Measurement of CP-violating asymmetries in $B^0 \to (\rho\pi)^0$ using a time-dependent Dalitz plot analysis of the BaBar dataset

CKM 2012 Workshop - Cincinnati, Ohio

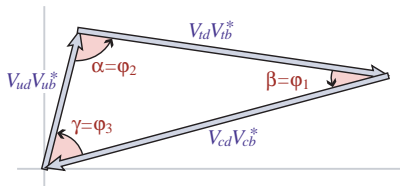**Tomonari Miyashita**

on behalf of the BaBar collaboration

Department of Physics
Stanford University

Sept 30th, 2012

GENERAL DESCRIPTION

- $B^0 \to \pi^+\pi^-\pi^0$ CPV measurement
- Dominated by $B^0 \to \rho^\pm\pi^\mp$
- Update of a 2007 BaBar analysis*
- Extracts information about α, and other parameters

[2011 PDG]

MOTIVATION

- Interference between the tree and penguin modes and decays with and without mixing allows sensitivity to α

- A precision measurement of α serves to further test the Standard Model through sensitivity to new physics in loops

- The use of a full Dalitz plot analysis reduces ambiguities found in analyses which ignore the interference regions

- Time-Dependent Dalitz Plot $\rho\pi$ measurement of α first proposed by Snyder and Quinn*
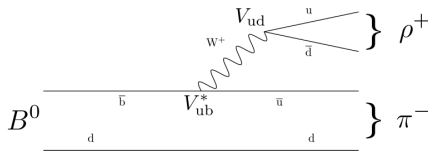
Tree Diagram
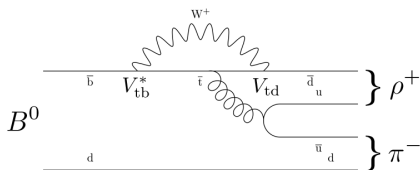
Penguin Diagram

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$$|\mathcal{A}_{3\pi}^{\pm}(\Delta t)|^2 = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\Big[|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2 \mp \left(|A_{3\pi}|^2 - |\overline{A}_{3\pi}|^2\right)\cos(\Delta m_d \Delta t) \pm$$

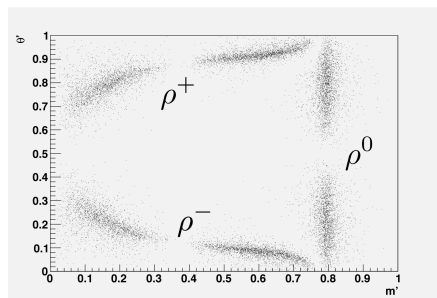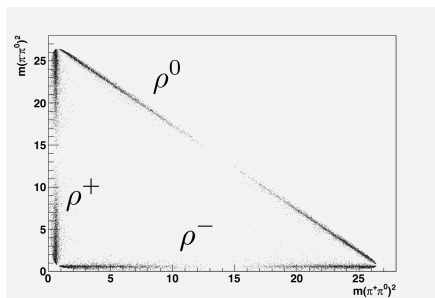$$2\text{Im}\left[\frac{q}{p}\,\overline{A}_{3\pi}A_{3\pi}^*\right]\sin(\Delta m_d \Delta t)\Big]$$

$$A_{3\pi} = f_+ A^+ + f_- A^- + f_0 A^0 \quad \text{for} \quad B^0 \to \pi^+\pi^-\pi^0$$

$$\overline{A}_{3\pi} = f_+ \overline{A}^+ + f_- \overline{A}^- + f_0 \overline{A}^0 \quad \text{for} \quad \overline{B}^0 \to \pi^+\pi^-\pi^0$$

$$f_\kappa(m, \theta_\kappa) \propto F_{\rho(770)}(m, \theta_\kappa) + a_{\rho'}e^{i\phi_{\rho'}}F_{\rho(1450)}(m, \theta_\kappa)$$
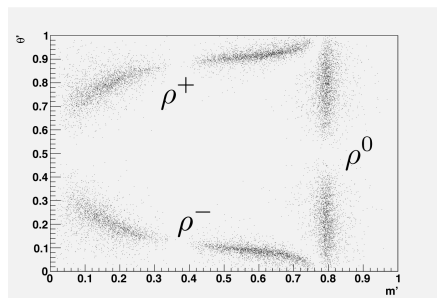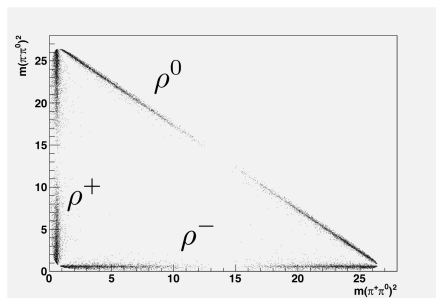
EXAMPLE TOY MC DALITZ PLOTS

- The Dalitz plot is transformed to cover a unit square before fitting

$\rho\pi$ Toy Dalitz Plot (Left) And Square Dalitz Plot (Right)

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- $m' \equiv \frac{1}{\pi} \arccos\left(2\frac{m_0 - m_0^{\min}}{m_0^{\max} - m_0^{\min}} - 1\right)$

$\rho\pi$ Toy Dalitz Plot (Left) And Square Dalitz Plot (Right)
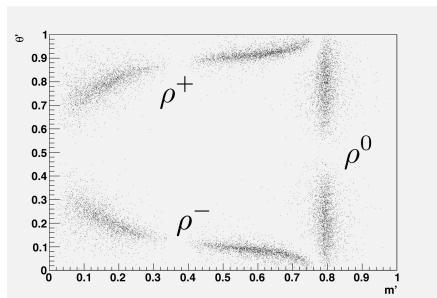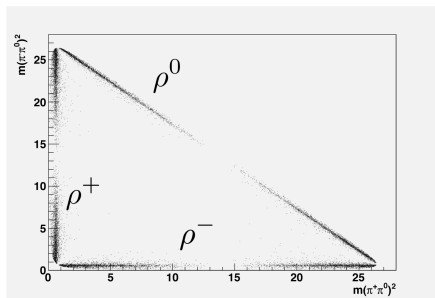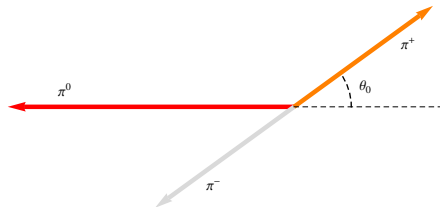
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KINEMATIC VARIABLES

- Loose cuts are applied using beam energy constraints

Kinematic Variables

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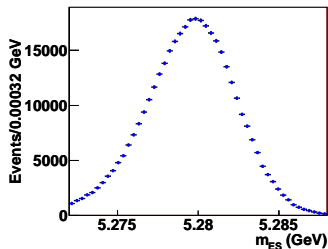
- $m_{ES} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - (p_B^*)^2}$

m_{ES} In Signal MC

m_{ES} In Data Recorded Below $B^0\overline{B}^0$ Threshold

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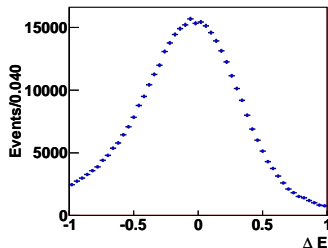
- $\Delta E = E_B^* - \frac{1}{2}\sqrt{s}$
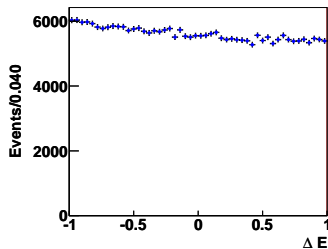
ΔE In Signal MC

ΔE In m_{ES} Sideband Data
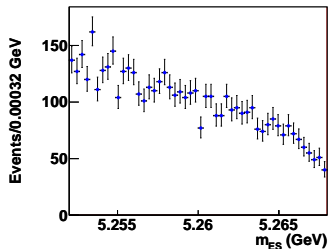
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- $m_{ES} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - (p_B^*)^2}$
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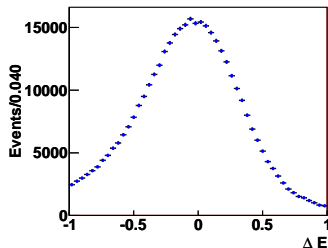
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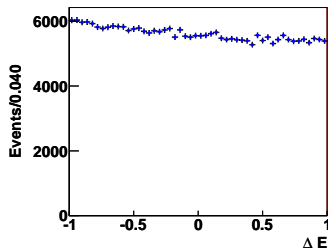
ΔE In m_{ES} Sideband Data

Multivariate Discriminator Optimization

- Discriminate signal from continuum ($q\bar{q}$ where $q = u, d, s, c$) using Neural Net discriminator
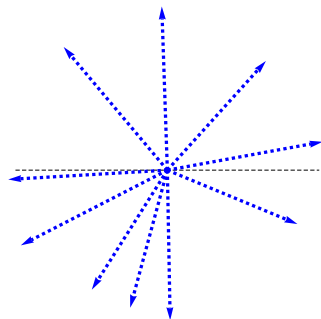
Signal-Like Event Shape
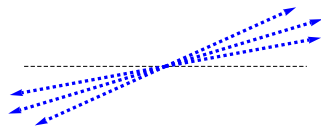
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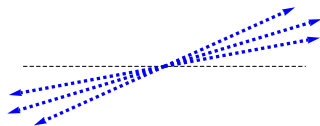
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  - Signal MC with full detector simulation
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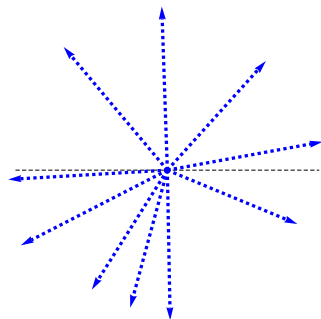
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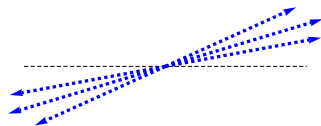
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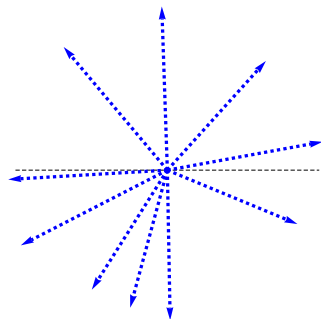- 4 Input variables provide sensitivity to event topology
- Trained using:
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- The NN is used for a loose selection cut and as a variable in the fit
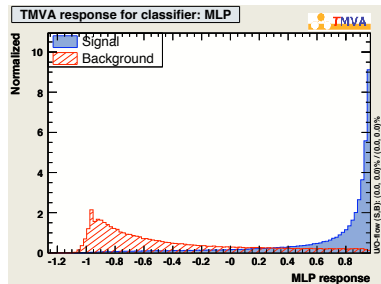
Dataset Overview

- Data modeled using four components
  - Correctly reconstructed signal
  - Misreconstructed signal
  - 26 B backgrounds
  - Continuum $q\bar{q}$ background
- Fixed and initial parameter values are obtained from fits to:
  - Fully simulated MC (for signal and B-bkgs)
  - Data collected below the $B^0\overline{B}^0$ threshold (for continuum)
  - A lower sideband in M_{ES} (for continuum)

Correctly Reconstructed Signal MC
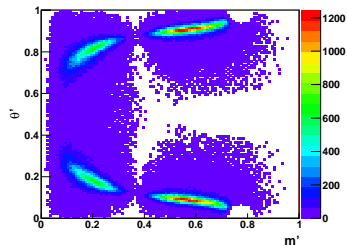
M_{ES} Sideband Data
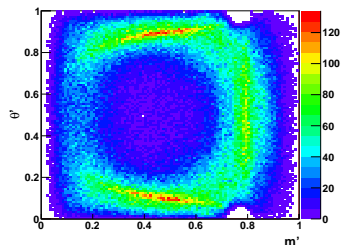
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m_{ES} In Signal MC
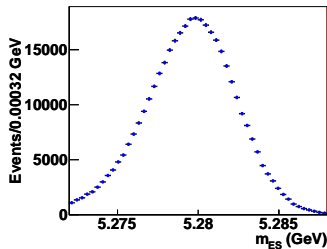
m_{ES} In Below $B^0\overline{B}^0$ Threshold Data
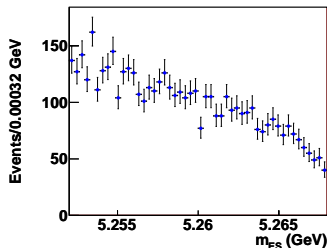
FITTING

- Data is fit using a multi-dimensional extended maximum likelihood approach with 6 input variables:
  - m_{ES}, ΔE
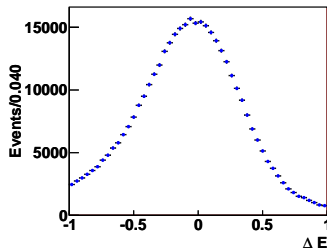
ΔE In Signal MC

ΔE In Sideband Data
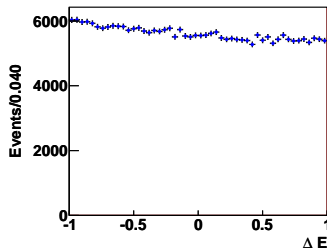
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NN_{out} In Signal MC
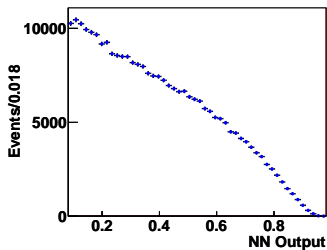
NN_{out} In Sideband Data

FITTING

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    - Time-Dependent SDP:

NN$_{out}$ In Signal MC

NN$_{out}$ In Sideband Data
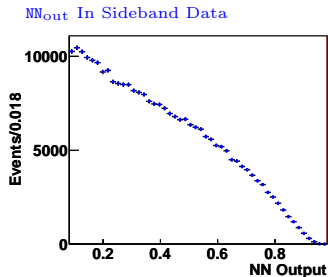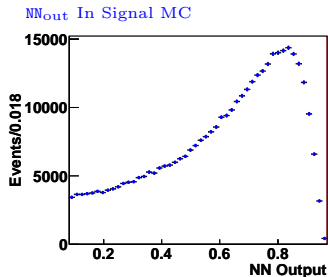
FITTING

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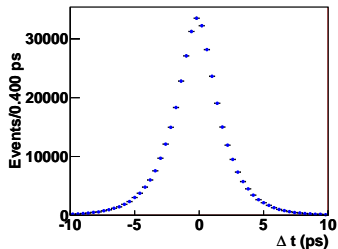
Δt In Signal MC

Δt In Sideband Data
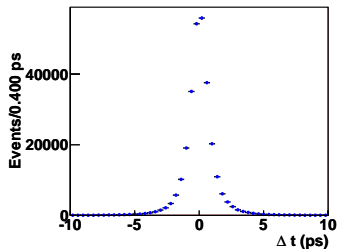
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Signal MC

M_{ES} Sideband Data

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  - Using a more sophisticated parameterization (26 free parameters) reduces ambiguities in the solution and provides gaussian errors, but solutions may be unphysical
- Ultimately, it was determined that the third parameterization (with 26 free parameters) is the most practical option

U/I Parameterization I

- In our parameterization, 26 "U and I" parameters are calculated from the ρ resonance amplitudes:

$$
\begin{aligned}
U_\kappa^\pm &= |A^\kappa|^2 \pm |\overline{A}^\kappa|^2 \\
U_{\kappa\sigma}^{\pm,\mathrm{Re(Im)}} &= \mathrm{Re(Im)}\left[A^\kappa A^{\sigma*} \pm \overline{A}^\kappa \overline{A}^{\sigma*}\right] \\
I_\kappa &= \mathrm{Im}\left[\overline{A}^\kappa A^{\kappa*}\right] \\
I_{\kappa\sigma}^{\mathrm{Re}} &= \mathrm{Re}\left[\overline{A}^\kappa A^{\sigma*} - \overline{A}^\sigma A^{\kappa*}\right] \\
I_{\kappa\sigma}^{\mathrm{Im}} &= \mathrm{Im}\left[\overline{A}^\kappa A^{\sigma*} + \overline{A}^\sigma A^{\kappa*}\right]
\end{aligned}
$$

- The time-dependent probability distribution may then be expressed in terms of these parameters as

$$
\begin{aligned}
|\mathcal{A}_{3\pi}^{\pm}(\Delta t)|^2 \quad = \quad & \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \Big[|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2 \mp \left(|A_{3\pi}|^2 - |\overline{A}_{3\pi}|^2 \right) \cos(\Delta m_d \Delta t) \pm \\
& 2\mathrm{Im}\left[\frac{q}{p} \overline{A}_{3\pi} A_{3\pi}^* \right] \sin(\Delta m_d \Delta t) \Big]
\end{aligned}
$$

- where

$$
|A_{3\pi}|^2 \pm |\overline{A}_{3\pi}|^2 = \sum_{\kappa \in [+,-,0]} |f_\kappa|^2 U_\kappa^{\pm} + 2 \sum_{\kappa < \sigma \in [+,-,0]} \left(\mathrm{Re}\left[f_\kappa f_\sigma^* \right] U_{\kappa\sigma}^{\pm,\mathrm{Re}} - \mathrm{Im}\left[f_\kappa f_\sigma^* \right] U_{\kappa\sigma}^{\pm,\mathrm{Im}} \right)
$$

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\mathrm{Im}\left[\frac{q}{p} \overline{A}_{3\pi} A_{3\pi}^* \right] = \sum_{\kappa \in [+,-,0]} |f_\kappa|^2 I_\kappa + \sum_{\kappa < \sigma \in [+,-,0]} \left(\mathrm{Re}\left[f_\kappa f_\sigma^* \right] I_{\kappa\sigma}^{\mathrm{Im}} + \mathrm{Im}\left[f_\kappa f_\sigma^* \right] I_{\kappa\sigma}^{\mathrm{Re}} \right)
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- The bootstrap technique allows one to estimate the uncertainty on parameters calculated from a single dataset
- This is done by sampling with replacement from the original dataset to generate N "bootstrapped" datasets
- If one fits to each dataset, then the covariances of the fit variables across all the "bootstrapped" datasets provide an estimate of the variables' uncertainties

- Bootstrapped estimate of the uncertainty on changes in U/I parameters between fits with and without the $\rho(1700)$, and the ratio of the mean change in the U/I parameters across all bootstrapped fits to their estimated uncertainties.

Parameter	$\sigma_{\Delta U}$	$\langle \Delta U \rangle / \sigma_{\Delta U}$
I_0	0.020	-0.36
I_-	0.022	0.41
I_{-0}^{Im}	0.42	-0.7
I_{-0}^{Re}	0.7	0.28
I_+	0.023	0.18
I_{+0}^{Im}	0.42	-0.11
I_{+0}^{Re}	0.7	0.26
I_{+-}^{Im}	0.9	-1.0
I_{+-}^{Re}	0.9	-0.6
U_0^-	0.029	0.29
U_0^+	0.017	0.5
$U_{-0}^{-,\mathrm{Im}}$	0.5	0.8
$U_{-0}^{-,\mathrm{Re}}$	0.34	0.5
$U_{-0}^{+,\mathrm{Im}}$	0.21	-0.8

Parameter	$\sigma_{\Delta U}$	$\langle \Delta U \rangle / \sigma_{\Delta U}$
$U_{-0}^{+,\mathrm{Re}}$	0.17	-0.8
U_-^-	0.034	0.19
U_-^+	0.05	0.15
$U_{+0}^{-,\mathrm{Im}}$	0.44	-0.10
$U_{+0}^{-,\mathrm{Re}}$	0.36	0.5
$U_{+0}^{+,\mathrm{Im}}$	0.16	0.5
$U_{+0}^{+,\mathrm{Re}}$	0.17	0.05
$U_{+-}^{-,\mathrm{Im}}$	0.8	-0.27
$U_{+-}^{-,\mathrm{Re}}$	0.7	0.15
$U_{+-}^{+,\mathrm{Im}}$	0.25	-0.22
$U_{+-}^{+,\mathrm{Re}}$	0.28	-0.21
U_+^-	0.06	-0.5

- The changes in the fit parameters are small relative to the uncertainty on the fit parameters and the estimated uncertainty on the changes in the fit parameters
- Therefore, we choose the covariance matrix of the bootstrapped changes as our systematic covariance matrix and don't correct for any bias
- A similar bootstrapped systematic study is performed to assess the systematic uncertainty associated with a non-resonant component, and the effect is found to be even smaller than for the $\rho(1700)$ systematic

Final Fit Results BaBar Preliminary

- From an on-resonance dataset containing 53,084 events, our multi-dimensional fit extracts 2,940±100 signal events and 46,750±220 continuum events.
- Goodness of fit is demonstrated by the figure below, which contains overlaid plots of the data used in the final fit and parameterized MC generated using the results of the final fit and equivalent to 10 times the data sample.
- The signal component of these plots is enhanced by a tight cut on the NN variable.
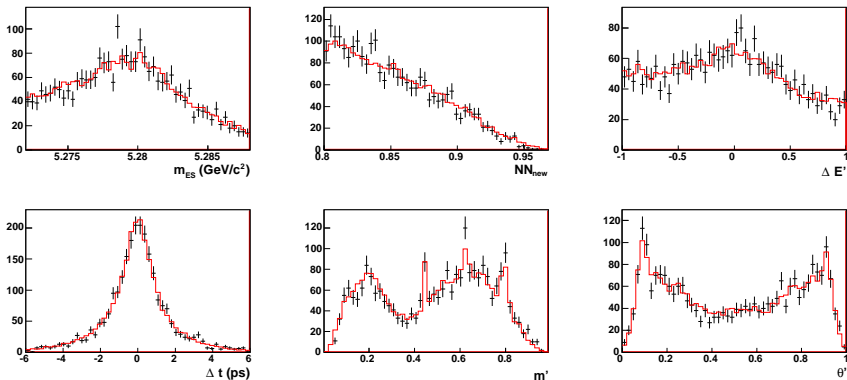
Agreement between data (points with error bars) and fit results (red)

FINAL U/I FIT RESULTS BABAR PRELIMINARY

- The U and I parameter values extracted from our final fit are given in the tables below along with stat and syst errors:

$$\langle \sigma_{\text{stat}}^{\text{new}}/\sigma_{\text{stat}}^{2007} \rangle = 0.47$$

Parameter	Final Fit Value
I_0	$-0.042 \pm 0.038 \pm 0.022$
I_-	$-0.00 \pm 0.06 \pm 0.03$
I_{-0}^{Im}	$-0.61 \pm 0.43 \pm 0.46$
I_{-0}^{Re}	$0.4 \pm 0.6 \pm 0.8$
I_+	$0.05 \pm 0.06 \pm 0.03$
I_{+0}^{Im}	$-0.04 \pm 0.36 \pm 0.43$
I_{+0}^{Re}	$0.5 \pm 0.5 \pm 0.7$
I_{+-}^{Im}	$-0.5 \pm 0.7 \pm 0.9$
I_{+-}^{Re}	$-0.6 \pm 0.8 \pm 1.0$
U_0^-	$0.04 \pm 0.05 \pm 0.03$
U_0^+	$0.225 \pm 0.030 \pm 0.020$
$U_{-0}^{-,\text{Im}}$	$0.53 \pm 0.44 \pm 0.52$
$U_{-0}^{-,\text{Re}}$	$0.49 \pm 0.35 \pm 0.37$
$U_{-0}^{+,\text{Im}}$	$-0.39 \pm 0.20 \pm 0.24$

Parameter	Final Fit Value
$U_{-0}^{+,\text{Re}}$	$-0.05 \pm 0.17 \pm 0.18$
U_-^-	$-0.27 \pm 0.10 \pm 0.06$
U_-^+	$1.22 \pm 0.07 \pm 0.05$
$U_{+0}^{-,\text{Im}}$	$0.10 \pm 0.29 \pm 0.45$
$U_{+0}^{-,\text{Re}}$	$0.30 \pm 0.32 \pm 0.38$
$U_{+0}^{+,\text{Im}}$	$0.41 \pm 0.16 \pm 0.17$
$U_{+0}^{+,\text{Re}}$	$0.01 \pm 0.15 \pm 0.19$
$U_{+-}^{-,\text{Im}}$	$1.1 \pm 0.5 \pm 0.8$
$U_{+-}^{-,\text{Re}}$	$-0.5 \pm 0.5 \pm 0.8$
$U_{+-}^{+,\text{Im}}$	$-0.07 \pm 0.26 \pm 0.26$
$U_{+-}^{+,\text{Re}}$	$-0.19 \pm 0.25 \pm 0.33$
U_+^-	$0.25 \pm 0.09 \pm 0.07$

U/I ROBUSTNESS STUDY I

- In order to judge the robustness with which the correct values of U and I parameters can be extracted by the fit framework, a series of studies were performed
- We perform fits to 25 toy MC datasets generated with expected "on-resonance" dataset statistics and a shared set of known parameter values (with $\alpha = 89°$), but different random seeds
- U and I parameter values are extracted in fits to each of these datasets and their agreement with the generated values is assessed

U/I Robustness Study II

- U/I Robustness Study Results:

<center>

<RMS #σ Diff From Gen Val>= 1.17

<Avg #σ Diff From Gen Val>= 0.04

</center>

Param	RMS #σ Diff From Gen Val	Avg #σ Diff From Gen Val
I_0	1.05	0.27
I_-	1.21	0.18
I_{-0}^{Im}	1.24	-0.10
I_{-0}^{Re}	1.29	0.09
I_+	1.01	-0.17
I_{+0}^{Im}	1.30	0.17
I_{+0}^{Re}	1.31	0.22
I_{+-}^{Im}	1.03	-0.08
I_{+-}^{Re}	1.09	-0.07
U_0^-	1.14	-0.12
U_0^+	1.09	0.29
$U_{-0}^{-,\mathrm{Im}}$	1.62	0.19
$U_{-0}^{-,\mathrm{Re}}$	1.29	-0.02
$U_{-0}^{+,\mathrm{Im}}$	1.50	-0.62

Param	RMS #σ Diff From Gen Val	Avg #σ Diff From Gen Val
$U_{-0}^{+,\mathrm{Re}}$	1.18	0.44
U_-^-	1.06	0.01
U_-^+	0.95	0.12
$U_{+0}^{-,\mathrm{Im}}$	1.49	0.20
$U_{+0}^{-,\mathrm{Re}}$	0.82	-0.08
$U_{+0}^{+,\mathrm{Im}}$	1.41	-0.13
$U_{+0}^{+,\mathrm{Re}}$	0.93	-0.03
$U_{+-}^{-,\mathrm{Im}}$	1.08	0.33
$U_{+-}^{-,\mathrm{Re}}$	1.25	-0.29
$U_{+-}^{+,\mathrm{Im}}$	1.13	-0.14
$U_{+-}^{+,\mathrm{Re}}$	1.12	0.44
U_+^+	0.88	0.08

QUASI-TWO-BODY FIT RESULTS I

- The U and I parameters extracted from our final fit may be used to calculate the values of quasi-two-body (Q2B) parameters often used in CP-violation analyses:

$$f_{Q_{\text{tag}}}^{\rho^\pm \pi^\mp}(\Delta t) = (1 \pm \mathcal{A}_{\rho\pi}) \frac{e^{-|\Delta t|/\tau}}{4\tau}$$
$$\times \left[1 + Q_{\text{tag}}(\mathcal{S} \pm \Delta \mathcal{S}) \sin(\Delta m_d \Delta t) \right.$$
$$\left. - Q_{\text{tag}}(\mathcal{C} \pm \Delta \mathcal{C}) \cos(\Delta m_d \Delta t) \right].$$

with

$$\mathcal{A}_{\rho\pi} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+},$$

and

$$\mathcal{C} = (\mathcal{C}^+ + \mathcal{C}^-)/2,$$
$$\Delta \mathcal{C} = (\mathcal{C}^+ - \mathcal{C}^-)/2,$$
$$\mathcal{S} = (\mathcal{S}^+ + \mathcal{S}^-)/2,$$
$$\Delta \mathcal{S} = (\mathcal{S}^+ - \mathcal{S}^-)/2.$$

where

$$\mathcal{C}^+ = \frac{U_+^-}{U_+^+}, \ \ \mathcal{C}^- = \frac{U_-^-}{U_-^+}, \ \ \mathcal{S}^+ = \frac{2I_+}{U_+^+}, \ \ \mathcal{S}^- = \frac{2I_-}{U_-^+},$$

QUASI-TWO-BODY FIT RESULTS II

- We can also use our fit results to extract the Q2B $B^0 \to \rho^0 \pi^0$ CP-violation parameters and decay fraction:

$$\mathcal{C}_{00} = \frac{U_0^-}{U_0^+},$$

$$\mathcal{S}_{00} = \frac{2I_0}{U_0^+},$$

$$f_{00} = \frac{U_0^+}{U_+^+ + U_-^+ + U_0^+}.$$

- All eight of these Q2B parameters are extracted in a χ^2 minimization using the full stat+syst covariance matrix for the relevant U and I parameters:

Param	Value	σ_{stat}	σ_{syst}	
$\mathcal{A}_{\rho\pi}$	-0.100	0.029	0.021	
$\mathcal{C}$	0.016	0.059	0.036	
$\Delta\mathcal{C}$	0.234	0.061	0.048	
$\mathcal{S}$	0.053	0.081	0.034	BaBar Preliminary
$\Delta\mathcal{S}$	0.054	0.082	0.039	
$\mathcal{C}_{00}$	0.19	0.23	0.15	
$\mathcal{S}_{00}$	-0.37	0.34	0.20	
f_{00}	0.092	0.011	0.008	

Quasi-Two-Body Fit Results III

- The parameters $\mathcal{A}_{\rho\pi}$ and $\mathcal{C}$ can be transformed into the direct CP-violation parameters $\mathcal{A}_{\rho\pi}^{+-}$ and $\mathcal{A}_{\rho\pi}^{-+}$ where
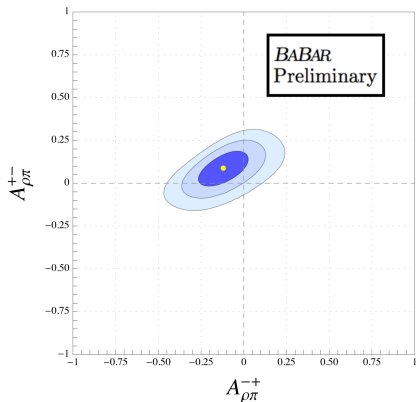
$$\mathcal{A}_{\rho\pi}^{+-} \equiv \frac{\Gamma(\overline{B}^0 \to \rho^-\pi^+) - \Gamma(B^0 \to \rho^+\pi^-)}{\Gamma(\overline{B}^0 \to \rho^-\pi^+) + \Gamma(B^0 \to \rho^+\pi^-)} = -\frac{\mathcal{A}_{\rho\pi} + \mathcal{C} + \mathcal{A}_{\rho\pi}\Delta\mathcal{C}}{1 + \Delta\mathcal{C} + \mathcal{A}_{\rho\pi}\mathcal{C}},$$

$$\mathcal{A}_{\rho\pi}^{-+} \equiv \frac{\Gamma(\overline{B}^0 \to \rho^+\pi^-) - \Gamma(B^0 \to \rho^-\pi^+)}{\Gamma(\overline{B}^0 \to \rho^+\pi^-) + \Gamma(B^0 \to \rho^-\pi^+)} = \frac{\mathcal{A}_{\rho\pi} - \mathcal{C} - \mathcal{A}_{\rho\pi}\Delta\mathcal{C}}{1 - \Delta\mathcal{C} - \mathcal{A}_{\rho\pi}\mathcal{C}}.$$

- These parameters are extracted in a 2-dimensional likelihood scan yielding:

$$\mathcal{A}_{\rho\pi}^{+-} = 0.09^{+0.05}_{-0.06} \pm 0.04,$$
$$\mathcal{A}_{\rho\pi}^{-+} = -0.12 \pm 0.08^{+0.04}_{-0.05}.$$

Quasi-Two-Body Fit Results IV

- In the 2D scan, the origin, corresponding to no direct CP violation, lies on the 96.0% confidence-level contour ($\Delta\chi^2 = 6.42$)

Quasi-Two-Body Robustness Study

- We assess the robustness with which the Q2B parameters are extracted using the same toy MC as used for the U/I robustness study
- Q2B Robustness Study Results:

Param	$\sqrt{\text{Variance}}/\langle\sigma\rangle$	Avg #σ Diff From Gen Val
$\mathcal{A}_{\rho\pi}$	0.94	-0.13
$\mathcal{C}$	1.15	0.06
$\Delta\mathcal{C}$	0.94	0.04
$\mathcal{S}$	1.11	0.03
$\Delta\mathcal{S}$	1.02	-0.20
$\mathcal{C}_{00}$	1.15	-0.10
$\mathcal{S}_{00}$	1.13	0.23
f_{00}	1.08	0.28

α Scan Technique

- Information about the unitarity triangle angle α is extracted in a likelihood scan based on our final U/I fit results and full stat+syst covariance matrix
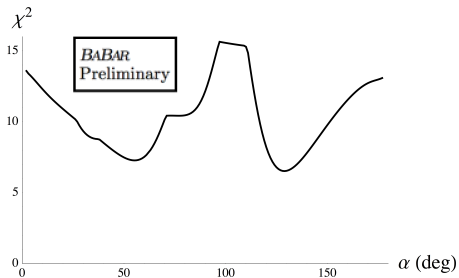- Perform a χ^2 minimization at each value of α from $(0 - 180)$ degrees using

$$\chi^2_{\alpha \text{ scan}} = \left[V^{\text{data}} - V^{\text{scan}} \right]^T (C^{\text{data}})^{-1} \left[V^{\text{data}} - V^{\text{scan}} \right]$$

- The variables that float in these fits are actually the tree and penguin amplitudes which are related to the ρ amplitudes by:

$$
\begin{aligned}
A^+ &= T^+ e^{-i\alpha} + P^+ \\
A^- &= T^- e^{-i\alpha} + P^- \\
A^0 &= T^0 e^{-i\alpha} + P^0 \\
\overline{A}^+ &= T^- e^{+i\alpha} + P^- \\
\overline{A}^- &= T^+ e^{+i\alpha} + P^+ \\
\overline{A}^0 &= T^0 e^{+i\alpha} + P^0
\end{aligned}
$$

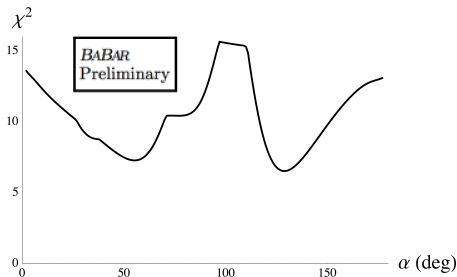- α scan results including stat+syst uncertainties:
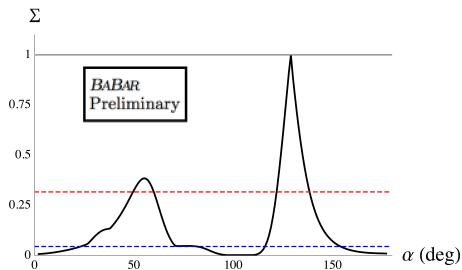

α Scan χ^2 Distribution

Final Fit α Scan Results

- α scan results including stat+syst uncertainties:
- (Σ is calculated as the integral of a χ^2 distribution with 1 degree of freedom from $\Delta\chi^2$ to ∞)
- Σ corresponds to what is commonly referred to as "$1 - $ C.L."
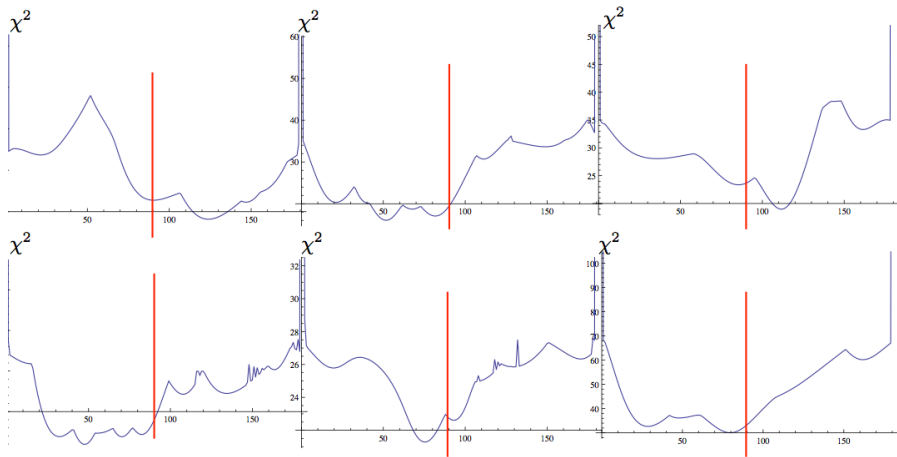

α Scan χ^2 Distribution

α Scan Σ Distribution
(See robustness studies for interpretation)

α Scan Robustness Studies I

- In order to judge the robustness with which the correct value of α can be extracted by the fit framework, a series of studies were performed
- We perform α scans using the 25 toy MC datasets from the previous robustness studies and assess the robustness with which the generated value of α ($89°$) is extracted
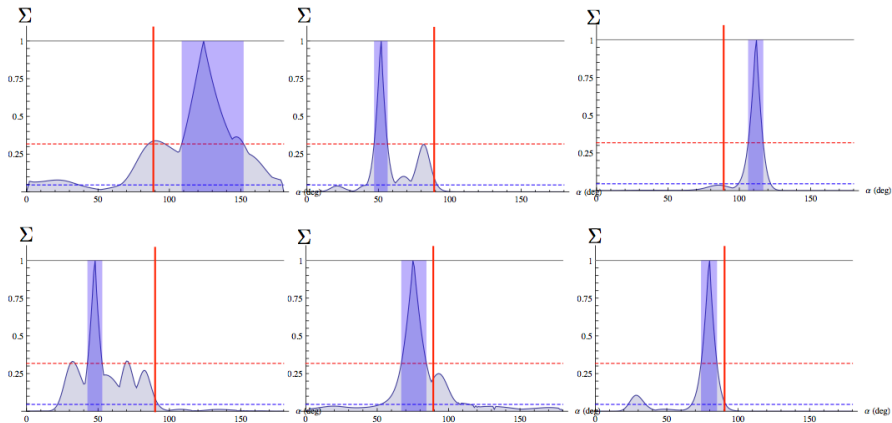
- Example α Scan χ^2 Distributions ($\alpha_{\text{gen}} = 89°$)

α Scan Robustness Studies III

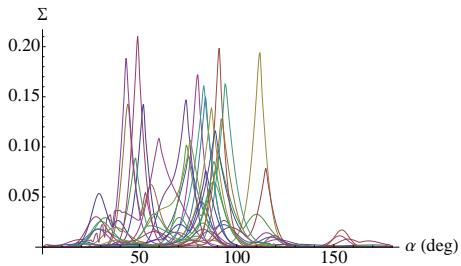- Example α Scan Σ Distributions ($\alpha_{\text{gen}} = 89°$)

α Scan Peak	Mean Error	#σ Between Gen and Fit Peaks
43	4.6	-9.0
44	5.0	-8.5
48	5.2	-8.2
49	4.6	-8.7
52	4.7	-8.2
53	12.8	-7.2
60	10.9	-2.1
74	6.5	-2.1
74	9.0	-2.9
75	8.8	-1.5
76	12.6	-1.7
80	5.6	-1.7
83	6.9	-0.94
84	5.9	-0.89
84	6.7	-0.72
87	7.1	-0.29
88	6.7	-0.14
89	8.4	0
91	9.2	0.22
91	4.5	0.43
92	6.9	0.48
94	6.3	0.89
112	5.3	3.9
115	5.4	4.6
124	21.7	2.3

- Overview of scan results using sum of normalized Σ scans
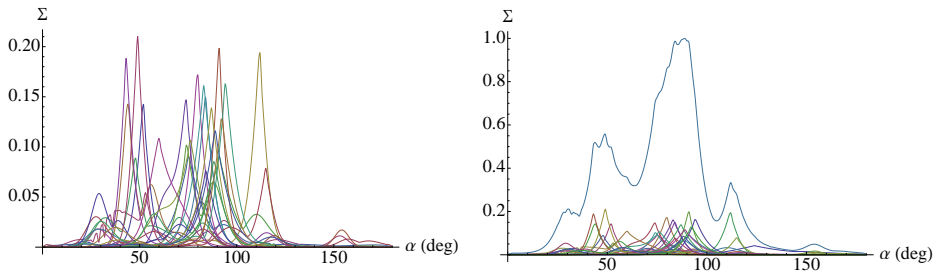
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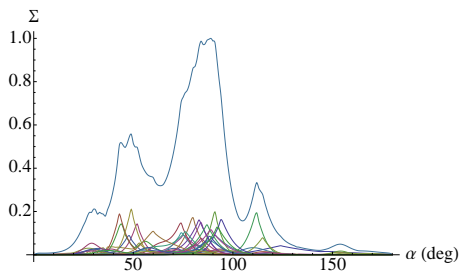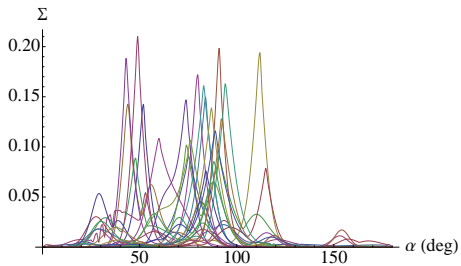
α Scan Robustness Studies V

- Overview of scan results using sum of normalized Σ scans
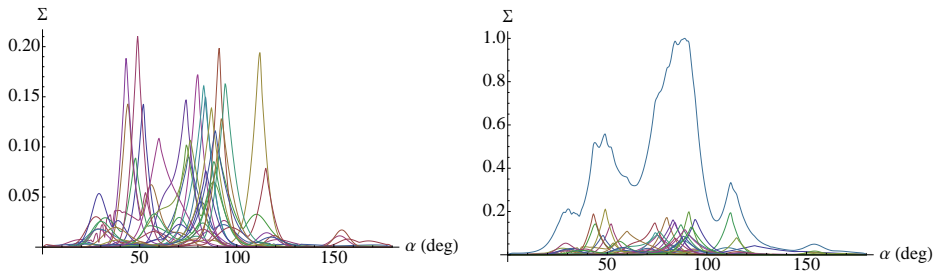

- The study indicates that with the current statistics, α scans may favor secondary solutions

α SCAN ROBUSTNESS STUDIES V

- Overview of scan results using sum of normalized Σ scans

- The study indicates that with the current statistics, α scans may favor secondary solutions
- When the signal to background ratio is increased, the fit becomes robust

- Due to isospin relations, information about the charged B decays $B^{\pm} \to \rho^{\pm,0}\pi^{0,\pm}$ may be used to constrain our scan of α
- (Red = isospin constrained scan / Dashed black = new scan from final fit)

α Scan χ^2 Distribution

Final Fit α Scan Results

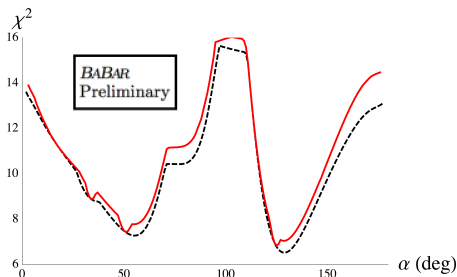- Due to isospin relations, information about the charged B decays $B^{\pm} \to \rho^{\pm,0} \pi^{0,\pm}$ may be used to constrain our scan of α
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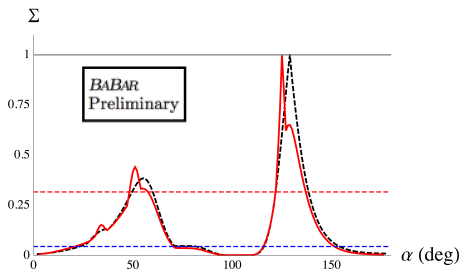
α Scan Σ Distribution

α Scan Comparison

- Interestingly, though our α scan disfavors the world average α value of $89°$, it is in good agreement with the existing CKMFitter world average for the $\rho\pi$ mode
- (Blue = world average / Dashed black = new scan from final fit)
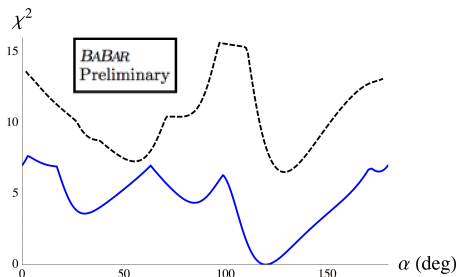
α Scan χ^2 Distribution
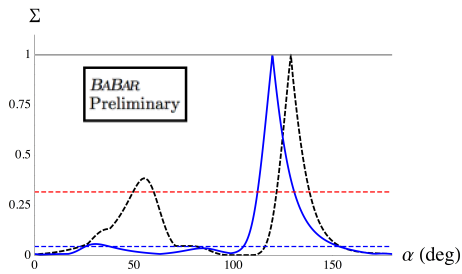
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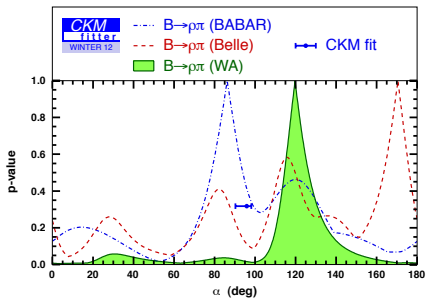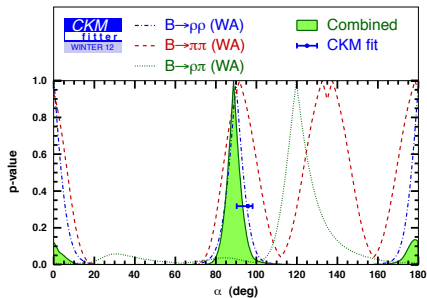 - This analysis would benefit greatly from increased sample sizes available at high-luminosity experiments.

BACKUP SLIDES

- CKMFitter overall world average and $\rho\pi$ world average α scans

$\rho\pi$ World Average α Scan

World Average α Scan

- Due to isospin relations, information about the charged B decays $B^{\pm} \to \rho^{\pm,0}\pi^{0,\pm}$ may be used to constrain our scan of α

α Scan - Isospin Constraints I

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- These relations result in four "constraint" equations while introducing only two new free parameters to the fit (the real and imaginary parts of a tree amplitude, T^{+0})
- The charged B measurements of interest are the branching fractions and asymmetries:

$$
\begin{aligned}
\mathcal{B}(\rho^+\pi^0) &= c(|A^{+0}|^2 + |A^{-0}|^2)\tau_{B+}, \\
\mathcal{B}(\rho^0\pi^+) &= c(|A^{0+}|^2 + |A^{0-}|^2)\tau_{B+}, \\
\mathcal{A}(\rho^+\pi^0) &= \frac{|A^{-0}|^2 - |A^{+0}|^2}{|A^{-0}|^2 + |A^{+0}|^2}, \\
\mathcal{A}(\rho^0\pi^+) &= \frac{|A^{0-}|^2 - |A^{0+}|^2}{|A^{0-}|^2 + |A^{0+}|^2},
\end{aligned}
$$

where

$$
\begin{aligned}
A^{\pm 0} &= \frac{q}{p}A(B^{\pm} \to \rho^{\pm}\pi^0), \\
A^{0\pm} &= \frac{q}{p}A(B^{\pm} \to \rho^0\pi^{\pm}).
\end{aligned}
$$

- Due to isospin symmetry,

$$A^+ + A^- + 2A^0 = e^{-2i\alpha}\left(\overline{A}^+ + \overline{A}^- + 2\overline{A}^0\right)$$
$$= \sqrt{2}(A^{+0} + A^{0+})$$
$$= \sqrt{2}e^{-2i\alpha}(A^{-0} + A^{0-}),$$

$$A^{+0} - A^{0+} - \sqrt{2}(A^+ - A^-) = e^{-2i\alpha}\left[A^{-0} - A^{0-} - \sqrt{2}(A^- - A^+)\right].$$

α Scan - Isospin Constraints II

- Due to isospin symmetry,

$$A^+ + A^- + 2A^0 = e^{-2i\alpha}\left(\overline{A}^+ + \overline{A}^- + 2\overline{A}^0\right)$$
$$= \sqrt{2}(A^{+0} + A^{0+})$$
$$= \sqrt{2}e^{-2i\alpha}(A^{-0} + A^{0-}),$$

$$A^{+0} - A^{0+} - \sqrt{2}(A^+ - A^-) = e^{-2i\alpha}\left[A^{-0} - A^{0-} - \sqrt{2}(A^- - A^+)\right].$$

- Based on these relations, we can parameterize the charged B amplitudes according to

$$\begin{aligned}
\sqrt{2}A^{+0} &= e^{-i\alpha}T^{+0} + P^+ - P^-, \\
\sqrt{2}A^{0+} &= e^{-i\alpha}(T^+ + T^- + 2T^0 - T^{+0}) - P^+ + P^-, \\
\sqrt{2}A^{-0} &= e^{+i\alpha}T^{+0} + P^+ - P^-, \\
\sqrt{2}A^{0-} &= e^{+i\alpha}(T^+ + T^- + 2T^0 - T^{+0}) - P^+ + P^-.
\end{aligned}$$

- For each of the four amplitudes $A^{\pm 0}$ and $A^{0\pm}$, we can calculate the value based on tree and penguin amplitudes or from measurements of charged B branching fractions and asymmetries

- Hence, for each of the four amplitudes, we add a gaussian constraint term to the χ^2 used in the α scan:

$$\left(\frac{|A_{\text{iter.}}| - |A_{\text{meas.}}|}{\sigma_{|A_{\text{meas.}}|}} \right)^2$$