short-distance D^0–$\bar{D}^0$ mixing

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Outline

- neutral meson mixing: an introduction
- assessing the short-distance picture
- mixing and SU(3) symmetry
- Conclusions

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Mixing and CPV in charm

- sets limits of some 10^3 TeV to the scale of various effective $\Delta C = 2$ operators
  - ▷ **e.g. Isidori, Nir, & Perez ('10)**
- sensitive to FCNC among weak isospin up quarks
  - ▷ **e.g. Gedalia, Grossman, Nir, & Perez ('09)**
- SM charm physics dominated by the first two generations, CPV small
  - ▷ **e.g. Falk, Grossman, Ligeti, Nir, & Petrov ('04); Alex Kagan @ FPCP 2011**

CPV in mixing enters at $\mathcal{O}\left(\frac{V_{cb}^* V_{ub}}{V_{cs}^* V_{us}}\right) \simeq 10^{-3}$.

CPV in charm is a promising channel for searches of Beyond-SM physics

large penguins partially compromise this feature

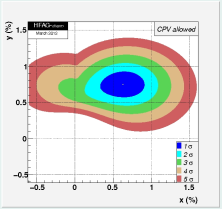
D^0–$\bar{D}^0$ oscillations

flavour oscillations due to non-zero mass and width splittings ΔM and $\Delta\Gamma$ between the stationary eigenstates

$$x = \frac{\Delta M}{\Gamma} = 0.63^{+0.19}_{-0.20}\,\%$$

$$y = \frac{\Delta\Gamma}{2\Gamma} = (0.75 \pm 0.12)\,\%$$

$$\phi = -10.1^{+9.5}_{-8.9}\,{}^{\circ}$$



▷ Y. Amhis et al. (HFAG collaboration), 1207.1158

- short distance approach: parton-level perturbation theory and heavy-quark expansion (operator product expansion in $1/m_c$)

  ▷ Georgi ('92); Ohl, Ricciardi, & Simmons ('93); Bigi & Uraltsev ('01); Golowich, Pakvasa, & Petrov ('07)

- long distance approach: sum over final states common to D^0 and $\bar{D}^0$ decays

  ▷ Wolfenstein ('85); Donoghue & al. ('86); Buccella, Lusignoli, & Pugliese ('96); Golowich & Petrov ('98); Falk, Grossman, Ligeti and Petrov ('02); Falk, Grossman, Ligeti, Nir, & Petrov ('04)

predictions are subject to substantial hadronic uncertainties in either framework

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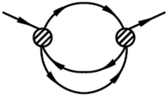
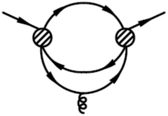
heavy quark expansion

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \left(\frac{\Lambda}{m_c}\right)^2 \mathcal{H}_2 + \left(\frac{\Lambda}{m_c}\right)^3 \mathcal{H}_3 + \left(\frac{\Lambda}{m_c}\right)^4 \mathcal{H}_4 + \dots$$

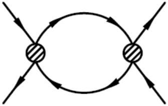
▷ **Beneke, Buchalla, & Dunietz ('96); Beneke, Buchalla, Greub, Lenz, & Nierste ('99); Ciuchini, Franco, Lubicz, Mescia, & Tarantino ('03); Beneke, Buchalla, Lenz, & Nierste ('03)**



- $D = 3$: $\mathcal{H}_0$ – spectator model quark decay: mean lifetime



- $D = 5$: $\mathcal{H}_2$ – kinetic and chromomagnetic operator



- $D = 6$: $\mathcal{H}_3$ – lifetime differences and mixing

OPE in B physics

- inclusive rates agree amazingly well

$$\frac{\tau\left(B_s\right)_{\text{exp}}}{\tau\left(B_d\right)} = 1.001 \pm 0.014, \qquad \frac{\tau\left(B_s\right)_{\text{SM}}}{\tau\left(B_d\right)} = 0.996 \ldots 1.000$$

▷ **LHCb & TeVatron combined 2012; Lenz & Nierste, 1102.4274 (2011)**

- first measurement ($> 5\sigma$) of $\Delta\Gamma(B_s)$ from LHCb at Moriond 2012

$$\frac{\Delta\Gamma\left(B_s\right)_{\text{exp}}}{\Delta\Gamma\left(B_s\right)_{\text{SM}}} = \frac{0.100 \pm 0.013}{0.087 \pm 0.021} = 1.15 \pm 0.32$$

▷ **LHCb & TeVatron combined 2012; Lenz & Nierste, 1102.4274 (2011)**

$\Delta\Gamma\left(B_s\right)$ is believed to be most sensitive to violations of quark hadron duality and receives substantial contributions from hadronic scale dynamics:

$$\begin{aligned}\Delta\Gamma\left(B_s\right) &= \Delta\Gamma^0\left(B_s\right) \times \left(1 + \delta^{\text{lattice}} + \delta^{\text{QCD}} + \delta^{\text{HQE}}\right) \\ &= 0.142\ \text{ps}^{-1}\left(1 - 0.14 - 0.06 - 0.19\right)\end{aligned}$$

$\longrightarrow$ the heavy-quark expansion works within 30% accuracy!

OPE in charm

the naive prediction for the $D^0 - \bar{D}^0$ width difference is way too small (missing a factor of 10^3)

$$\boxed{|y| \equiv |\Gamma_{12}| \cdot \tau(D^0) \simeq 10^{-6}}$$

▷ **MB, Lenz, Riedl, & Rohrwild ('10)**

→ maybe Λ/m_c is not small enough to expand in?

maybe QCD does not converge at the charm threshold?

OPE in charm

- heavy-quark expansion is an expansion in hadronic scale over energy released in decay modes generating the transition

  energy releases in dominant decays are not quite different for the D system

B_s^0	$\rightarrow$	$D_s^+ D_s^-$	1.4 GeV
D^0	$\rightarrow$	$\pi\,\pi$	1.6 GeV
D^0	$\rightarrow$	$K\,K$	0.9 GeV

  The expansion parameter in $\Delta\Gamma\,(B_s)$ turned out to be $\sim 1/5$, corresponing to an effective hadronic scale significantly below 1 GeV.

- no breakdown of the perturbative approach: NLO $\lesssim 50\%$, $\mathcal{O}(1/m_c) \lesssim 30\%$

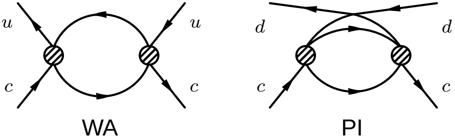
  ▷ **MB, Lenz, Riedl, & Rohrwild 1011.5608 (2010)**

- charm hadron lifetimes do not vanish in the limit of SU(3) $\rightsquigarrow$ no GIM interference

  new physics and higher dimension contributions cannot be large

The $D^+ - D^0$ lifetime difference

- the $1/m_c$-leading contribution is the spectator model quark decay
- weak interaction with light quarks gives rise to the $D^+–D^0$ lifetime difference

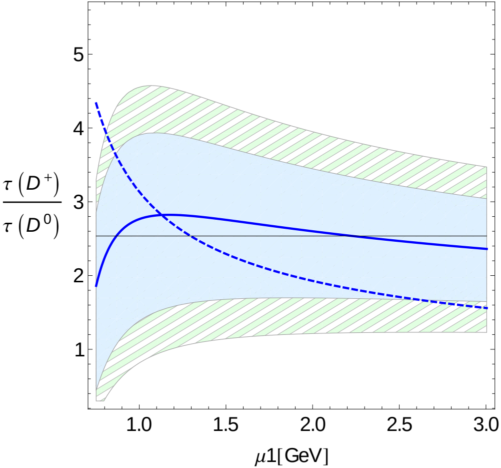


very similar to the $\Delta C = 2$ transition generating mass and width splittings: $\tau(D^+)/\tau(D^0)$ can test the heavy-quark expansion in the same order in $1/m_c$

$$\left.\frac{\tau\left(D^+\right)}{\tau\left(D^0\right)}\right|_{\text{exp}} = 2.536 \pm 0.019$$

$$\left.\frac{\tau\left(D^+\right)}{\tau\left(D^0\right)}\right|_{\text{SM}} = 2.8 \pm 1.5^{\text{(hadronic)}} {}^{+0.3}_{-0.7}{}^{\text{(scale)}} \pm 0.2^{\text{(exp)}}$$

The $D^+ - D^0$ lifetime difference



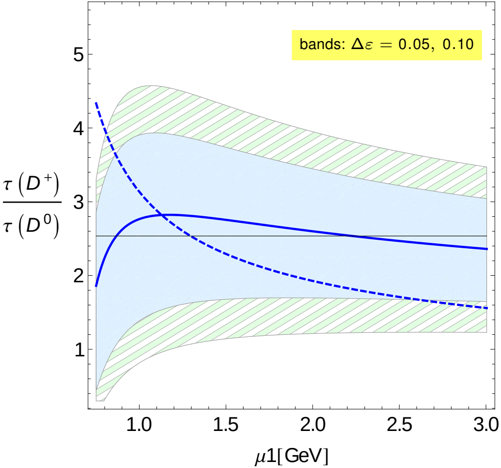
very good overall agreement

reasonable size of QCD corrections at the charm threshold

NLO significantly reduces the scale dependence

I expect sizeable corrections from dimension seven (yet to be done)

The $D^+ - D^0$ lifetime difference



huge hadronic uncertainties due to matrix elements of $\Delta C = 0$ operators (taken in vacuum saturation)

$$\begin{aligned} Q^q &= (\bar{c}\,q)_{\mathrm{V-A}}\,(\bar{q}\,c)_{\mathrm{V-A}} \\ Q^q_{\mathrm{S}} &= (\bar{c}\,q)_{\mathrm{S-P}}\,(\bar{q}\,c)_{\mathrm{S+P}} \\ T^q &= \left(\bar{c}\,T^a\,q\right)_{\mathrm{V-A}}\left(\bar{q}\,T^a\,c\right)_{\mathrm{V-A}} \\ T^q_{\mathrm{S}} &= \left(\bar{c}\,T^a\,q\right)_{\mathrm{S-P}}\left(\bar{q}\,T^a\,c\right)_{\mathrm{S+P}} \end{aligned}$$

$$\begin{aligned} \left\langle D^+ \right| Q^u - Q^d \left| D^+ \right\rangle &= f_D^2\,M_D^2\,B_1\,, \\ \left\langle D^+ \right| Q^u_{\mathrm{S}} - Q^d_{\mathrm{S}} \left| D^+ \right\rangle &= f_D^2\,M_D^2\,B_2\,, \\ \left\langle D^+ \right| T^u - T^d \left| D^+ \right\rangle &= f_D^2\,M_D^2\,\varepsilon_1\,, \\ \left\langle D^+ \right| T^u_{\mathrm{S}} - T^d_{\mathrm{S}} \left| D^+ \right\rangle &= f_D^2\,M_D^2\,\varepsilon_2\,. \end{aligned}$$

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Neutral meson oscillations

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\hat{M} - \frac{\mathrm{i}}{2}\hat{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

3 parameters for mixing & CPV: $|M_{12}|$, $|\Gamma_{12}|$, $\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$

translate into 3 mixing-related observables...

- mass and decay width differences

$$\boxed{\begin{aligned} \Delta M &= M_{\mathrm{H}} - M_{\mathrm{L}}, \\ \Delta\Gamma &= \Gamma_{\mathrm{L}} - \Gamma_{\mathrm{H}} \end{aligned}}$$

$$\Delta M = 2|M_{12}|$$

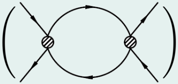
$$\Delta\Gamma = 2|\Gamma_{12}| \ \operatorname{sgn}\cos\phi$$

- flavour-specific CP asymmetries

$$\boxed{a_f = \frac{\Gamma\left(\bar{D}(t) \to f\right) - \Gamma\left(D(t) \to \bar{f}\right)}{\Gamma\left(\bar{D}(t) \to f\right) + \Gamma\left(D(t) \to \bar{f}\right)}}$$

$$a_f = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi \ll 1$$

SU(3) symmetry and GIM mechanism

$\Gamma_{12} = \mathrm{Im}$ 

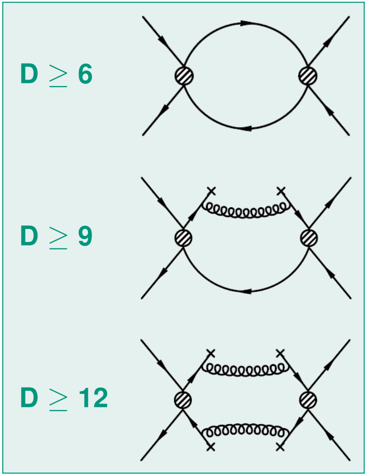
CKM couplings induce a hierarchy in $\lambda \simeq 0.2255$:

$$\Gamma_{12}\left(D^0\right) = -\mathcal{O}\left(\lambda^2\right) \underbrace{\boxed{\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right)}}_{m_s^4/m_c^4} + \mathcal{O}\left(\lambda^6\right) \underbrace{\boxed{\left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right)}}_{m_s^2/m_c^2} + \mathcal{O}\left(\lambda^{10}\right) \underbrace{\boxed{\Gamma_{12}^{dd}}}_{\sim 1}$$

SU(3) amplitudes interfere to (almost) zero in the limit of flavour symmetry
The CKM-leading part scales with the 4th power of the SU(3) breaking parameter m_s/m_c.

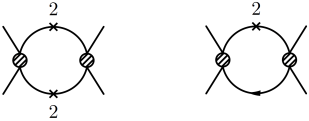
—> D mesons mix slowly due to residual SU(3) symmetry

The meson's soft QCD background



interference within SU(3) multiplets:

$$-\lambda^2 \underset{(m_s/m_c)^4}{\boxed{\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd}\right)}} + \lambda^6 \underset{(m_s/m_c)^2}{\boxed{\left(\Gamma_{12}^{ds} - \Gamma_{12}^{dd}\right)}} - \lambda^{10} \boxed{\Gamma_{12}^{dd}}$$



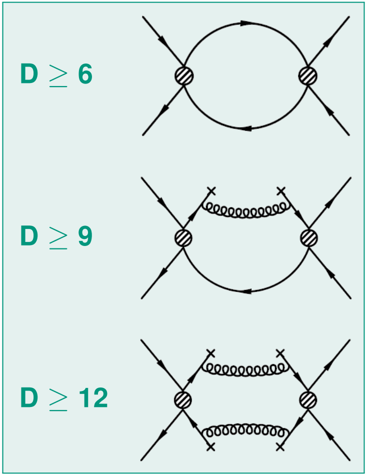
If an internal momentum is $\lesssim \Lambda_{\mathrm{QCD}}$, the intermediate state couples to the meson's soft QCD substructure.

SU(3) breaking arises from the hadron state: albeit subleading in $1/m_c$, the amplitude carries less powers of m_s and can actually dominate the OPE.

▷ **Georgi ('92); Ohl, Ricciardi, & Simmons ('93); Bigi & Uraltsev ('01)**

→ cutting one internal line may lift one order of SU(3) suppression

Hadronic matrix elements at $D = 9$



SU(3) breaking from non-perturbative soft QCD dynamics, enters the OPE through hadronic matrix elements of 6-quark operators

factorization limit ($\sim 1/N_c$)

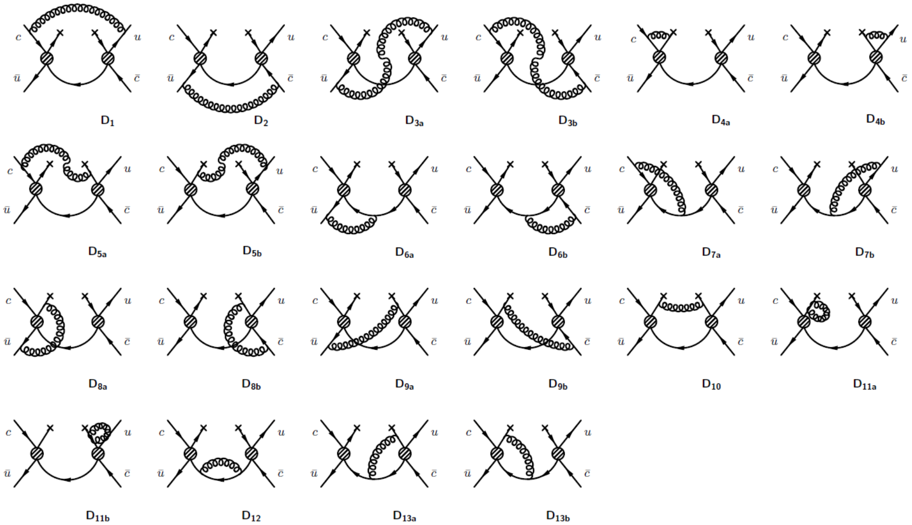
▷ **MB & Alex Lenz**

model the mesons's sea quark content with the vacuum condensate, neglecting higher excitations in the meson state

the quark field operators from the intermediate state are taken to be saturated with vacuum

→ the matrix elements of the remaining 4-quark operators are known from the lattice

Diquark condensate intermediate states



Width splitting

- SU(3) cancellations softer in the condensate contribution

$$\Gamma_{12} = -\lambda_s^2 \boxed{\underset{1.15\, m_s^4/m_c^4}{\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd}\right)}} + 2\lambda_s\lambda_b \boxed{\underset{-2.75\, m_s^2/m_c^2}{\left(\Gamma_{12}^{ds} - \Gamma_{12}^{dd}\right)}} - \lambda_b^2 \boxed{\underset{1.96}{\Gamma_{12}^{dd}}}$$

$$\delta\,\Gamma_{12} = -\lambda_s^2 \boxed{\underset{0.43\, m_s^3/m_c^3}{\delta\,\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd}\right)}} + 2\lambda_s\lambda_b \boxed{\underset{0.19\, m_s/m_c}{\delta\,\left(\Gamma_{12}^{ds} - \Gamma_{12}^{dd}\right)}} - \lambda_b^2 \boxed{\underset{0}{\delta\,\Gamma_{12}^{dd}}}$$

×13 ×0.66

- flavour symmetry breaking:

$$\Gamma_{12}^{ss}/\text{ps}^{-1} = 1.908 + 0.036 \qquad (+1.9\%)$$

$$\Gamma_{12}^{sd}/\text{ps}^{-1} = 1.935 + 0.018 \qquad (+0.9\%)$$

$$\Gamma_{12}^{dd}/\text{ps}^{-1} = 1.962 + 0$$

$$y = (0.86 + 7.3) \cdot 10^{-6}$$

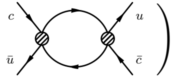
×8.5

▷ **MB, Lenz, & Nierste**

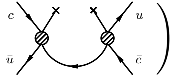
→ additional SU(3) breaking induced from the soft QCD background

mass splitting and CPV

$$M_{12} = \mathrm{Re}\Big(\quad \Big)$$



$$\delta M_{12} = \mathrm{Re}\Big(\quad \Big)$$



weak phase in the SD-Hamiltonian

$$\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \simeq \mathcal{O}(1)$$

Maybe there is some mechanism to break the remaining GIM interference.
(e.g. by cutting the second line, non-factorisable contributions, . . .)

If the effect is able to push x and y up to the observed values, then $\phi = 10^{-3}$ is within reach (this is pure speculation!)

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summary & outlook

- we investigated $SU(3)$ breaking effects at higher orders in the HQE
- assuming factorisation of sea quark operators, we find an $\mathcal{O}(10)$ enhancement in $\Delta\Gamma$ at operator dimension nine
- In the $D^+ - D^0$ lifetime difference ($D = 6$ at NLO) we find surprisingly good agreement, yet with huge hadronic uncertainties.
- as regards CPV. . .

We see an $\mathcal{O}(1)$ weak phase in the SD Hamiltonian.

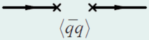
If HQE works and it does not behave completely unexpected, 1‰ of indirect CPV can not be excluded.

Outline

backup slides

Diquark condensate intermediate states

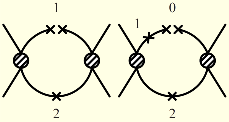
QCD vacuum condensation:

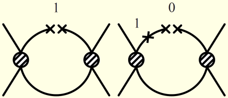


$$= \langle \underset{\sim}{0} \,|\, : q(x) \otimes \bar q(0) : \,|\, \underset{\sim}{0} \rangle = -\frac{\langle \bar q q\rangle}{4N_\mathrm{c}} \times \mathbb{1}_\mathrm{c} \left(\mathbb{1}_\mathrm{D} - \frac{i\, m}{d} \not{x}\right)$$

SU(3) breaking expected from a single condensate insertion competes with $\times 4\pi\alpha_s \frac{\langle \bar s s\rangle}{m_c^3} \simeq 0.3$

$$\delta\Gamma_{12} = -\lambda_s^2 \underset{\sim\, (m_s/m_c)^3}{\boxed{\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd}\right)}} + 2\lambda_s\lambda_b \underset{\sim\, m_s/m_c}{\boxed{\left(\Gamma_{12}^{ds} - \Gamma_{12}^{dd}\right)}} - \lambda_b^2 \boxed{\Gamma_{12}^{dd}}$$





- cutting one line, we have gained one power of m_s
- one factor m_s is intrinsic to the matrix element of the b-quark operator