Extraction of γ from three-body B decays

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Work done in collaboration with M. Imbeault and D. London.
Work in progress!

Outline

- Motivation
- Theory
- Results
- Summary and Conclusions

Motivation

Conventional γ extraction methods: GLW, ADS $\rightarrow$ two-body $B^{\pm}$ decays. Very clean weak phase information however sensitive to a small ratio of interfering amplitudes. Solve this problem using multi-body D decays.

Giri et al. , PRD **68***, 054018*

Can we do any better with three-body B decays?

Extract γ using three-body B decays: cross check for SM! Great, even if not viable with current statistics!

Issues:

- Indirect CPV measurement requires a final CP eigenstate: not all three-body final states are useful. *E.g.,* $K_S\pi^+\pi^-$ *is not a CP eigenstate.*
- How do we treat decay amplitudes with two different weak phase terms?

These issues were addressed by London et al.
Details: D. London's talk tomorrow (Oct 1, 8:30 am, WG V)

Theory Goal

In order to extract weak-phase information from three-body B decays:

- Use multiple three-body decays
- Relate them using flavor-SU(3) symmetry
- Estimate by how much flavor SU(3) is broken (Work under progress)

N. Rey-Le Lorier and D. London, PRD **85**, *016010*

The purpose of this talk is to show a case where such an extraction actually works!

Which decays to consider?

Three-body $\overline{b} \to \overline{s}$ transitions : 3 each involving $B^{0,+} \to K(\pi\pi, KK)$.

Decays involving $1+$ final state π^0s : difficult to observe.
B^+ decays don't have indirect CPV information.
Finally, under SU(3) symmetry some information is redundant.

In the end we chose the following three-body transitions:
(data taken from *BaBar* papers)

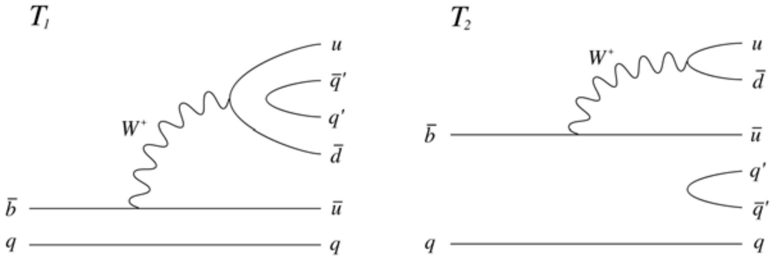
- $B^0 \to K_S\pi^+\pi^-$: *PRD* **80**, *112001.*
- $B^0 \to K^+\pi^-\pi^0$: *PRD* **83**, *112010.*
- $B^0 \to K_S K^+ K^-$: *PRD* **85**, *112010.*
- $B^0 \to K_S K_S K_S$: *PRD* **85**, *054023.*
- $B^+ \to K^+\pi^+\pi^-$: To estimate a one-parameter SU(3) breaking.

Three-body Diagrammatics

In two-body $\overline{b} \to \overline{s}$ transitions : T', C', P'_{tc}, P'_{uc}, P'_{EW} and P'^{C}_{EW}
(Ignoring weak-exchange topologies)

Two-body transition $\to$ three-body transition : pop a $q\bar{q}$ pair

Two choices of popping : between two non spectators (add subscript 1), or between two quarks one of which is the spectator (add subscript 2).



Three-body topologies : T'_1, T'_2, C'_1, C'_2, ...

Flavor-SU(3) symmetry

Under SU(3) the three-body final states involve 3 identical particles.

Flavor SU(3) imposes EWP – Tree relations :

$$P'_{EW1} = \kappa T'_1 \qquad P'_{EW2} = \kappa T'_2$$
$$P'^{C}_{EW1} = \kappa C'_1 \qquad P'^{C}_{EW2} = \kappa C'_2$$

where $\kappa \approx 0.5$: a function of Wilson coefficients and CKM elements.

M. Imbeault, N. Rey-Le Lorier and D. London, PRD **84**, *034041*

Caveat: The EWP-Tree relations hold only for fully-symmetric final states.

Construct fully-symmetric amplitudes using all possible permutations of particles.

Amplitudes

Relevant (fully-symmetric) three-body amplitudes in terms of diagrams:

$$
\begin{aligned}
A(B^0 \to K_0 K^0 \overline{K}^0)_{\rm sym} &= P'_{uc} e^{i\gamma} - P'_{tc} + \kappa \left(\frac{2}{3} T'_1 + \frac{1}{3} C'_1 + \frac{2}{3} C'_2 \right) \\
\sqrt{2} A(B^0 \to K^+ K^0 K^-)_{\rm sym} &= -(T'_2 + C'_1 + P'_{uc}) e^{i\gamma} + P'_{tc} \\
&\quad + \kappa \left(\frac{1}{3} T'_1 - \frac{1}{3} C'_1 + \frac{2}{3} C'_2 \right) \\
&= \sqrt{2} A(B^+ \to K^+ \pi^+ \pi^-) \\
2A(B^0 \to K^+ \pi^0 \pi^-)_{\rm sym} &= (T'_1 + C'_2) e^{i\gamma} - \kappa (T'_2 + C'_1) \\
\sqrt{2} A(B^0 \to K^0 \pi^+ \pi^-)_{\rm sym} &= -(T'_1 + C'_1 + P'_{uc}) e^{i\gamma} + P'_{tc} \\
&\quad + \kappa \left(\frac{1}{3} T'_1 + \frac{2}{3} C'_1 - \frac{1}{3} C'_2 \right)
\end{aligned}
$$

Number of theory parameters can be considerably reduced by considering combinations of diagrams to give "effective diagrams"!

Effective diagrams

Experimentally, CPV is not measured in $B^0 \to 3K_S \Rightarrow P'_{uc} = 0$

Amplitudes in terms of effective diagrams:

$$\begin{aligned}
A(B^0 \to K^0 K^0 \overline{K}^0)_{\rm sym} &= a \\
\sqrt{2}A(B^0 \to K^+ K^0 K^-)_{\rm sym} &= -ce^{i\gamma} - a + \kappa b \\
2A(B^0 \to K^+ \pi^0 \pi^-)_{\rm sym} &= be^{i\gamma} - \kappa c \\
\sqrt{2}A(B^0 \to K^0 \pi^+ \pi^-)_{\rm sym} &= -de^{i\gamma} - a + \kappa d
\end{aligned}$$

4 effective parameters a, b, c and d :

$$a = -P'_{tc} + \kappa\left(\frac{2}{3}T'_1 + \frac{1}{3}C'_1 + \frac{1}{3}C'_2\right)$$

$$b = T'_1 + C'_2 \qquad c = T'_2 + C'_1 \qquad d = T'_1 + C'_1$$

Unknowns : 4 (magnitudes) + 3 (relative strong phases) + $\gamma = 8$

Dalitz plots

Amplitudes are dependent on momenta : known from Dalitz plots

Isobar model : isobar coefficients and phases taken from Experiments:

$$\mathcal{A}_{\rm DP} = \mathcal{N}_{\rm DP} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}) \qquad \mathcal{N}_{\rm DP}^2 = \Gamma_{\rm ex} / \int_{\rm DP} |\mathcal{A}|^2 d\Pi$$

$$s_{12} + s_{23} + s_{13} = M_B^2 + M_1^2 + M_2^2 + M_3^2 \qquad s_{ij} = (p_i + p_j)^2$$

Fully symmetrized amplitudes :

$$\mathcal{A}_{\rm sym} = \frac{1}{\sqrt{6}}(\mathcal{A}(s_{12}, s_{13}) + \mathcal{A}(s_{13}, s_{12}) + \mathcal{A}(s_{12}, s_{23}) + \mathcal{A}(s_{23}, s_{12}) + \mathcal{A}(s_{23}, s_{13}) + \mathcal{A}(s_{13}, s_{23}))$$

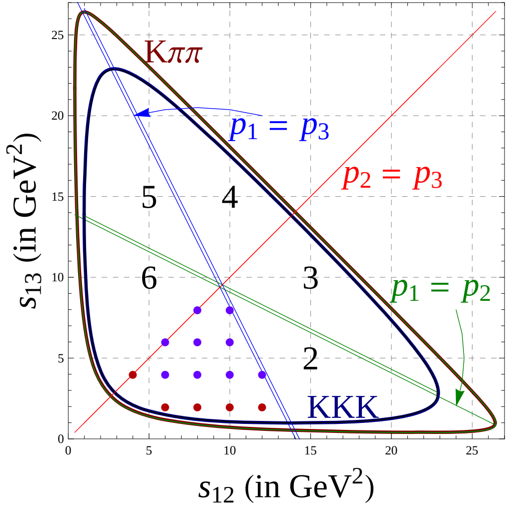
Observables constructed for a χ^2 fit :

$$X_{\rm DP} = |\mathcal{A}_{\rm sym}|^2 + |\overline{\mathcal{A}}_{\rm sym}|^2$$ *CP averaged branching fraction*

$$Y_{\rm DP} = |\mathcal{A}_{\rm sym}|^2 - |\overline{\mathcal{A}}_{\rm sym}|^2$$ *Direct CP asymmetry*

$$Z_{\rm DP} = {\rm Im}\left(\mathcal{A}_{\rm sym}^* \overline{\mathcal{A}}_{\rm sym}\right)$$ *Indirect CP asymmetry*

Dalitz plots



- Use one-sixth of the inner-most Dalitz plot
- 9 observables per point :
  $B^0 \to K^0\pi^+\pi^-$: X, Y, Z
  $B^0 \to K^0K^+K^-$: X, Y, Z
  $B^0 \to K^+\pi^-\pi^0$: X, Y
  $B^0 \to K^0K^0\overline{K}^0$: X, ~~Y~~, ~~Z~~
- 8 unknowns :
  4 magnitudes
  3 relative strong phases
  γ
- χ^2 fit to extract γ.

Results

Preliminary analysis with 14 points:

(s_{12}, s_{13}) (in GeV^2)	γ (in deg)	(s_{12}, s_{13}) (in GeV^2)	γ (in deg)
(4, 4)	–	(8, 8)	80^{+13}_{-14}
(6, 2)	–	(10, 2)	–
(6, 4)	73^{+12}_{-18}	(10, 4)	86^{+12}_{-14}
(6, 6)	80^{+10}_{-15}	(10, 6)	79^{+12}_{-15}
(8, 2)	–	(10, 8)	81^{+10}_{-13}
(8, 4)	80^{+12}_{-15}	(12, 2)	–
(8, 6)	78^{+13}_{-14}	(12, 4)	88^{+14}_{-19}

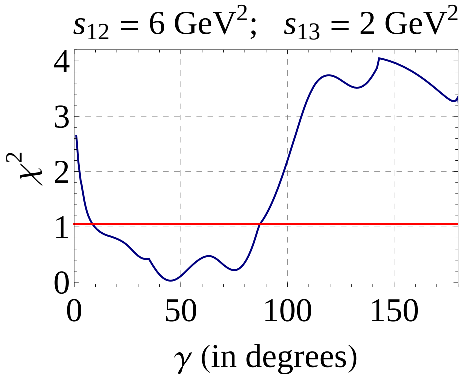
$$\gamma = \left(81^{+4}_{-5}\ (avg.) \pm 4\ (std.\ dev.)\right)^\circ$$

Multiple overlapping solutions : Discrete ambiguity

Average of GLW, ADS and Dalitz analyses (PDG 2012) : $\gamma = (68^{+11}_{-10})^\circ$

χ^2 vs γ : boundary point

$\chi^2_{\rm min}$ as a function of γ for one of the boundary points:

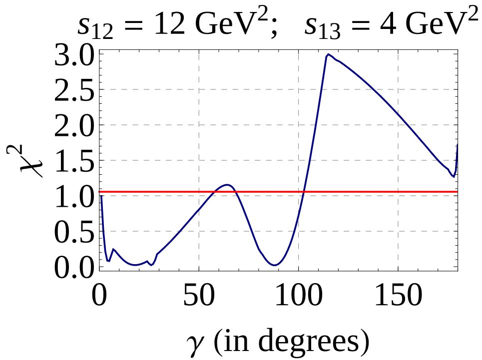


- Multiple indistinguishable solutions.
- Large error bars.
- Low χ^2 for a solution close to 45°.
- Is this a sign of larger SU(3) breaking closer to the boundary?

We have excluded such points at this stage, because of the presence of multiple indistinguishable solutions.

χ^2 vs γ : interior point 1

$\chi^2_{\rm min}$ as a function of γ for one of the interior points:



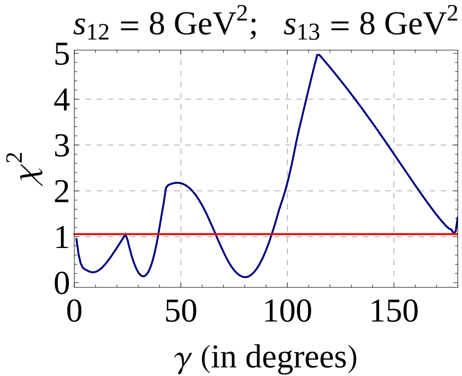
- Multiple discrete solutions.
- Overlapping solutions with $\gamma \leq 30^\circ$.
- Cleaner solution close to $\gamma \approx 80^\circ$.
- $(\Delta\chi^2)_{\rm min} = 1$ determines the (1σ) error bars.

Improvements possible: a) Reduce experimental errors on isobar parameters; b) Measure CP asymmetries in $B^0 \to K^0 K^0 \overline{K}^0$.

Here we have chosen the solution with γ closer to 80°. At this stage, it is difficult to distinguish the solutions with $\gamma \leq 30^\circ$.

χ^2 vs γ : interior point 2

$\chi^2_{\rm min}$ as a function of γ for one of the boundary points:



- Three distinct solutions : $\gamma = 9^\circ, 32^\circ, 80^\circ$
- $\gamma \approx 10^\circ, 30^\circ$ are not distinguishable for other points
- A more detailed analysis will have to be performed to reach a sensible conclusion.

We only include the $\gamma \approx 80^\circ$ solution from such points at this stage.

A complete analysis will aim at distinguishing different solutions for all interior as well as boundary points.

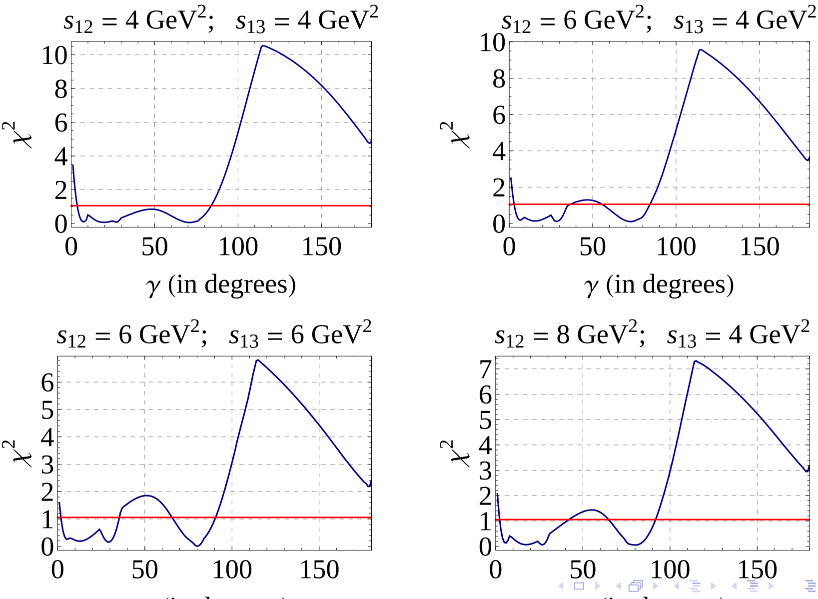
Caveats

- Correlation between isobar coefficients may effect the error analysis
- Normalization ($\mathcal{N}_{\rm DP}$) treated as a constant without error : In principle, $\mathcal{N}_{\rm DP}$ implicitly depends on isobar coefficients
- M_π, M_K and other resonance masses break SU(3) : results from points near resonances or the boundary are susceptible to large SU(3) breaking
- A one-parameter SU(3) breaking may alleviate some of these issues
- 14 points chosen for ease of preliminary fit. However, many more points can be chosen depending on statistics available for the Dalitz plots
- An analysis of the complete experimental data set may be able to avoid many of these issues $\Rightarrow$ improve the analysis significantly and hopefully decrease the error bars on γ even further
- Successful in obtaining a modest value for γ using three-body B decays, but much work still remains to be done

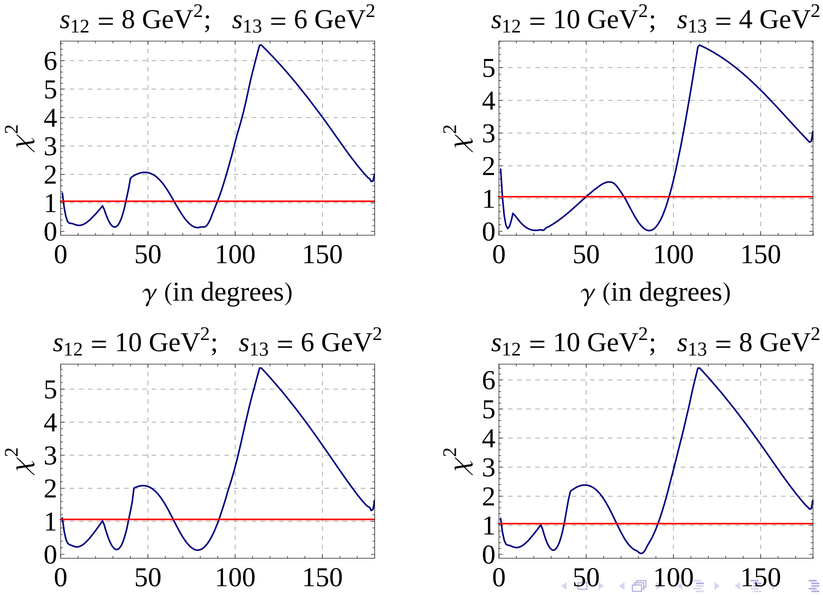
Summary and Conclusions

- A combination of multiple three-body B decays can actually be used to extract γ! Analysis possible with available *BaBar* and BELLE data
- This technique of γ extraction augments other independent ways of measuring γ from two-body decays : Good test for the SM value of γ
- Each point on the Dalitz plot provides an independent source for γ!
- Although there are discrete ambiguities, we find a clean solution : $\gamma = \left(81^{+4}_{-5}\ (avg.) \pm 4\ (std.\ dev.)\right)^\circ$
- We hope to resolve multiple solutions with $\gamma \approx 30^\circ$ in the near future
- Tricky issue of SU(3) breaking : one-parameter SU(3) breaking may lead to more clues
- We hope that in the future, a complete analysis can be done by an experimental collaboration using the data sets on all relevant three-body processes

χ^2 vs γ : interior points



χ^2 vs γ : interior points



χ^2 vs γ : boundary points