Single top and top pair production

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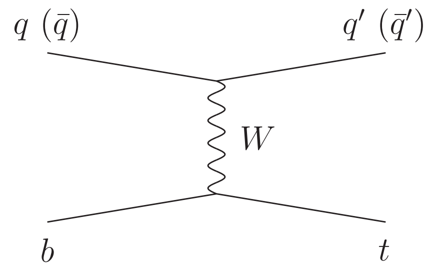
- **Single top and $t\bar{t}$ production channels**
- **Higher-order two-loop corrections**
- **NNLL resummation and NNLO expansions**
- **t-channel and s-channel production**
- **tW^-, tH^-, and FCNC processes**
- **$t\bar{t}$ cross section at LHC and Tevatron**
- **Top p_T and rapidity distributions**

Top production partonic processes at LO

Single top quark production

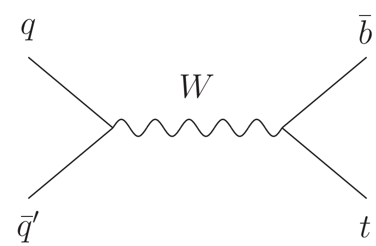
- **t channel: $qb \rightarrow q't$ and $\bar{q}b \rightarrow \bar{q}'t$**

dominant at Tevatron and LHC



- **s channel: $q\bar{q}' \rightarrow \bar{b}t$**

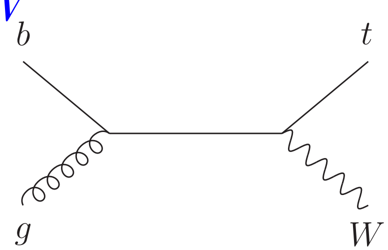
small at Tevatron and LHC

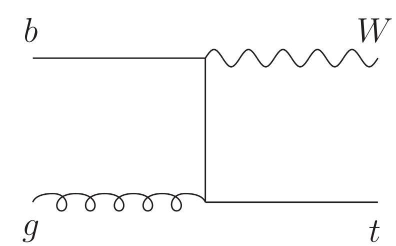


- **associated tW production: $bg \rightarrow tW^-$**

very small at Tevatron, significant at LHC

Related process: $bg \rightarrow tH^-$



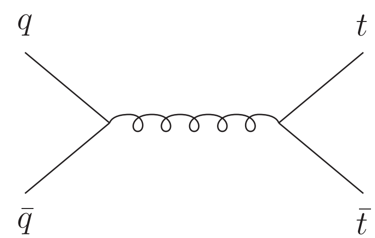


Top production partonic processes at LO

Top-antitop pair production

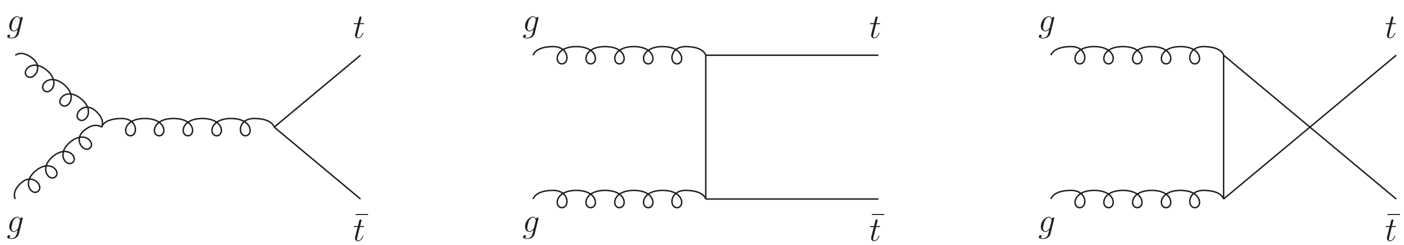
- $q\bar{q} \longrightarrow t\bar{t}$

dominant at Tevatron



- $gg \longrightarrow t\bar{t}$

dominant at LHC



Higher-order corrections

QCD corrections significant for single top and top pair production

Soft-gluon corrections from emission of soft (low-energy) gluons

Soft corrections: $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ with $k \le 2n-1$, s_4 distance from threshold

Soft-gluon corrections are dominant near threshold

Resum these soft corrections - factorization and RGE

Complete results at NNLL–two-loop soft anomalous dimension

NK, PRD 82, 114030 (2010); PRD 84, 011504 (2011) ($t\bar{t}$)

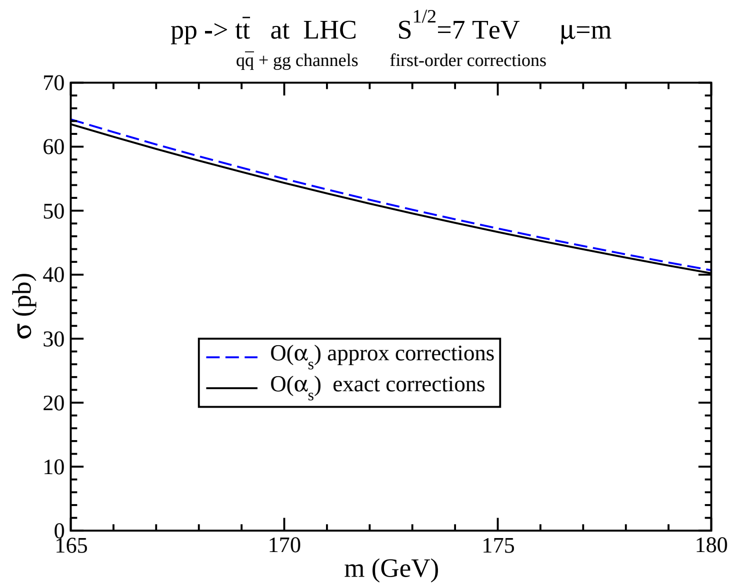
NK, PRD81, 054028 (2010); PRD 82, 054018 (2010); PRD 83,091503 (2011) (single top)

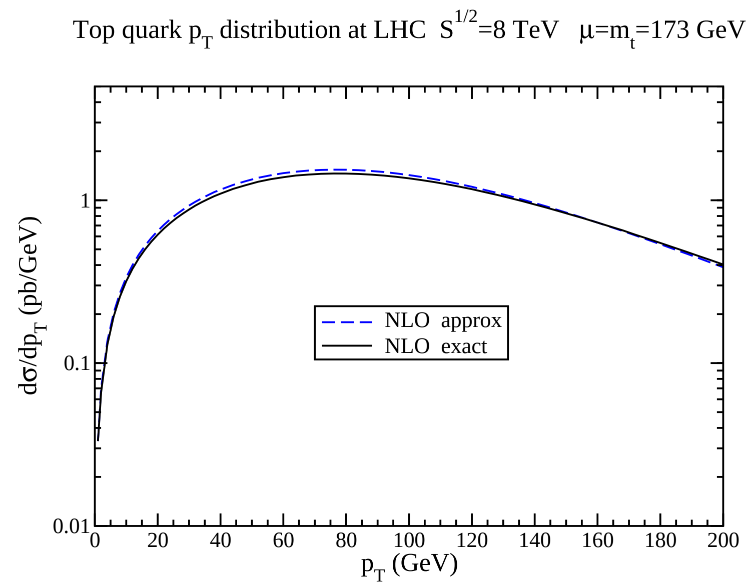
Approximate NNLO cross section from expansion of resummed cross section

This is the only calculation for partonic threshold at the double differential cross section level using the standard moment-space resummation in pQCD

Threshold approximation

Approximation works very well not only for Tevatron but also for LHC energies because partonic threshold is still important





only 1% difference between first-order approximate and exact corrections
→ less than 1% difference between NLO approximate and exact cross sections
Also true for differential distributions

For best prediction add NNLO approximate corrections to exact NLO cross section

Factorization and Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

$\sigma = \left(\prod \psi\right) H_{IL} \, S_{LI} \, \left(\prod J\right)$ **H: hard-scattering function**

S: soft-gluon function

Use RGE to evolve soft-gluon function

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA} (\Gamma_S)_{AI}$$

Γ_S is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants s, t, u

Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$

$$\times \, \mathrm{tr} \left\{ H\left(\alpha_s\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S\left(\alpha_s(\mu)\right)\right] \right\}$$

collinear and soft radiation from incoming partons

$$E_i(N_i) = \int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{ \int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_i\left(\alpha_s(\lambda s)\right) + D_i\left[\alpha_s((1-z)^2 s)\right] \right\}$$

purely collinear: replace $\frac{z^{N-1}-1}{1-z}$ **by** $-z^{N-1}$

factorization scale μ_F **dependence controlled by**
$\gamma_{i/i} = -A_i \ln \tilde{N}_i + \gamma_i$

Noncollinear soft gluon emission controlled by the soft anomalous dimension Γ_S

determine Γ_S **from coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams**

Γ_S **is process-dependent; calculated at two loops**

We are resumming $\ln^k N$ **- we can expand to fixed order and invert to get** $\ln^k(s_4/m_t^2)/s_4$

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not p + m}{2p\cdot k + i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k + i\epsilon}$$

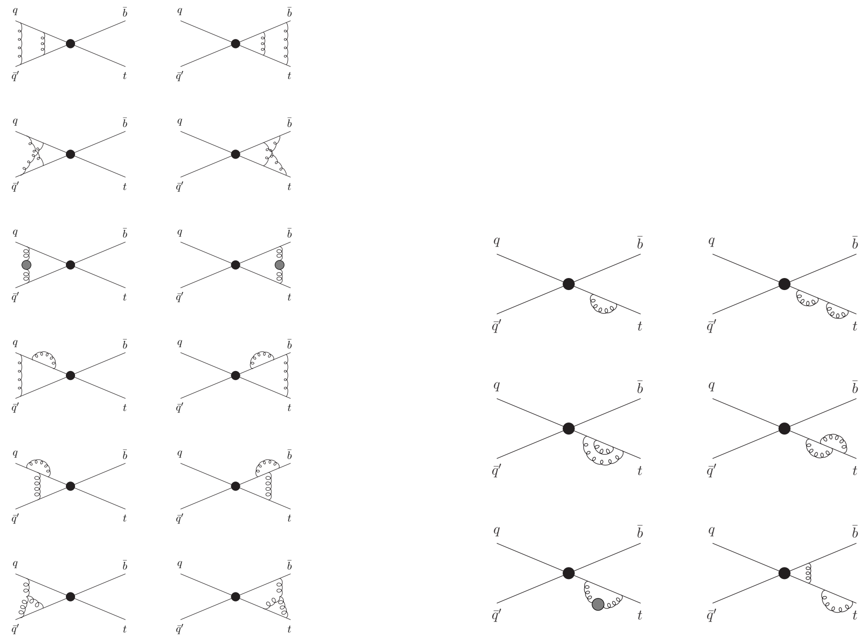
Perform calculations in momentum space and Feynman gauge

Complete two-loop results for

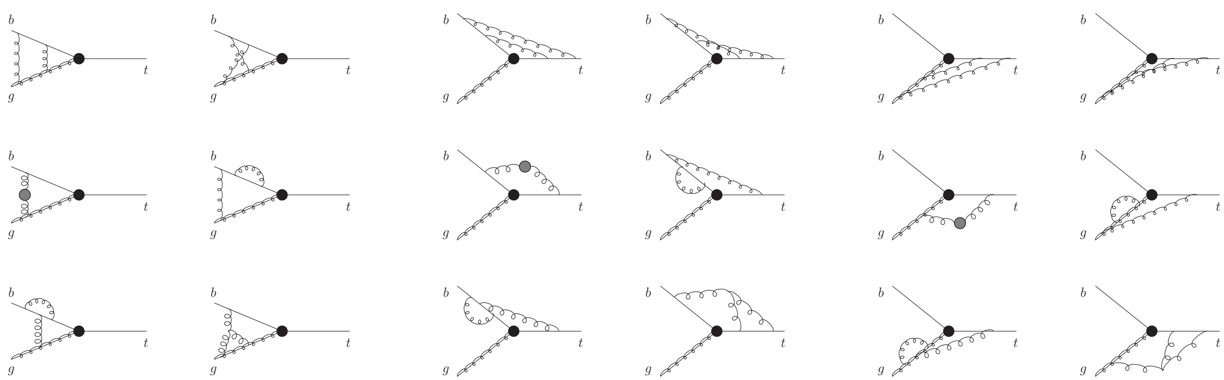
- **soft (cusp) anomalous dimension for** $e^+e^- \rightarrow t\bar{t}$
- $t\bar{t}$ **hadroproduction**
- t**-channel single top production**
- s**-channel single top production**
- $bg \rightarrow tW^-$ **and** $bg \rightarrow tH^-$
- **direct photon production**
- W **and** Z **production at large** p_T

Typical two-loop eikonal diagrams

Single-top s-channel



tW **production**



NNLO approximate cross sections

NLO expansion

Define $\mathcal{D}_k(s_4) \equiv \left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3\, \mathcal{D}_1(s_4) + c_2\, \mathcal{D}_0(s_4) + c_1\, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, \mathcal{D}_0(s_4) + T_1^c\, \delta(s_4)]$$

where $c_3 = \sum_i 2\, A_i^{(1)} - \sum_j A_j^{(1)}$ **and** $c_2 = c_2^\mu + T_2$**, with** $c_2^\mu = -\sum_i A_i^{(1)} \ln\left(\frac{\mu_F^2}{M^2}\right)$

and

$$T_2 = \sum_i \left[-2\, A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(1)} - A_i^{(1)} \ln\left(\frac{M^2}{s}\right)\right] + \sum_j \left[B_j^{(1)} + D_j^{(1)} - A_j^{(1)} \ln\left(\frac{M^2}{s}\right)\right]$$

Also $A^c = \mathrm{tr}\left(H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right)$

$c_1 = c_1^\mu + T_1$**, with**

$$c_1^\mu = \sum_i \left[A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

NNLO expansion

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \, \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j \frac{\beta_0}{8} A_j^{(1)} \right] \mathcal{D}_2(s_4) \right.$$

$$+ \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + \sum_i 2A_i^{(2)} - \sum_j A_j^{(2)} + \sum_j \frac{\beta_0}{4} B_j^{(1)} \right] \mathcal{D}_1(s_4)$$

$$+ \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{s}\right) - \sum_i \frac{\beta_0}{2} A_i^{(1)} \ln^2\left(\frac{-t_i}{M^2}\right) \right.$$

$$+ \sum_i [\left(-2A_i^{(2)} + \frac{\beta_0}{2} D_i^{(1)}\right) \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(2)} + \frac{\beta_0}{8} A_i^{(1)} \ln^2\left(\frac{\mu_F^2}{s}\right) - A_i^{(2)} \ln\left(\frac{\mu_F^2}{s}\right)]$$

$$\left. \left. + \sum_j [B_j^{(2)} + D_j^{(2)} - \left(A_j^{(2)} + \frac{\beta_0}{4}(B_j^{(1)} + 2D_j^{(1)})\right) \ln\left(\frac{M^2}{s}\right) + \frac{3\beta_0}{8} A_j^{(1)} \ln^2\left(\frac{M^2}{s}\right)] \right] \mathcal{D}_0(s_4) \right\}$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c \, \mathcal{D}_2(s_4) + \left[\left(2c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c \right] \mathcal{D}_1(s_4) \right.$$

$$\left. + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right)\right) A^c + c_2 T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right] \mathcal{D}_0(s_4) \right\}$$

where

$$F^c = \mathrm{tr}\left[H^{(0)}\left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)}\left(\Gamma_S^{(1)}\right)^2 + 2H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)}\Gamma_S^{(1)}\right]$$

$$G^c = \mathrm{tr}\left[H^{(1)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(1)} S^{(0)}\Gamma_S^{(1)} + H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(1)} + H^{(0)} S^{(1)}\Gamma_S^{(1)} + H^{(0)}\Gamma_S^{(2)\,\dagger} S^{(0)} + H^{(0)} S^{(0)}\Gamma_S^{(2)}\right]$$

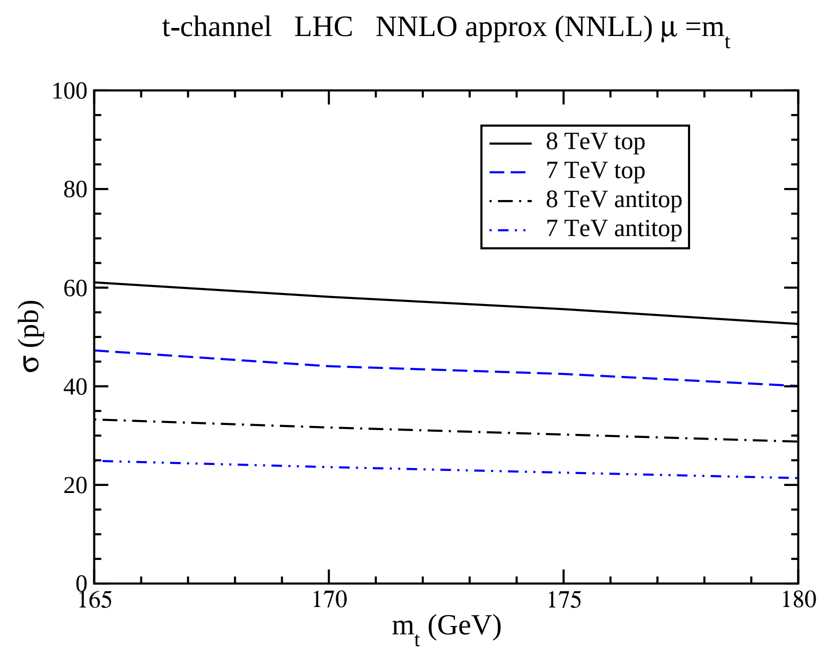
and c_3, c_2, c_1, etc are from the NLO expansion

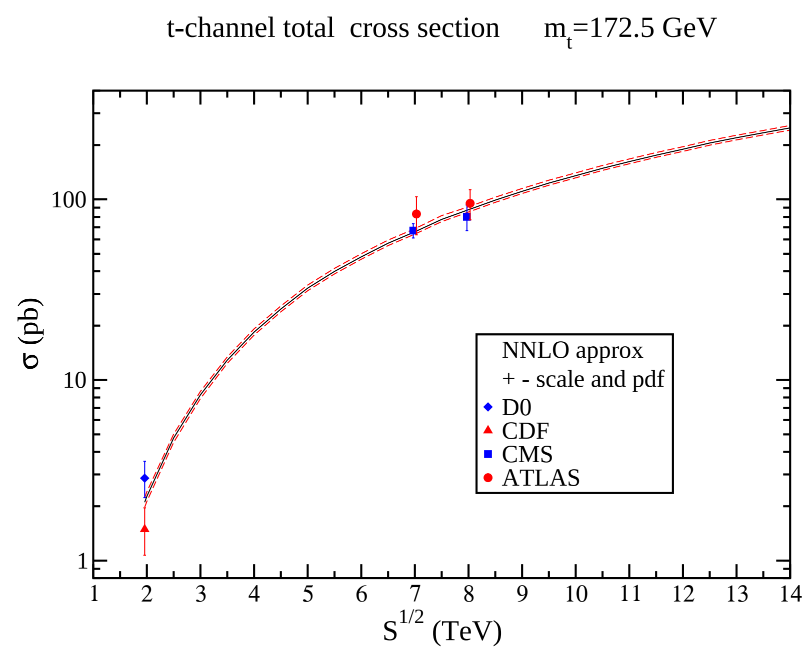
Two-loop universal quantities $A^{(2)}$, $B^{(2)}$, $D^{(2)}$ known

Two-loop process-dependent $\Gamma_S^{(2)}$ recently calculated for several processes

Additional purely collinear corrections of the form $(\ln^k N)/N$ provide very small contributions and do not improve the threshold approximation

Single top t-channel cross sections at LHC





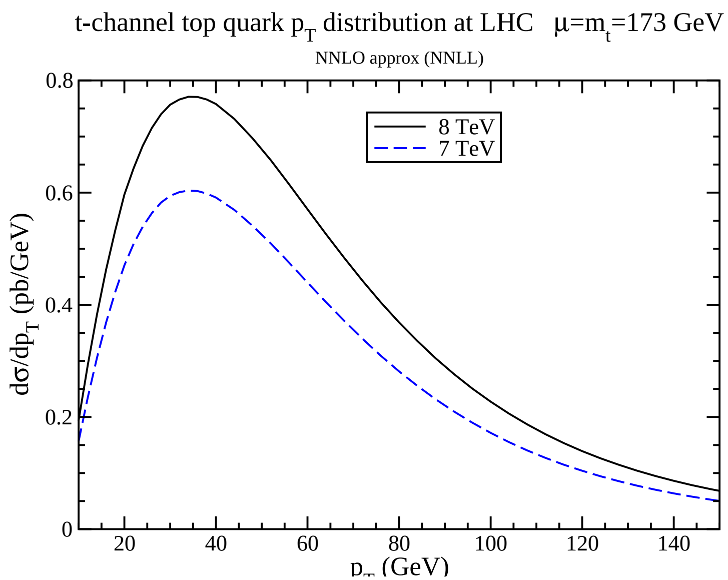
$m_t = 173$ **GeV**

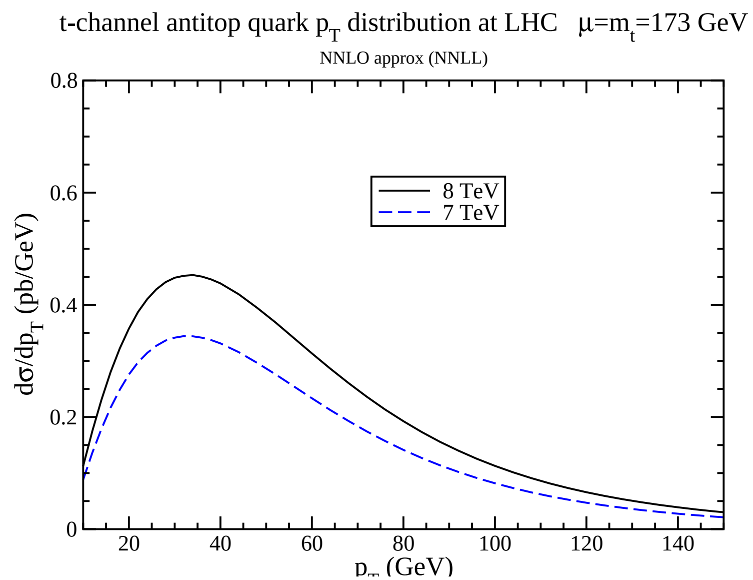
LHC	t	$\bar{t}$	Total (pb)
7 TeV	$43.0^{+1.6}_{-0.2} \pm 0.8$	$22.9 \pm 0.5^{+0.7}_{-0.9}$	$65.9^{+2.1}_{-0.7}{}^{+1.5}_{-1.7}$
8 TeV	$56.4^{+2.1}_{-0.3} \pm 1.1$	$30.7 \pm 0.7^{+0.9}_{-1.1}$	$87.2^{+2.8}_{-1.0}{}^{+2.0}_{-2.2}$
14 TeV	$154^{+4}_{-1} \pm 3$	$94^{+2}_{-1}{}^{+2}_{-3}$	$248^{+6}_{-2}{}^{+5}_{-6}$

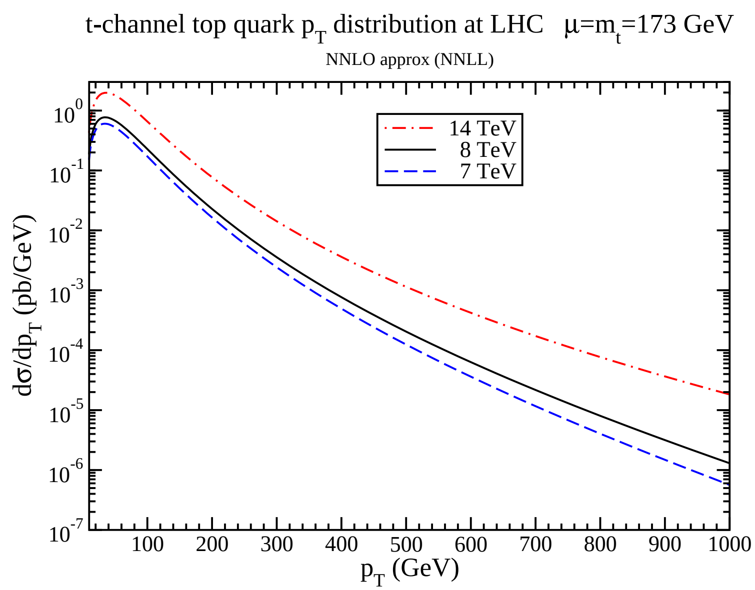
$\pm$ scale $\pm$ pdf errors with MSTW2008 NNLO pdf 90% CL

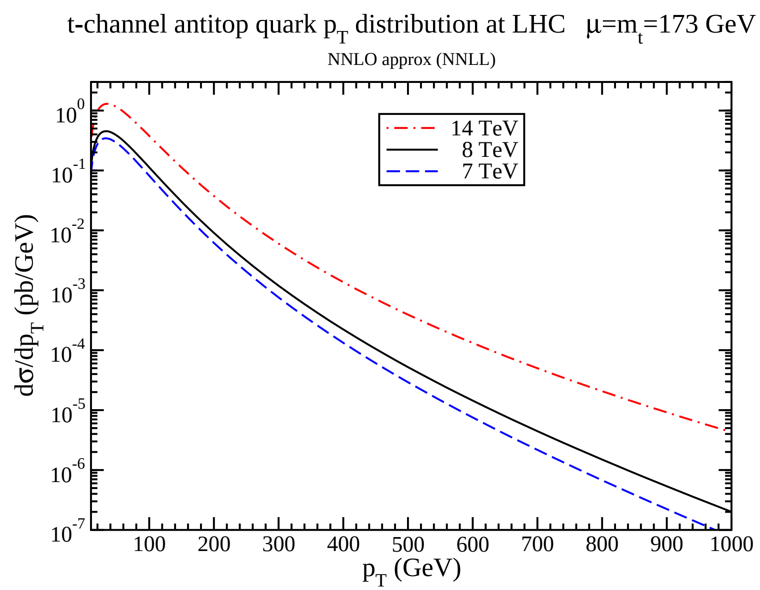
ratio $\sigma(t)/\sigma(\bar{t}) = 1.88^{+0.11}_{-0.09}$ at 7 TeV - compares well with ATLAS result $1.81^{+0.23}_{-0.22}$

t-channel top and antitop p_T distributions at LHC

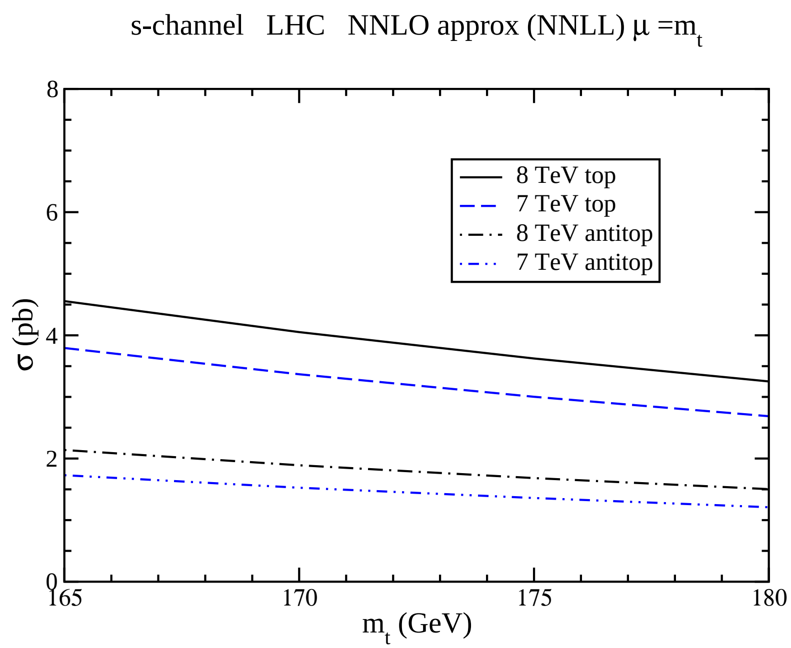


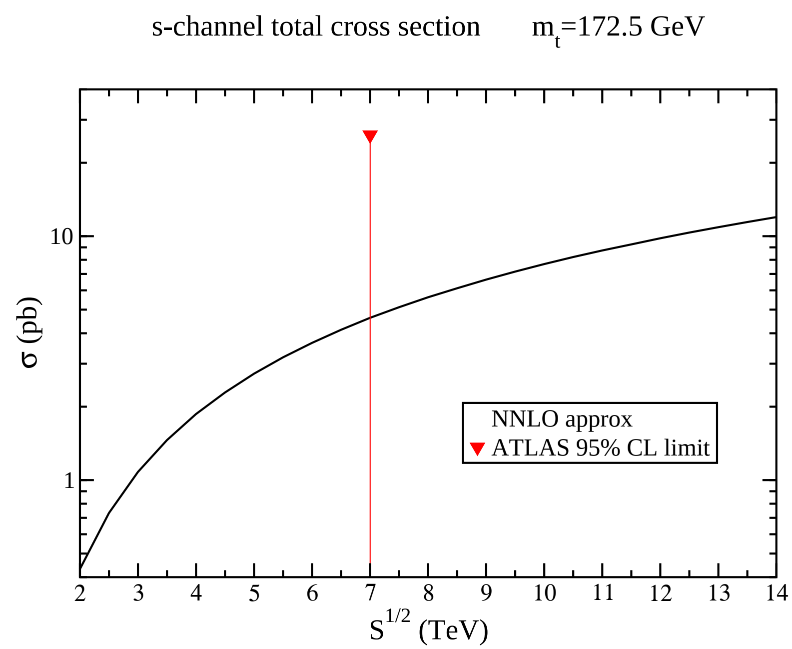






Single top s-channel cross sections at LHC



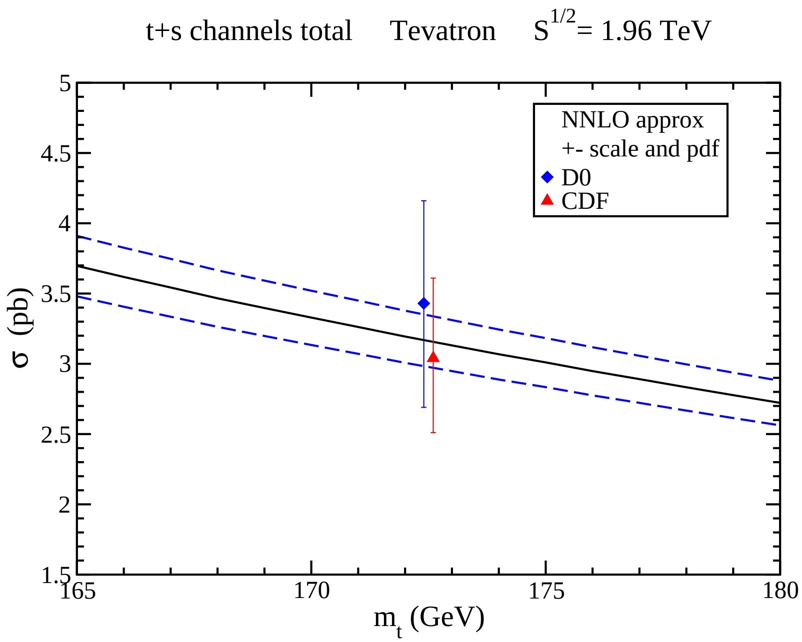


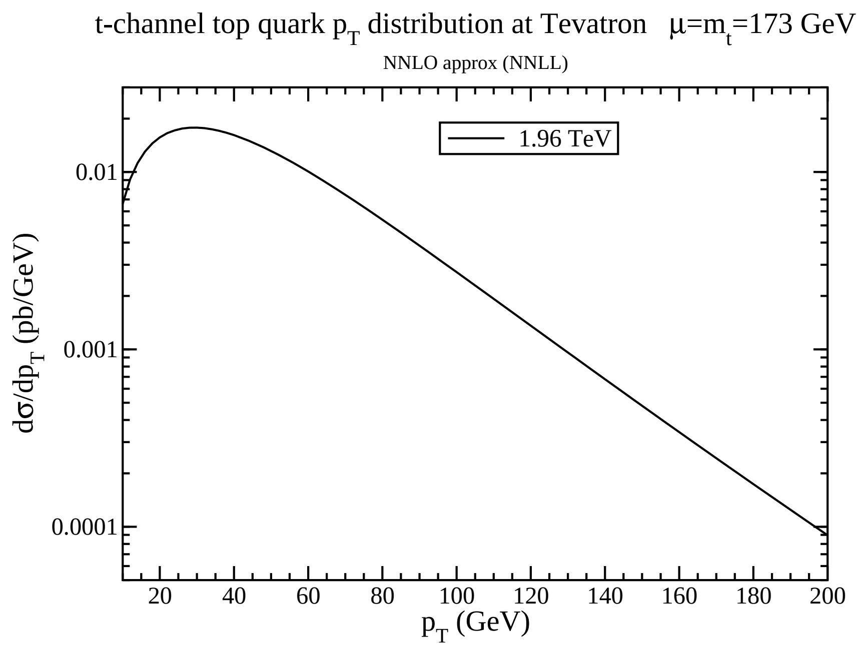
$m_t = 173$ **GeV**

LHC	t	$\bar{t}$	Total (pb)
7 TeV	$3.14 \pm 0.06^{+0.12}_{-0.10}$	$1.42 \pm 0.01^{+0.06}_{-0.07}$	$4.56 \pm 0.07^{+0.18}_{-0.17}$
8 TeV	$3.79 \pm 0.07 \pm 0.13$	$1.76 \pm 0.01 \pm 0.08$	$5.55 \pm 0.08 \pm 0.21$
14 TeV	$7.87 \pm 0.14^{+0.31}_{-0.28}$	$3.99 \pm 0.05^{+0.14}_{-0.21}$	$11.86 \pm 0.19^{+0.45}_{-0.49}$

NNLO approx: enhancement over NLO (same pdf) is $\sim 10\%$

t- and s-channel single top production at Tevatron

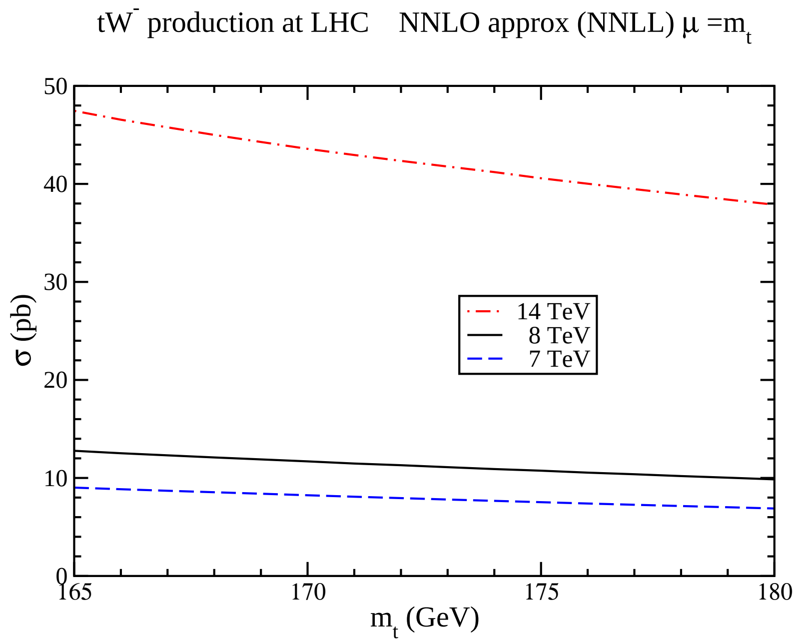


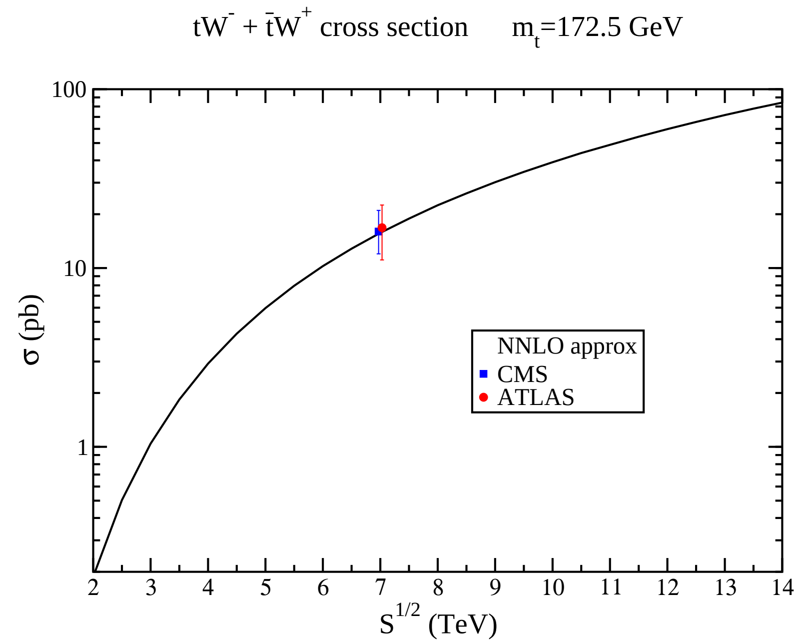


$m_t = 173$ **GeV**

Tevatron	Total (pb) at 1.96 TeV
t-channel	$2.08^{+0.00}_{-0.04} \pm 0.12$
s-channel	$1.05^{+0.00}_{-0.01} \pm 0.06$
$t+s$ sum	$3.13^{+0.00}_{-0.05} \pm 0.18$

Associated tW^- production at the LHC





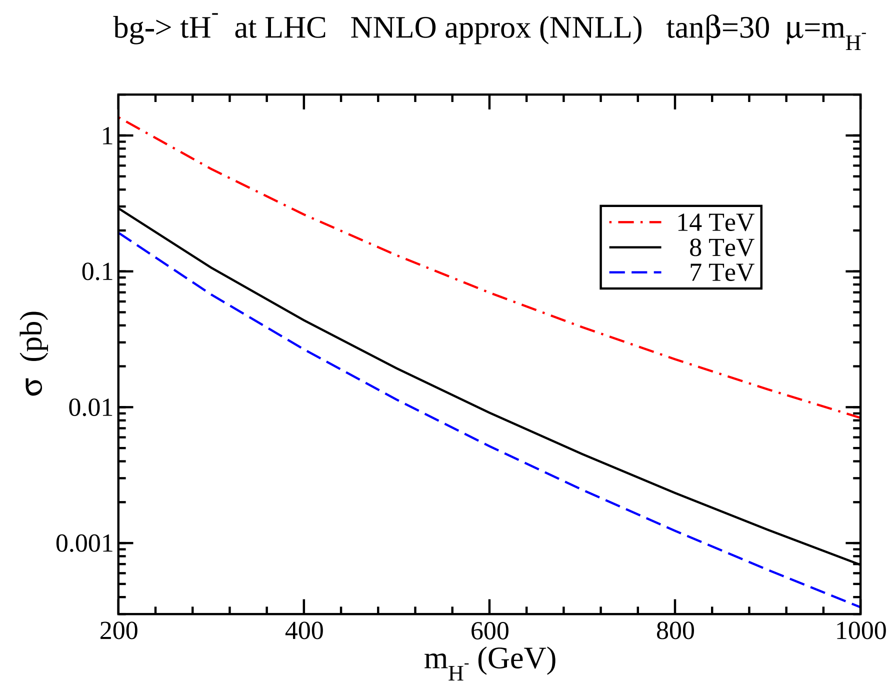
$m_t = 173$ **GeV**

LHC	tW^- (pb)
7 TeV	$7.8 \pm 0.2^{+0.5}_{-0.6}$
8 TeV	$11.1 \pm 0.3 \pm 0.7$
14 TeV	$41.8 \pm 1.0^{+1.5}_{-2.4}$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

Cross section for $\bar{t}W^+$ **production is identical**

Associated production of a top quark with a charged Higgs



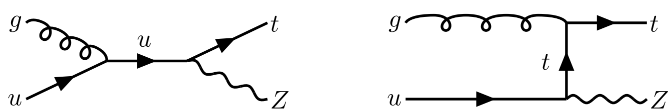
NNLO approx corrections increase NLO cross section by ~ 15 to $\sim 20\%$

FCNC processes

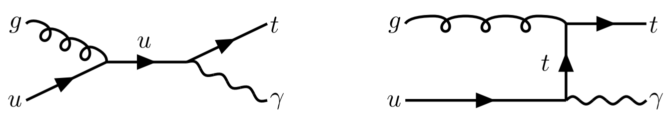
Single-top production via flavor-changing neutral currents
Anomalous couplings in Lagrangian, e.g.

$$\Delta\mathcal{L}^{eff} = \frac{1}{\Lambda}\,\kappa_{tqV}\,e\,\bar{t}\,\sigma_{\mu\nu}\,q\,F_V^{\mu\nu} + h.c.$$

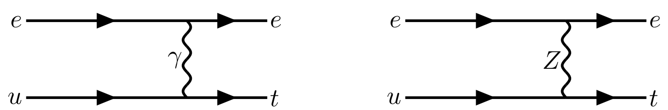
$gu \rightarrow tZ$



$gu \rightarrow t\gamma$



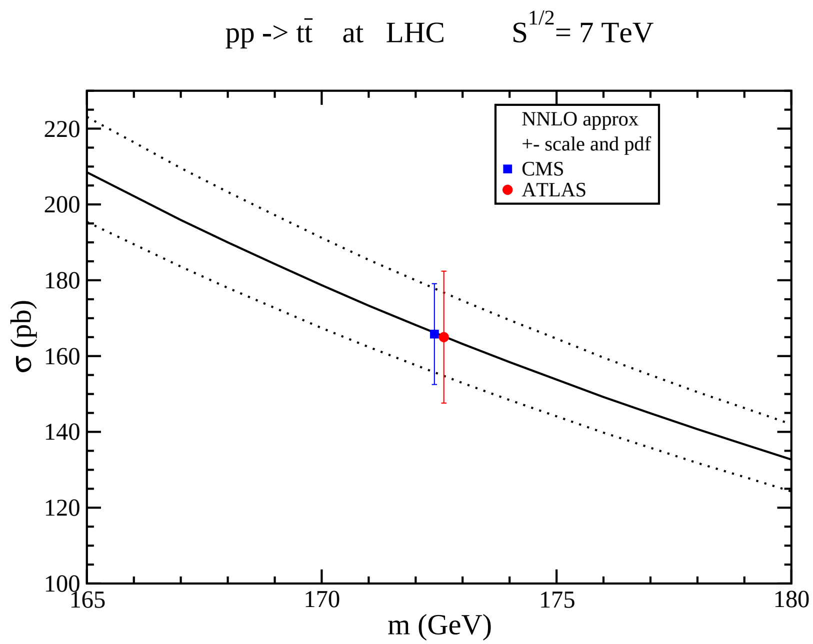
$eu \rightarrow et$

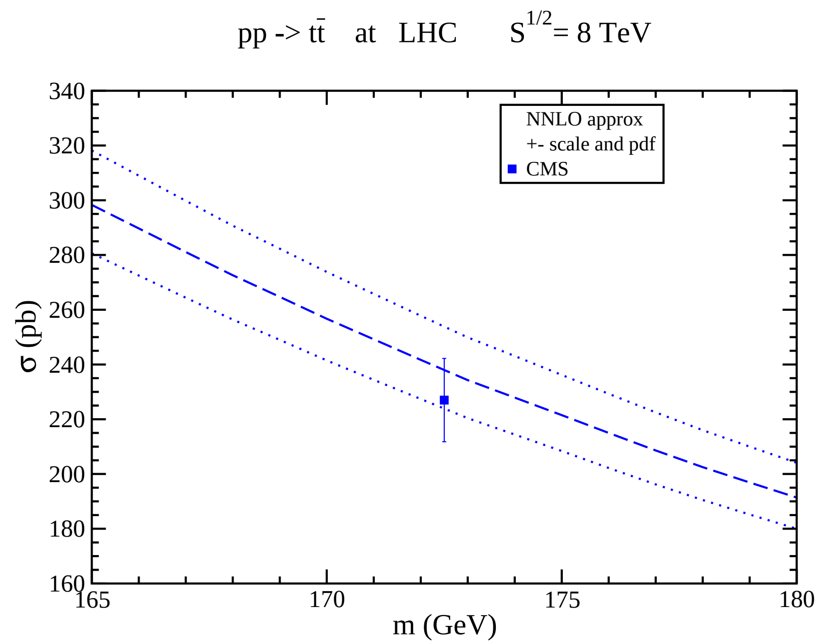


decrease of scale dependence, significant corrections over Born at Tevatron and HERA (e.g. 15 to 20% for $eu \rightarrow et$ at HERA)

Future studies for LHC energies and other couplings

$t\bar{t}$ cross section at the LHC





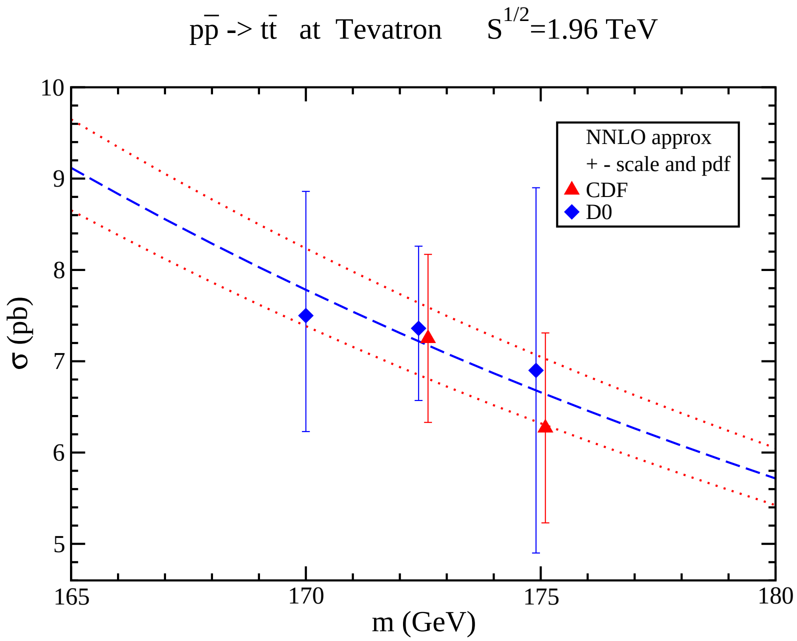
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) = 163^{+7}_{-5} \pm 9 \text{ pb}$$

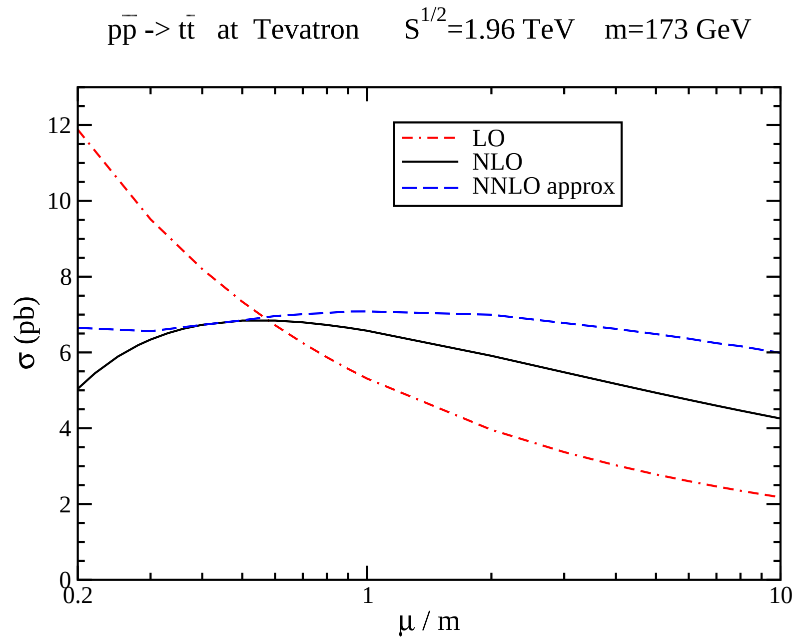
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 8\,{\rm TeV}) = 234^{+10}_{-7} \pm 12 \text{ pb}$$

$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV}, 14\,{\rm TeV}) = 920^{+50}_{-39}{}^{+33}_{-35} \text{ pb}$$

NNLO approx: enhancement over NLO (same pdf) is 7.6% at 7 TeV; 7.8% at 8 TeV; 8.0% at 14 TeV

$t\bar{t}$ cross section at the Tevatron





$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\,1.96\,{\rm TeV}) \quad = \quad 7.08^{+(0.20)}_{-0.24}\,{}^{+0.36}_{-0.27}\ {\rm pb}$$

scale pdf

NNLO approx: 7.8% enhancement over NLO
scale dependence greatly reduced

Differences between various resummation/NNLO approx approaches

NNLL resummed versus NNLO approx from NNLL resummation

Mellin-moment space vs SCET

Total vs differential cross section

Name	Observable	Soft limit
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = s + t_1 + u_1 \to 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}} d\theta$	$(1-z) = 1 - M_{t\bar{t}}^2/s \to 0$
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/s} \to 0$

For differential calculations, further differences arise from how the relation $s + t_1 + u_1 = 0$ is used in the plus-distribution coefficients, how subleading terms are treated, damping factors, etc.

see N. Kidonakis and B.D. Pecjak, Eur. Phys. J C 72, 2084 (2012) for details and review

All results presented here are NNLO approx from Mellin-space NNLL resummation for the double-differential cross section in 1PI kinematics

Comparison of various resummation/NNLO approx approaches

Tevatron 1.96 TeV, scale uncertainty included;
pdf uncertainty not shown - same for all if same assumptions are used
use $m_t = 173$ GeV unless otherwise indicated

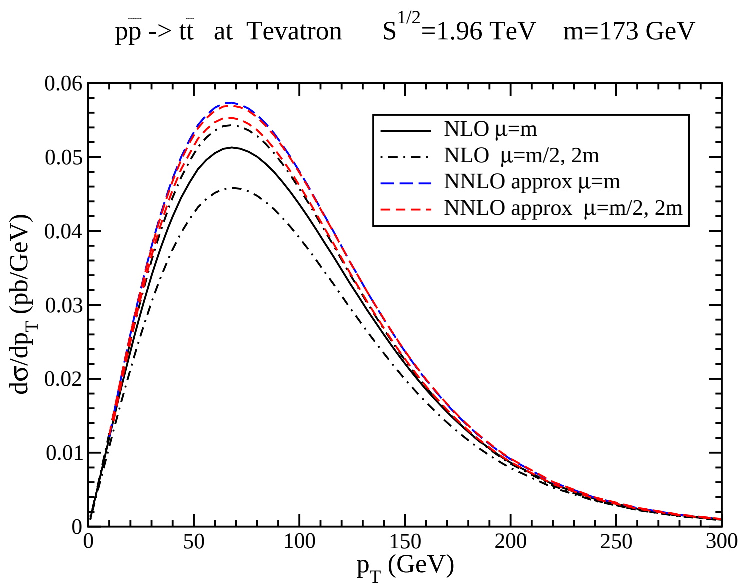
NLO $6.74^{+0.36}_{-0.76}$
Kidonakis, PRD 82, 114030 (2010) $7.08^{+0.20}_{-0.24}$
Aliev *et al*, **CPC 182, 1034 (2011)** $7.13^{+0.31}_{-0.39}$
Ahrens *et al*, **PLB 703, 135 (2011)** $6.65^{+0.08}_{-0.41}$
Beneke *et al*, **NPB 855, 695 (2012)** $(m_t = 173.3)$ $7.22^{+0.31}_{-0.47} \to 7.29$ **at** $m_t = 173$
Cacciari *et al*, **PLB 710, 612 (2012)** $(m_t = 173.3)$ $6.72^{+0.24}_{-0.41} \to 6.78$ **at** $m_t = 173$

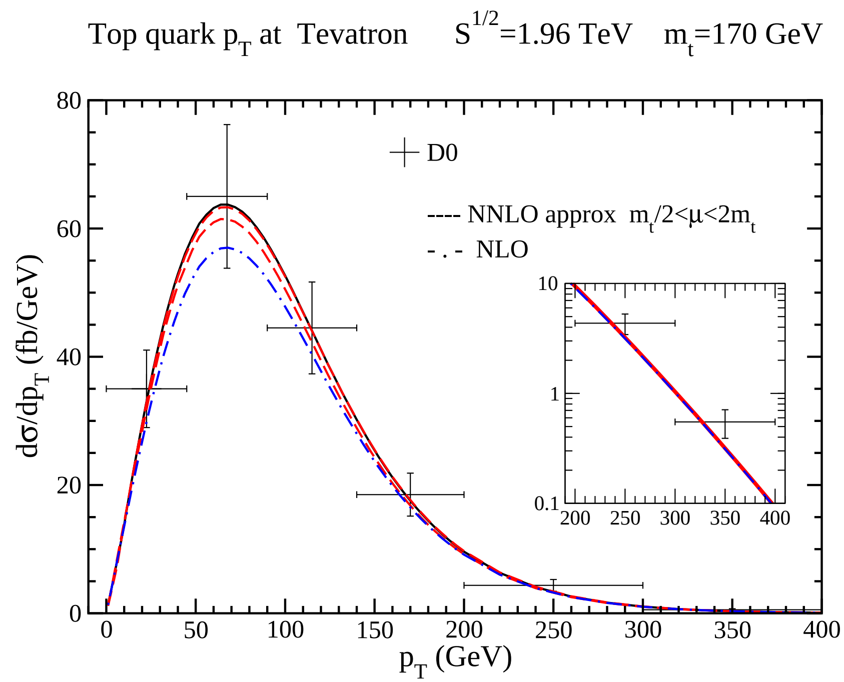
[See also Moch *et al* (2012) $7.27^{+0.41}_{-0.46}$ threshold + high-energy terms
Brodsky & Wu (2012) $(m_t = 172.9)$ $7.626 \to 7.602$ at $m_t = 173$ PMC]

partly exact NNLO (exact for $q\bar{q}$ plus approx for gg)
Barnreuther *et al* **(2012)** $(m_t = 173.3)$ $7.005^{+0.202}_{-0.310} \to 7.07$ **at** $m_t = 173$

The PRD 82 result is very close to the partly exact NNLO:
7.08 **vs** 7.07 **with similar scale uncertainty**
This was expected from comparison of 1PI and PIM results in 2003
(PRD 68, N. Kidonakis & R. Vogt; see also discussion in PRD82)
Once NNLO fully known, add approximate NNNLO
(see N. Kidonakis PRD 73,034001 (2006) for early NNNLO results)

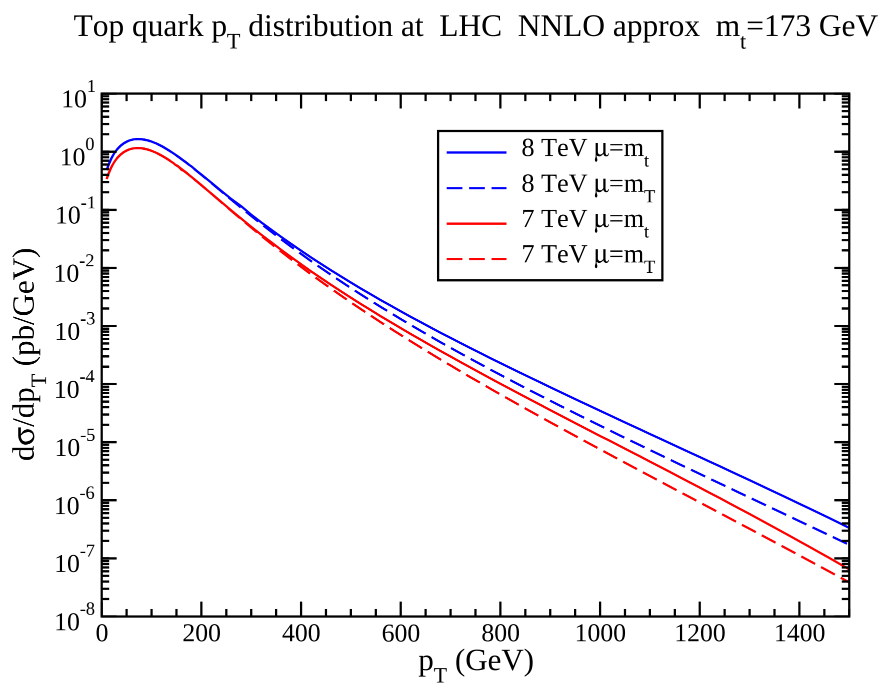
Top quark p_T distribution at Tevatron

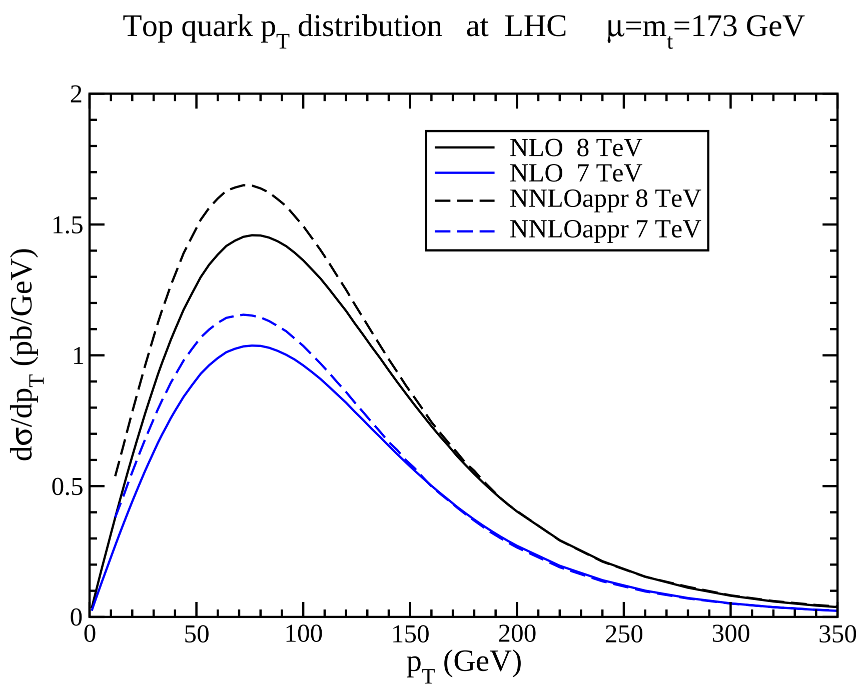




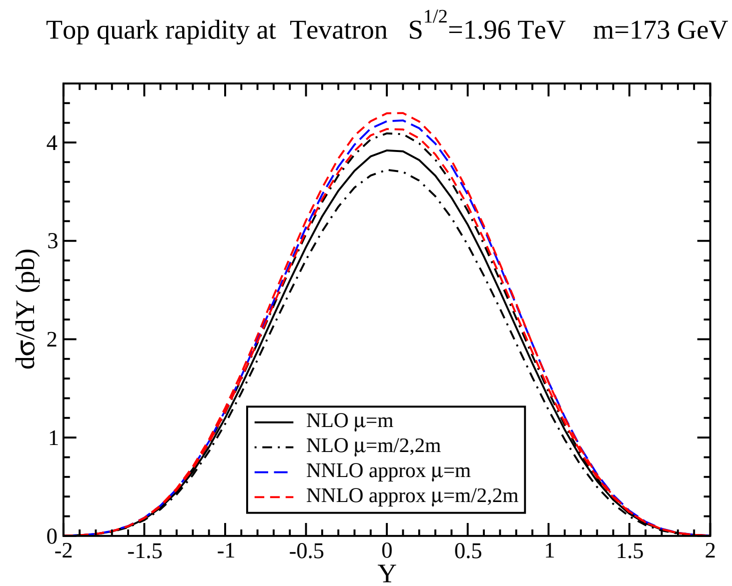
Excellent agreement of NNLO approx results with D0 data

Top quark p_T distribution at the LHC





Top quark rapidity distribution at Tevatron



Top Forward-backward asymmetry

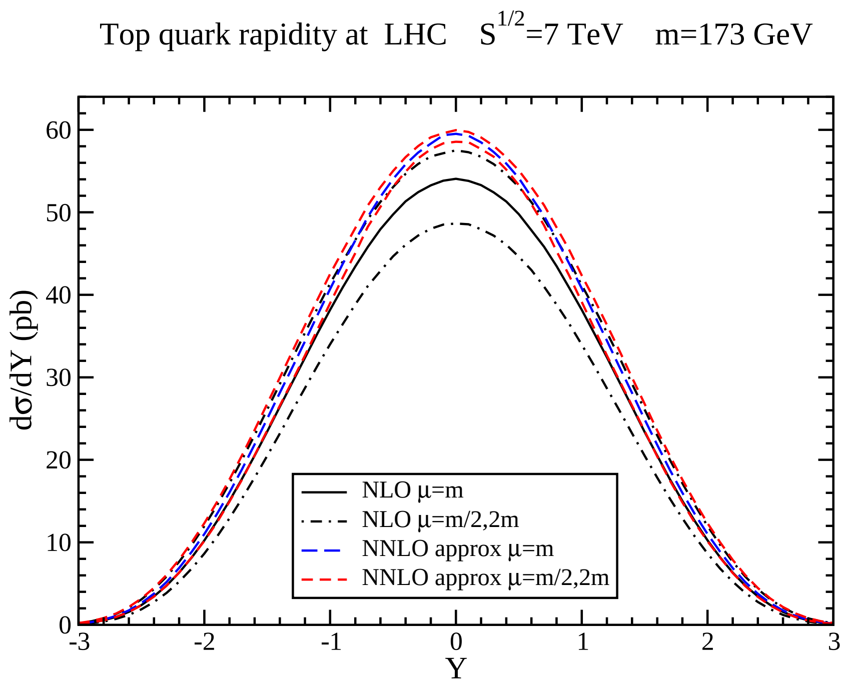
$$A_{\rm FB} = \frac{\sigma(Y>0)-\sigma(Y<0)}{\sigma(Y>0)+\sigma(Y<0)}$$

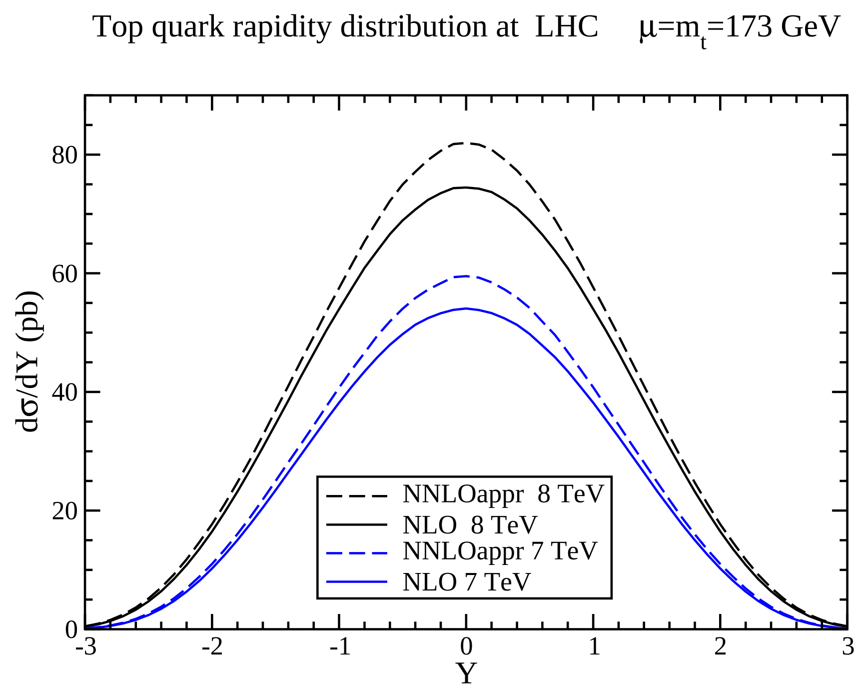
Asymmetry significant at the Tevatron

Theoretical result at Tevatron: $A_{\rm FB} = 0.052^{+0.000}_{-0.006}$

smaller than observed values

Top quark rapidity distribution at LHC





Summary

- NNLL resummation for single top and top pair production
- single top cross sections and p_T distributions
- $t\bar{t}$ production cross section
- top quark p_T and rapidity distributions
- NNLO approx corrections are significant at the LHC and the Tevatron
- good agreement with LHC and Tevatron data
- future work on more differential distributions and on NNNLO soft-gluon corrections