrow 3K^0_S(\pi^+\pi^-)$	$K^0_S K^0_S K^0_L$	no	2.4	0.71
	$K^0_S K^0_S K^{*0}$	no	27.5	9.55
	$K^0_S K^0_S K^+$	no	11.5	4.27
	$B^0 \rightarrow$ {neutral generic decays}	yes	not applicable	21.7
	$B^+ \rightarrow$ {charged generic decays}	yes	not applicable	15.5
$B^0 \rightarrow 2K^0_S(\pi^+\pi^-)K^0_S(\pi^0\pi^0)$	$K^0_S K^0_S K^0_L$	no	2.4	0.67
	$K^0_S K^0_S K^{*0}$	no	27.5	5.3
	$K^0_S K^0_L K^{*0}$	no	27.5	0.3
	$K^0_S K^0_S K^+$	no	11.5	2.9
	$K^0_S K^0_S K^{*+}$	no	27.5	7.2
	$B^0 \rightarrow$ {neutral generic decays}	yes	not applicable	73.6
	$B^+ \rightarrow$ {charged generic decays}	yes	not applicable	73.8

Species	$3K^0_S(\pi^+\pi^-)$	$2K^0_S(\pi^+\pi^-)K^0_S(\pi^0\pi^0)$
Signal	201^{+16}_{-15}	62^{+13}_{-12}
Continuum	3086^{+56}_{-54}	7086^{+85}_{-83}
B^+B^- bkg	-54^{+29}_{-24}	45^{+34}_{-30}
$B^0\overline{B}^0$ bkg	9^{+31}_{-30}	4^{+38}_{-29}

TD analysis

$B^0 \to K^0_S K^0_S K^0_S$

B decay vertex and K^0_S reconstruction

There are no charged particles coming from the primary vertex: no direct tracks

Reconstruct vertex using **charged pions** from **K_s^0** decay

Ensure quality of vertex by using only events where both pions of at least one K_s^0 have hits in the strips in both dimensions in the vertex detector (SVT).

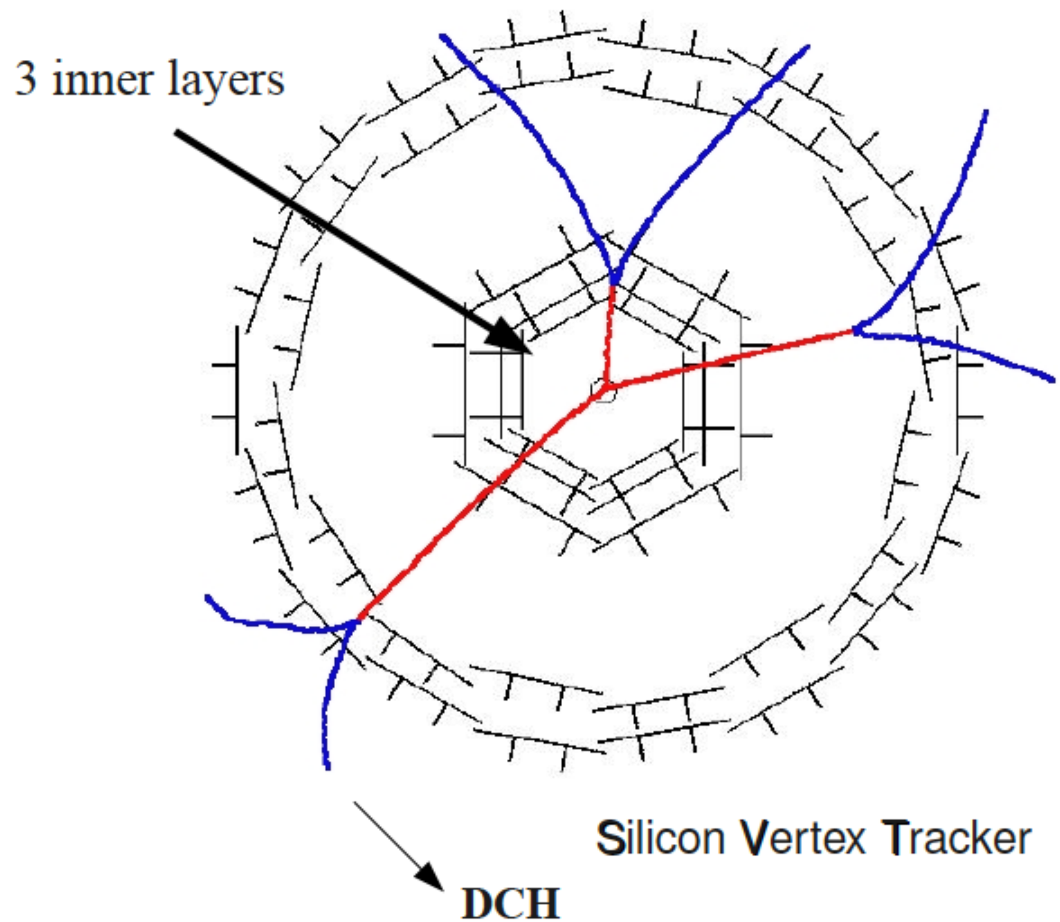
Good quality K_S:

hits in both dimensions in 3 inner layers (class 1)

hits in both dimensions but not in 3 inner layers (class 2)

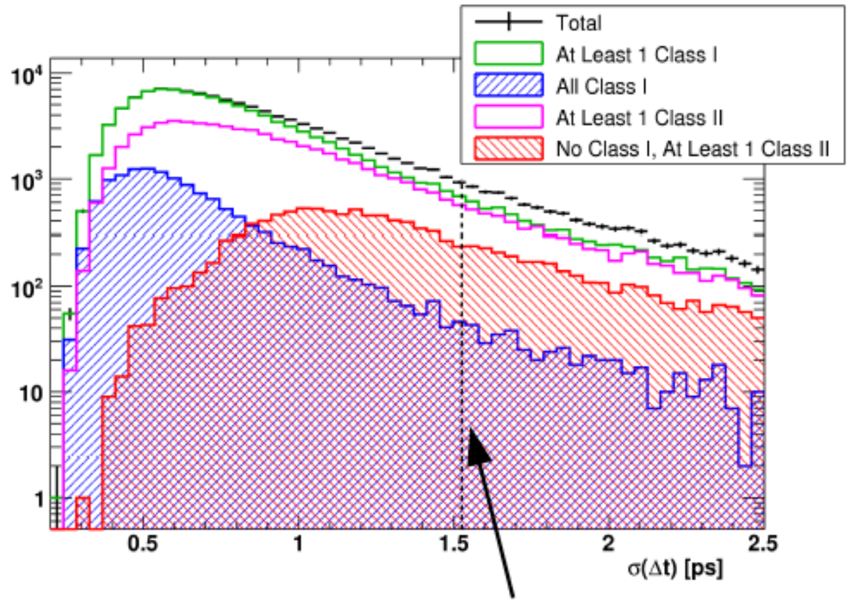
Bad quality K_S:

hits in only one dimension or no hits (classes 3 and 4).



TD analysis

B decay vertex, K^0_S reconstruction, Δt measurement



Average B lifetime

Δt PDF is convoluted event by event with resolution function.

This function takes into account the error $\sigma\Delta t$.
Rule of thumb: An average (signal) event contributes to the measurement when the error is smaller than the lifetime of the B.

Tail, core and outlier **G**(aussians)

$$R(\delta t;\sigma\Delta t)=(1-f_{tail}-f_{out})G(\delta t, s^b_{core}\sigma\Delta t, s^\sigma_{core}\sigma\Delta t)+f_{tail}G(\delta t, s^b_{tail}\sigma\Delta t, s^\sigma_{tail}\sigma\Delta t)+f_{out}G(\delta t, s^b_{out}, s^\sigma_{out})$$

Usually the resolution function can be taken from B→cc̄K(*) analyses, as when there are **direct charged tracks** from the signal B decay, it is tag-side dominated. Here we take it from **simulation** and assign a **systematic uncertainty for simulation-data differences** (see later).

TD analysis: systematic uncertainties

$B^0 \rightarrow K^0_S K^0_S K^0_S$

Source	$\mathcal{S}$	$\mathcal{C}$
MC_{stat}	0.002	0.001
B_{reco}	0.004	0.003
B-bkg	0.032	0.012
MC-Data: Δt	0.045	0.027
MC-Data: Discr. Vars	0.021	0.004
Fit Bias	0.022	0.018
Vetoes	0.006	0.004
Misc	0.004	0.015
Sum	0.064	0.038

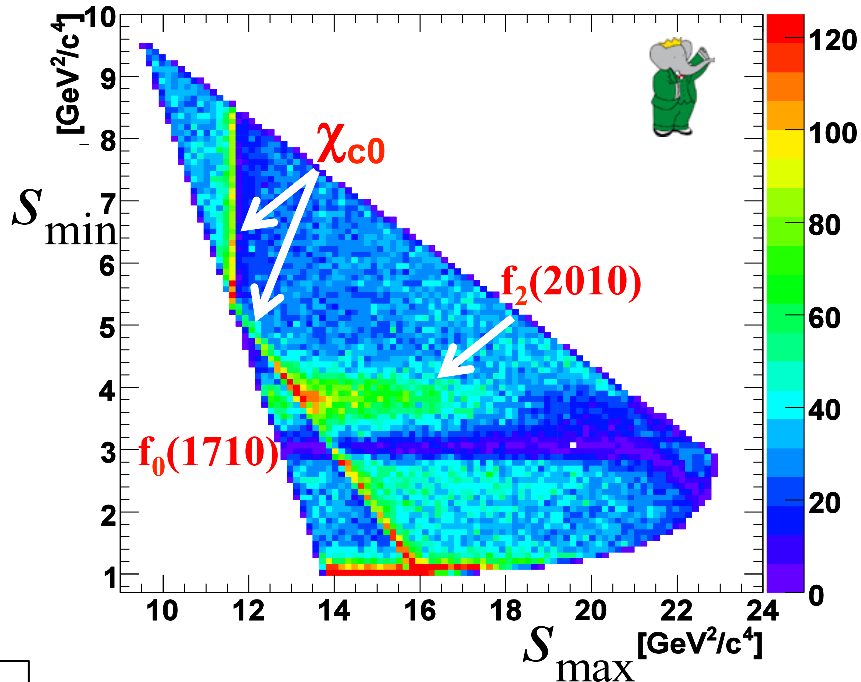
Results of the amplitude analysis

$B^0 \to K_S K_S K_S$

	Fit Fraction (FF)	
$f_0(980)K_S^0$	Fit Fraction (FF)	$0.44^{+0.20}_{-0.19}$
	Significance [σ]	3.0
$f_0(1710)K_S^0$	FF	$0.07^{+0.07}_{-0.03}$
	Significance [σ]	3.3
$f_2(2010)K_S^0$	FF	$0.09^{+0.03}_{-0.03}$
	Significance [σ]	3.3
NR	FF	$2.16^{+0.36}_{-0.37}$
	Significance [σ]	8.0
$\chi_{c0}K_S^0$	FF	$0.07^{+0.04}_{-0.02}$
	Significance [σ]	3.9
	Total FF	$2.84^{+0.71}_{-0.66}$

Huge destructive interference

!



5 (+inclusive) Branching fractions ($\mathcal{B}$)

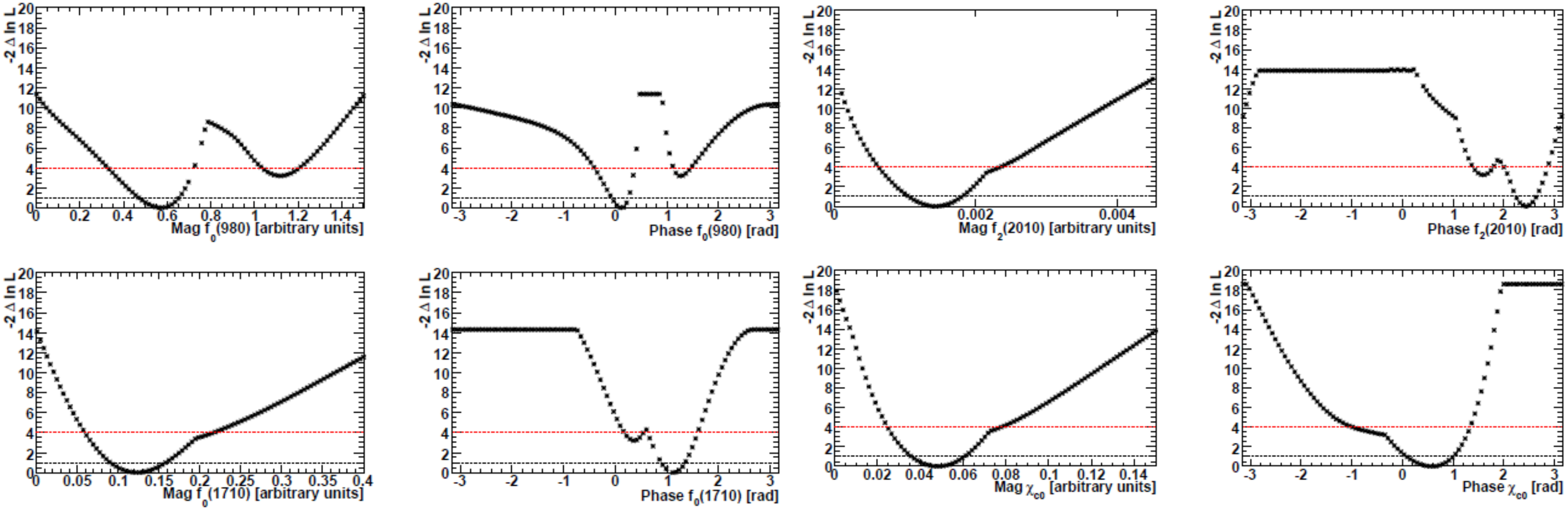
Mode	$\mathcal{B}$ [$\times 10^{-6}$]
Inclusive $B^0 \to K_S^0 K_S^0 K_S^0$	$6.19 \pm 0.48 \pm 0.15 \pm 0.12$
$f_0(980)K_S^0$, $f_0(980) \to K_S^0 K_S^0$	$2.7^{+1.3}_{-1.2} \pm 0.4 \pm 1.2$
$f_0(1710)K_S^0$, $f_0(1710) \to K_S^0 K_S^0$	$0.50^{+0.46}_{-0.24} \pm 0.04 \pm 0.10$
$f_2(2010)K_S^0$, $f_2(2010) \to K_S^0 K_S^0$	$0.54^{+0.21}_{-0.20} \pm 0.03 \pm 0.52$
NR, $K_S^0 K_S^0 K_S^0$	$13.3^{+2.2}_{-2.3} \pm 0.6 \pm 2.1$
$\chi_{c0}K_S^0$, $\chi_{c0} \to K_S^0 K_S^0$	$0.46^{+0.25}_{-0.17} \pm 0.02 \pm 0.21$

PDG: $6.2 \pm {}^{1.2}_{1.1}$

Consistent with other measurements

Likelihood scans

$B^0 \to K^0_S K^0_S K^0_S$



Signal model

TABLE I. Parameters of the DP model used in the fit. Values are given in MeV($/c^2$) unless specified otherwise. All parameters are taken from Ref. [14], except for the $f_0(980)$ parameters, which are taken from Ref. [15].

Resonance	Parameters	Line shape
$\phi(1020)$	$m_0 = 1019.455 \pm 0.020$ $\Gamma_0 = 4.26 \pm 0.04$	RBW
$f_0(980)$	$m_0 = 965 \pm 10$ $g_\pi = (0.165 \pm 0.018)\mathrm{GeV}^2/c^4$ $g_K/g_\pi = 4.21 \pm 0.33$	Flatté
$f_0(1500)$	$m_0 = 1505 \pm 6$ $\Gamma_0 = 109 \pm 7$	RBW
$f_0(1710)$	$m_0 = 1720 \pm 6$ $\Gamma_0 = 135 \pm 8$	RBW
$f_2'(1525)$	$m_0 = 1525 \pm 5$ $\Gamma_0 = 73^{+6}_{-5}$	RBW
NR decays		See text
χ_{c0}	$m_0 = 3414.75 \pm 0.31$ $\Gamma_0 = 10.3 \pm 0.6$	RBW

Interference fit fractions

Other B $\rightarrow$ 3K modes

TABLE XVIII. Values of the interference fit fractions FF_{jk} for $B^+ \to K^+K^-K^+$, solution I. The diagonal terms FF_{jj} are the ordinary fit fractions FF_j, which sum to 272%. The NR component is split into S-wave and P-wave parts for these calculations. Values are given in percent.

	$\phi(1020)$	$f_0(980)$	$f_0(1500)$	$f_2'(1525)$	$f_0(1710)$	χ_{c0}	NR (S wave)	NR (P wave)
$\phi(1020)$	12.9	−0.1	0.0	0.0	0.1	−0.0	−7.4	8.2
$f_0(980)$		27.2	−4.7	−0.0	−5.4	−1.0	−0.8	−3.7
$f_0(1500)$			2.1	0.0	2.3	0.1	3.1	−0.8
$f_2'(1525)$				2.0	0.1	−0.0	−0.0	0.7
$f_0(1710)$					3.2	−0.1	−13.5	4.9
χ_{c0}						3.2	3.3	−1.8
NR (S wave)							151.4	−155.0
NR (P wave)								69.4

Interference fit fractions

TABLE XX. Values of the interference fit fractions FF_{jk} for $B^+ \rightarrow K^0_S K^0_S K^+$, for the global minimum. The diagonal terms FF_{jj} are the ordinary fit fractions FF_j, which sum to 345%. Values are given in percent.

	$f_0(980)$	$f_0(1500)$	$f'_2(1525)$	$f_0(1710)$	χ_{c0}	NR (S wave)
$f_0(980)$	139.0	−19.2	0.0	−12.4	−1.0	−217.0
$f_0(1500)$		4.0	−0.0	4.1	0.2	9.5
$f'_2(1525)$			5.7	−0.0	−0.0	−0.0
$f_0(1710)$				4.5	0.1	−9.2
χ_{c0}					5.0	−0.0
NR (S wave)						186.5

TABLE XXI. Values of the interference fit fractions FF_{jk} for $B^0 \rightarrow K^+ K^- K^0_S$, for the global minimum. The diagonal terms FF_{jj} are the ordinary fit fractions FF_j, which sum to 188%. The NR component is split into S-wave and P-wave parts for these calculations. Values are given in percent.

	$\phi(1020)$	$f_0(980)$	$f_0(1500)$	$f'_2(1525)$	$f_0(1710)$	χ_{c0}	NR (S wave)	NR (P wave)
$\phi(1020)$	13.1	0.0	0.0	0.0	0.0	0.0	0.0	0.2
$f_0(980)$		26.3	0.1	−0.0	14.4	−0.7	−81.2	0.0
$f_0(1500)$			2.1	−0.0	5.3	−0.1	−0.7	0.0
$f'_2(1525)$				0.5	−0.0	0.0	0.0	0.0
$f_0(1710)$					16.7	−0.2	−27.0	0.0
χ_{c0}						3.4	1.6	0.0
NR (S wave)							114.5	0.0
NR (P wave)								11.7

More results

Other B $\rightarrow$ 3K modes

TABLE VIII. Branching fractions (neglecting interference), CP asymmetries, and CP-violating phases [see Eq. (11)] for $B^+ \to K^+K^-K^+$. The $\mathcal{B}(B^+ \to RK^+)$ column gives the branching fractions to intermediate resonant states, corrected for secondary branching fractions obtained from Ref. [14]. Central values and uncertainties are obtained from solution I. In addition to quoting the overall NR branching fraction, we quote the S-wave and P-wave NR branching fractions separately.

Decay mode	$\mathcal{B}(B^+ \to K^+K^-K^+) \times FF_j(10^{-6})$	$\mathcal{B}(B^+ \to RK^+)(10^{-6})$	A_{CP} (%)	$\Delta\phi_j$ (deg)
$\phi(1020)K^+$	$4.48 \pm 0.22^{+0.33}_{-0.24}$	$9.2 \pm 0.4^{+0.7}_{-0.5}$	$12.8 \pm 4.4 \pm 1.3$	$23 \pm 13^{+4}_{-5}$
$f_0(980)K^+$	$9.4 \pm 1.6 \pm 2.8$		$-8 \pm 8 \pm 4$	$9 \pm 7 \pm 6$
$f_0(1500)K^+$	$0.74 \pm 0.18 \pm 0.52$	$17 \pm 4 \pm 12$		
$f_2'(1525)K^+$	$0.69 \pm 0.16 \pm 0.13$	$1.56 \pm 0.36 \pm 0.30$	$14 \pm 10 \pm 4$	$-2 \pm 6 \pm 3$
$f_0(1710)K^+$	$1.12 \pm 0.25 \pm 0.50$			
$\chi_{c0}K^+$	$1.12 \pm 0.15 \pm 0.06$	$184 \pm 25 \pm 14$		$-4 \pm 13 \pm 2$
NR	$22.8 \pm 2.7 \pm 7.6$		$6.0 \pm 4.4 \pm 1.9$	0 (fixed)
NR (S wave)	$52^{+23}_{-14} \pm 27$			
NR (P wave)	$24^{+22}_{-12} \pm 27$			

Decay mode	$\mathcal{B}(B^+ \to K^0_S K^0_S K^+)$ $\times FF_j(10^{-6})$	$\mathcal{B}(B^+ \to RK^+)(10^{-6})$
$f_0(980)K^+$	$14.7 \pm 2.8 \pm 1.8$	
$f_0(1500)K^+$	$0.42 \pm 0.22 \pm 0.58$	$20 \pm 10 \pm 27$
$f_2'(1525)K^+$	$0.61 \pm 0.21^{+0.12}_{-0.09}$	$2.8 \pm 0.9^{+0.5}_{-0.4}$
$f_0(1710)K^+$	$0.48^{+0.40}_{-0.24} \pm 0.11$	
$\chi_{c0}K^+$	$0.53 \pm 0.10 \pm 0.04$	$168 \pm 32 \pm 16$
NR (S wave)	$19.8 \pm 3.7 \pm 2.5$	

TABLE X. Branching fractions (neglecting interference) for $B^+ \to K^0_S K^0_S K^+$. The $\mathcal{B}(B^+ \to RK^+)$ column gives the branching fractions to intermediate resonant states, corrected for secondary branching fractions obtained from Ref. [14]. Central values and uncertainties are for the global minimum only. See the text for discussion of the variations between the local minima.

Likelihood scans

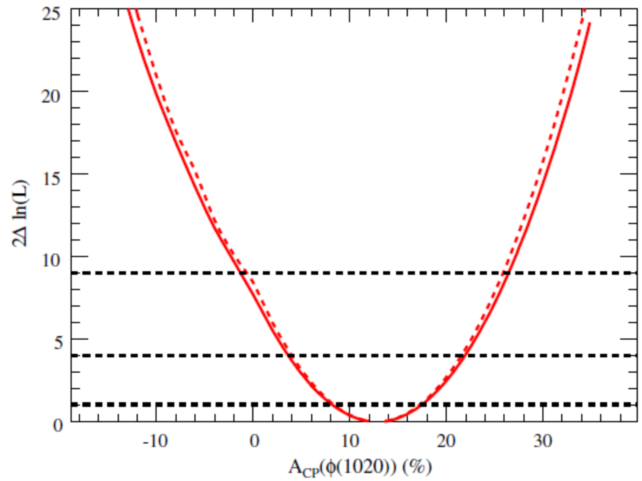


FIG. 9 (color online). Scan of $2\Delta\ln\mathcal{L}$, with (solid line) and without (dashed line) systematic uncertainties, as a function of $A_{CP}(\phi(1020))$ in $B^+ \to K^+K^-K^+$.

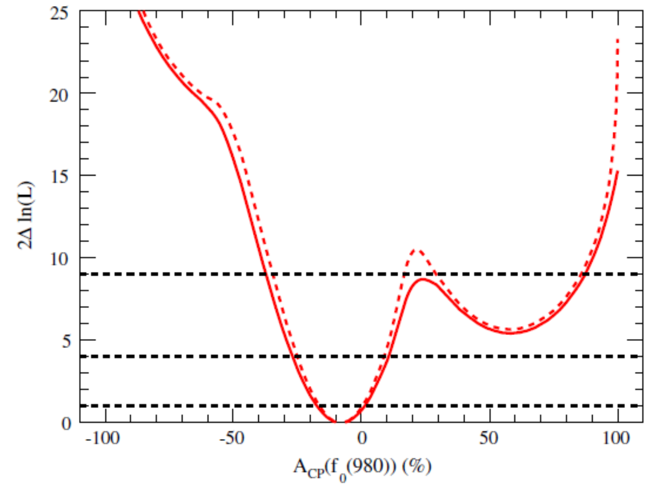


FIG. 10 (color online). Scan of $2\Delta\ln\mathcal{L}$, with (solid line) and without (dashed line) systematic uncertainties, as a function of $A_{CP}(f_0(980))$ in $B^+ \to K^+K^-K^+$.

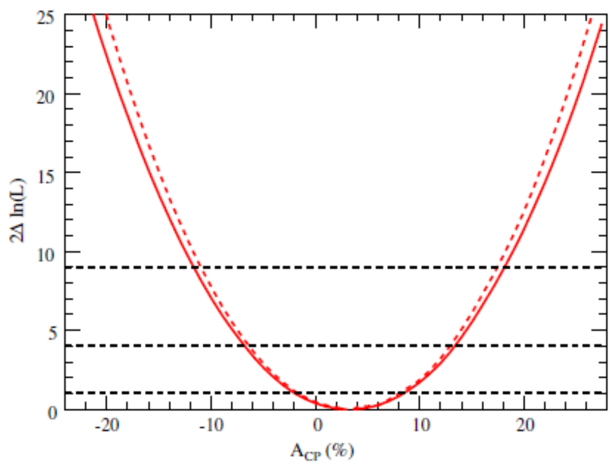
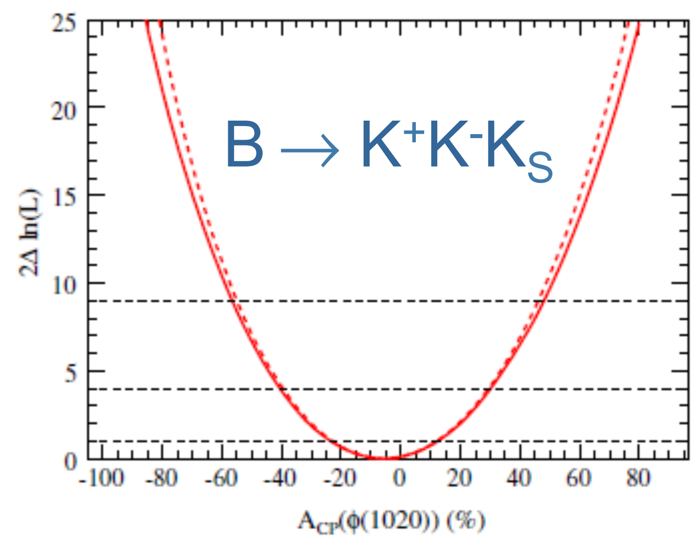


FIG. 14 (color online). Scan of $2\Delta\ln\mathcal{L}$, with (solid line) and without (dashed line) systematic uncertainties, as a function of A_{CP} in $B^+ \to K_S^0K_S^0K^+$.



Results ($B^+ \to K^+K^-K^+$; $K_SK_SK^+$)

Other $B \to 3K$ modes

$K^+K^-K^+$

N_{sig} = **5269±84** (Purity = **43%**)

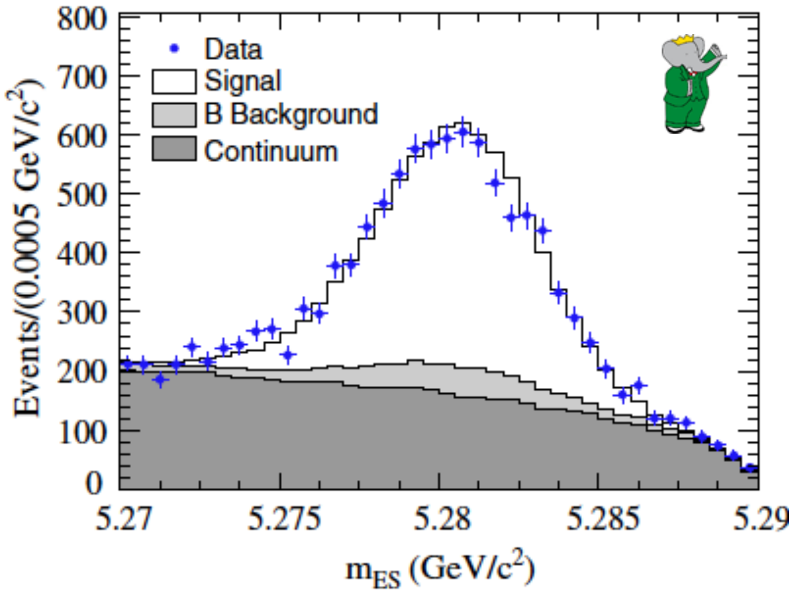
BF = $(33.4 \pm 0.5 \pm 0.9) \times 10^{-6}$ [$\chi_{c0}K$ excluded]

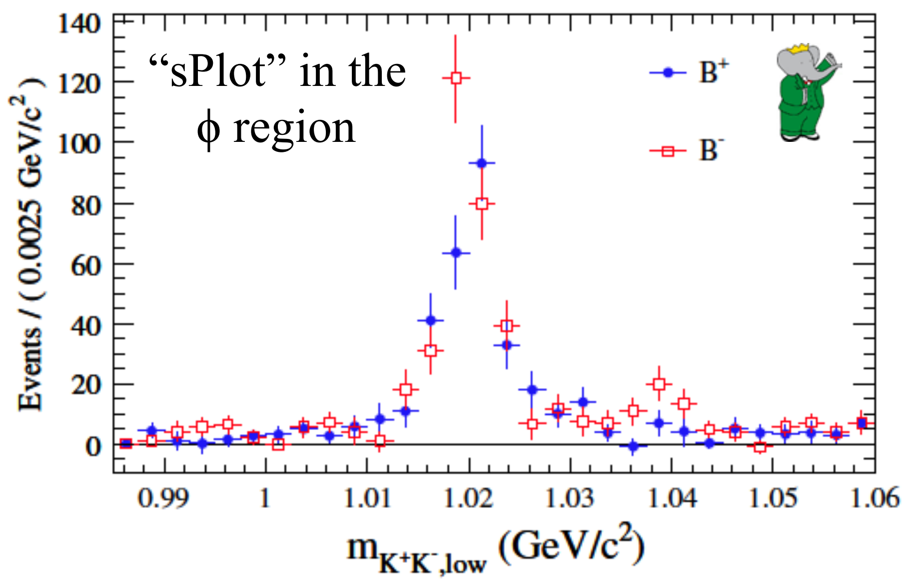
A_{CP}(inclusive) = $(-1.7^{+1.9}_{-1.4} \pm 1.4)\%$

$A_{CP}(\phi K) = (12.8 \pm 4.4 \pm 1.3)\%$

(**2.8σ from 0**, SM: ~ 0 - 4.7%)

Beneke, Neubert, Nucl.Phys B675,333 (QCDF) ; Li, Mishima, PRD 74, 094020 (pQCD)





$K_SK_SK^+$

N_{sig} = 632±28 (Purity = 20%)

BF = $(10.1 \pm 0.5 \pm 0.3) \times 10^{-6}$ [$\chi_{c0}K$ excluded]

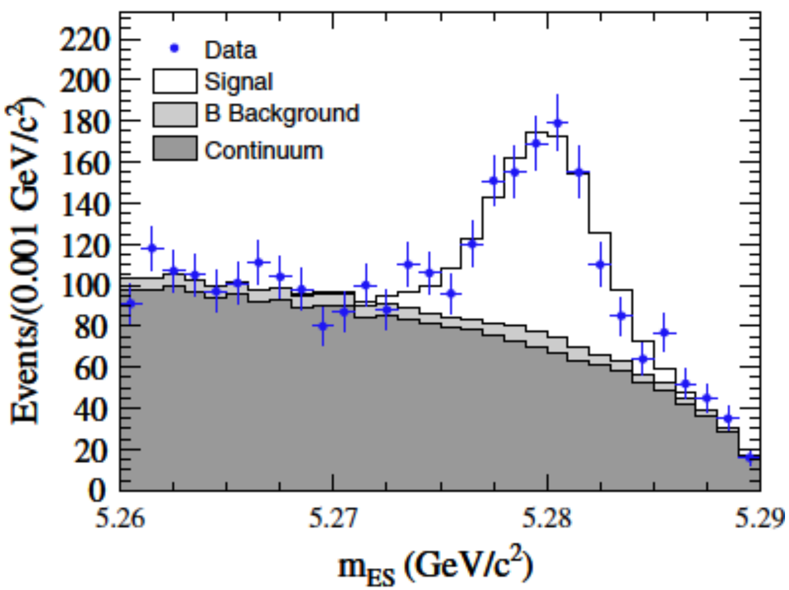
$A_{CP} = (4 \pm 5 \pm 2)\%$

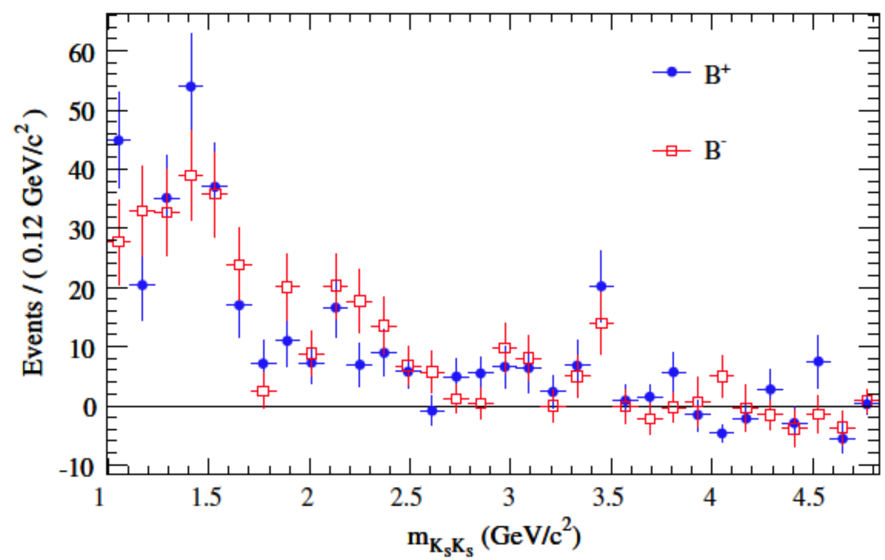
Results ($B^0 \to K_S K_S K^+$)

N_{sig} = 632±28 (Purity = 20%)

BF = $(10.1\pm0.5\pm0.3)\times10^{-6}$ [χ_{c0}K excluded]

A_{CP} = (4±5±2)%





Systematic uncertainties

Other B $\rightarrow$ 3K modes

TABLE XV. Summary of systematic uncertainties for $B^+ \rightarrow K^+K^-K^+$ parameters. Errors on phases, A_{CP}'s, and branching fractions are given in degrees, percent, and units of 10^{-6}, respectively.

Parameter	Line shape	Fixed PDF params	Other	Add resonances	Fit bias	Total
$\Delta\phi(\phi(1020))$	3	1	0	2	2	4
$\Delta\phi(f_0(980))$	2	1	0	6	1	6
$\Delta\phi(f_2'(1525))$	1	0	0	3	1	3
$\Delta\phi(\chi_{c0})$	1	1	0	1	1	2
$A_{CP}(\phi(1020))$	0.2	0.2	1.0	0.3	0.7	1.3
$A_{CP}(f_0(980))$	3	1	1	2	1	4
$A_{CP}(f_2'(1525))$	1	1	1	3	1	4
A_{CP} (NR)	1.1	0.4	1.0	0.8	0.7	1.9
$\mathcal{B}(\phi(1020))$	0.20	0.04	0.11	0.14	0.08	0.29
$\mathcal{B}(f_0(980))$	1.2	0.1	0.3	2.5	0.4	2.8
$\mathcal{B}(f_0(1500))$	0.06	0.02	0.02	0.52	0.02	0.52
$\mathcal{B}(f_2'(1525))$	0.05	0.01	0.02	0.07	0.10	0.13
$\mathcal{B}(f_0(1710))$	0.08	0.04	0.03	0.49	0.05	0.50
$\mathcal{B}(\chi_{c0})$	0.01	0.01	0.03	0.02	0.04	0.06
$\mathcal{B}$ (NR)	1.0	0.2	0.5	7.4	0.3	7.6
$\mathcal{B}$ (NR (S wave))	13	2	1	23	2	27
$\mathcal{B}$ (NR (P wave))	10	2	1	25	3	27
$\mathcal{B}$ (total)	0.0	0.2	0.8	0.1	0.4	0.9
$\mathcal{B}$ (charmless)	0.0	0.2	0.8	0.1	0.3	0.9

Systematic uncertainties

Other B → 3K modes

TABLE XVI. Summary of systematic uncertainties for $B^+ \to K_S^0 K_S^0 K^+$ parameters. Errors on A_{CP} and branching fractions are given in percent and units of 10^{-6}, respectively.

Parameter	Line shape	Fixed PDF params	Other	Add resonances	Fit bias	Total
A_{CP}	0	0	1	0	1	2
$\mathcal{B}(f_0(980))$	1.4	0.3	0.3	1.0	0.4	1.8
$\mathcal{B}(f_0(1500))$	0.05	0.03	0.01	0.57	0.04	0.58
$\mathcal{B}(f_2'(1525))$	0.06	0.02	0.02	0.07	0.03	0.10
$\mathcal{B}(f_0(1710))$	0.06	0.04	0.01	0.02	0.08	0.11
$\mathcal{B}(\chi_{c0})$	0.01	0.01	0.01	0.00	0.03	0.04
$\mathcal{B}$ (NR (S wave))	1.3	0.6	0.4	2.0	0.2	2.5
$\mathcal{B}$ (total)	0.0	0.2	0.2	0.0	0.0	0.3
$\mathcal{B}$ (charmless)	0.0	0.2	0.2	0.0	0.0	0.3

TABLE XVII. Summary of systematic uncertainties for $B^0 \to K^+ K^- K_S^0$ parameters. Errors on angles, A_{CP}'s, and branching fractions are given in degrees, percent, and units of 10^{-6}, respectively.

Parameter	Line shape	Fixed PDF params	Other	Add resonances	Fit bias	Total
$\beta_{\rm eff}(\phi(1020))$	2	1	0	2	0	2
$\beta_{\rm eff}(f_0(980))$	1	1	0	4	0	4
$\beta_{\rm eff}$ (other)	0.7	0.4	0.2	0.8	0.4	1.2
$A_{CP}(\phi(1020))$	2	2	2	2	3	5
$A_{CP}(f_0(980))$	6	3	2	5	2	9
A_{CP} (other)	1	1	1	2	1	3
$\mathcal{B}(\phi(1020))$	0.13	0.05	0.08	0.05	0.03	0.18
$\mathcal{B}(f_0(980))$	1.3	0.3	0.1	2.0	0.1	2.4
$\mathcal{B}(f_0(1500))$	0.04	0.02	0.02	0.10	0.03	0.12
$\mathcal{B}(f_2'(1525))$	0.02	0.01	0.00	0.15	0.02	0.16
$\mathcal{B}(f_0(1710))$	0.3	0.1	0.1	0.4	0.1	0.5
$\mathcal{B}(\chi_{c0})$	0.02	0.02	0.02	0.01	0.04	0.06
$\mathcal{B}$ (NR(total))	2	1	1	8	1	9
$\mathcal{B}$ (NR (S wave))	2	1	1	8	1	8
$\mathcal{B}$ (NR (P wave))	0.1	0.2	0.1	0.3	0.1	0.4
$\mathcal{B}$ (total)	0.0	0.4	0.7	0.0	0.1	0.8
$\mathcal{B}$ (charmless)	0.1	0.4	0.6	0.0	0.2	0.8

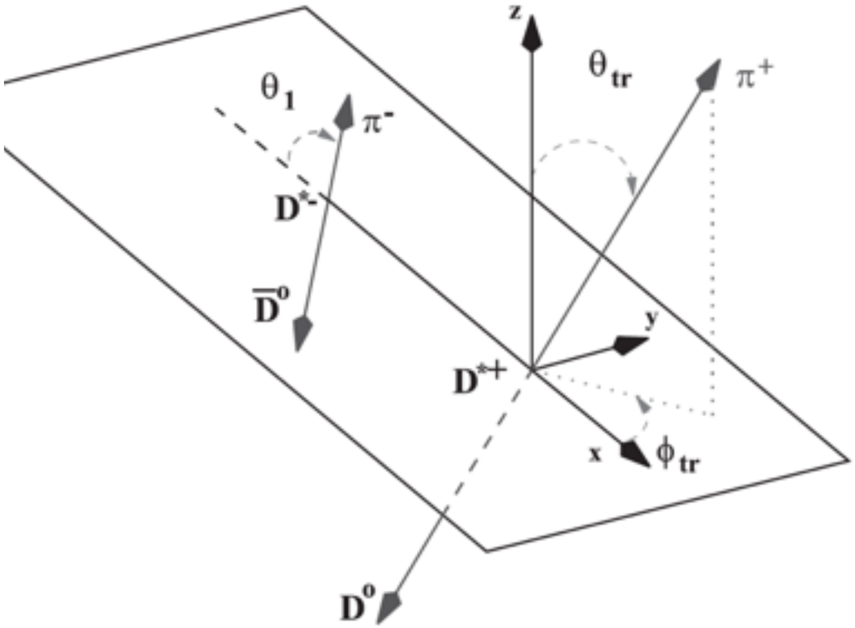
Full TD amplitude and $R_\perp$

$B^0 \to D^{*+}D^{*-}$

$$\frac{1}{\Gamma}\frac{d^4\Gamma}{d\cos\theta_1 d\cos\theta_{tr} d\phi_{tr} dt} = \frac{9}{16\pi}\frac{1}{|A_0|^2+|A_\parallel|^2+|A_\perp|^2}\times$$
$$\Big\{2\cos^2\theta_1\sin^2\theta_{tr}\cos^2\phi_{tr}|A_0|^2$$
$$+\sin^2\theta_1\sin^2\theta_{tr}\sin^2\phi_{tr}|A_\parallel|^2$$
$$+\sin^2\theta_1\cos^2\theta_{tr}|A_\perp|^2$$
$$-\sin^2\theta_1\sin 2\theta_{tr}\sin\phi_{tr}\,\mathrm{Im}(A_\parallel^* A_\perp)$$
$$+\frac{1}{\sqrt{2}}\sin 2\theta_1\sin^2\theta_{tr}\sin 2\phi_{tr}\,\mathrm{Re}(A_0^* A_\parallel)$$
$$-\frac{1}{\sqrt{2}}\sin 2\theta_1\sin 2\theta_{tr}\cos\phi_{tr}\,\mathrm{Im}(A_0^* A_\perp)\Big\}\,, \qquad (2)$$

where A_k, with $k = \parallel, 0, \perp$, represent time-dependent amplitudes given by

$$A_k(t) = \frac{\sqrt{2}A_k(0)}{1+|\lambda_k|^2}e^{-imt}e^{-t/2\tau_{B^0}}$$
$$\times\left(\cos\frac{\Delta m_d t}{2} + i\eta_{CP}^k\lambda_k\sin\frac{\Delta m_d t}{2}\right). \qquad (3)$$



$$R_\perp = \frac{|A_\perp^0|^2}{|A_0^0|^2+|A_\parallel^0|^2+|A_\perp^0|^2}$$

CP = **+1** for $A_\parallel$, A_0

CP = **-1** for $A_\perp$

PDFs

$B^0 \to D^{*+}D^{*-}$

- Overall PDF for the on-Peak sample is the sum of three components

$$P_{\rm on} = f_{B\bar{B}}\,\underbrace{[\underbrace{f_{\rm sig}P_{\rm sig}}_{\text{signal}} + \underbrace{(1-f_{\rm sig})P_{\rm comb}}_{B\bar{B}\text{ combinatoric}}]}_{B\bar{B}} + \underbrace{(1-f_{B\bar{B}})P_{q\bar{q}}}_{\text{continuum}} \qquad P_{\rm off} = P_{q\bar{q}}$$

- Each component is the product of a kinematical and a Δt part

$$P_i(m_{\rm rec}, F, \Delta t, \sigma_{\Delta t}, S_{\rm tag}) = \underbrace{\mathcal{M}_i(m_{\rm rec})\,\mathcal{F}_i(F)}_{\text{“KIN”}}\,\underbrace{T_i'(\Delta t, \sigma_{\Delta t}, S_{\rm tag})}_{\text{“}\Delta t\text{”}}$$

- Δt PDF: $T_i'(\Delta t, \sigma_{\Delta t}, S_{\rm tag}) = \int d\Delta t_{\rm true}\, T_i(\Delta t_{\rm true}, S_{\rm tag})\, \mathcal{R}_i(\Delta t - \Delta t_{\rm true}, \sigma_{\Delta t})$

- Signal Δt:

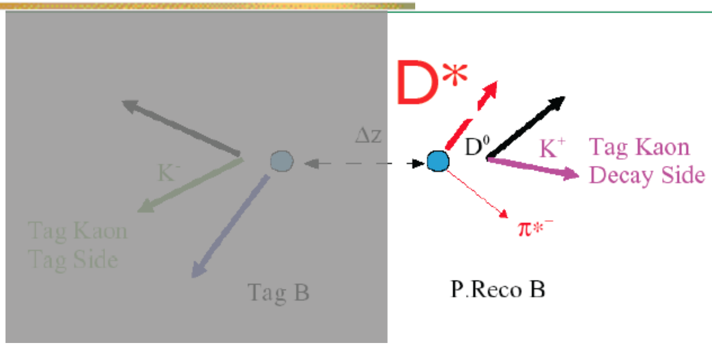
$$T_{\rm sig} = \frac{1}{4\tau_b} e^{-|\Delta t_{\rm true}|/\tau_b} \cdot \Big\{(1 - S_{\rm tag}\,\Delta\omega(1-\alpha)) + S_{\rm tag}\,(1-2\omega)\,(1-\alpha) \cdot [C\cos(\Delta m_d \Delta t_{\rm true}) + S\,\sin(\Delta m_d \Delta t_{\rm true})]\Big\}$$

$$S = -\frac{2\Im m(\lambda)}{1+|\lambda|^2} \qquad C = \frac{1-|\lambda|^2}{1+|\lambda|^2} \qquad \lambda = \frac{q}{p}\frac{\bar{A}}{A}$$

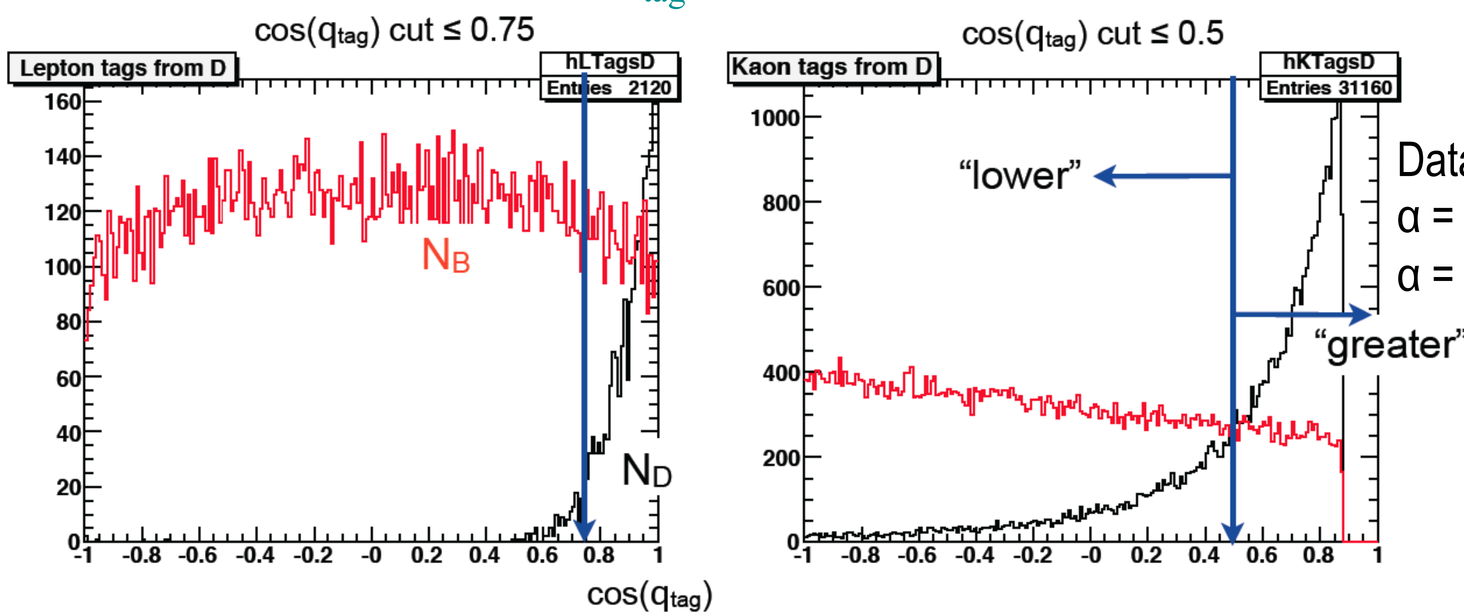
Mis-tag due to unreconstructed D^0 tracks

$B^0 \rightarrow D^{*+}D^{*-}$

- Partial reconstruction introduces an additional dilution D = (1-α), where α is the fraction of tags coming from the missing D^0
- This fraction can be obtained from data with some input from signal MC
- Can be reduced with a cut on the cosine of the opening angle between the tagging track and the missing D0 direction θ_{tag}



$$T_{\rm sig} = \frac{1}{4\tau_b}\, e^{-|\Delta t_{\rm true}|/\tau_b} \cdot \Big\{1 - S_{\rm tag}\,\Delta\omega(1-\alpha) + \\ + S_{\rm tag}\,(1-2\omega)\,(1-\alpha) \\ \cdot [C\cos(\Delta m_d \Delta t_{\rm true}) + S\,\sin(\Delta m_d \Delta t_{\rm true})]\Big\}$$



Data driven extraction:
α = 0.12 ±0.04 (K tag)
α = 0.00 ±0.02 (l tag)

Figure 2.1: Signal Monte Carlo distributions of $\cos(\vartheta_{\rm tag})$ for tracks from the missed D^0 (black) and from the other B^0 (red); lepton tags on the left, kaon tags on the right.

Event yields

$B^0 \to D^{*+}D^{*-}$

	# of events kaon tag	# of events lepton tag
on-peak	61179	20855
off-peak	1025	51
continuum	$9814 \pm 307 \pm 196$	$488 \pm 68 \pm 10$
$B\overline{B}$	51365 ± 364	20367 ± 69
N_{sig}	1129 ± 218	3843 ± 397

$B^0 \to D^{*+}D^{*-}$

Systematic uncertainties

TABLE V: Systematic uncertainties evaluated for C and S. Uncertainties in the top section are independent for kaon and lepton tags, those in the bottom section are correlated.

Systematic source	kaon tags C	kaon tags S	lepton tags C	lepton tags S
Kinematic fit parameters	0.013	0.034	0.023	0.057
Continuum Δt fit parameters	0.002	0.001	–	–
Signal s_w	0.0002	0.0007	–	–
$B\overline{B}$ combinatorial s_w	0.017	0.0007	0.001	0.005
Signal tag side (ω)	0.012	0.045	0.002	0.002
Mistag difference ($\Delta\omega$)	0.007	0.0004	0.007	0.0009
Signal *CP* side (α_{D^0})	0.006	0.017	0.002	0.002
Peaking background	0.0002	0.0003	0.0002	0.00004
Fit bias (MC statistics)	0.011	0.018	0.012	0.019
Tag interference from DCSD	0.030	0.002	–	–
B^0 lifetime variation	0.0002	0.002	0.0003	0.004
Δm_d variation	0.0003	0.001	0.0004	0.002
SVT misalignment	0.003	0.007	0.002	0.004
Boost uncertainty	0.002	0.006	0.005	0.007
Total	0.042	0.062	0.028	0.061