The electroweak contribution to top pair production: cross-sections and asymmetries

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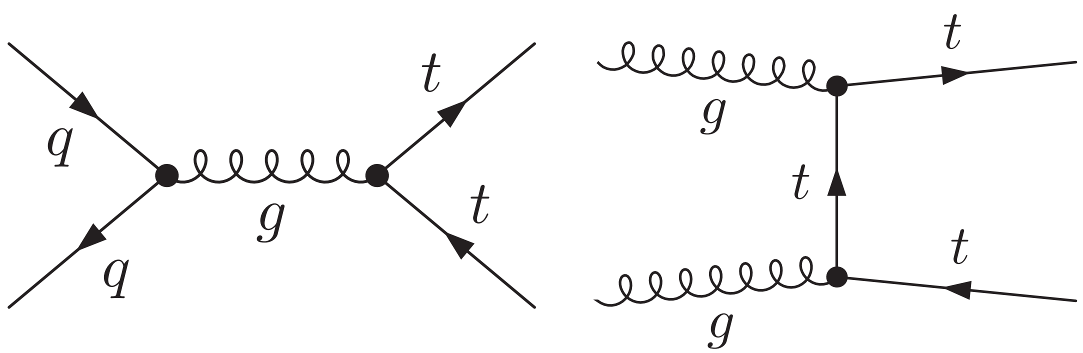
Hadronic top quark pair production is a "QCD process"



LO $\mathcal{O}(\alpha_s^2)$

NLO $\mathcal{O}(\alpha_s^3)$: *Nason et al. '89, Beenakker et al. '91, Mangano, Nason, Ridolfi '92 Frixione et al. '95*

complete NNLO $q\bar{q} \to t\bar{t}$:
Baernreuther, Czakon, Mitov '12

beyond NLO : *Beneke, Falgari, Schwinn '09 Czakon, Mitov, Sterman '09, Kidonakis '10 Ahrens, Ferroglia et al. '10 and* ***many*** *others*

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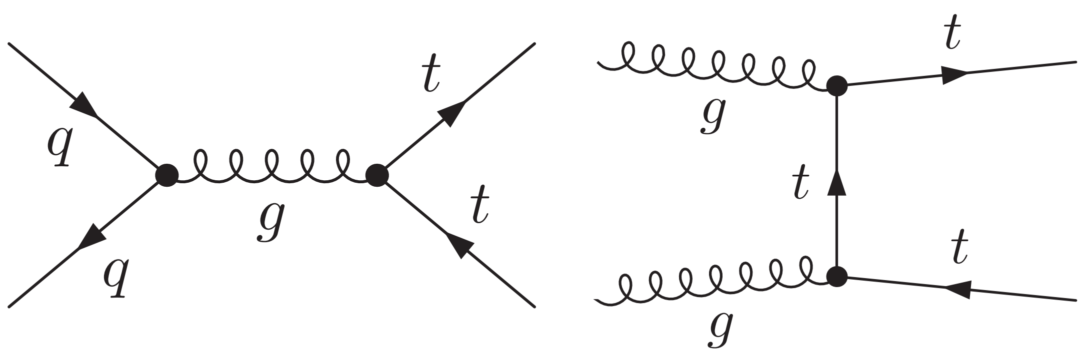

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The EW contribution to top pair production

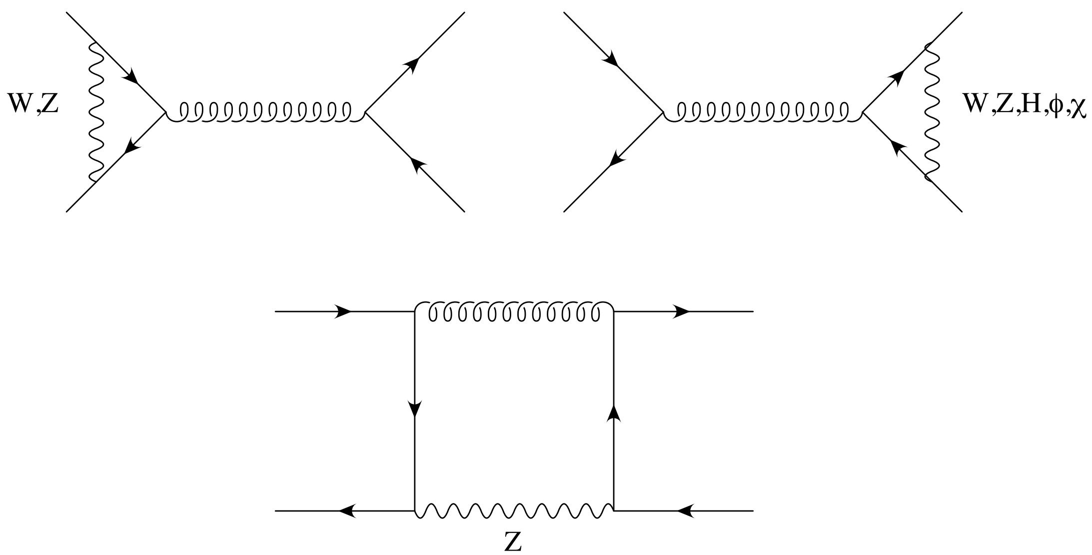
- Cross section, charge asymmetry
- LHC, Tevatron
- $gg \to t\bar{t}$, $q\bar{q} \to t\bar{t}$
- Differential and integrated quantities
- $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha_s^2\alpha)$, ...
- QED, Weak

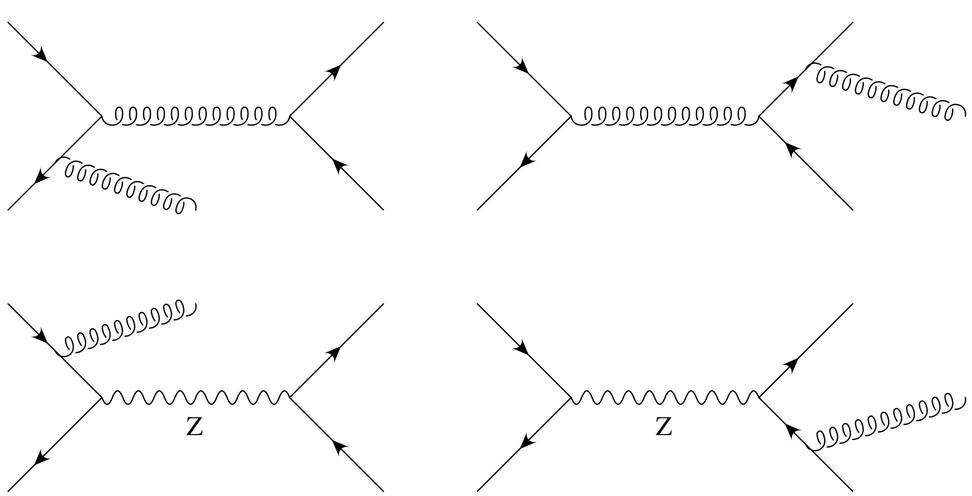
Weak corrections to the cross section

Beenakker et al '94, Kuhn, Scharf, Uwer '06, Bernreuther, Fuecker, Si '06
Moretti, Nolten, Ross '06,

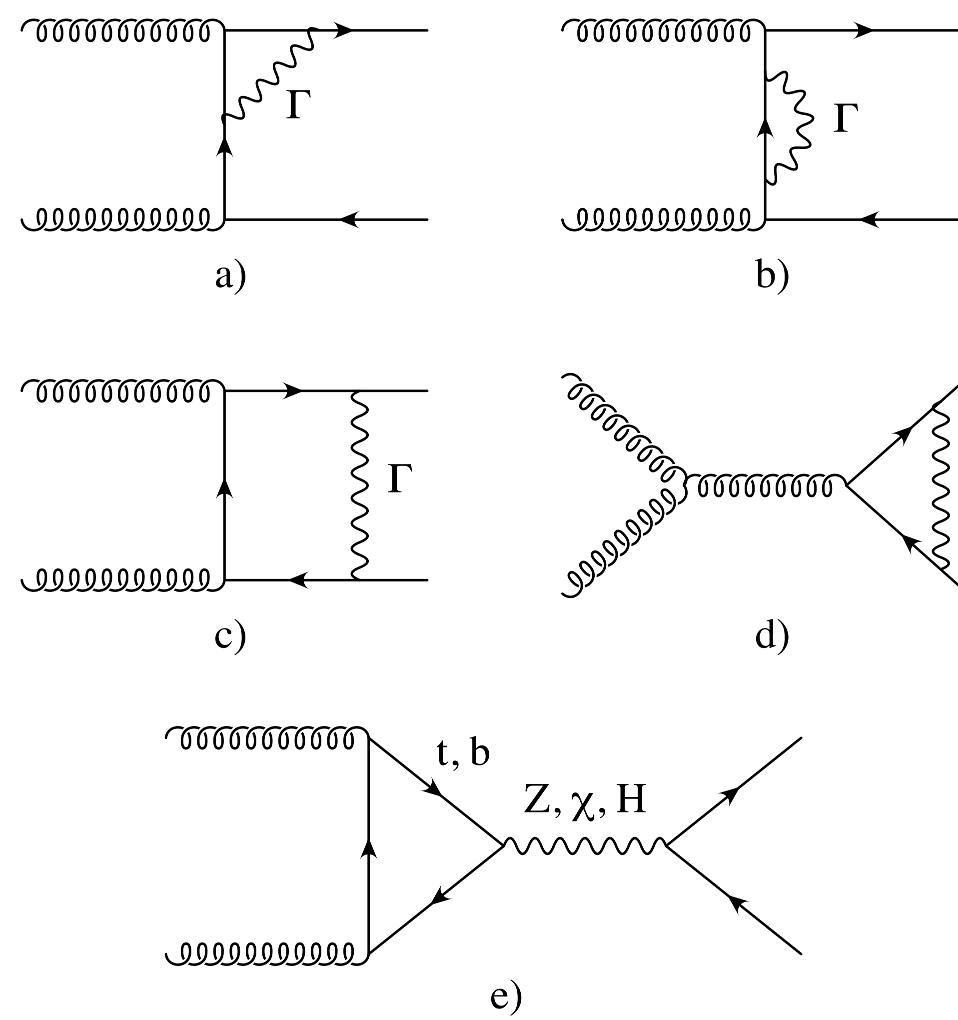
$\mathcal{O}(\alpha_s^2 \alpha)$

$q\bar{q} \to t\bar{t}$

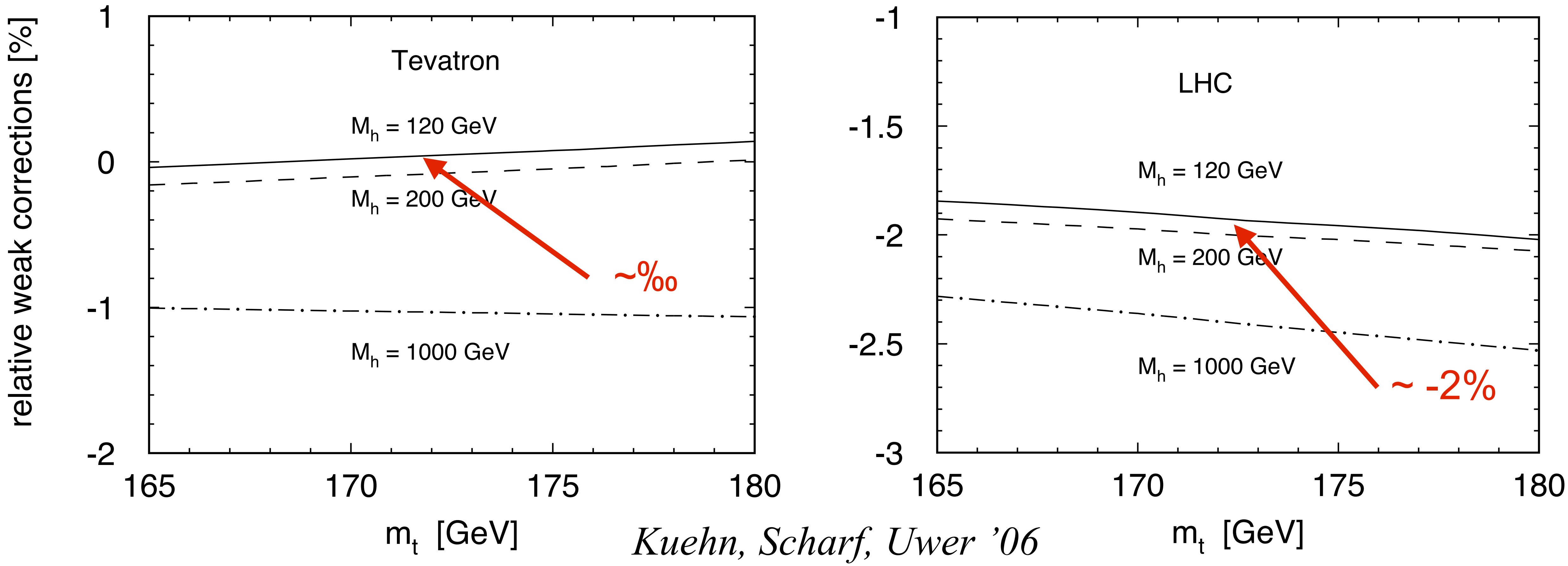




$gg \to t\bar{t}$



Weak corrections to the cross section



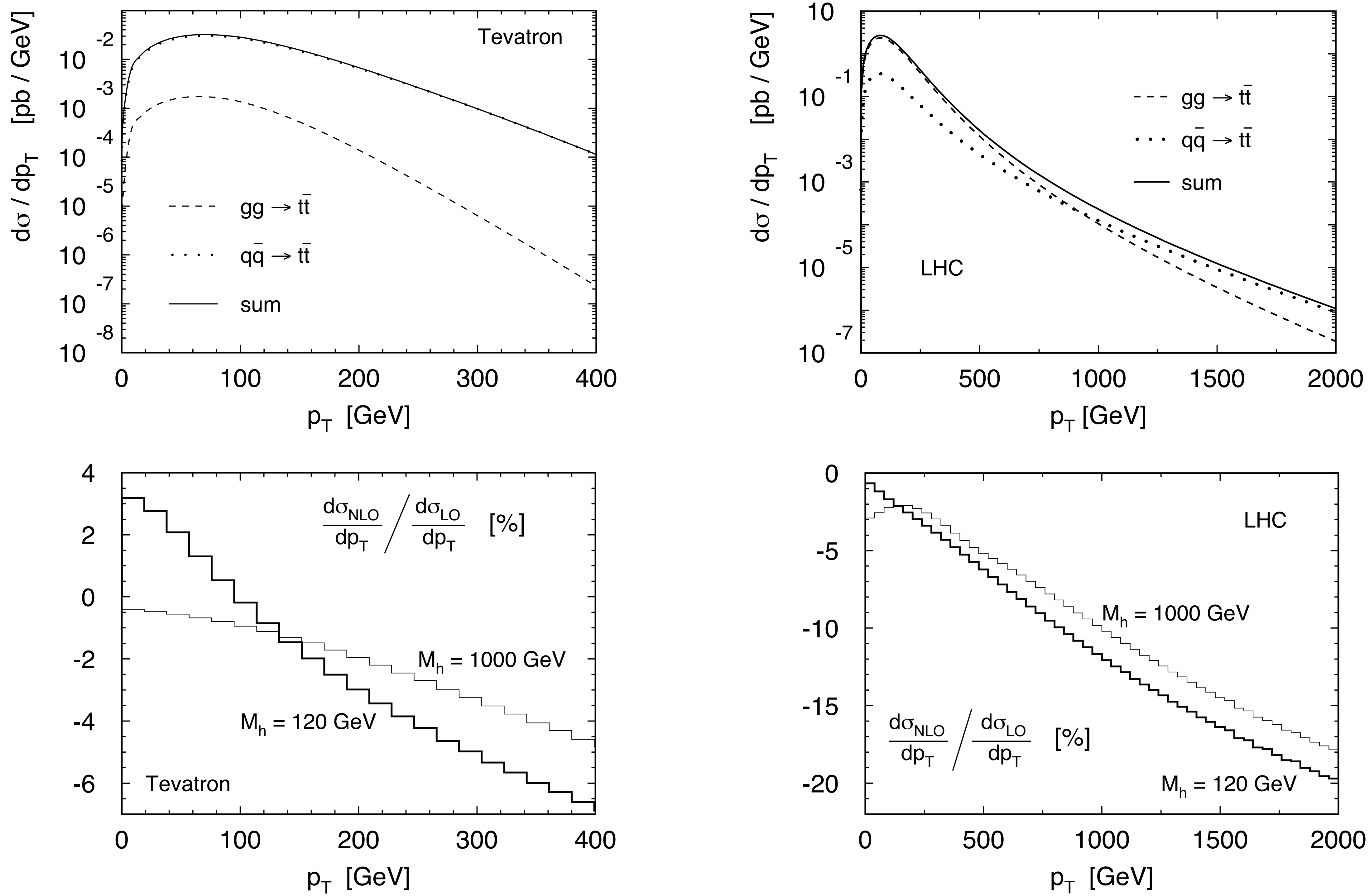
Kuehn, Scharf, Uwer '06

	Tevatron	LHC (7 TeV)	LHC (14 TeV)
$\sigma_{\rm LO}$	$4.49^{+1.71+0.24}_{-1.15-0.19}$	84^{+29+4}_{-20-5}	$495^{+148+19}_{-107-24}$
$\sigma_{\rm NLL}$	$5.07^{+0.37+0.28}_{-0.36-0.18}$	112^{+18+5}_{-14-5}	$598^{+108+19}_{-94\ -19}$
$\sigma_{\rm NLO,\ leading}$	$5.49^{+0.78+0.31}_{-0.78-0.20}$	134^{+16+7}_{-17-7}	761^{+64+25}_{-75-26}
$\sigma_{\rm NLO}$	$5.79^{+0.79+0.33}_{-0.80-0.22}$	133^{+21+7}_{-19-7}	$761^{+105+26}_{-101-27}$
$\sigma_{\rm NLO+NNLL}$	$6.30^{+0.19+0.31}_{-0.19-0.23}$	149^{+7+8}_{-7-8}	821^{+40+24}_{-42-31}
$\sigma_{\rm NNLO,\ approx}$ (scheme A)	$6.14^{+0.49+0.31}_{-0.53-0.23}$	146^{+13+8}_{-12-8}	821^{+71+27}_{-65-29}
$\sigma_{\rm NNLO,\ approx}$ (scheme B)	$6.05^{+0.43+0.31}_{-0.50-0.23}$	139^{+9+7}_{-9-7}	773^{+47+25}_{-50-27}

Ahrens, Ferroglia et al.'11

Errors from scale variation and PDFs are bigger than EW corrections

Differential distributions



Kuhn, Scharf, Uwer '06

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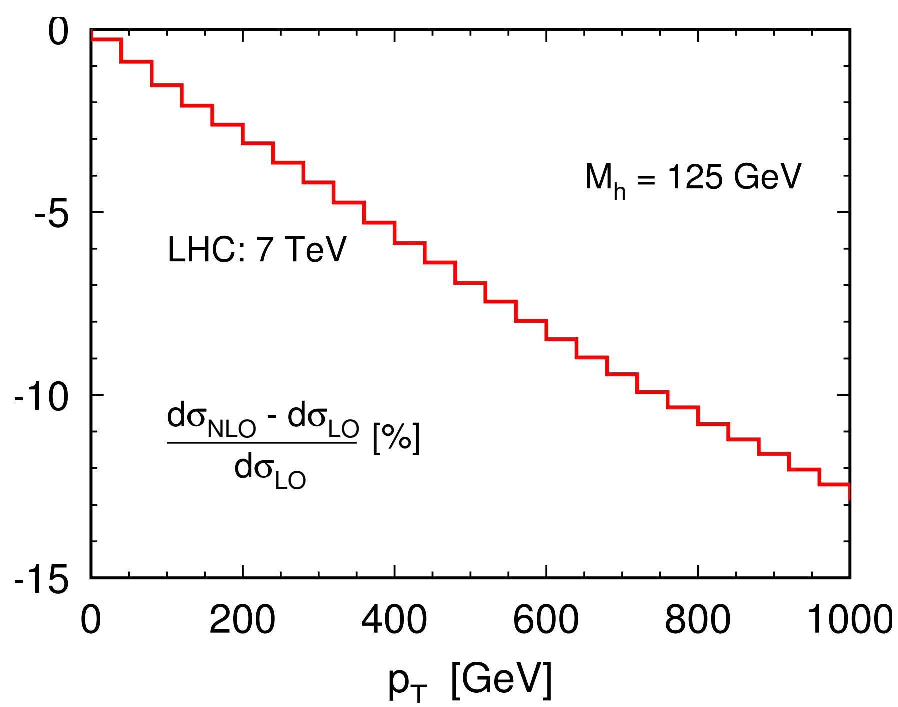
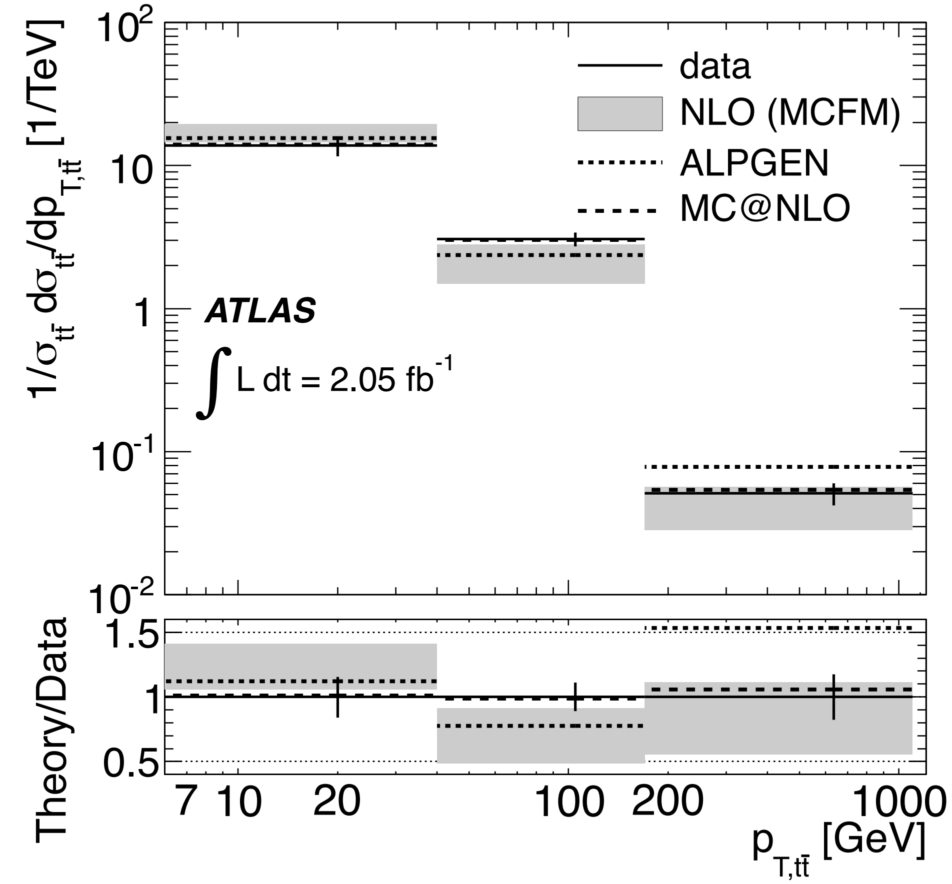
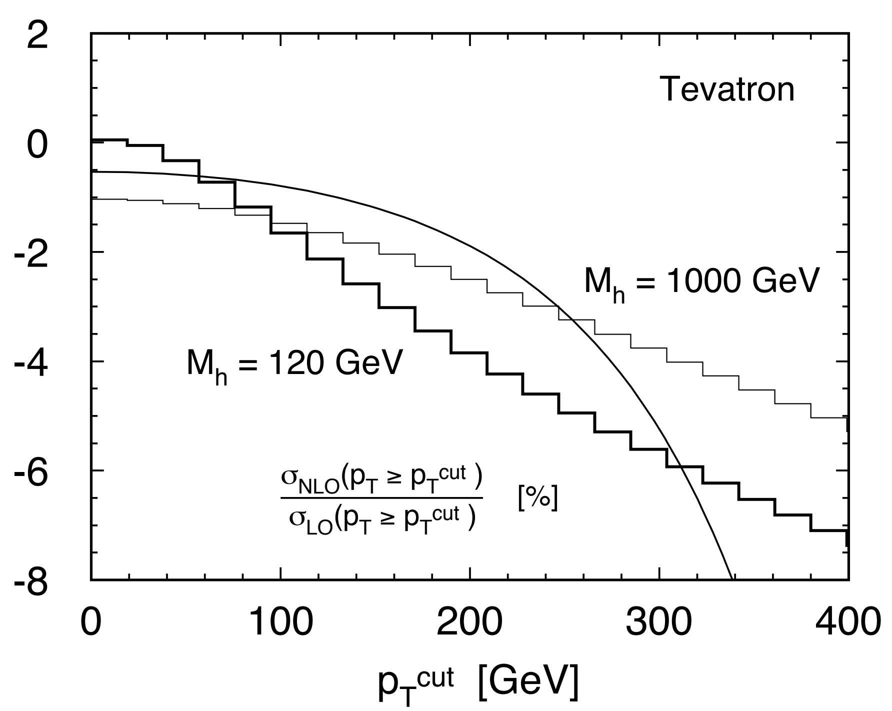
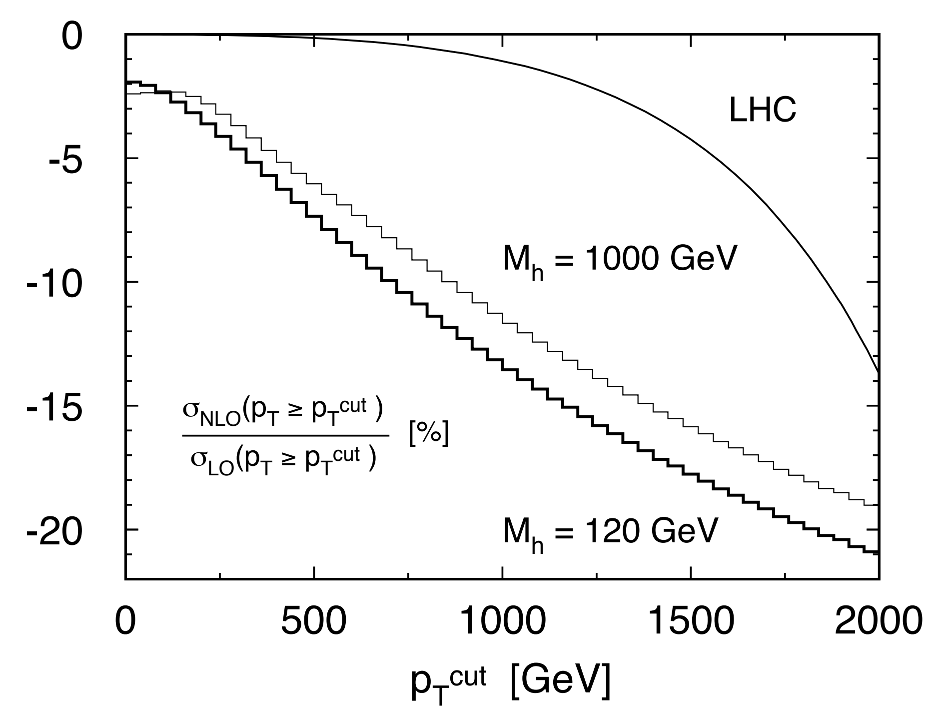


Talk of A.Scharf, CERN, 17-9-2012





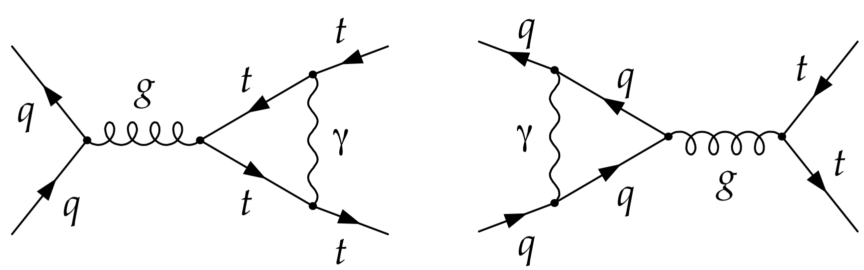


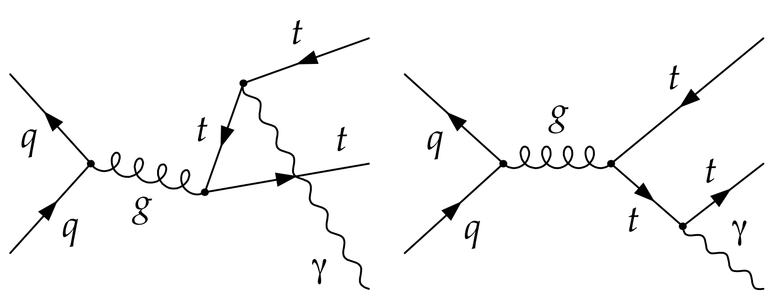
QED corrections to the cross section

Hollik, Kollar '07

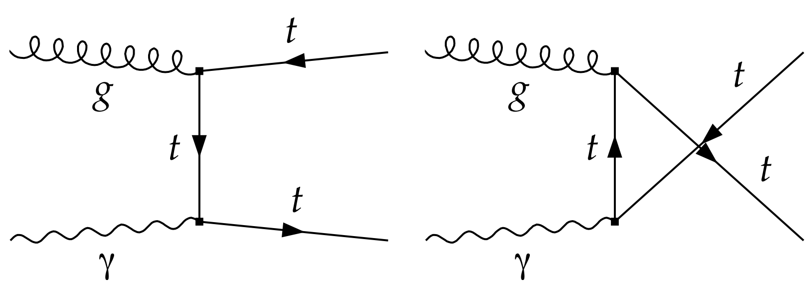
$\mathcal{O}(\alpha_s^2\alpha)$

$q\bar{q} \to t\bar{t}$

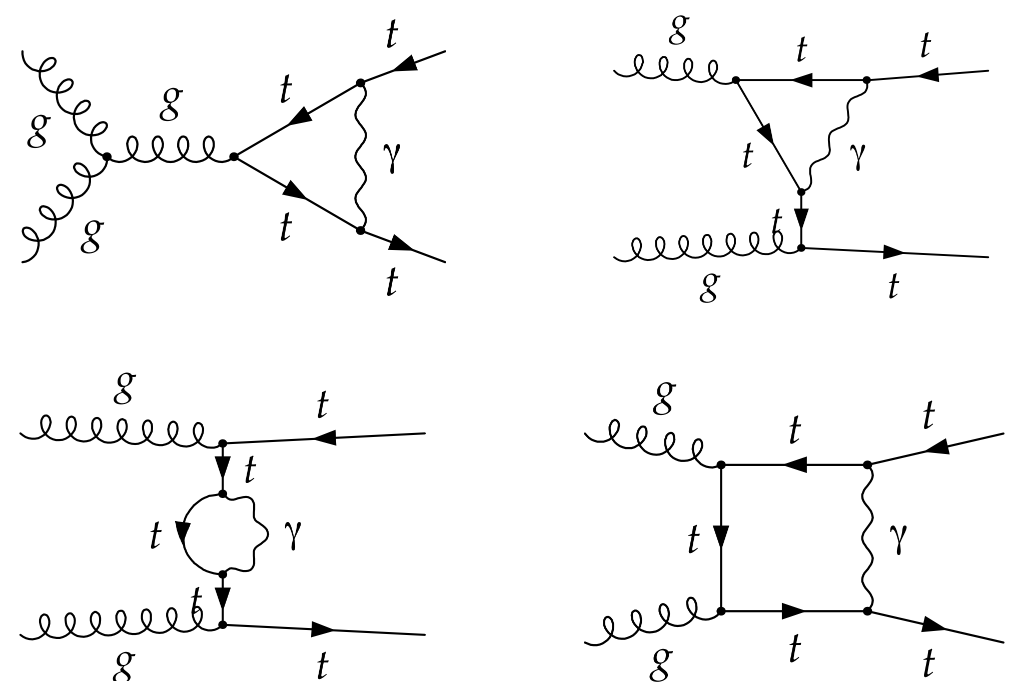


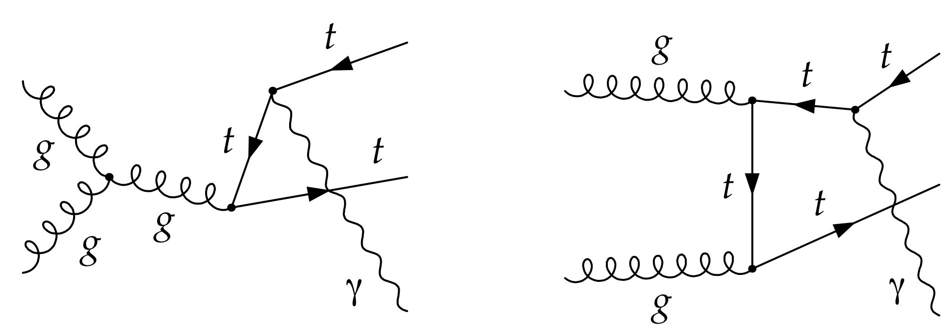


$g\gamma \to t\bar{t}$



$gg \to t\bar{t}$





QED corrections to the cross section

Process	σ_{tot} without cuts [pb]	
	Born	correction
$u\bar{u}$	3.411	-0.117
$d\bar{d}$	0.5855	-2.89×10^{-3}
$s\bar{s}$	8.063×10^{-3}	-1.21×10^{-5}
$c\bar{c}$	2.044×10^{-3}	-5.06×10^{-5}
gg	0.4128	3.17×10^{-3}
$g\gamma$		0.0154
$p\bar{p}$	4.420	-0.102

Tevatron

~ -2%

Process	σ_{tot} without cuts [pb]	
	Born	correction
$u\bar{u}$	34.25	-1.41
$d\bar{d}$	21.61	-0.228
$s\bar{s}$	4.682	-0.0410
$c\bar{c}$	2.075	-0.0762
gg	407.8	2.08
$g\gamma$		4.45
pp	470.4	4.78

LHC 14 TeV

~ 1%

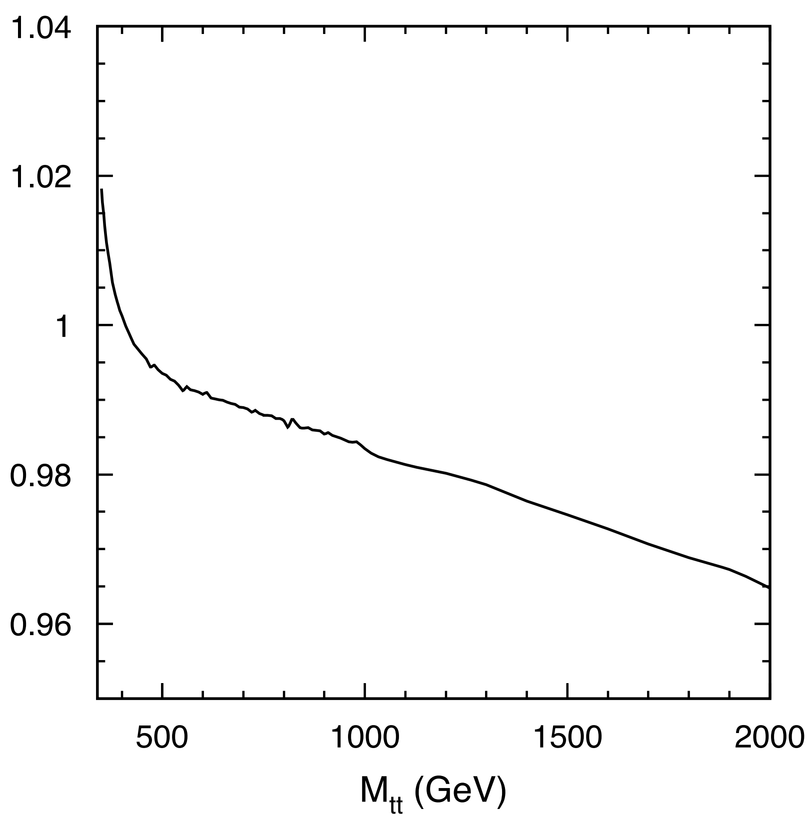
Hollik, Kollar '07

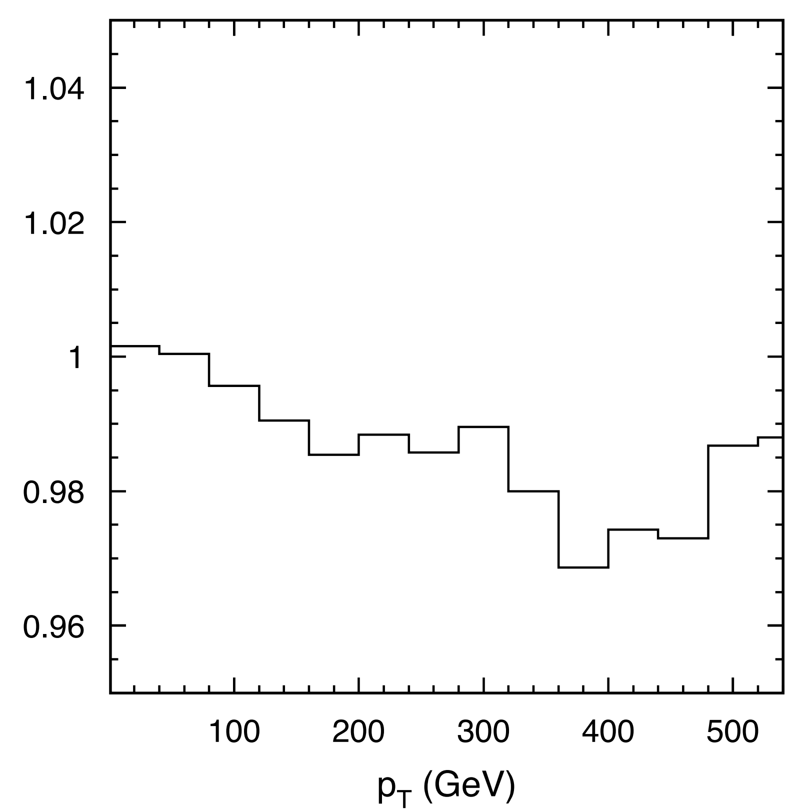
The dominant contribution at LHC comes from $g\gamma \to t\bar{t}$, only one set of PDFs (MRST2004QED) has photon PDF.

Electroweak corrections to the cross section

$\alpha_s^2\alpha$	α^2	$q\bar{q} \to t\bar{t}$
	$\alpha_s^2\alpha$	$g\gamma \to t\bar{t}$ $\quad gg \to t\bar{t}$
$\alpha_s\alpha$	α^2	$b\bar{b} \to t\bar{t}$
$\alpha_s\alpha^2$	$\alpha_s^2\alpha$	$gq\,(\bar{q}) \to t\bar{t}q\,(\bar{q}) \qquad (q = u,d,s,c,b)$

Bernreuther, Si '10





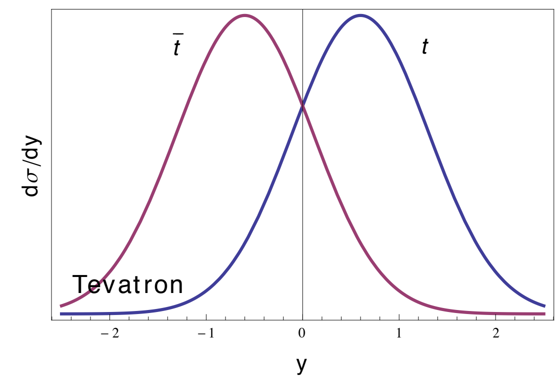
Ratio $(\mathrm{NLO}_{QCD}+\mathrm{electroweak})/\mathrm{NLO}_{QCD}$

Asymmetries

Tevatron

$$A_{t\bar{t}} = \frac{N(y_t > y_{\bar{t}}) - N(y_{\bar{t}} > y_t)}{N(y_t > y_{\bar{t}}) + N(y_{\bar{t}} > y_t)}$$

$$A_{\mathrm{lab}} = \frac{N(y_t > 0) - N(y_t < 0)}{N(y_t > 0) + N(y_t < 0)}$$

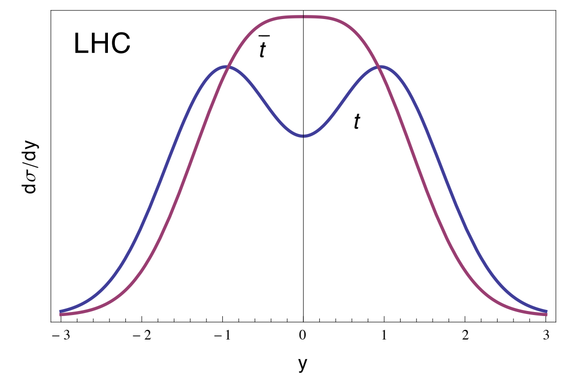


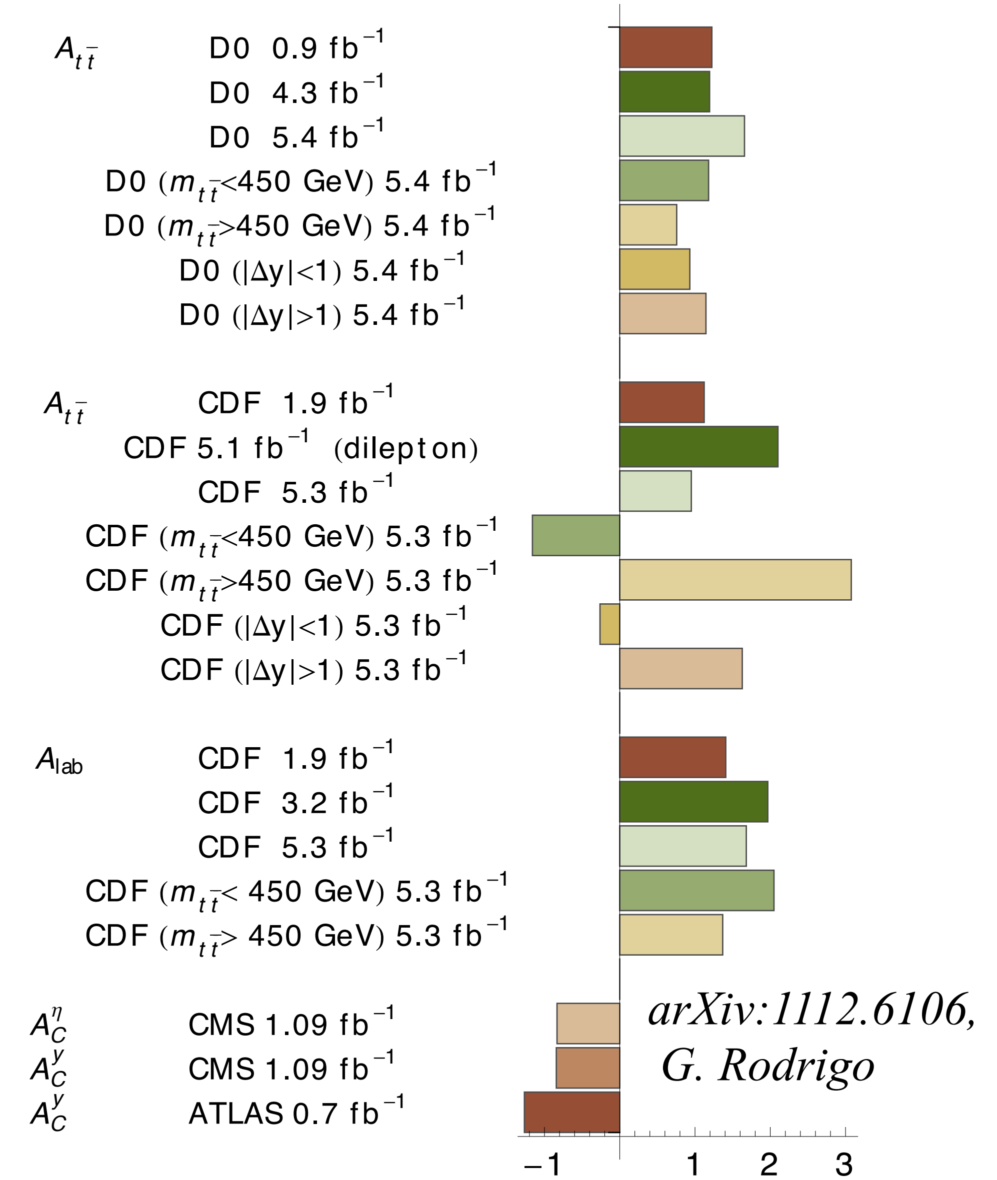
LHC

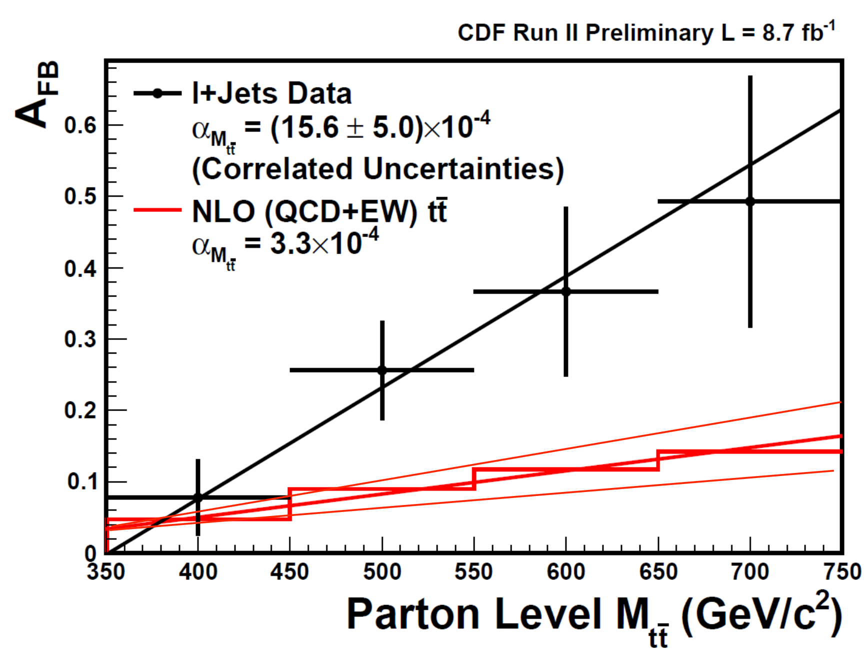
$$A_C^{\eta} = \frac{N(\Delta_\eta > 0) - N(\Delta_\eta < 0)}{N(\Delta_\eta > 0) + N(\Delta_\eta < 0)}$$

$$A_C^{y} = \frac{N(\Delta_y > 0) - N(\Delta_y < 0)}{N(\Delta_y > 0) + N(\Delta_y < 0)}$$

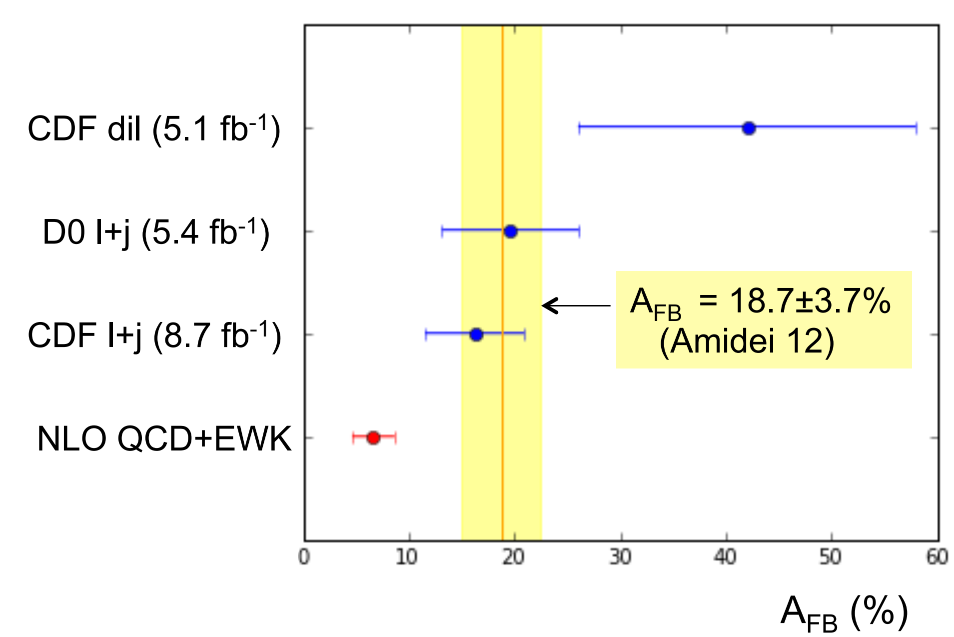
$$\Delta_\eta = |\eta_t| - |\eta_{\bar{t}}| \qquad \Delta_y = |y_t| - |y_{\bar{t}}|$$

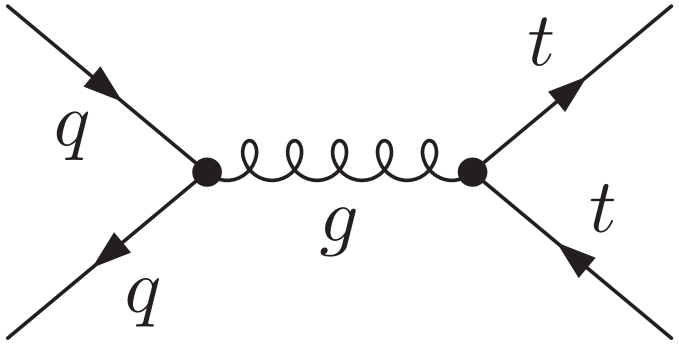






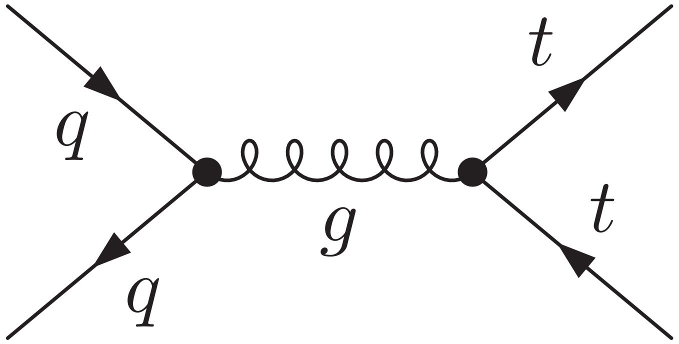
Many different measurements: 3σ deviations in high invariant mass region for CDF.





At LO partonic processes are not asymmetric.
QCD produces the asymmetry only at NLO!
NLO in the cross-section, LO in A_{FB}

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \cdots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \cdots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$



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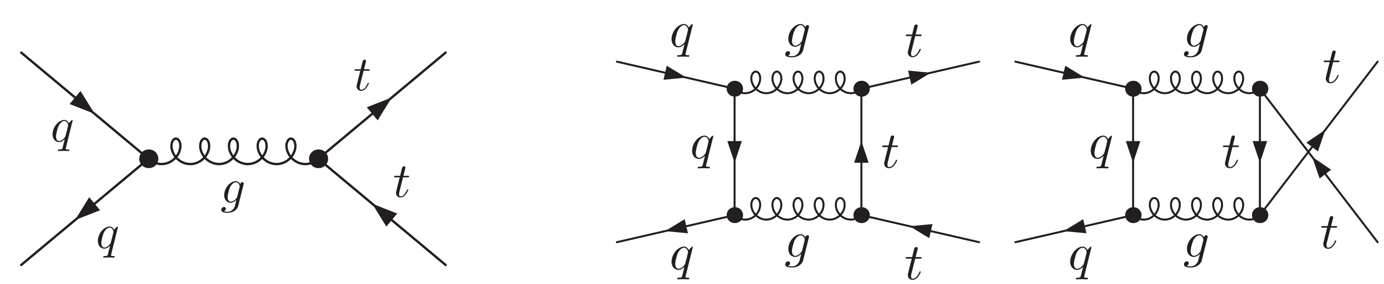
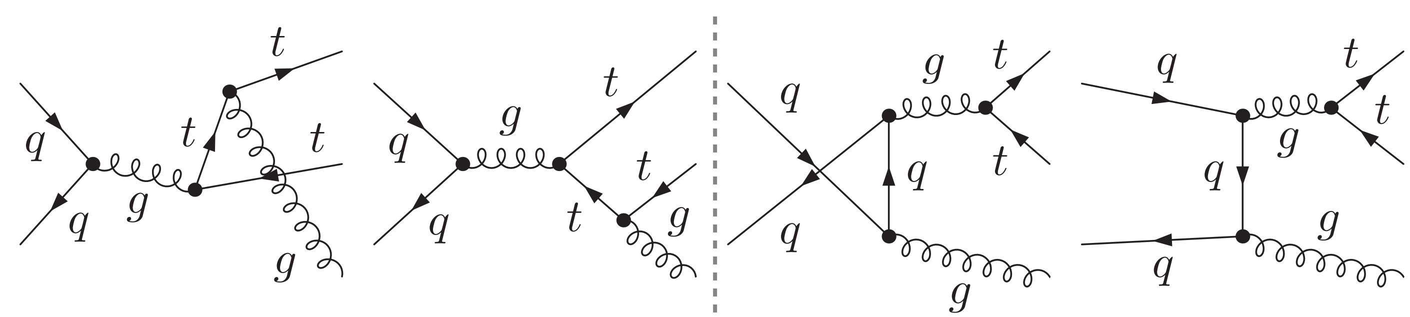
gg initial state doesn't contribute to Tevatron and LHC asymmetry numerator!
q-qbar QCD contribution only from interaction between initial and final state!

$$\alpha_s \frac{N_1}{D_0}$$

VIRTUAL (Only Boxes)
NO UV, NO Coll. Div.
Only IR



REAL
Only interference of initial and final gluon emission is asymmetric.



Final

Initial

Kuhn, Rodrigo '99

$\alpha \frac{\tilde{N}_1}{D_0}$

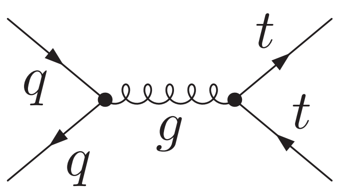
It's useful to divide electroweak contribution into QED (photon) and weak (Z) part.

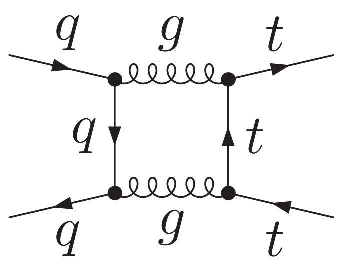
QED

QED can be easily obtained from QCD calculation and the substitution of one gluon into one photon in the squared amplitudes.

$q\bar{q} \to t\bar{t}$

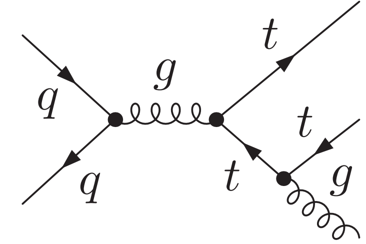
$\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha_s^3)}$

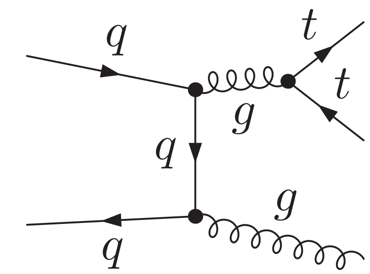




$q\bar{q} \to t\bar{t}g$

$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^3)}$





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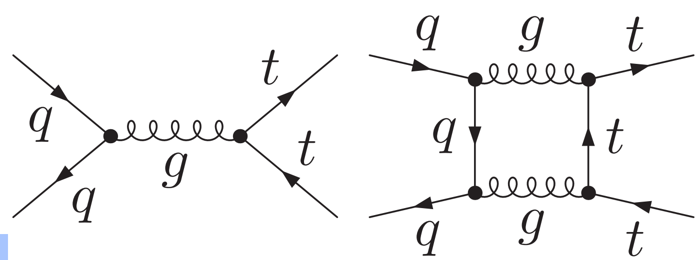
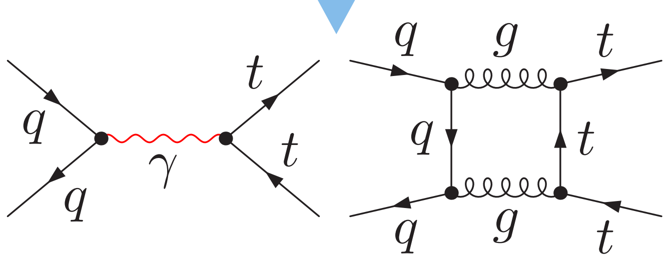
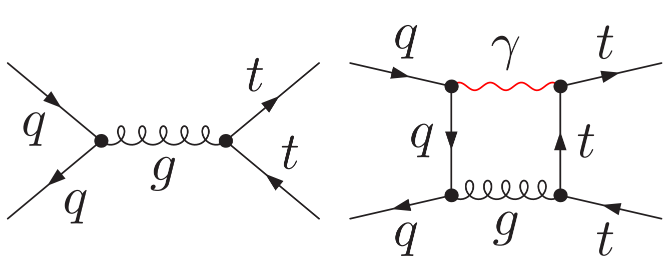
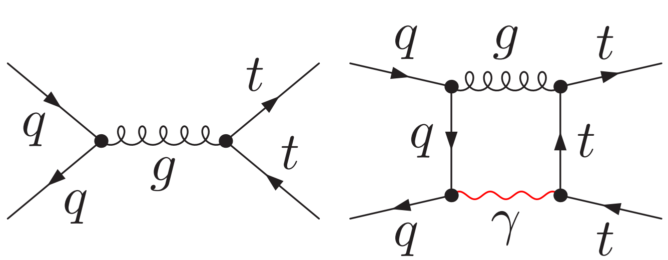


#(QED diagrams)
=
3 #(QCD diagrams)



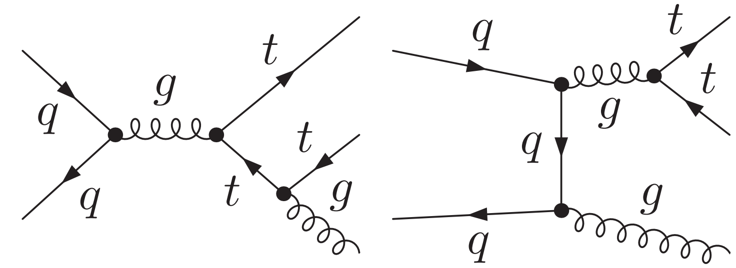
$\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$



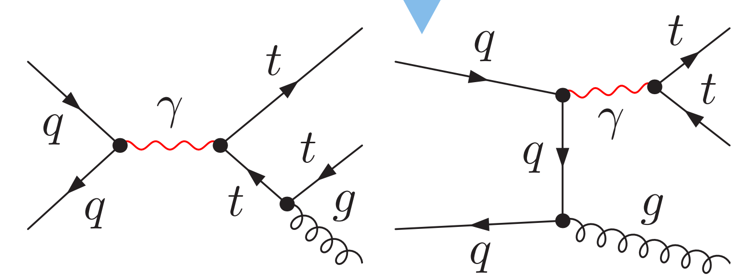


$q\bar{q} \to t\bar{t}g$

$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^3)}$

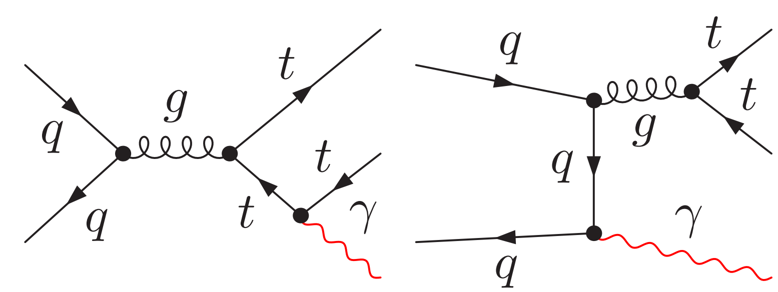


$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$



$q\bar{q} \to t\bar{t}\gamma$

$\overline{|\mathcal{M}^{t\bar{t}\gamma}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$



DIFFERENCES:
Only couplings and color factor!

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \cdots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \cdots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

Hollik, D.P. '11

$$R_{QED}(Q_q) = \frac{\alpha \tilde{N}_1^{QED}}{\alpha_s N_1} = Q_q Q_t \frac{36}{5} \frac{\alpha}{\alpha_s}$$

QED correction can be obtained from QCD × R_{QED}

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \cdots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \cdots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

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QED correction can be obtained from QCD × R_{QED}

Weak

The same diagrams as QED part, but $\gamma \rightarrow Z$.

Z is not massless → If we write Weak=QCD × R_{Weak}.
R_{Weak} does not depend only on couplings and color factor

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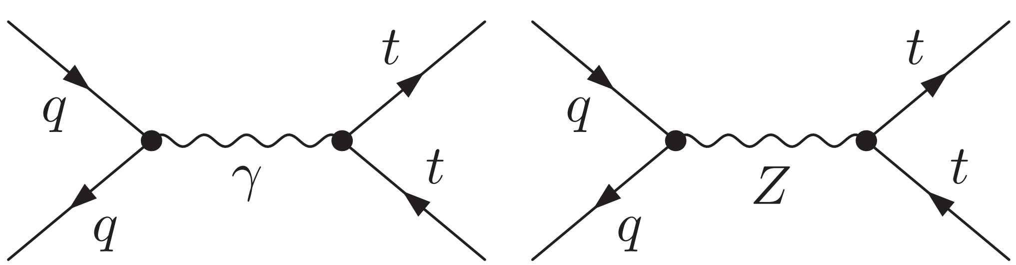
$$\frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$



Different couplings for different chiralities produce asymmetric terms in the cross-section

$$\frac{d\sigma_{asym}}{d\cos\theta} = 2\pi\alpha^2 \cos\theta \left(1 - \frac{4m_t^2}{s}\right) \left[\kappa \frac{Q_q Q_t A_q A_t}{(s - M_Z^2)} + 2\kappa^2 A_q A_t V_q V_t \frac{s}{(s - M_Z^2)^2}\right]$$

Forward-backward asymmetry

(a) $A_{FB}^{t\bar{t}}$

$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s^3)$	$u\bar{u}$	7.01%	6.29%	5.71%
$\mathcal{O}(\alpha_s^3)$	$d\bar{d}$	1.16%	1.03%	0.92%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$u\bar{u}$	1.35%	1.35%	1.35%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$d\bar{d}$	-0.11%	-0.11%	-0.11%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$u\bar{u}$	0.16%	0.16%	0.16%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$d\bar{d}$	-0.04%	-0.04%	-0.04%
$\mathcal{O}(\alpha^2)$	$u\bar{u}$	0.18%	0.23%	0.28%
$\mathcal{O}(\alpha^2)$	$d\bar{d}$	0.02%	0.03%	0.03%
tot	$p\bar{p}$	9.72%	8.93%	8.31%

$R_{QED}^{u\bar{u}} = (0.192, 0.214, 0.237)$

$R_{QED}^{d\bar{d}} = (-0.096, -0.107, -0.119)$

- R_{QED} depend only on the renormalization scale, not on A_{FB} definitions and cuts

$\mathcal{O}(\alpha_s^2\alpha)$ QED is the dominant contribution of the electroweak corrections. It is stable under factorization and renormalization scale variation.

u and d have different charges: contributions of opposite sign for $\mathcal{O}(\alpha_s^2\alpha)$

Forward-backward asymmetry

$$R_{EW}^{t\bar{t}} = \frac{N^{t\bar{t}}_{\mathcal{O}(\alpha_s^2\alpha)+\mathcal{O}(\alpha^2)}}{N^{t\bar{t}}_{\mathcal{O}(\alpha_s^3)}} = (0.190, 0.220, 0.254)$$

$$R_{EW}^{p\bar{p}} = \frac{N^{p\bar{p}}_{\mathcal{O}(\alpha_s^2\alpha)+\mathcal{O}(\alpha^2)}}{N^{p\bar{p}}_{\mathcal{O}(\alpha_s^3)}} = (0.186, 0.218, 0.243)$$

$$R_{EW}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (0.200, 0.232, 0.266)$$

$$R_{EW}^{t\bar{t}}(|\Delta y| > 1) = (0.191, 0.216, 0.246)$$

EW corrections to A_{FB} depends on fac/ren scale, and very slightly on A_{FB} definitions and cuts.

Forward-backward asymmetry

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EW corrections to A_{FB} are more important than EW corrections to total cross-section. 2 Reasons:

-LO total cross section $\mathcal{O}(\alpha_s^2)$, LO numerator of A_{FB} $\mathcal{O}(\alpha_s^3)$

-The dominant EW contribution, $\mathcal{O}(\alpha_s^2\alpha)$ QED, to the A_{FB} comes from boxes: 3 times the number of diagrams of QCD case.
QED contribution to total cross-section comes “from vertex corrections”: same number of diagrams of QCD case.

Charge asymmetry

At LHC same partonic processes, but different partonic luminosities.

Gluon-gluon luminosity is larger, so asymmetry is smaller.
Gluon-quark initial state starts to be "interesting".

The ratio of integrated luminosities $u\bar{u}/d\bar{d}$ at Tevatron is 4:1, at LHC 2:1. Cancellation between QED contributions is bigger. EW contribution at LHC in general is smaller (between 15% and 20% of QCD contribution).

$$R_{QED}(Q_q) = \frac{\alpha \tilde{N}_1^{QED}}{\alpha_s N_1} = Q_q Q_t \frac{36}{5} \frac{\alpha}{\alpha_s}$$

$\sqrt{s}$		$M_c = 2m_t$	0.5 TeV	0.7 TeV	1 TeV
7 TeV	QCD: $A_C^{\Delta\|y\|}$ (%)	1.07 (4)	1.27 (4)	1.68 (4)	2.06 (5)
	QCD + EW: $A_C^{\Delta\|y\|}$ (%)	1.23 (5)	1.48 (4)	1.95 (4)	2.40 (6)
8 TeV	QCD: $A_C^{\Delta\|y\|}$ (%)	0.96 (4)	1.14 (4)	1.48 (4)	1.85 (4)
	QCD + EW: $A_C^{\Delta\|y\|}$ (%)	1.11 (4)	1.33 (5)	1.73 (5)	2.20 (5)
		$M_c = 2m_t$	0.5 TeV	1 TeV	2 TeV
14 TeV	QCD: $A_C^{\Delta\|y\|}$ (%)	0.58 (3)	0.74 (3)	1.11 (5)	1.72 (10)
	QCD + EW: $A_C^{\Delta\|y\|}$ (%)	0.67 (4)	0.86 (5)	1.32 (8)	2.12 (10)

Bernreuther, Si '12

CONCLUSION

The electroweak contribution to total cross-section is still smaller than QCD uncertainty. It could be seen in differential distribution, with high luminosity.

Total electroweak contribution to the asymmetries is not negligible and increases QCD result by a factor ~ 1.2 (Tevatron), ~ 1.15 (LHC)

EW cannot explain $A_{FB}(M_{INV}>450$ GeV), but new models cannot forget its contribution when they try to fill the gap between theory (SM) and experiment.

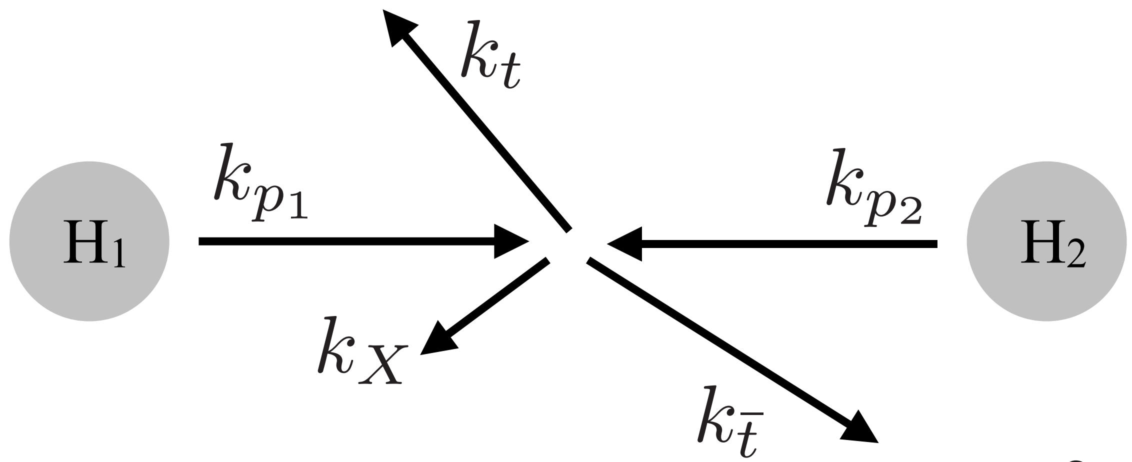
THANK YOU FOR THE ATTENTION!

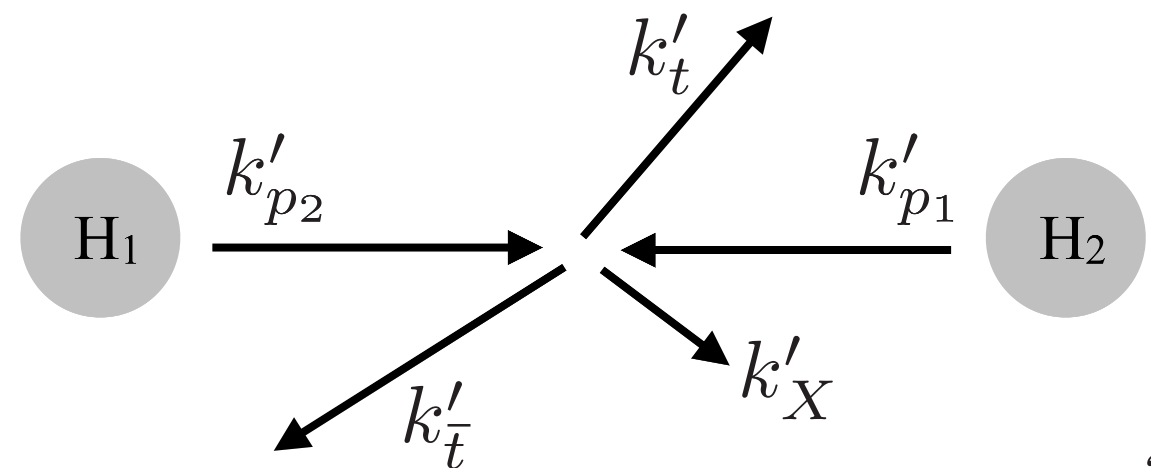
EXTRA SLIDES

Hadronic process = partonic process ⊗ PDF

$$\sigma(H_1 H_2 \to t\bar{t} + X) = \sigma(p_1 p_2 \to t\bar{t} + X) \otimes \left[f_{p_1,H_1}(x_1) f_{p_2,H_2}(x_2) + f_{p_1,H_2}(x_1) f_{p_2,H_1}(x_2) \right]$$

Partonic process can be produced in two different directions





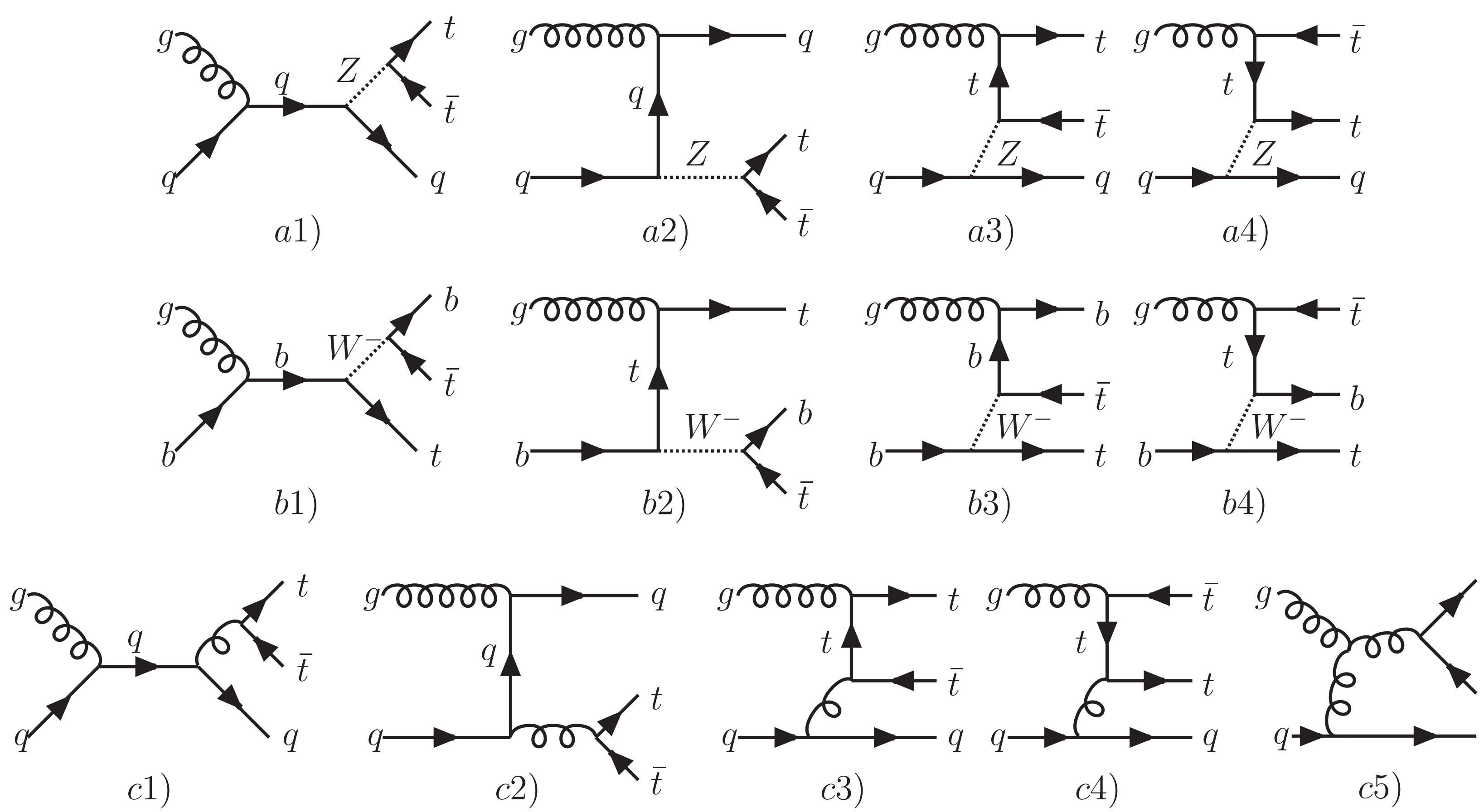
$$\left| \mathcal{M}_{p_1 p_2 \to t\bar{t}+X}(k_{p_1}, k_{p_2}, k_t, k_{\bar{t}}, k_X) \right|^2 = \left| \mathcal{M}_{p_1 p_2 \to t\bar{t}+X}(k'_{p_1}, k'_{p_2}, k'_t, k'_{\bar{t}}, k'_X) \right|^2$$

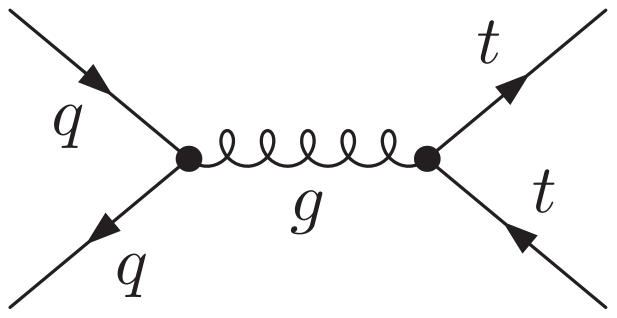
$A_{FB} \neq 0$ ⟶ $f_{p_1,H_1}(x_1) f_{p_2,H_2}(x_2) \neq f_{p_1,H_2}(x_1) f_{p_2,H_1}(x_2)$

At LHC $H_1 = H_2 \longrightarrow A_{FB} = 0$

At Tevatron only processes with p_1 or p_2 = (up, antiup, down, antidown) can produce asymmetric terms!

Diagrams contributing to orders $\alpha_s\alpha^2$, $\alpha_s^2\alpha$ of $gq\,(\bar{q}) \to t\bar{t}q\,(\bar{q})$ $\quad (q = u, d, s, c, b)$





At LO partonic processes are not asymmetric.
QCD produces the asymmetry only at NLO!
NLO in the cross-section, LO in A_{FB}

ONLY LO IS AVAILABLE! ← NOT AVAILABLE, NOT TRIVIAL

$$A_{FB} = \frac{N}{D} = \frac{\alpha_s^3 N_1 + \alpha_s^4 N_2 + \cdots}{\alpha_s^2 D_0 + \alpha_s^3 D_1 + \cdots} = \frac{\alpha_s}{D_0}(N_1 + \alpha_s(N_2 - N_1 D_1/D_0)) + \cdots$$

QCD only at LO, but there is also electroweak theory. $\mathcal{O}(\alpha_s \alpha) = 0$

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \cdots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \cdots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

$\alpha_s^2 D_0$ is the LO cross section, now we see the terms in N

(a) $A_{FB}^{t\bar{t}}$

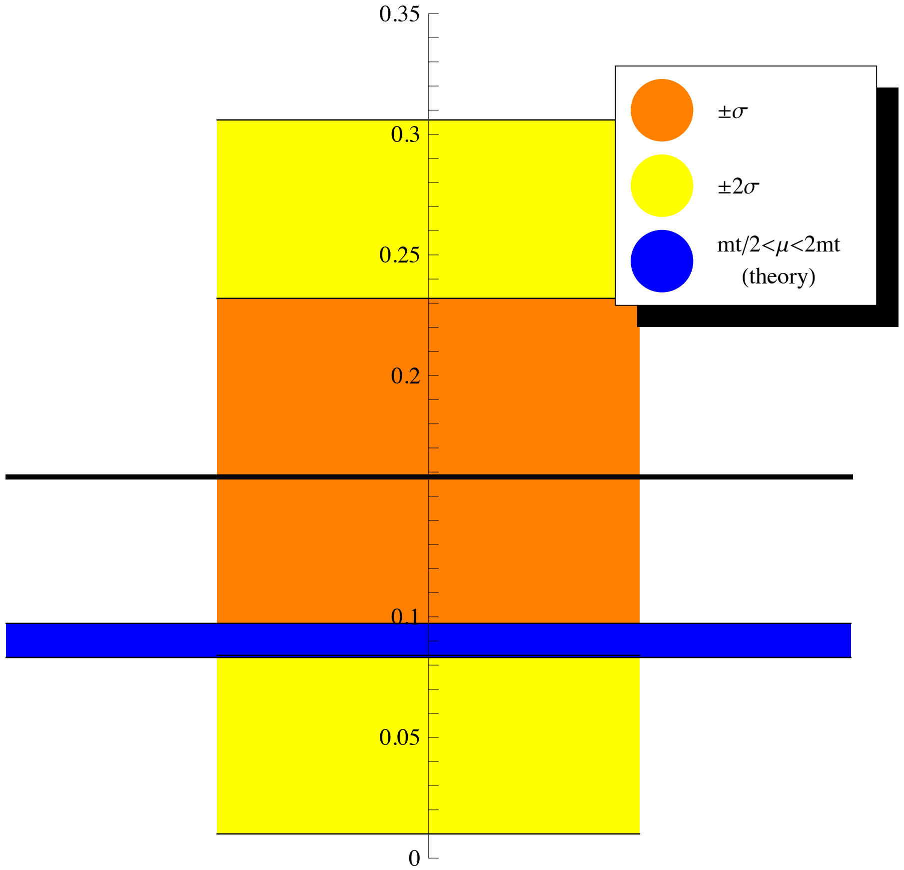
$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s)$	$u\bar{u}$	7.01%	6.29%	5.71%
$\mathcal{O}(\alpha_s)$	$d\bar{d}$	1.16%	1.03%	0.92%
$\mathcal{O}(\alpha)_{QED}$	$u\bar{u}$	1.35%	1.35%	1.35%
$\mathcal{O}(\alpha)_{QED}$	$d\bar{d}$	-0.11%	-0.11%	-0.11%
$\mathcal{O}(\alpha)_{weak}$	$u\bar{u}$	0.16%	0.16%	0.16%
$\mathcal{O}(\alpha)_{weak}$	$d\bar{d}$	-0.04%	-0.04%	-0.04%
$\mathcal{O}(\alpha^2/\alpha_s^2)$	$u\bar{u}$	0.18%	0.23%	0.28%
$\mathcal{O}(\alpha^2/\alpha_s^2)$	$d\bar{d}$	0.02%	0.03%	0.03%
tot	$p\bar{p}$	9.72%	8.93%	8.31%

(b) $A_{FB}^{p\bar{p}}$

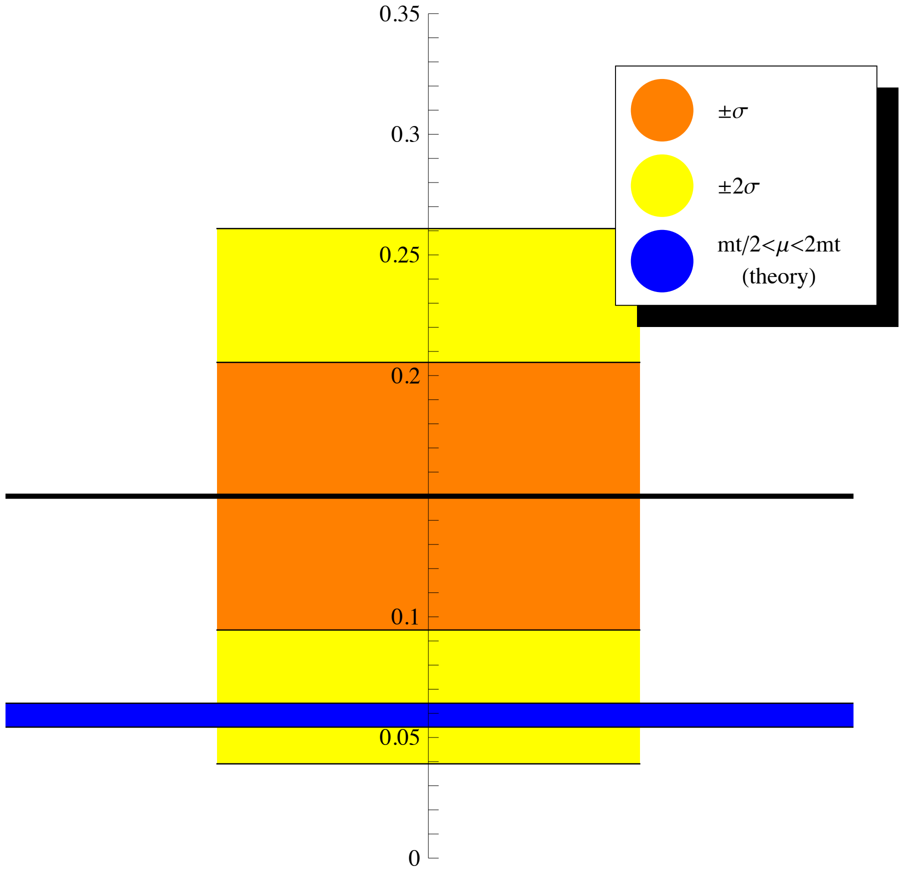
$A_{FB}^{p\bar{p}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s)$	$u\bar{u}$	4.66%	4.19%	3.78%
$\mathcal{O}(\alpha_s)$	$d\bar{d}$	0.75%	0.66%	0.59%
$\mathcal{O}(\alpha)_{QED}$	$u\bar{u}$	0.90%	0.90%	0.90%
$\mathcal{O}(\alpha)_{QED}$	$d\bar{d}$	-0.07%	-0.07%	-0.07%
$\mathcal{O}(\alpha)_{weak}$	$u\bar{u}$	0.10%	0.10%	0.10%
$\mathcal{O}(\alpha)_{weak}$	$d\bar{d}$	-0.03%	-0.03%	-0.03%
$\mathcal{O}(\alpha^2/\alpha_s^2)$	$u\bar{u}$	0.11%	0.14%	0.17%
$\mathcal{O}(\alpha^2/\alpha_s^2)$	$d\bar{d}$	0.01%	0.02%	0.02%
tot	$p\bar{p}$	6.42%	5.92%	5.43%

$$R_{QED}^{u\bar{u}} = (0.192, 0.214, 0.237)$$

$$R_{QED}^{d\bar{d}} = (-0.096, -0.107, -0.119)$$



(a) $A_{FB}^{t\bar{t}}$



(b) $A_{FB}^{p\bar{p}}$

a) at 1σ
b)inside 2σ

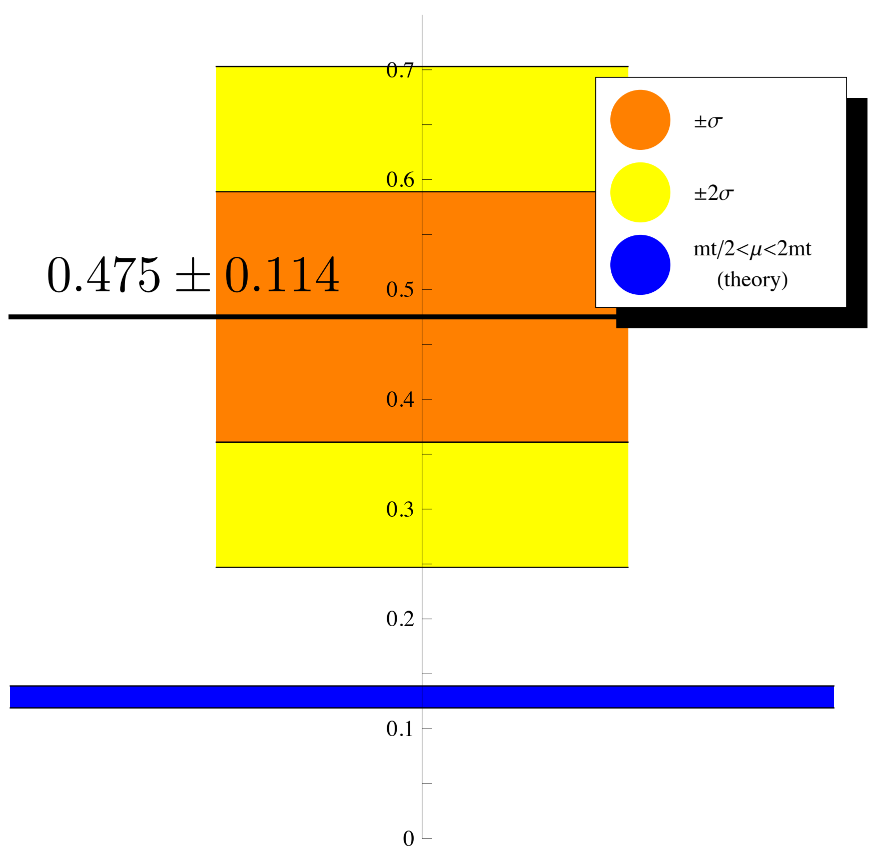
$$\frac{(A_{FB}^{t\bar{t}})^{EW}}{(A_{FB}^{t\bar{t}})^{QCD}} = (0.190, 0.220, 0.254) \qquad \frac{(A_{FB}^{p\bar{p}})^{EW}}{(A_{FB}^{p\bar{p}})^{QCD}} = (0.186, 0.218, 0.243)$$

(a) $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV})$

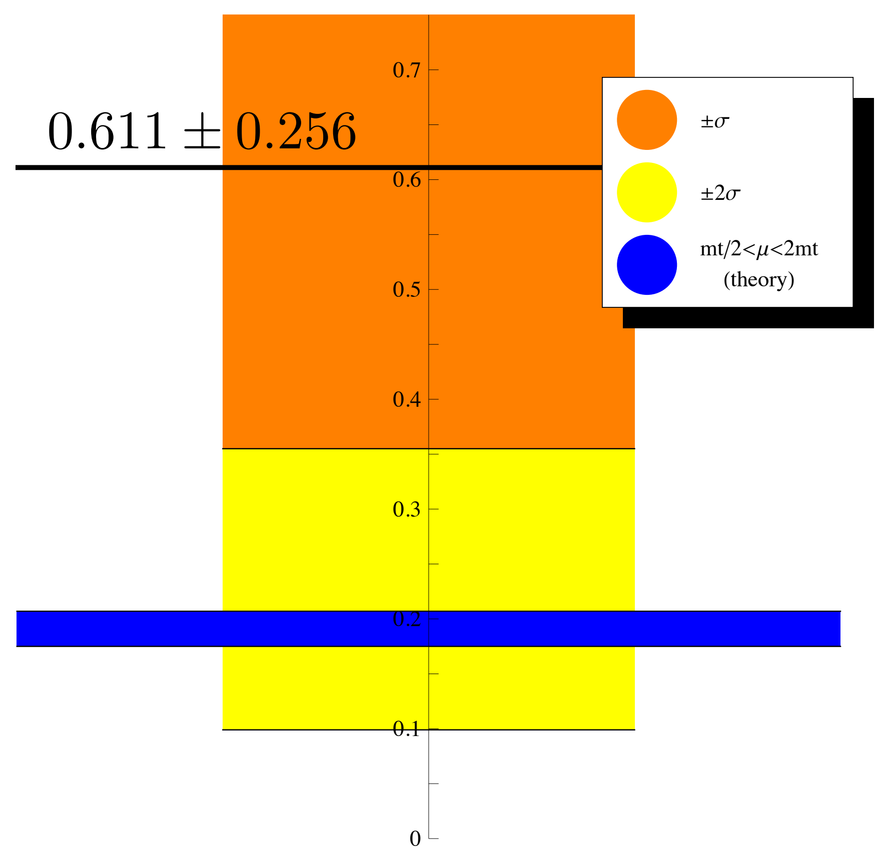
$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s^3)$	$u\bar{u}$	10.13%	9.10%	8.27%
$\mathcal{O}(\alpha_s^3)$	$d\bar{d}$	1.44%	1.27%	1.14%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$u\bar{u}$	1.94%	1.95%	1.96%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$d\bar{d}$	-0.14%	-0.14%	-0.14%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$u\bar{u}$	0.28%	0.28%	0.28%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$d\bar{d}$	-0.05%	-0.05%	-0.05%
$\mathcal{O}(\alpha^2)$	$u\bar{u}$	0.26%	0.33%	0.41%
$\mathcal{O}(\alpha^2)$	$d\bar{d}$	0.03%	0.03%	0.04%
tot	$p\bar{p}$	13.90%	12.77%	11.91%

(b) $A_{FB}^{t\bar{t}}(|\Delta y| > 1)$

$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s^3)$	$u\bar{u}$	15.11%	13.72%	12.41%
$\mathcal{O}(\alpha_s^3)$	$d\bar{d}$	2.28%	2.02%	1.84%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$u\bar{u}$	2.90%	2.94%	2.94%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$d\bar{d}$	-0.22%	-0.22%	-0.22%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$u\bar{u}$	0.25%	0.25%	0.26%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$d\bar{d}$	-0.09%	-0.09%	-0.08%
$\mathcal{O}(\alpha^2)$	$u\bar{u}$	0.35%	0.45%	0.55%
$\mathcal{O}(\alpha^2)$	$d\bar{d}$	0.04%	0.05%	0.06%
tot	$p\bar{p}$	20.70%	19.12%	17.75%



(a) $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV})$



(b) $A_{FB}^{t\bar{t}}(|\Delta y| > 1)$