Theory of Rare Kaon Decays

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- Minimal Flavour violation
- Short distance dominated $K \to \pi\bar{\nu}\nu$
- $K_L \to \pi^0 e^+ e^-$, the related channels $K \to \pi\gamma\gamma$ and $K_S \to \pi^0 e^+ e^-$
- CP violation in $K \to \pi\pi\gamma$, $K \to \pi\pi ee$
- Conclusions

Flavour Problem

- the SM Yukawa structure

$$\mathcal{L}^{Y}_{SM} \;=\; \bar{Q} Y_D D H + \bar{Q} Y_U U H_c + \bar{L} Y_E E H \;+\; \text{h.c.}$$

FCNC

$$\mathcal{H}^{SM}_{\Delta F=2} \sim \frac{G_F^2 M_W^2}{16\pi^2}\left[\frac{(V^*_{td} m_t^2 V_{tb})^2}{v^4}(\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m_t^2 V_{ts})^2}{v^4}(\bar{d}_L \gamma^\mu s_L)^2\right] + \text{charm}$$

- Supersymmetry must be broken

$$-\mathcal{L}_{soft} = \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \bar{\tilde{U}} a_u \tilde{Q}\; H_u + \ldots$$

- $m_Q^2, m_L^2, a_u, \ldots$ matrices in flavour space additional (to $Y_{u,d,l}$) non-trivial structures

Theory

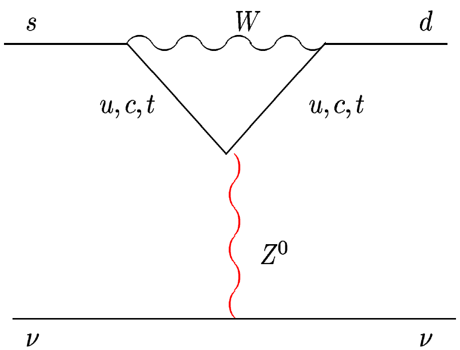
- There is a symmetry that New Physics must obey to satisfy FCNC -constraints

$$G_F = \overbrace{U(3)_Q \otimes U(3)_U \otimes U(3)_D \otimes U(3)_L \otimes U(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

- Technicolour Chivukula, Georgi
- susy Hall,Randall
- Gauge mediation Dine,Nelson, Shirman; Giudice, Rattazzi

$K \to \pi\nu\overline{\nu}$

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \;\times \left[\sum_{q=c,t} V_{qs}^* V_{qd}\, m_q^2\right]$$



$$\sim \qquad \left[A^2\lambda^5\,(1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM: $\underbrace{V-A \otimes V-A}_{\Downarrow}$ Littenberg

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow \; J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

$K^+ \to \pi^+ \nu \overline{\nu}$

Brod,CKM2010, Straub, Gorbhan

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_\mathrm{c}}{\lambda} \left(P_c + \delta P_{c,u} \right) + \frac{\mathrm{Re}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 \right]$$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%± 2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^{\pm}) = (8.22 \pm 0.27 \pm 0.29) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^{\pm}) = \left(1.73^{+1.15}_{-1.05}\right) \times 10^{-10}$

K_L

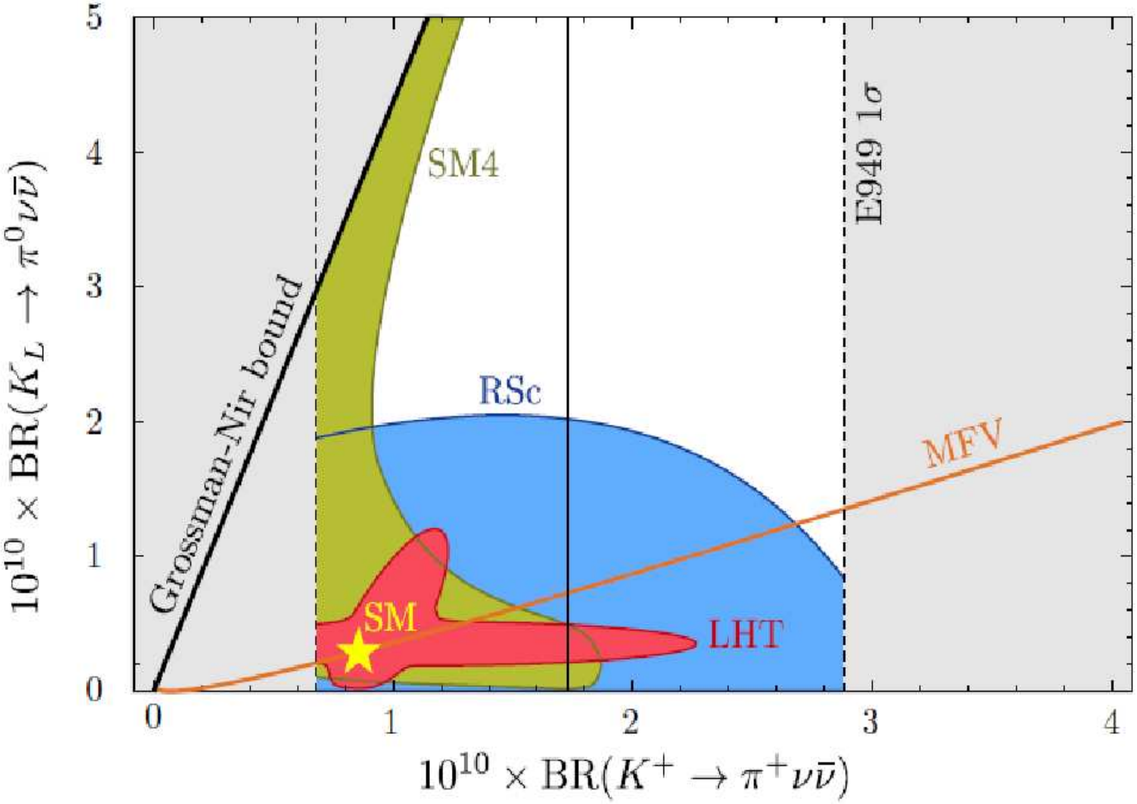
$B(K_L) = (2.43 \pm 0.25 \pm 0.06) \times 10^{-11}$ vs

E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\overline{s}d\overline{\nu}\nu$ Grossman-Nir

$$B(K_L) \le \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\mathrm{E949}} \le 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

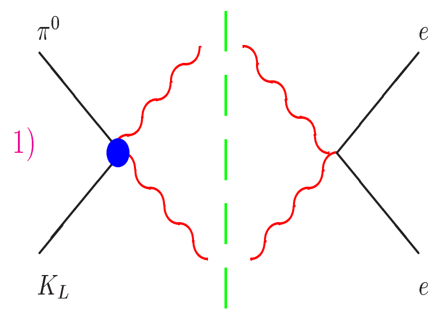
P326 , KOTO, ORKA



Straub, CKM 2010 workshop (arXiv:1012.3893v2)

$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ at 90% CL KTeV

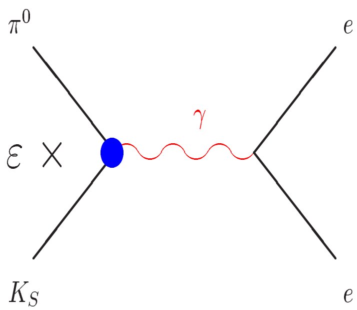


CP conserving NA48

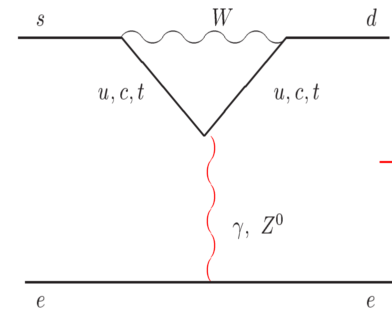
Br$(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon\ \times$



$+$ 3)



$\longrightarrow$ Im λ_t

$\lambda_t = V_{td} V_{ts}^*$

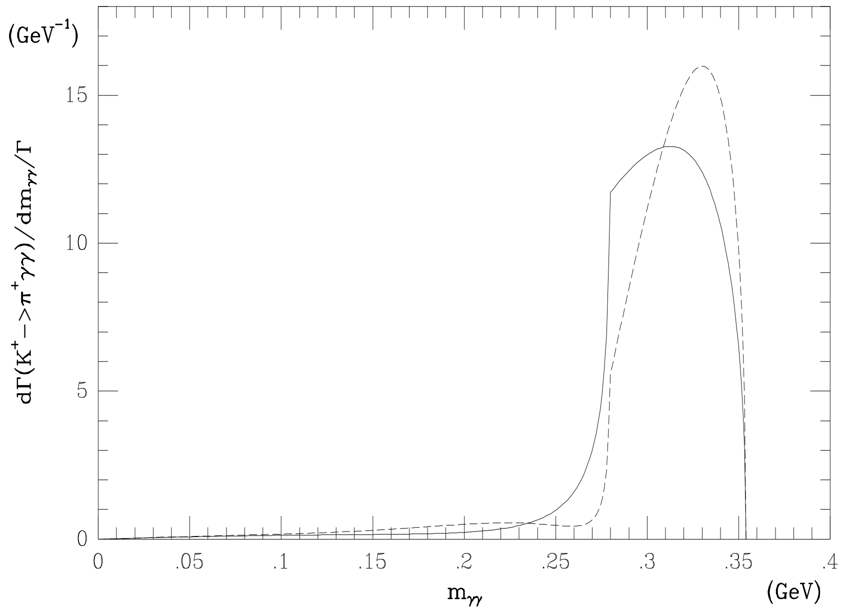
$\uparrow$ $\mathrm{B}(K_s \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\ a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\ a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12}$$

$$[17.7 \pm \qquad 9.5 + \qquad 4.7] \quad \cdot 10^{-12}$$

$K^+ \to \pi^+\gamma\gamma$



	$\hat{c}$
———	0
– – –	-2.3

BNL 787 (96) got 31 events Br$\sim (6 \pm 1.6) \cdot 10^{-7}$ $\hat{c} = 1.8 \pm 0.6$

NA48/2 - NA62 preliminary $B = (1.01 \pm 0.07) \cdot 10^{-6}$ and $\hat{c} = 2.00 \pm 0.3$

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} +$ c $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.44 \pm 0.07)10^{-5}$ PDG	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

$K^+ \to \pi^+\pi^0\gamma$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

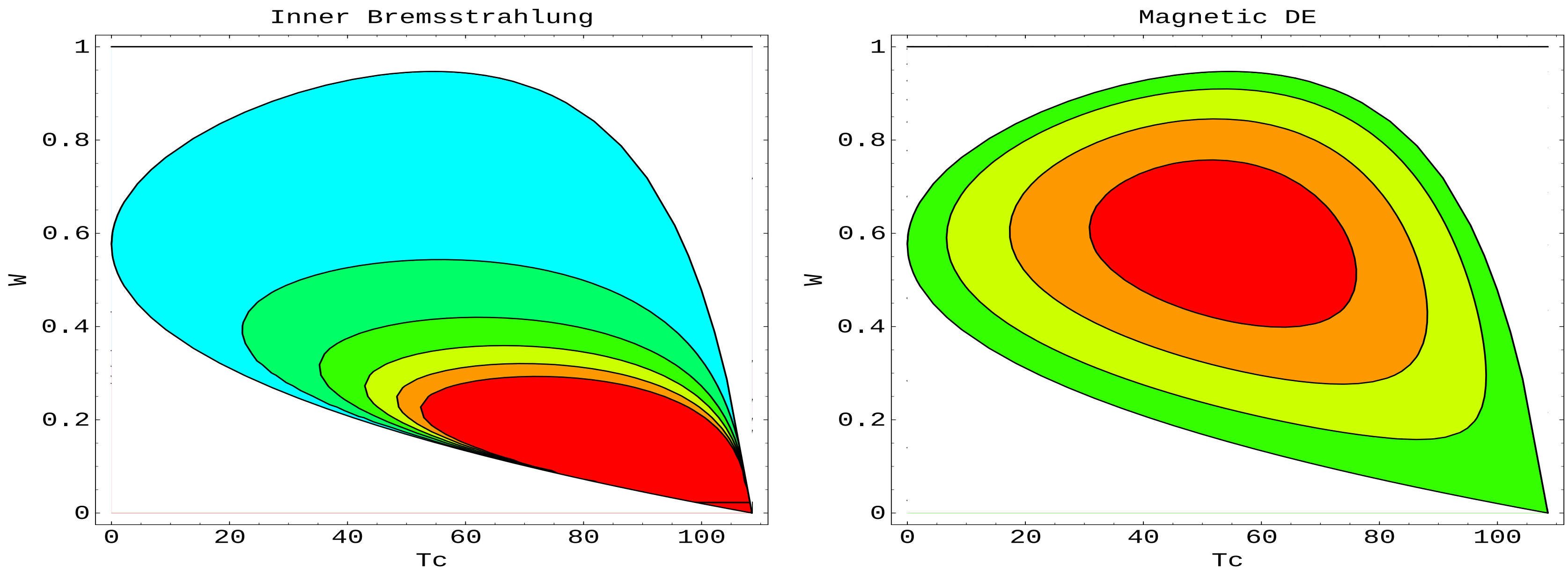
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

$$W^2 = (q\cdot p_K)(q\cdot p_+)/(m_\pi^2 m_K^2)$$

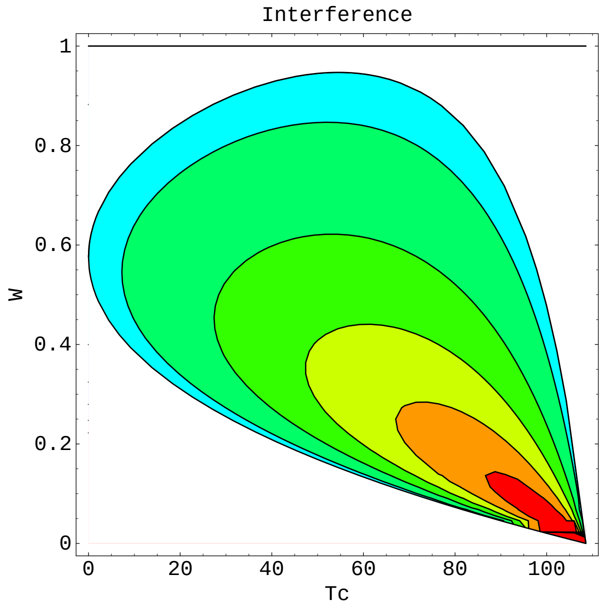
$$A = A(K^+ \to \pi^+\pi^0)$$

$K^+ \to \pi^+\pi^0\gamma\ W - T_c$ **Dalitz plot**

Integrating over T_c deviations from IB measured



CP asymmetry $K^+ \to \pi^+\pi^0\gamma$



Dalitz plot analysis crucial

SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

NP $\leq \mathcal{O}(10^{-4})$ Colangelo et al.

NA48/2 $< 1.5 \cdot 10^{-3}$ at 90% CL

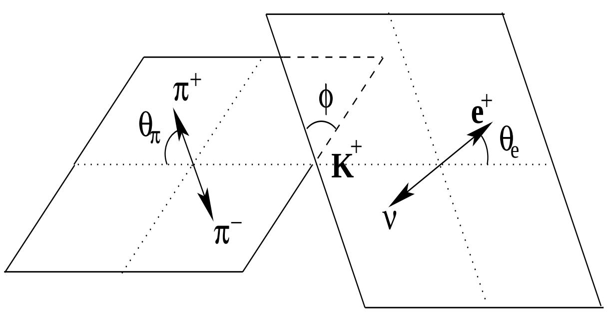
BUT NOT in the interesting interf. kin. region (statistics)

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

Cabibbo Maksymowicz

$$\frac{G_F}{\sqrt{2}} V_{us}\ \bar{e}\gamma^\mu(1-\gamma^5)\nu\ H_\mu(p_1,p_2,q)$$

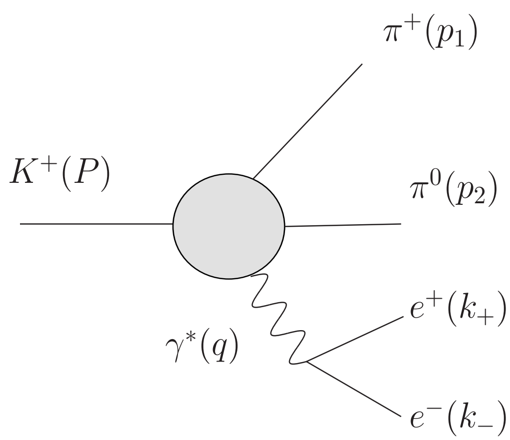
$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta . \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



- crucial to measure $\sin\delta \Longrightarrow$ interf F_3
- Look angular plane asymmetry

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$

Sehgal et al; Savage,Wise et al

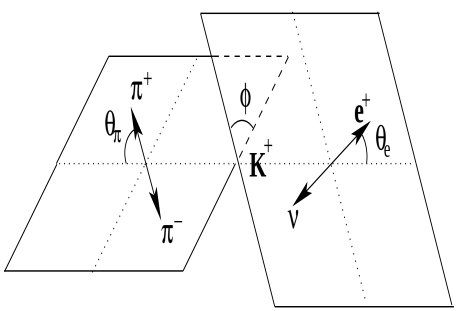


- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\ e\ H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$

$$\frac{d^5\Gamma}{dE^*_\gamma dT^*_c dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi$$
$$+\mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell$$
$$+\mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$

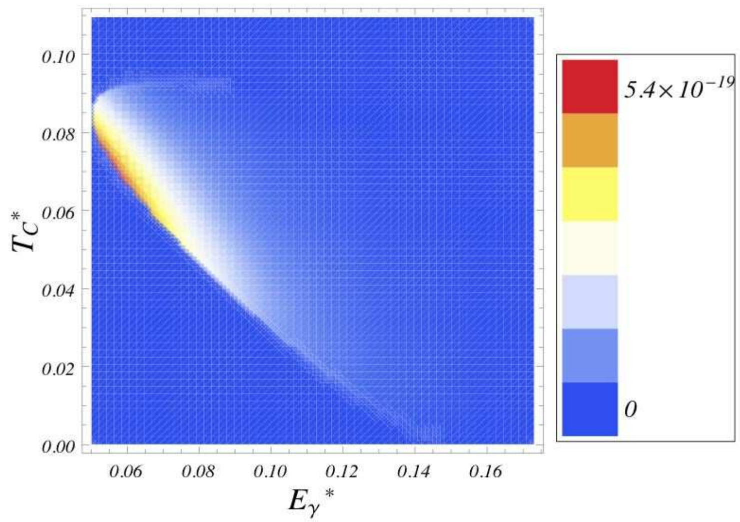


- $\mathcal{A}_{8,9}$, odd in θ_ℓ
- $\frac{\Re(E_B\ M^*)}{|E_B|^2 + |M|^2}$ is maximal,
- $\mathcal{A}_{5,6,7}$ interf. with axial leptonic current

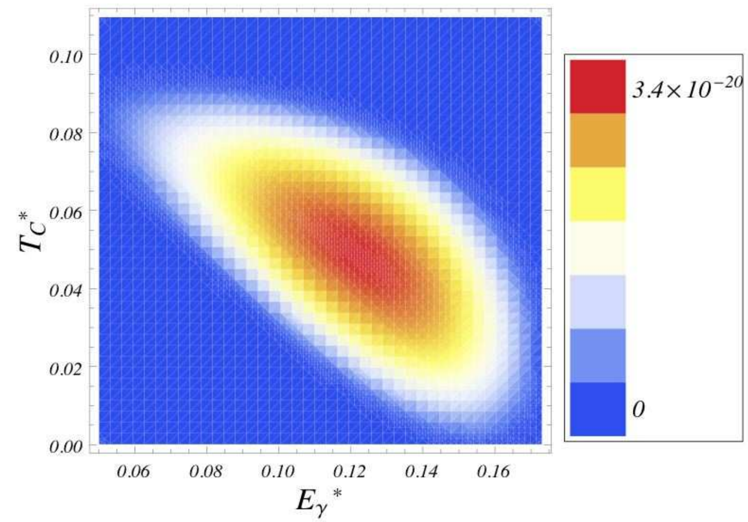
$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+e^-$ **Cappiello, Cata,G.D. and Gao,**

- the asymm. , $\frac{\Re(E_B\ M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B >> M$:
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6}\ \sim\ 50\ B(K^+)_M$
- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



IB



DE

How to extract SD from $K^+ \to \pi^+\pi^0 e^+e^-$

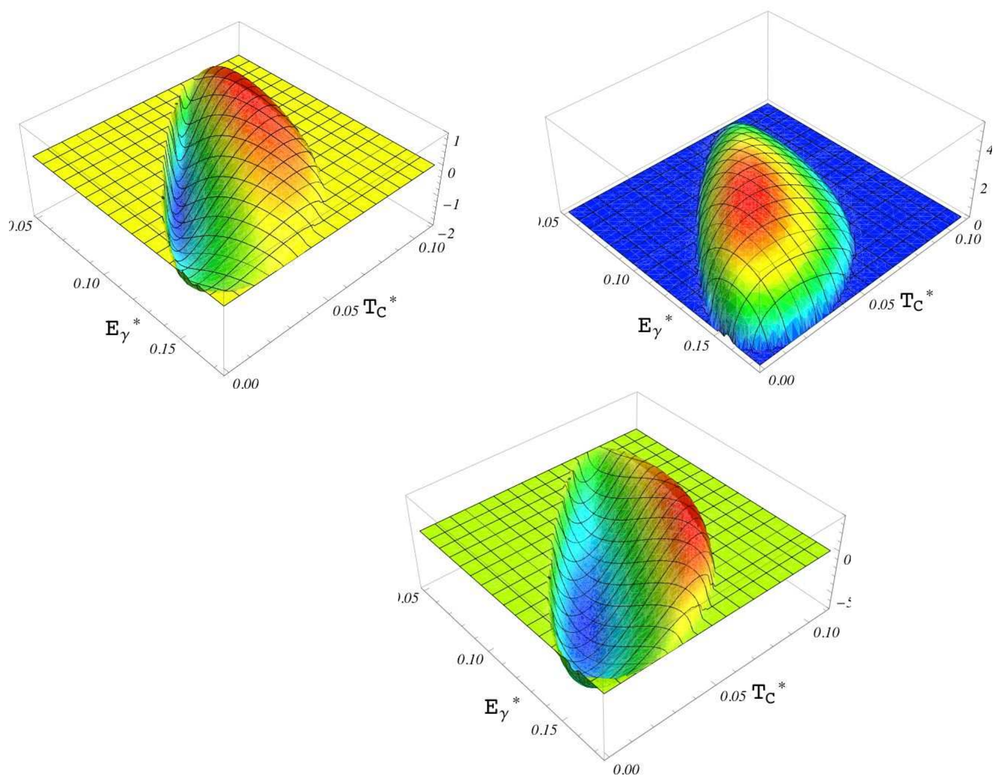
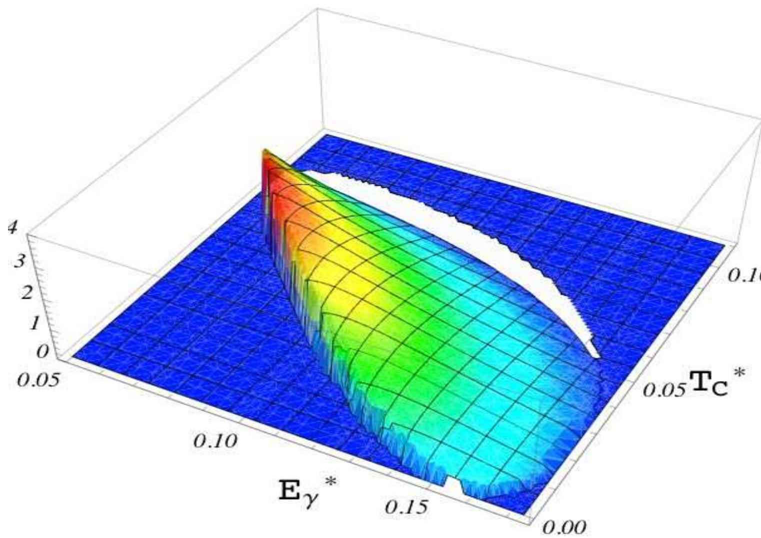
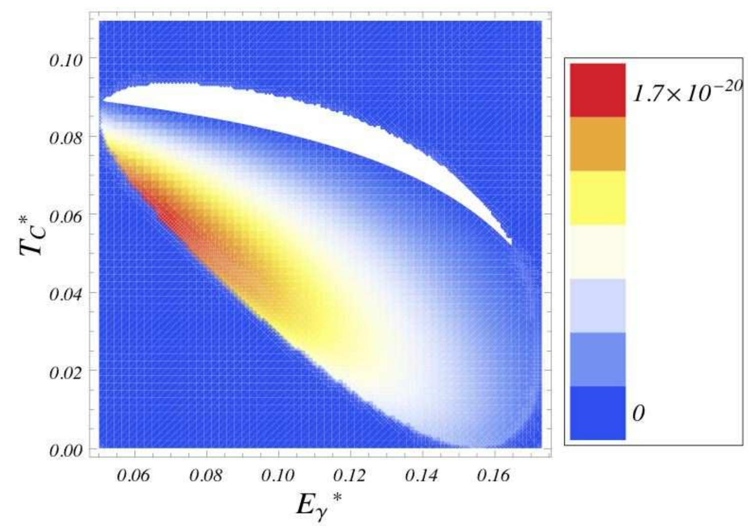


Figure 1: Dalitz plot for the contributions to $A_P^{(S)}$ (in arbitrary units) at $q^2 = (50\ \text{MeV})^2$.

Novel CP violation contributions (compared to $A_{CP}(K^+ \to \pi^+\pi^0\gamma)$

$$A_{CP} = \frac{\Gamma(K^+ \to \pi^+\pi^0 e^+e^-) - \Gamma(K^- \to \pi^-\pi^0 e^+e^-)}{\Gamma(K^+ \to \pi^+\pi^0 e^+e^-) + \Gamma(K^- \to \pi^-\pi^0 e^+e^-)}$$





Conclusions

- Match the promised experimental accuracy/statistics
- Look alternative channels
- NP models!
- Improve QCD determinations matrix elements