# Exclusive vs. Inclusive

## (Perspectives & provocations for discussions)

*Zoltan Ligeti*

- Past: history, introduction etc.
- Present: where tensions are / are not
- Future: lot more progress will take place

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- Past: history, introduction etc.
- Present: where tensions are / are not
- Future: lot more progress will take place

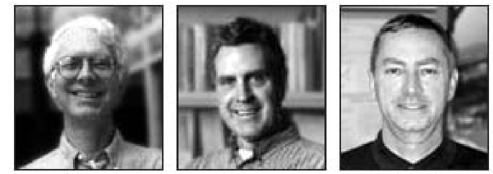
Disclaimers: I do not have many answers, will try to lay out some questions

The views expressed here are solely mine, will change depending on future data

Apologies for the many missing citations

# $V_{cb}$ — the beginning

- 1983: Long $B$ meson lifetime discovered $\Rightarrow |V_{cb}|$ is small



Bill Ford John Jaros Nigel Lockyer

- 1987: $B$ meson mixing discovered [ARGUS] ($\Rightarrow$ heavy $m_t$ and all that)

  Timescale of oscillation and decay comparable: $\Delta m/\Gamma \simeq 0.77$ (and $\Delta\Gamma \ll \Gamma$)

- Crucial to allow experimental study of $CP$ violation in $B$ system

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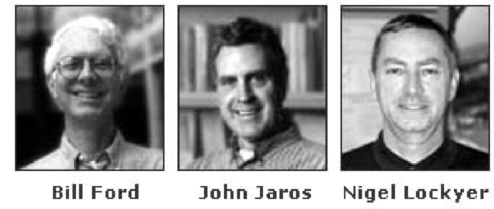


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- If $|V_{cb}|$ were as large as $|V_{us}|$, probably BaBar & Belle would not have been built, this conference would not exist, etc. $\Rightarrow$ one should thank $|V_{cb}|$ to be here...

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# $V_{ub}$ — the beginning

ARGUS, PLB **234** (1990) 409, Received 28 Nov 1989 $(201+69\,\mathrm{pb}^{-1})$

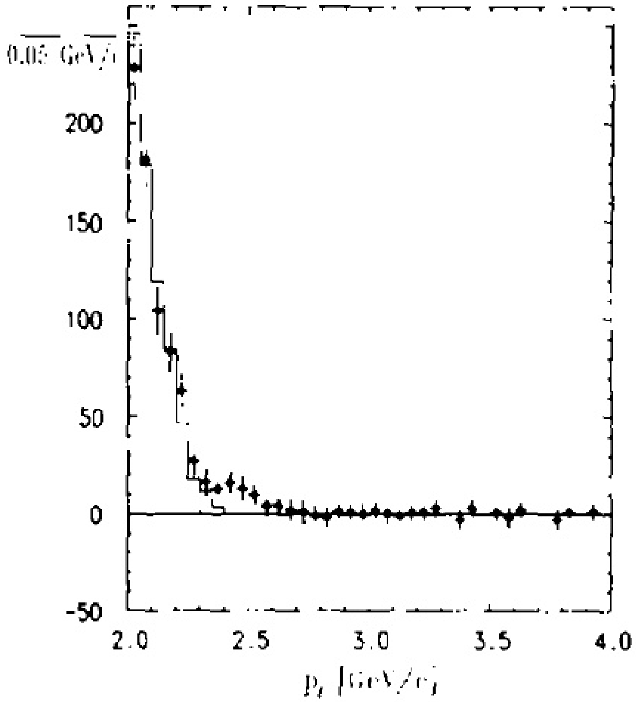



Fig. 5. Combined lepton momentum spectrum for direct $\Upsilon(4S)$ decays: the histogram is a $b \to c$ contribution normalized in the region 2.0–2.3 GeV/$c$.

CLEO, PRL **64** (1990) 16, Received 8 Nov 1989 $(212+101\,\mathrm{pb}^{-1})$

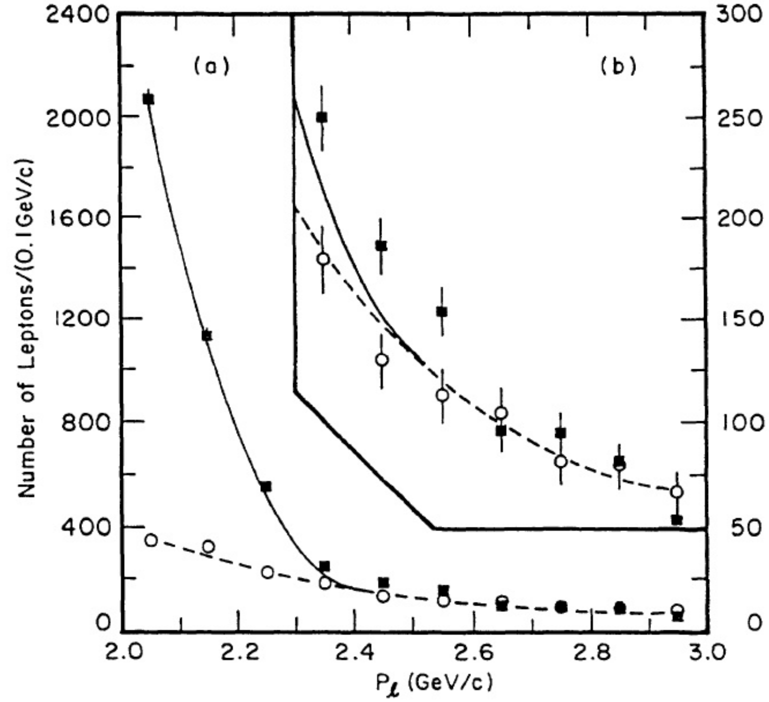



FIG. 1. Sum of the $e$ and $\mu$ momentum spectra for ON data (filled squares), scaled OFF data (open circles), the fit to the OFF data (dashed line), and the fit to the OFF data plus the $b \to c l \nu$ yield (solid line). Note the different vertical scales in (a) and (b).

"If interpreted as a signal of $b \to u$ coupling … $|V_{ub}/V_{cb}|$ of about $10\%$."

"$|V_{ub}/V_{cb}|$ … is approximately $0.1$; it is sensitive to the theoretical model."

# Status just before Babar & Belle

- BaBar & Belle increased the data used to measure $|V_{cb}|$ and $|V_{ub}|$ by a factor $\sim 10^2$

  The next generation of $e^+e^-$ $B$ factories will do the "same" (on a log scale)

- What did we expect before Babar & BELLE? [ZL, Kaon 99, hep-ph/9908432]

## 4 Conclusions

The present status of $|V_{cb}|$ and $|V_{ub}|$ is approximately

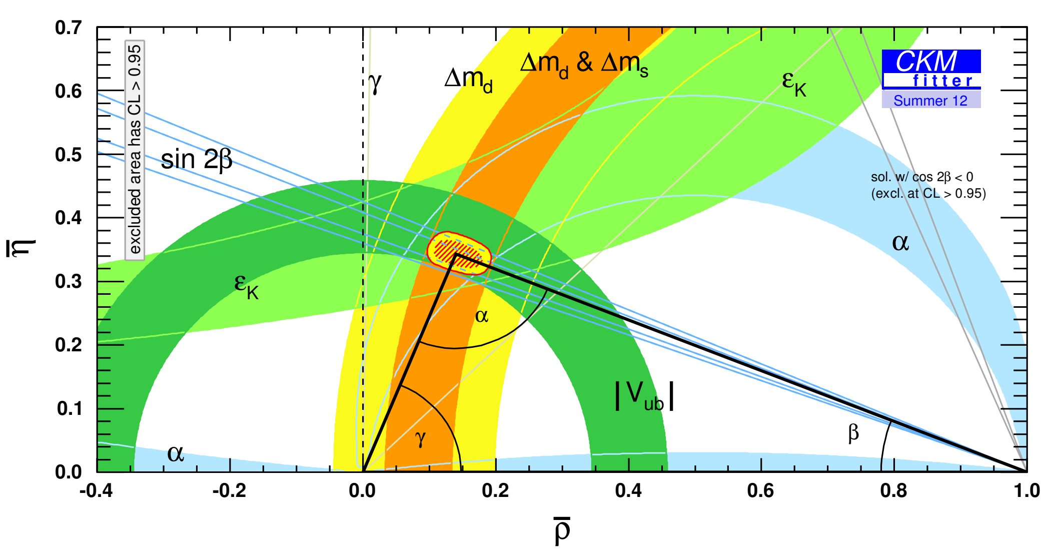
$$|V_{cb}| = 0.040 \pm 0.002\,, \qquad |V_{ub}/V_{cb}| \simeq 0.090 \pm 0.025\,. \qquad (12)$$

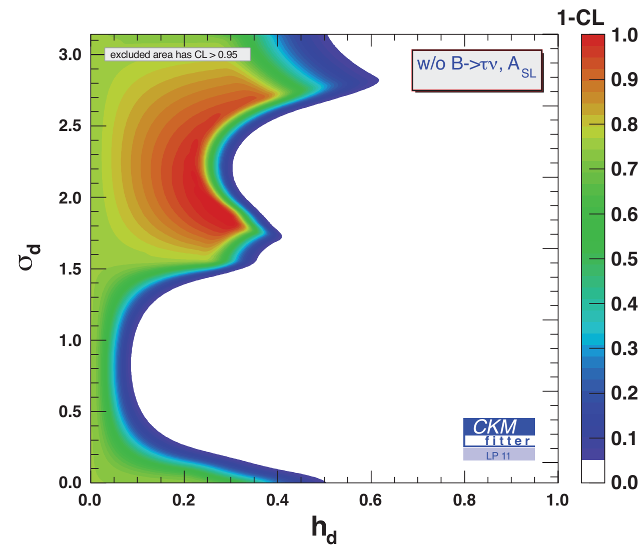
The central value and error of $|V_{cb}|$ comes from first principles, and the uncertainty in both its exclusive and inclusive determination is of order $1/m_Q^2$. On the other hand, the above error on $|V_{ub}|$ is somewhat ad hoc, since it is still estimated relying on phenomenological models.

"Within the next 3–5 years, in my opinion, an optimistic scenario is roughly as follows. The theoretical error of $|V_{cb}|$ might be reduced to 2–3%. . . . At the same time, the theoretical error of $|V_{ub}|$ might be reduced to about 10%."

- Could be worse...

# Other impacts of $|V_{cb}|$ and $|V_{ub}|$




- Error of $|V_{cb}|$ is large part of the uncertainty in the $\epsilon_K$ constraint, and in $K \to \pi\nu\bar\nu$
- $|V_{ub}|$ is the dominant uncertainty of the side opposite to $\beta$, crucial to constrain NP
- Same theoretical methods used for FCNC decays: $B \to X\gamma$, $X\ell^+\ell^-$, $X\nu\bar\nu$
- Both $|V_{cb}|$ and $|V_{ub}|$: persistent tensions between inclusive & exclusive
- Q: What would it take to conclude that there is unambiguous evidence for NP?

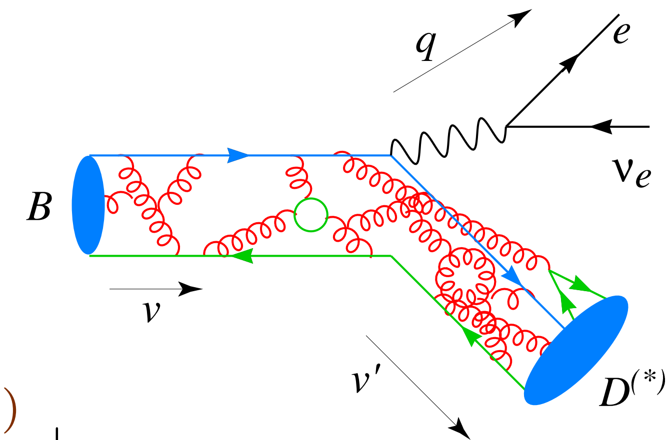
$$|V_{cb}|$$

# $|V_{cb}|$ from $B \to D^{(*)}\ell\bar{\nu}$

- Heavy Quark Symmetry: brown muck only sees $v \to v'$ (not $m_b \to m_c$ or $\vec{s}_b \to \vec{s}_c$)

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar{\nu})}{\mathrm{d}w} = (\dots)\,(w^2-1)^{3/2(1/2)}\,|V_{cb}|^2\,\mathcal{F}^2_{(*)}(w)$$

$w \equiv v \cdot v'$ — Isgur-Wise function $+ \dots$



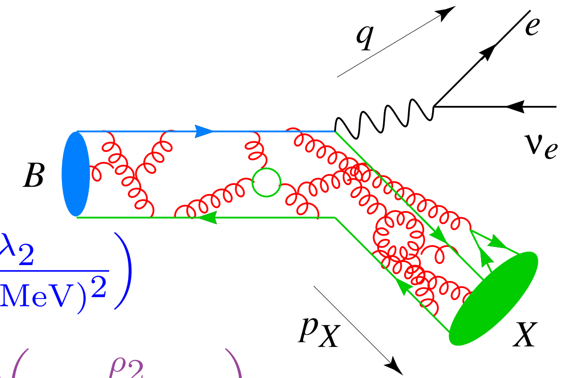

$$\mathcal{F}(1) = 1_{\rm Isgur-Wise} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = 1_{\rm Isgur-Wise} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\rm Luke}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$

- Lattice QCD: $\mathcal{F}_*(1) = 0.908 \pm 0.017,\ \ \mathcal{F}(1) = 1.074 \pm 0.024$ [arXiv:1011.2166, hep-lat/0409116]
- Need to know shape [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert], recently LQCD calc. [1111.0677]
- Need some understanding of decays to higher mass $X_c$ states (backgrounds)
- Data: $|V_{cb}\,\mathcal{F}_*(1)| = (35.90 \pm 0.45) \times 10^{-3},\ \ |V_{cb}\,\mathcal{F}(1)| = (42.64 \pm 1.53) \times 10^{-3}$ [HFAG]

# Classic application of OPE: inclusive $|V_{cb}|$

- Want to determine $|V_{cb}|$ from $B \to X_c \ell \bar\nu$:




$$\Gamma(B \to X_c \ell \bar\nu) = \frac{G_F^2\, |V_{cb}|^2}{192\pi^3} (4.7\,\mathrm{GeV})^5\, (0.534) \times$$

$$\Bigg[ 1 - 0.22 \left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) - 0.011 \left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right)^2 - 0.052 \left(\frac{\lambda_1}{(500\,\mathrm{MeV})^2}\right) - 0.071 \left(\frac{\lambda_2}{(500\,\mathrm{MeV})^2}\right)$$

$$- 0.006 \left(\frac{\lambda_1 \Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) + 0.011 \left(\frac{\lambda_2 \Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) - 0.006 \left(\frac{\rho_1}{(500\,\mathrm{MeV})^3}\right) + 0.008 \left(\frac{\rho_2}{(500\,\mathrm{MeV})^3}\right)$$

$$+ 0.011 \left(\frac{\mathcal{T}_1}{(500\,\mathrm{MeV})^3}\right) + 0.002 \left(\frac{\mathcal{T}_2}{(500\,\mathrm{MeV})^3}\right) - 0.017 \left(\frac{\mathcal{T}_3}{(500\,\mathrm{MeV})^3}\right) - 0.008 \left(\frac{\mathcal{T}_4}{(500\,\mathrm{MeV})^3}\right)$$

$$+ 0.096\epsilon - 0.030\epsilon^2_{\mathrm{BLM}} + 0.015\epsilon \left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) + \ldots \Bigg]$$

Corrections: $\mathcal{O}(\Lambda/m)$: $\sim 20\%$, $\mathcal{O}(\Lambda^2/m^2)$: $\sim 5\%$, $\mathcal{O}(\Lambda^3/m^3)$: $\sim 1 - 2\%$,
$\mathcal{O}(\alpha_s)$: $\sim 10\%$, Unknown terms: $< 2\%$

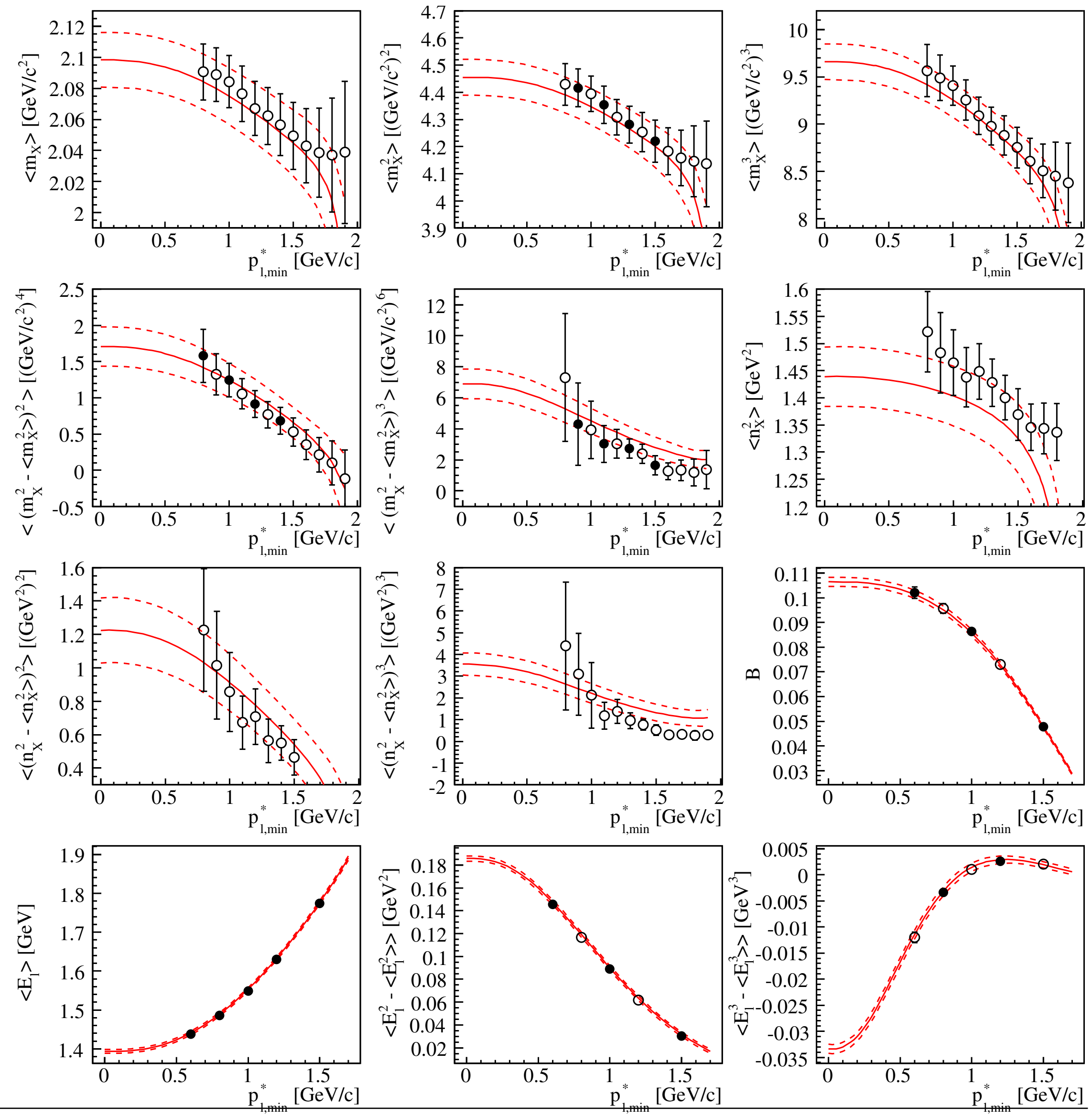
- Fit $\mathcal{O}(100)$ observables: test theory + determine $|V_{cb}|$ & hadronic matrix elements

- Precision field: $\sigma(|V_{cb}|) \sim 2\%$ ! Also important for $\epsilon_K$ (error $\propto |V_{cb}|^4$) and $K \to \pi\nu\bar\nu$

# The data...

- No significant evidence for deviations from quark-hadron duality

[BaBar, arXiv:0908.0415, similar results from Belle]



# Some comments on $|V_{cb}|$

- I feel that the lack of understanding of the non-$D^{(*,**)}$ contributions is worrisome
- The $\frac{3}{2} \gg \frac{1}{2}$ (narrow $D_1, D_2^*$, broad $D_0^*, D_1^*$) rule relies on saturation by lowest states
  Can radially excited states be important and not mess up $E_\ell$ spectrum?
- Modelling continuum only by Goity–Roberts (can one make up another model?)
- Role of $s\bar{s}$ popping? $B \to D_s^{(*)} K \ell \bar{\nu}$, $D^{(*)} \phi \ell \bar{\nu}$, etc. Possibly large impact for $|V_{ub}|$?
- Do we fully appreciate correlated impact on moments, $m_b$, $|V_{cb}|$ & $|V_{ub}|$ inclusive?

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- Recent updates reduce tension somewhat; I can imagine that inclusive and exclusive $|V_{cb}|$ will converge, by improving things we know we can with more data

# $|V_{ub}|$

More severe tensions, but jury is still out:

$$|V_{ub}|_{\mathrm{incl-BLL}} = (4.62 \pm 0.35) \times 10^{-3}$$
$$|V_{ub}|_{\mathrm{incl-BLNP}} = (4.40 \pm 0.32) \times 10^{-3}$$
$$|V_{ub}|_{\mathrm{incl-GGOU}} = (4.39 \pm 0.24) \times 10^{-3}$$

$$|V_{ub}|_{\pi\ell\bar{\nu}-\mathrm{LQCD}} = (3.4 \pm 0.5) \times 10^{-3}$$
$$|V_{ub}|_{\tau\nu} = ?$$
SM fit: $(3.5 \pm 0.2) \times 10^{-3}$

BLL seemed to be an outlier ($m_X - q^2$, local OPE, $\alpha_s^2\beta_0$); recent $\alpha_s^2$ calculations in SCET region expected to enhance BLNP by $5-10\%$ (being implemented)

# $|V_{ub}|$ from exclusive decays

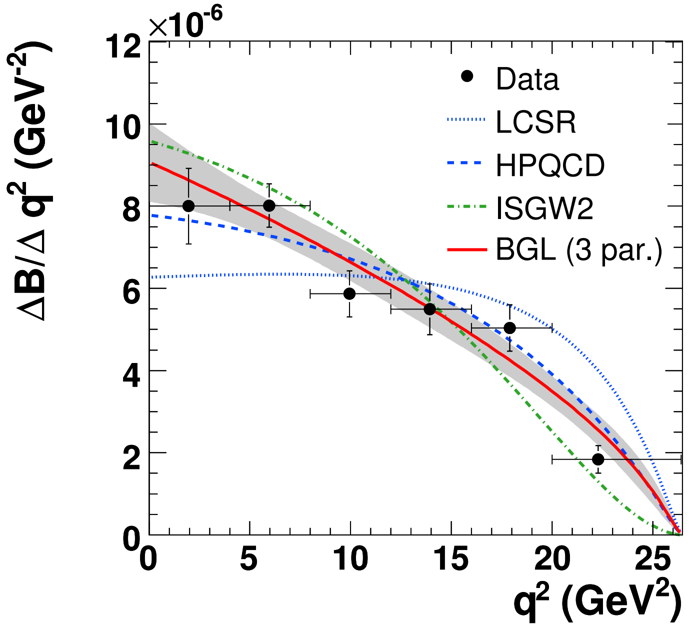
- Less constraints from heavy quark symmetry than in $B \to D^{(*)}\ell\bar\nu$
- $B \to \ell\bar\nu$: BaBar – Belle tension; measures $f_B \times |V_{ub}|$ — need $f_B$ (lattice QCD)
- $B \to \pi\ell\bar\nu$:

$$\frac{\mathrm{d}\Gamma(\overline{B}^0 \to \pi^+\ell\bar\nu)}{\mathrm{d}q^2} = \frac{G_F^2|\vec{p}_\pi|^3}{24\pi^3}\,|V_{ub}|^2\,|f_+(q^2)|^2$$

Determination of $f_+(q^2)$ in the hands of lattice QCD

... under better control at large $q^2$

Continuum theory input: some constraint on shape



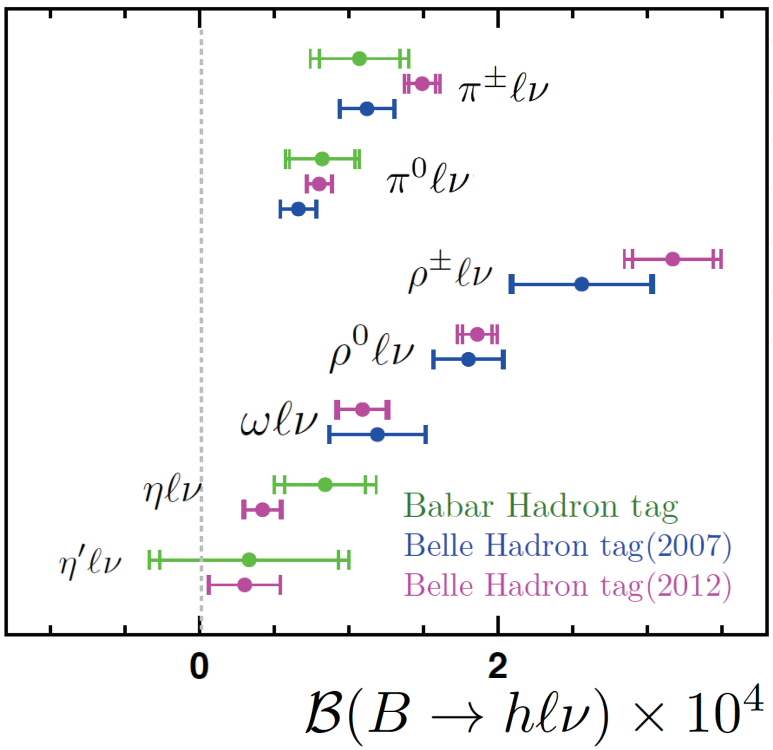

[BaBar, arXiv:1005.3288]

- $B \to \rho\ell\bar\nu$ is harder (sizable $\Gamma_\rho/m_\rho$), $B \to \eta^{(\prime)}\ell\bar\nu$ is even harder
- Expect $B \to \pi\ell\bar\nu$ to dominate $|V_{ub}|_{\rm excl}$ for the foreseeable future

# Latest Belle results (1)

- Need large statistics full reco results to make definitive conclusions [Yook @ ICHEP]

## Branching Ratios of the $B \to h\ell\nu$




$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu_l) = (1.49 \pm 0.09 \pm 0.07) \times 10^{-4}$
$\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu_l) = (0.80 \pm 0.08 \pm 0.04) \times 10^{-4}$
$\mathcal{B}(B^0 \to \rho^- \ell^+ \nu_l) = (3.17 \pm 0.27 \pm 0.18) \times 10^{-4}$
$\mathcal{B}(B^+ \to \rho^0 \ell^+ \nu_l) = (1.86 \pm 0.10 \pm 0.09) \times 10^{-4}$
$\mathcal{B}(B^+ \to \omega \ell^+ \nu_l) = (1.09 \pm 0.16 \pm 0.08) \times 10^{-4}$
$\mathcal{B}(B^+ \to \eta \ell^+ \nu_l) = (0.42 \pm 0.12 \pm 0.05) \times 10^{-4}$
$\mathcal{B}(B^+ \to \eta' \ell^+ \nu_l) < 0.57 \times 10^{-4}$ @ $90\%CL$.

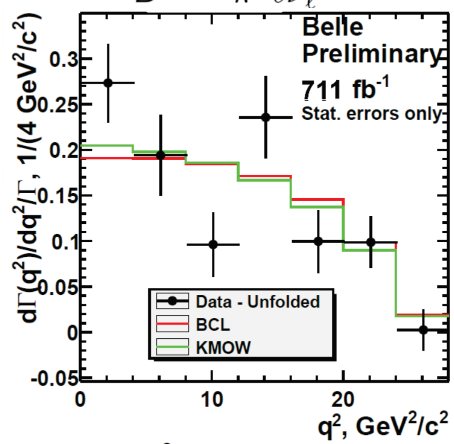
[Belle Preliminary Results]

→ **Significantly improved branching ratios compared to the past results.**

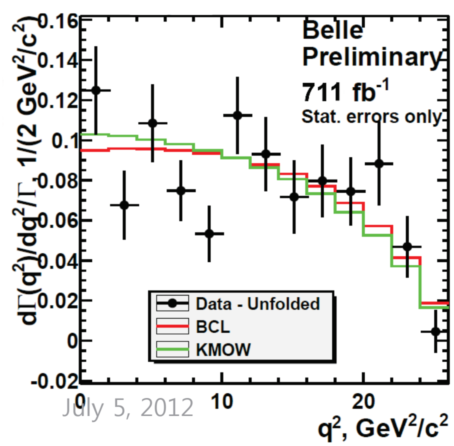
# Latest Belle results (2)

## Values of $|V_{ub}|$ from $\mathcal{B}(B \to \pi \ell \nu)$

$B^+ \to \pi^0 \ell \bar{\nu}_\ell$




$B^0 \to \pi^+ \ell \bar{\nu}_\ell$




($|V_{ub}|$(CKM fitter 2012) = $[3.14^{+0.21}_{-0.10}] \times 10^{-3}$) *Belle preliminary*

| $X_u l\nu$ | Theory | $q^2$[GeV$^2$] | $\lvert V_{ub}\rvert \times 10^3$ |
|---|---|---|---|
| $\pi^0 l\nu$ | KMOW[1] | <12 | $3.30 \pm 0.22 \pm 0.09^{+0.35}_{-0.30}$ |
| | Ball/Zwicky[2] | <16 | $3.62 \pm 0.20 \pm 0.10^{+0.60}_{-0.40}$ |
| | FNAL[3] | >16 | $3.30 \pm 0.30 \pm 0.09^{+0.36}_{-0.30}$ |
| | HPQCD[4] | >16 | $3.45 \pm 0.31 \pm 0.09^{+0.58}_{-0.38}$ |
| $\pi^+ l\nu$ | KMOW[1] | <12 | $3.38 \pm 0.14 \pm 0.09^{+0.36}_{-0.32}$ |
| | Ball/Zwicky[2] | <16 | $3.57 \pm 0.13 \pm 0.09^{+0.59}_{-0.39}$ |
| | FNAL[3] | >16 | $3.69 \pm 0.22 \pm 0.09^{+0.41}_{-0.34}$ |
| | HPQCD[4] | >16 | $3.86 \pm 0.23 \pm 0.10^{+0.66}_{-0.44}$ |

Statistical
Experimental Systematic
Theoretical

[1] PRD 83 (2011) 094031 } LCSR
[2] PRD 71 (2005) 014015 } LCSR
[3] PRD 79 (2009) 054507 } Lattice QCD
[4] PRD 73 (2006) 074502 } Lattice QCD

Calculation of $|V_{ub}|$ from different theory input for each $q^2$ range.



- I'll call $|V_{ub}|_{\rm excl}$ "done": hadronic reco + two LQCD fermion formulations used

# Some comments on $|V_{ub}|$ exclusive

- Inclusive / exclusive difference seems more worrisome than for $|V_{cb}|$
  (less quark-hadron duality concerns? weak annihilation doesn't seem huge, etc.)

- For $B \to \pi \ell \bar{\nu}$, theory is in the hand of lattice QCD

- Importance of parameterizations of $B \to \pi \ell \nu$ form factor?
  Eventually one $q^2$ bin will become competitive...

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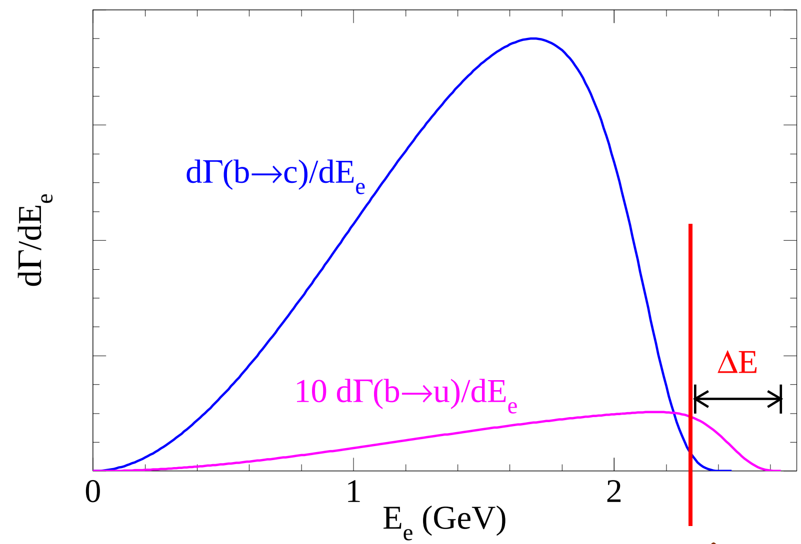
- Can LHCb help to pin down $|V_{ub}|$?
  - How well is $\overline{B}_s \to K^+ \mu \bar{\nu}$ measurable? Easier on the lattice than $\overline{B} \to \pi \ell \bar{\nu}$
  - Could $\overline{B}_d \to \pi^+ \mu \bar{\nu}$ at LHCb be competitive with $\overline{B} \to \pi \ell \bar{\nu}$ at $e^+e^-$?

  How good can the $q^2$ resolution get?

# The challenge of inclusive $|V_{ub}|$ measurements

- Total rate calculable with $\sim 4\%$ uncertainty, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$
- To remove the huge charm background $(|V_{cb}/V_{ub}|^2 \sim 100)$, need phase space cuts

  Can enhance pert. and nonpert. corrections




- Instead of being constants, the hadronic parameters are functions (like PDFs)

  Leading order: universal & related to $B \to X_s \gamma$;
  $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$: several new unknown functions

- Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ & determine its shape
- Shape in the endpoint region is determined by $b$ quark PDF in $B$ ["shape function"]
  Related to $B \to X_s \gamma$ photon spectrum at lowest order [Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

# Regions of $B \to X_s\gamma$ photon spectrum

- Important both for $|V_{ub}|$ and constraining NP
- Peak around $E_\gamma \sim 2.3\,\text{GeV}$ $\quad (m_B - 2E_\gamma \sim 0.8\,\text{GeV})$

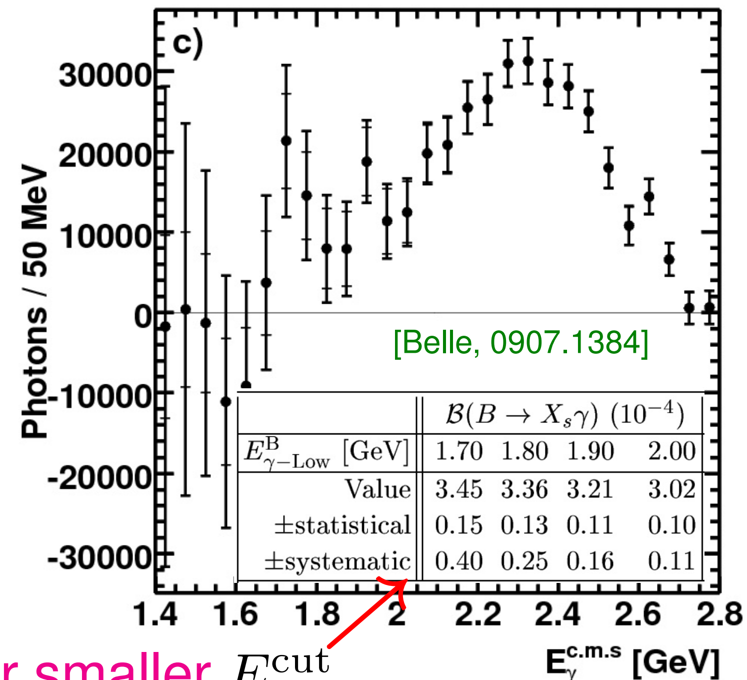
Three cases: 1) $\Lambda_{\text{QCD}} \sim m_B - 2E_\gamma \ll m_B$ ["SCET"]

2) $\Lambda_{\text{QCD}} \ll m_B - 2E_\gamma \ll m_B$ ["MSOPE"]

3) $\Lambda_{\text{QCD}} \ll m_B - 2E_\gamma \sim m_B$

Expansions and theory uncertainties differ in the $3$ regions

Neither 1) nor 2) is fully appropriate




- Experimental systematic error rapidly increases for smaller $E_\gamma^{\text{cut}}$

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- Current practice: Compare rate extrapolated to $1.6\,\text{GeV}$ with theoretical prediction

Con: (i) extrapolation uses theory, so comparison of theory and data is effectively done at the measured values; (ii) best use of the most precise measurements?

# $B \to X_u \ell \bar\nu$ is more complicated

- "Natural" kinematic variables: $p_X^\pm = E_X \mp |\vec p_X|$ (ratio is "jettiness" of hadrons)

  $B \to X_s\gamma$: $p_X^+ = m_B - 2E_\gamma$ & $p_X^- \equiv m_B$ — independent variables in $B \to X_u \ell \bar\nu$

- Three cases: 1) $\Lambda \sim p_X^+ \ll p_X^-$ } SF region

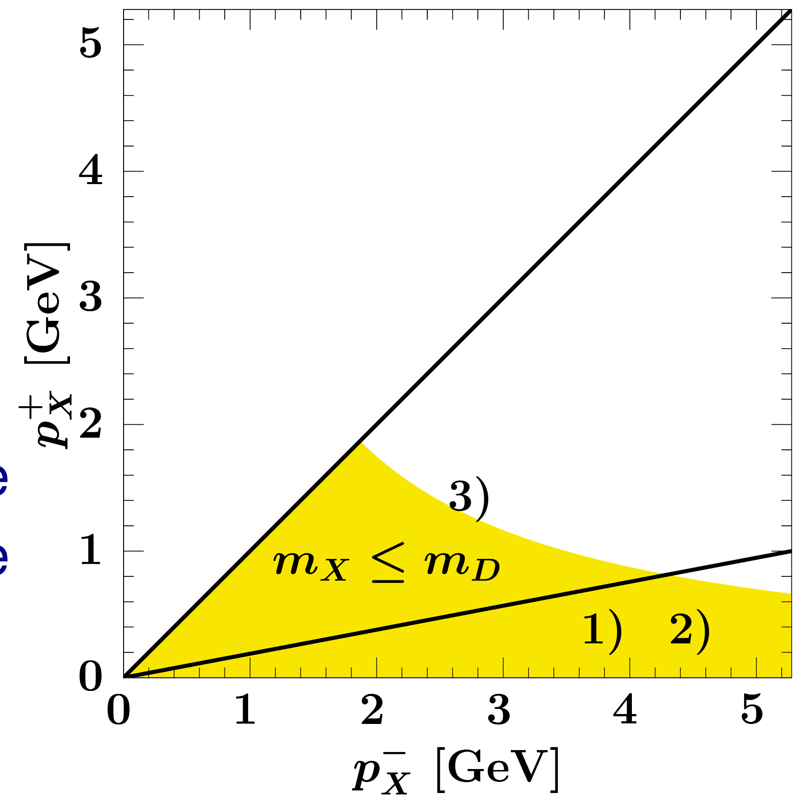
  2) $\Lambda \ll p_X^+ \ll p_X^-$ } SF region

  3) $\Lambda \ll p_X^+ \sim p_X^-$ local OPE region

  Want to make no assumptions how $p_X^-$ compares to $m_B$

- $B \to X_u \ell \bar\nu$: 3-body final state, appreciable rate in region 3), where hadronic final state not jet-like

  E.g., $m_X^2 < m_D^2$ does not imply $p_X^+ \ll p_X^-$




- Existing results based on theory in one region, extrapolated / modeled to rest



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# Past theoretical approaches

- BLNP [Bosch *et al.*] — based on SCET region
  - factorization & resummation in shape function region treated correctly
  - crossing into local OPE region not model independent
  - tied to “shape function” scheme
- DGE [Andersen & Gardi] — based on SCET region + perturbative model for the SF
  - SCET region treated correctly; motivated by renormalon resummation
- GGOU [Gambino *et al.*] — based on local OPE region + SF smearing
  - no resummation in SCET region
  - tied to “kinetic” scheme
- BLL [Bauer, ZL, Luke] — based on local OPE at large $q^2$ (but expansion scale is smaller)
  - combine $q^2$ and $m_X$ cuts, such that SF effect is kept small
- Shape function independent relations [Leibovich, Low, Rothstein; Hoang, ZL, Luke; Lange, Neubert, Paz; Lange]
  - beautiful at leading order, less so when $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ included

# SIMBA — advantages of a global fit

- Optimally combine all information, $B \to X_u \ell \bar{\nu}$, $B \to X_s \gamma$, etc.
  Consistently treat uncertainties and their correlations (exp, theo, parameters)

- Simultaneously determine:
  - Overall normalization: $\mathcal{B}(B \to X_s \gamma)$, $|V_{ub}|$
  - Parameters: $m_b$, shape function(s)

- Utilize all measurements:
  - Different $B \to X_s \gamma$ spectra, or partial rates
  - Different $B \to X_u \ell \bar{\nu}$ spectra, or partial rates
  - Include other constraints on $m_b$, $\lambda_1$, etc.
  - Eventually use or predict $B \to X_s \ell^+ \ell^-$

- Same strategy as for $|V_{cb}|$, just a lot more complicated...

# The shape function ($b$ quark PDF in $B$)

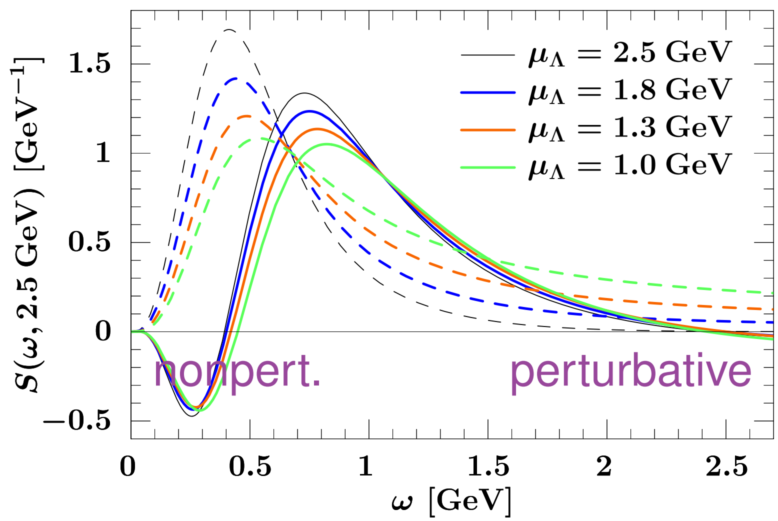
- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

  Even if $S(\omega, \mu_\Lambda)$ has exponentially small tail, RGE running gives long tail and divergent moments

  $$S(\omega, \mu_i) = \int \mathrm{d}\omega'\, U_S(\omega - \omega', \mu_i, \mu_\Lambda)\, S(\omega', \mu_\Lambda)$$

  [Balzereit, Mannel, Kilian]

  Constraint: moments (OPE) + $B \to X_s \gamma$ shape




Model $\begin{cases} S & \text{(dash)} \\ F & \text{(solid)} \end{cases}$ run to $2.5\mathrm{GeV}$

- Derive: $S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega - k, \mu_\Lambda) F(k)$

  [ZL, Stewart, Tackmann, 0807.1926]

  – Consistent setup at any order, in any scheme

  – Stable results for varying $\mu_\Lambda$ (SF modeling scale, part of uncertainty, often ignored)

  – Similar to how all matrix elements are defined [e.g., $B_K(\mu) = \widehat{B}_K \times [\alpha_s(\mu)]^{2/9}(1 + \ldots)$]

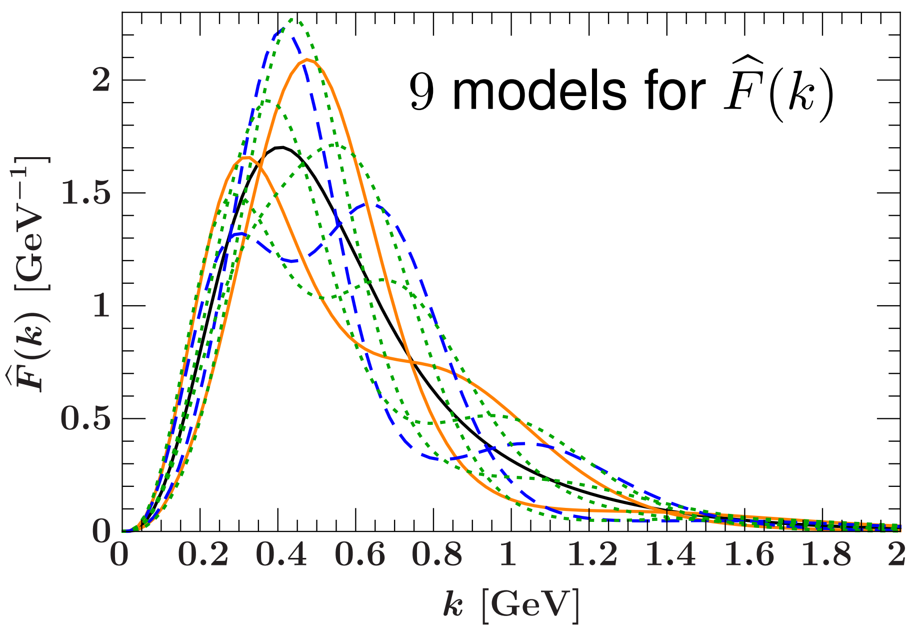
- Consistent to impose moment constraints on $F(k)$, but not on $S(\omega, \mu_\Lambda)$ w/o cutoff

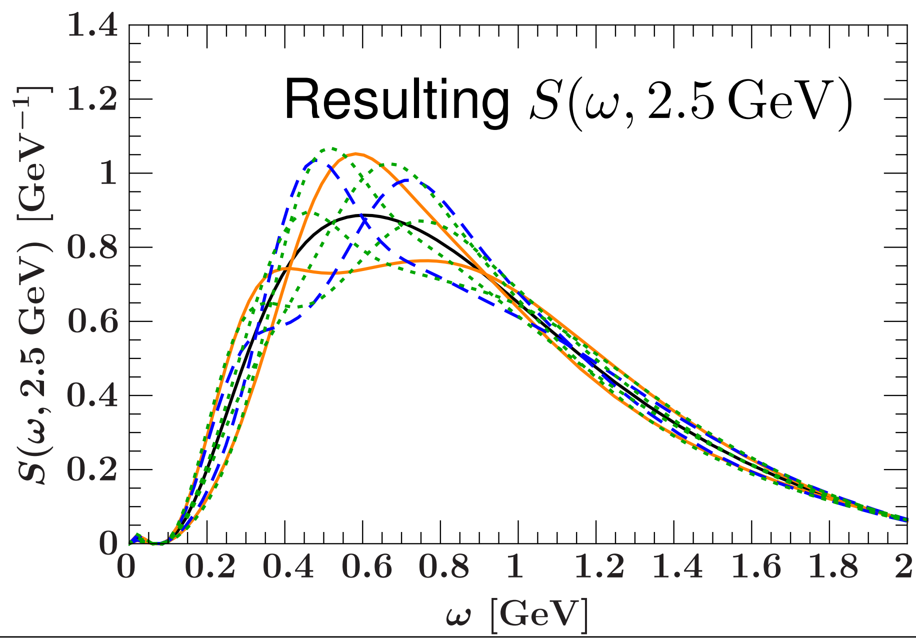
# Shape function: the bottom line

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, \widehat{C}_0(\omega - k, \mu_\Lambda)\, \widehat{F}(k)$$

$\widehat{F}$: nonperturbative
determines peak region
well-defined moments
fit from data

$\widehat{C}_0$: perturbative
generates tail consistent with RGE
divergent moments
calculable



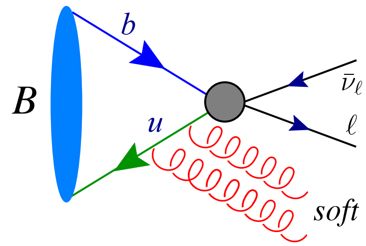



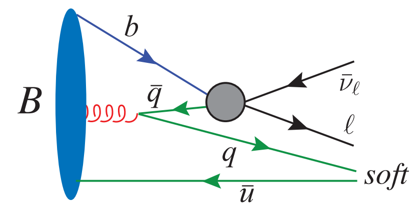
# Weak annihilation

- Hard to estimate: $(16\pi^2)\,(\Lambda_{\rm QCD}^3/m_b^3)\,\varepsilon$, centered near $q^2 = m_B^2$ and $E_\ell = m_B/2$

$$\langle B|(\bar b\gamma^\mu P_L u)\,(\bar u\gamma_\mu P_L b)|B\rangle = \frac{f_B^2\, m_B}{8}\, B_1$$

$$\langle B|(\bar b P_L u)\,(\bar u P_L b)|B\rangle = \frac{f_B^2\, m_B}{8}\, B_2$$




Overall shift vs. splitting between $B^\pm$ and $B^0$

Factorization + vacuum saturation: $B_{1,2} = \begin{cases} 1\,, & B^\pm \\ 0\,, & B^0 \end{cases}$ assume $\varepsilon \equiv B_1 - B_2 \sim 0.1$

Rate: $\Gamma_{\rm WA} = \dfrac{G_F^2 m_b^2 |V_{ub}|^2}{12\pi}\, f_B^2\, m_B (B_2 - B_1) \sim 3\%$ of $\Gamma(B \to X_u \ell\bar\nu)$ [Voloshin, hep-ph/0106040]

- Enters all $|V_{ub}|$ measurements, enhanced by $(m_b/m_c)^3 \sim 30$ in $D_{u,d,s}$ decays

$\Gamma(D^0 \to X\ell\bar\nu) \approx \Gamma(D^\pm \to X\ell\bar\nu)$ to $\lesssim 3\%$, recently $\Gamma(D_s \to X\ell\bar\nu)$ [CLEO-c, arXiv:0912.4232]

No evidence that WA is bigger when light quark in operator = spectator flavor

- Probably a smaller effect in the determination of $|V_{ub}|$ than typically assumed

[ZL, Luke, Manohar, arXiv:1003.1351; Gambino & Kamenik, arXiv:1004.0114]

# Some comments on $|V_{ub}|$ inclusive

- Is $\{$in/ex$\}$clusive tension a nuisance or tip of an iceberg? (right-handed currents?)

- Qualitatively better analyses are possible than those implemented so far
  - Fitting $F(k)$ instead of modeling $S(\omega, \mu)$
  - Designer orthonormal functions — reduce role of shape function modeling
  - Fully consistent combination of all phase space regions
  - Decouple SF shape variation from $m_b$ variation

- Inclusive $|V_{cb}|$ uses a combined fit; clearly the right method for $|V_{ub}|$ as well
  Combine all $B \to X_s\gamma$, $X_u\ell\bar{\nu}$, $X_c\ell\bar{\nu}$ data to constrain short distance physics & SFs
  Need spectra & correlations; so far we had to rely more on Belle than BaBar data

- Recently $\Gamma(D_s \to X\ell\bar{\nu})$ gave some indication of what the resolution is not

- $|V_{ub}|$ is tricky: to draw conclusions about new physics, we'll want $\geq 2$ extractions with different uncertainties to agree well (inclusive, exclusive, leptonic)

# Final remarks

# If all else fails: “Grinstein-type double ratios”

- Continuum theory may be competitive using HQS + chiral symmetry suppression

- $\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ [Grinstein ’93]

- $\dfrac{f^{(B\to\rho\ell\bar\nu)}}{f^{(B\to K^*\ell^+\ell^-)}} \times \dfrac{f^{(D\to K^*\ell\bar\nu)}}{f^{(D\to\rho\ell\bar\nu)}}$ or $q^2$ spectra — accessible soon? [ZL, Wise; Grinstein, Pirjol]

  $D\to\rho\ell\bar\nu$ data still consistent with no $SU(3)$ breaking in form factors [ZL, Stewart, Wise]

  Could lattice QCD do more to pin down the corrections?

- $\dfrac{\mathcal{B}(B\to\ell\bar\nu)}{\mathcal{B}(B_s\to\ell^+\ell^-)} \times \dfrac{\mathcal{B}(D_s\to\ell\bar\nu)}{\mathcal{B}(D\to\ell\bar\nu)}$ — very clean... by $\sim 2020$? [ZL, Ringberg ’03]

- $\dfrac{\mathcal{B}(B_u\to\ell\bar\nu)}{\mathcal{B}(B_d\to\mu^+\mu^-)}$ — uses only isospin... around $\sim 2025$? [Grinstein, CKM ’06]

The theoretically cleanest $|V_{ub}|$ I know... Need lots of LHCb and Super-$B$ data...

# Remember the $\Lambda_b$ lifetime...?

- Many people thought it was a serious challenge to theory for 20 years

PDG (1996): $\tau_{\Lambda_b} = (1.14 \pm 0.08)$ ps (first time $\sigma_{\rm WA} < 0.1$ ps)

PDG (2006): $\tau_{\Lambda_b} = (1.230 \pm 0.074)$ ps

PDG (2008): $\tau_{\Lambda_b} = (1.383^{+0.049}_{-0.048})$ ps

PDG (2010): $\tau_{\Lambda_b} = (1.391^{+0.038}_{-0.037})$ ps

PDG (2012): $\tau_{\Lambda_b} = (1.425 \pm 0.032)$ ps

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CDF: $\tau_{\Lambda_b} = (1.593^{+0.089}_{-0.085})$ ps [hep-ex/0609021] $\tau_{\Lambda_b} = (1.537 \pm 0.051)$ ps [arXiv:1012.3138]

ATLAS: $\tau_{\Lambda_b} = (1.449 \pm 0.040)$ ps [arXiv:1207.2284] [waiting for LHCb]

- We might never really know why, but "old" measurements not using fully reconstructed hadronic decays will probably be quite far from future averages

[There are examples of strongly time-dependent theory predictions — will leave it for another talk]

# Tremendous progress

- 10 years of BaBar and Belle data taking gave $\sim 100$ times earlier $(e^+e^-)$ data sets
- In some $V_{xb}$ results, progress may have seemed slower than expected, however:
  - The errors have become a lot more meaningful (both experiment & theory)
  - Better control of some theoretical assumptions (incl. lattice QCD progress)
  - Better control of experimental systematics
  - More cross-checks (theory + experiments)
  - More challenging methods used, to reduce model dependence
- It is clear that progress can / will continue
  - Mature field, still, promising experimental and theoretical ideas keep emerging
  - Much of the $B$ reco results are statistics limited
- I guess that at CKM 2020 people will mostly discuss $V_{xb}$ results with full reco data

# Conclusions

- Current status of $|V_{ub}|$ and $|V_{cb}|$ are unsettled (PDG in 2008 started to inflate error)
  (I do not think it's something as simple as $m_b$)
- Improving $|V_{xb}|$ will be important to better constrain new physics in $B^0 - \overline{B}^0$ mixing
- Recent measurements: $\tau_{\Lambda_b}$ by CDF / ATLAS and $\Gamma(D_s \to X\ell\bar{\nu})$ by CLEO-c
  taught us about what the resolution is not
- Qualitatively better inclusive $|V_{ub}|$ analysis possible than those implemented so far
- Hope to see analyses with different uncertainties to agree (incl., excl., leptonic)
- The "$B$ reco era": qualitatively new and powerful tool to go after this physics

  A compelling reason to want $\sim 100$ times larger data sets: Super-KEKB, Super-B

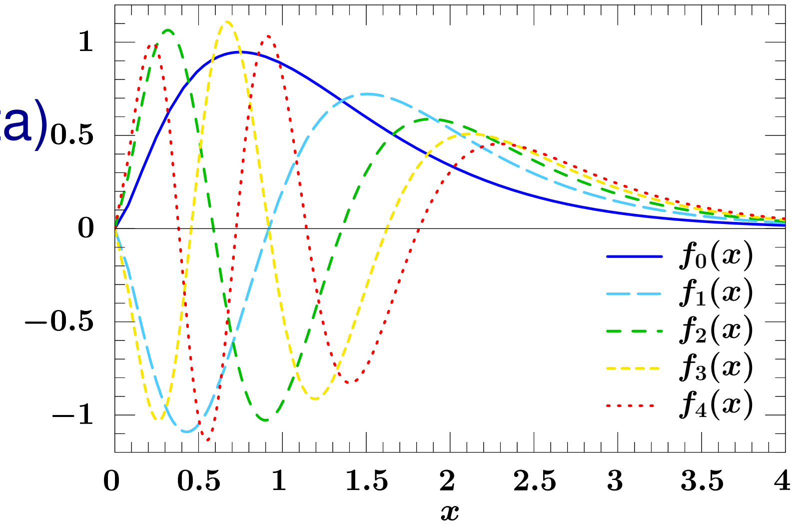

**Backup slides**

# Designer orthonormal functions

- Devise suitable orthonormal basis functions
  (earlier: fit parameters of model functions to data)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum c_n f_n(x)\right]^2,$$ $n$ th moment $\sim \Lambda_{\rm QCD}^n$

$$f_n(x) \sim P_n[y(x)]$$ $\leftarrow$ Legendre polynomials

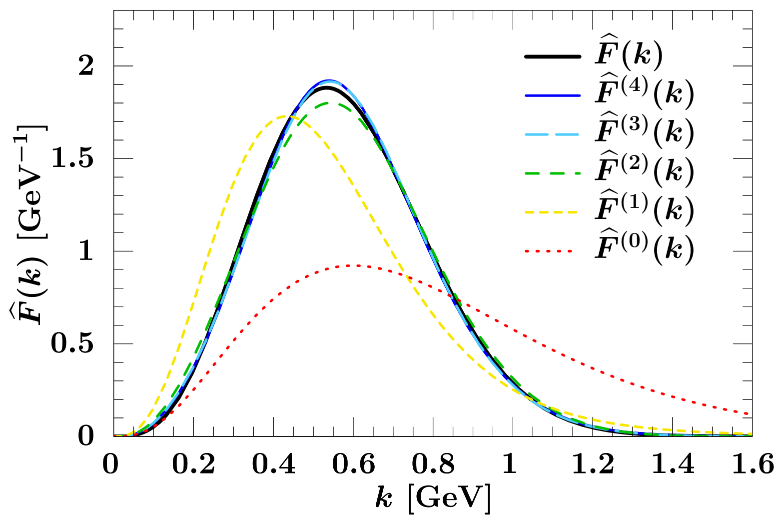



- Approximating a model shape function

Better to add a new term in an orthonormal basis than a new parameter to a model:

– less parameter correlations

– errors easier to quantify




"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

# Attempts for $\tau_{\Lambda_b}$, reasonable or not...

[hep-ph/9604202]

FAILURE OF LOCAL DUALITY IN INCLUSIVE NON-LEPTONIC HEAVY FLAVOUR DECAYS

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ABSTRACT

We argue that there is strong experimental evidence in the data of $b$- and $c$-decays that the pattern of power suppressed corrections predicted by the short distance expansion, the heavy quark effective theory and the assumption of local duality is not correct for the non-leptonic inclusive widths. The data indicate instead the presence of $1/m$ corrections that should be absent in the above theoretical framework. These corrections can be simply described by replacing the heavy quark mass by the mass of the decaying hadron in the $m^5$ factor in front of all the non-leptonic widths.

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4. Conclusion

We have presented a number of experimental facts that, in our opinion, make rather clear that $\Gamma_{NL}$ for charm and beauty decay approximately scale with the fifth power of hadron masses apart from corrections of order $1/m^2$ or smaller. These facts are the ratio of the $\Lambda_b$ and $B$ lifetimes, the value of $B_{SL}(B)$ and the charm lifetimes. This conclusion is at variance with the predictions of the short distance operator expansion approach augmented by the heavy quark effective theory. In fact, according to this theory, the relevant mass in the rate should be a universal quark mass and no corrections of order $1/m$ should be present once this mass is used. On the contrary the hadron mass differs from the quark mass by non-universal terms of

- Close to willing to throw out most of what we (thought we) knew about QCD