Lattice form factors and $|V_{cb}|$

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CKM 2012
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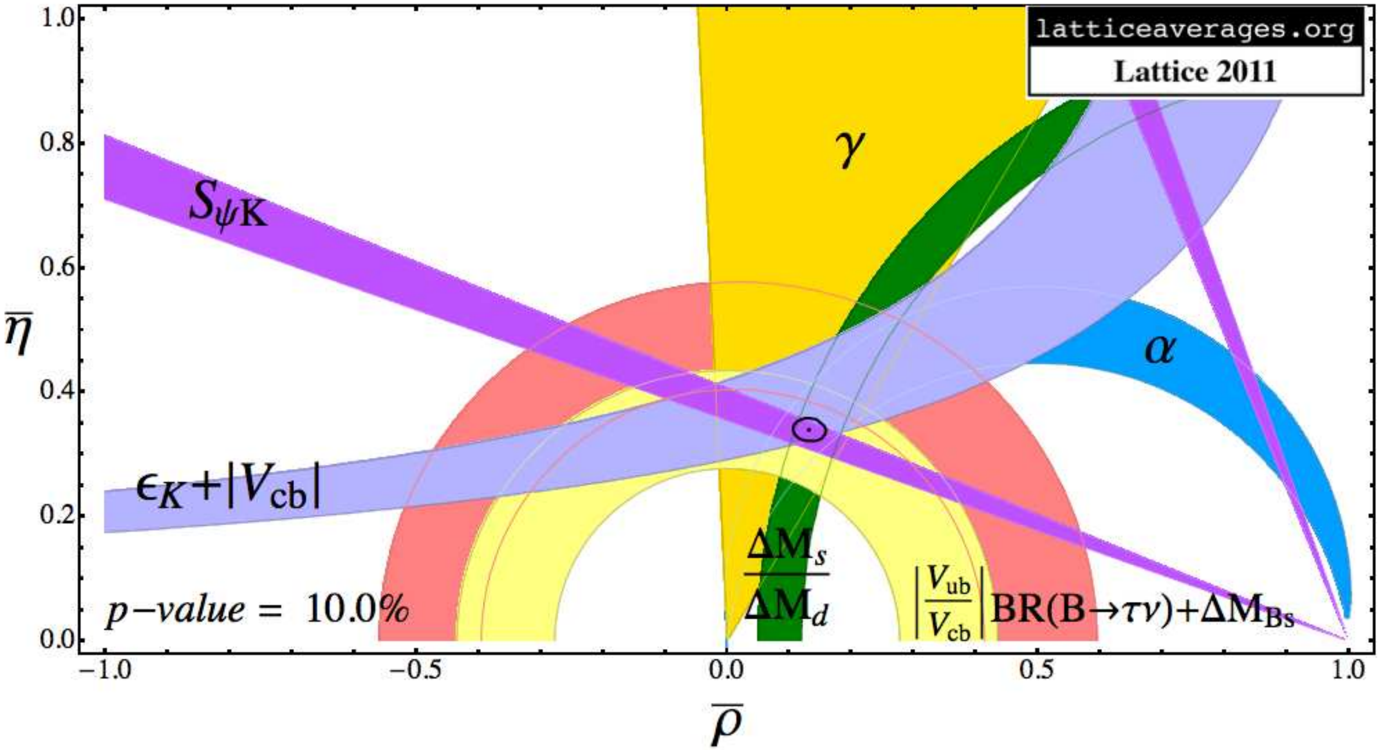
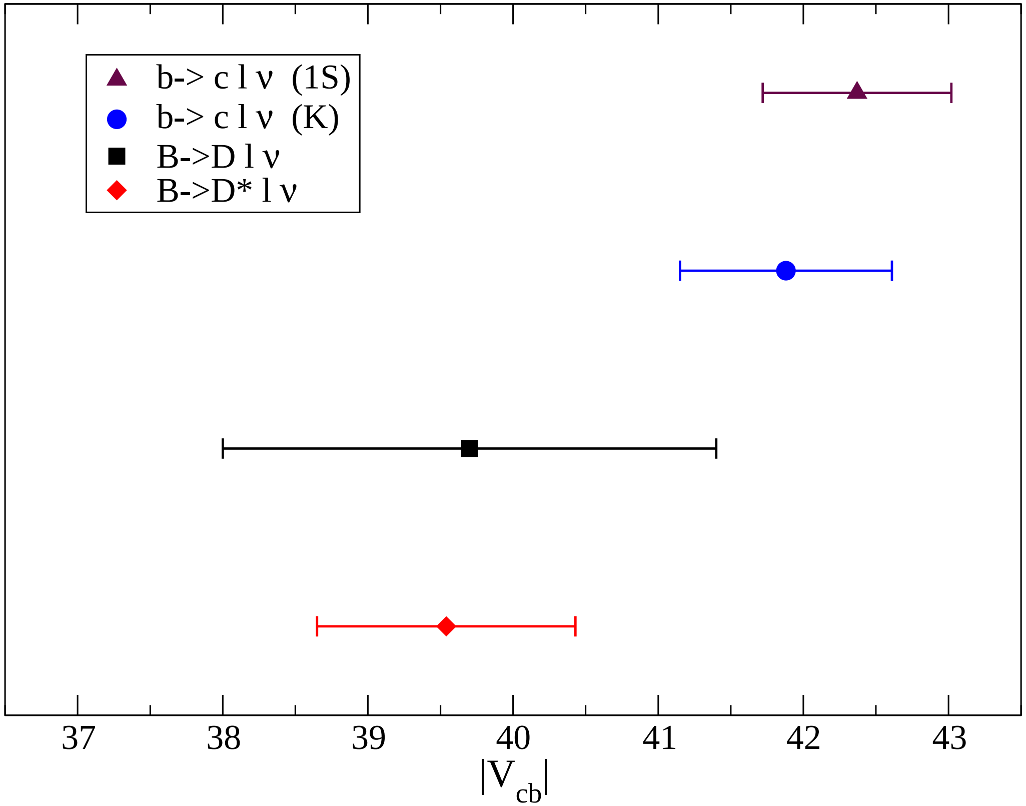
UT triangle



$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

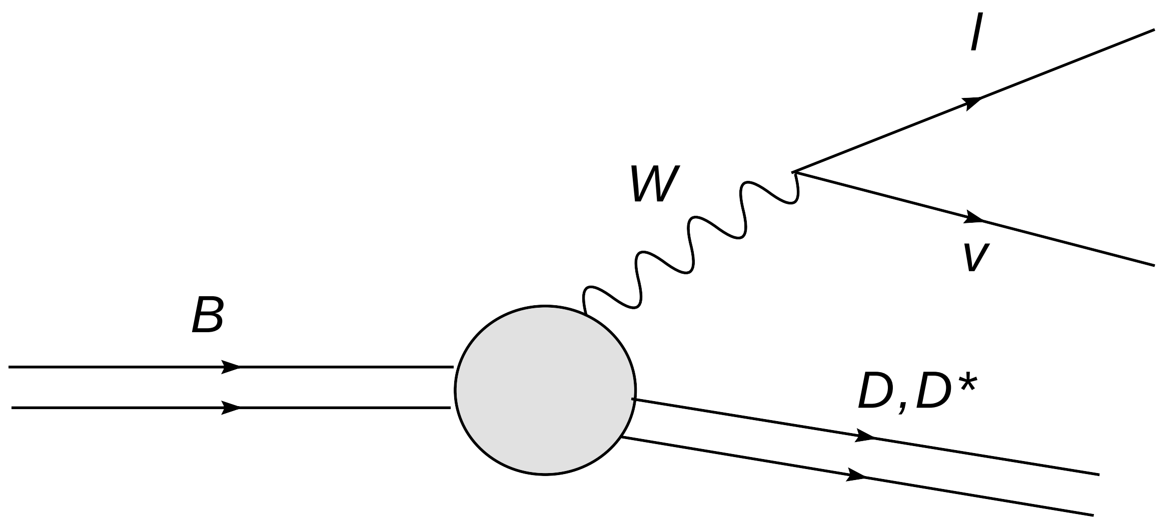
$A = |V_{cb}|/\lambda^2$. Error in $B_K \sim 2\%$, error in $|V_{cb}| \sim 2\%$.

Exclusive vs Inclusive



Discrepancy between exclusive (LQCD) and inclusive $|V_{cb}|$ is around 2σ.

Charmed B semileptonic decays



Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Lattice Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Finite volume effects

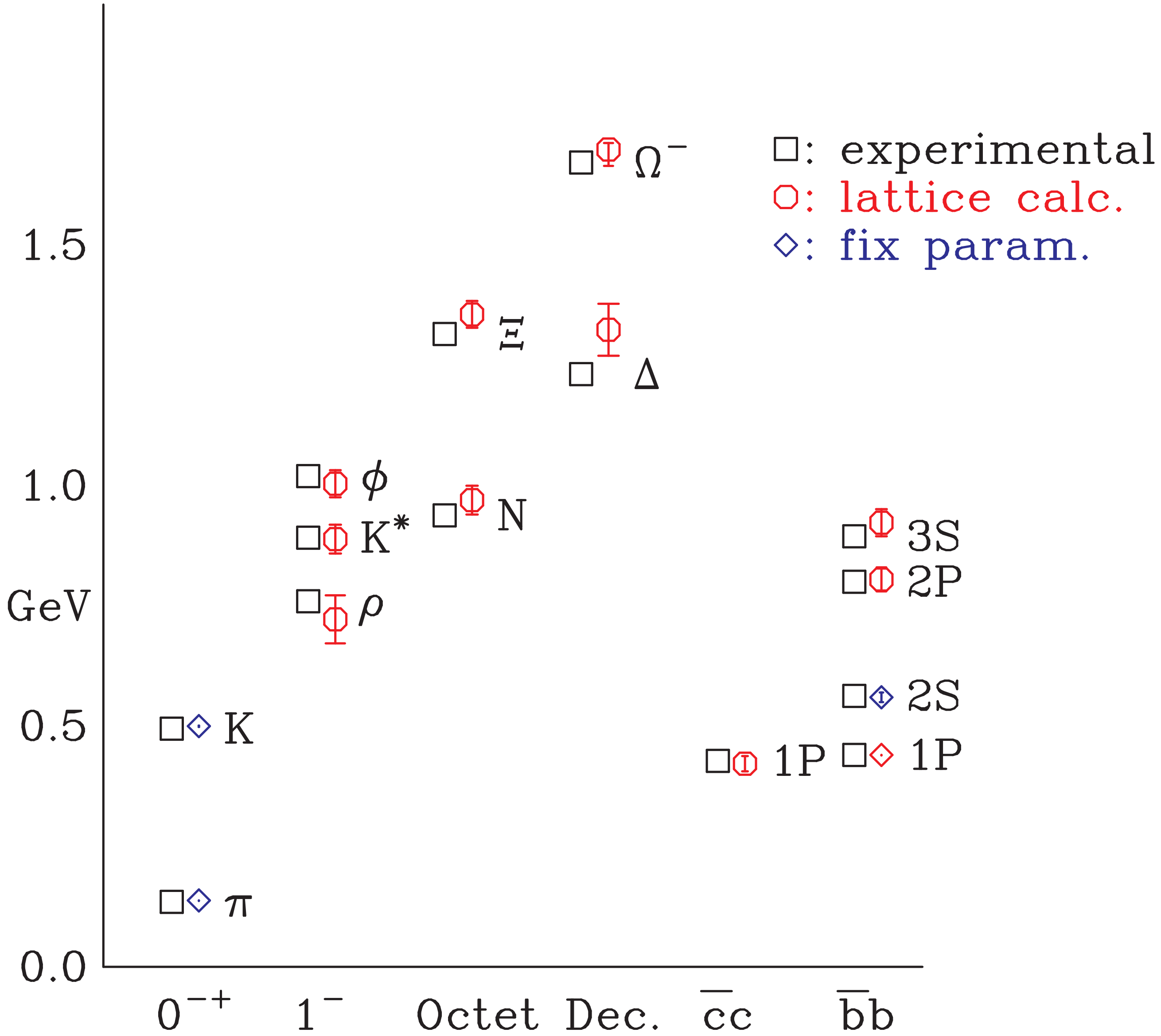
4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

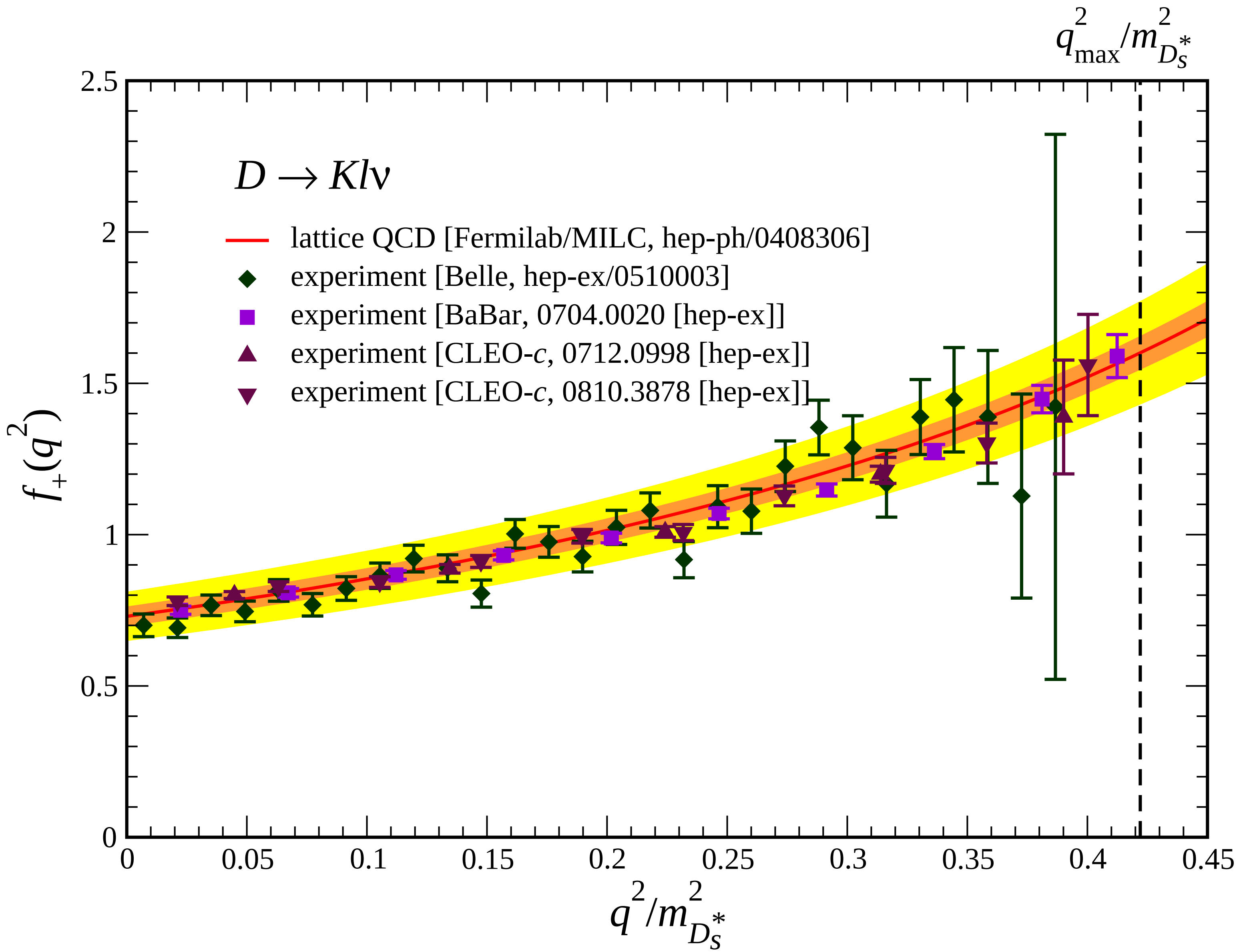
6.) Quenching. Uncontrolled!

Light Hadron spectrum

MILC and HPQCD, Rev.Mod.Phys. 82:1349 (2010)



Prediction of form factor



Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated using the OPE and perturbation theory (see previous talk!).
- Exclusive $B \to D\ell\nu$: expect legacy results from b-factories soon. To minimize errors, lattice must go to non-zero recoil.
- Exclusive $B \to D^*\ell\nu$ is very clean theoretically at zero-recoil. Still requires an extrapolation of experimental data. Unquenched lattice results at non-zero recoil are desirable.

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B\to D^*}(w)|^2 \qquad (1)$$

where $\mathcal{G}(w)|\mathcal{F}_{B\to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

History of lattice $B \to D^* \ell \nu$

2001, Quenched calculation, Hashimoto for FNAL, PRD 66:014503, 2002.

2008, Quenched calculation (non-zero recoil), de Divitis, et. al., NPB 807:373-395.

2008, Unquenched 2+1 staggered sea, JL for FNAL/MILC, PRD 79:014506.

2010, update: JL for FNAL/MILC, talk at CKM2010

2012, update: JL for FNAL/MILC this talk

- full MILC data set
- improved statistics and finer lattice spacings
- improved tunings and inconsistencies removed compared to 2008

Heavy quarks on the lattice

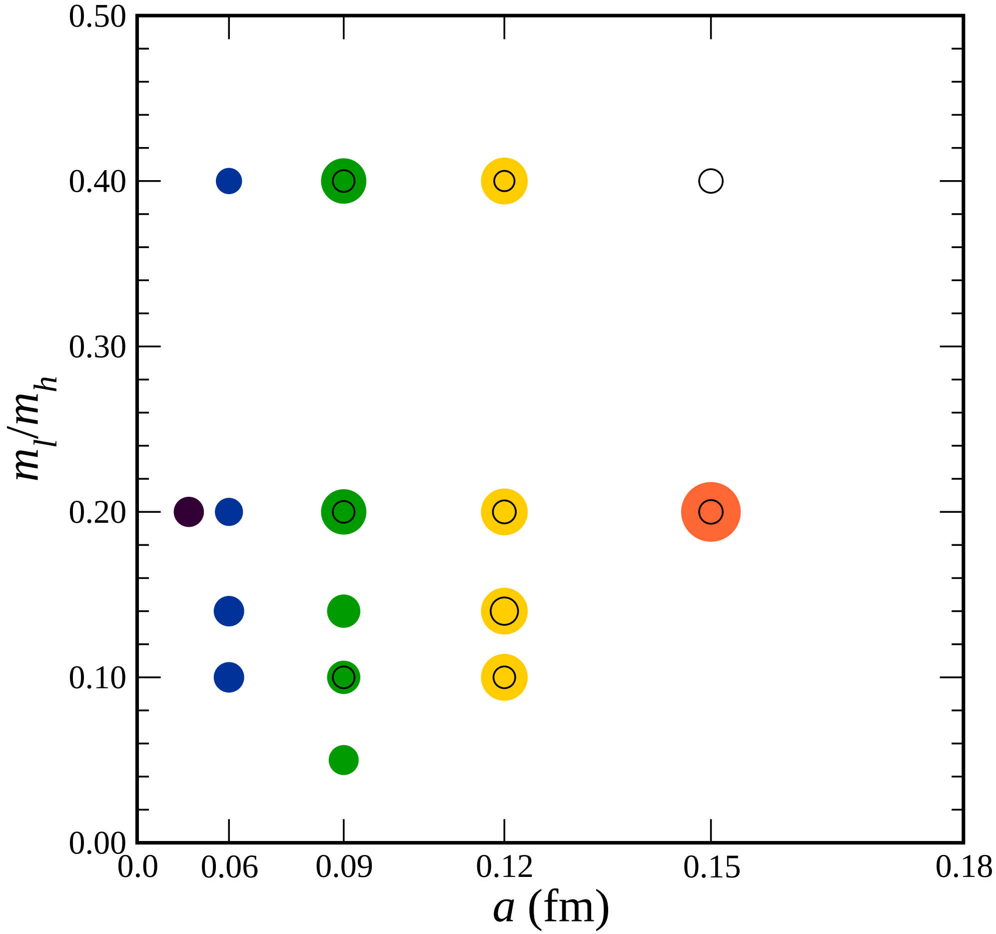
The lattice cut-off is smaller than the heavy quark masses for realistic lattices.
A solution: heavy quark effective theory(HQET)

Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

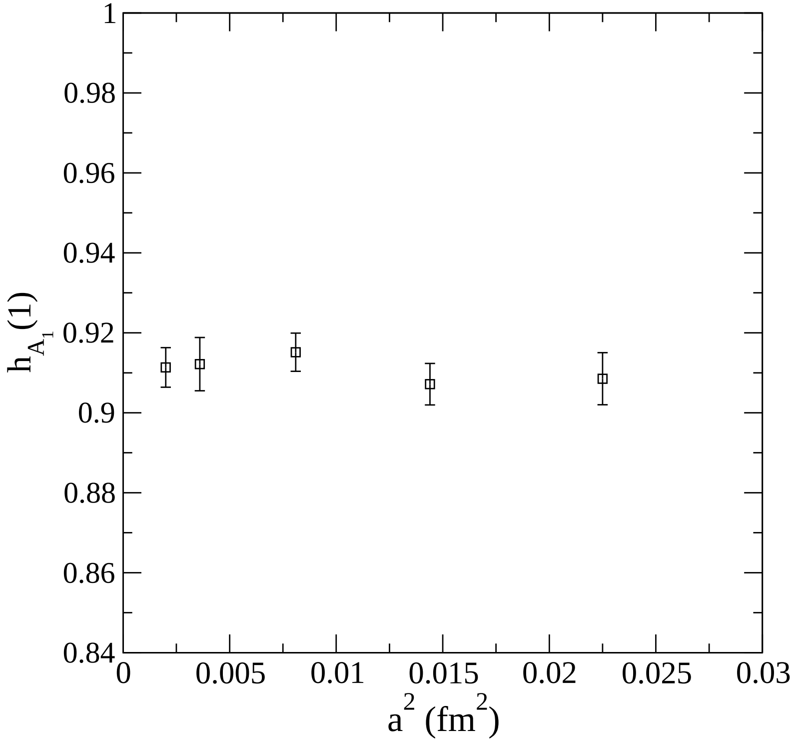
- Requires tuning parameters of the lattice action. Can be systematically improved by adding higher dimensional operators to the action.
- The currents and 4-quark operators must also be matched to continuum QCD. Typically this is done using lattice perturbation theory.
- For $B \to D^{(*)}\ell\nu$ at zero-recoil, because of Luke's Theorem the heavy-quark errors are higher order, so that the Fermilab action is highly improved for these special quantities.

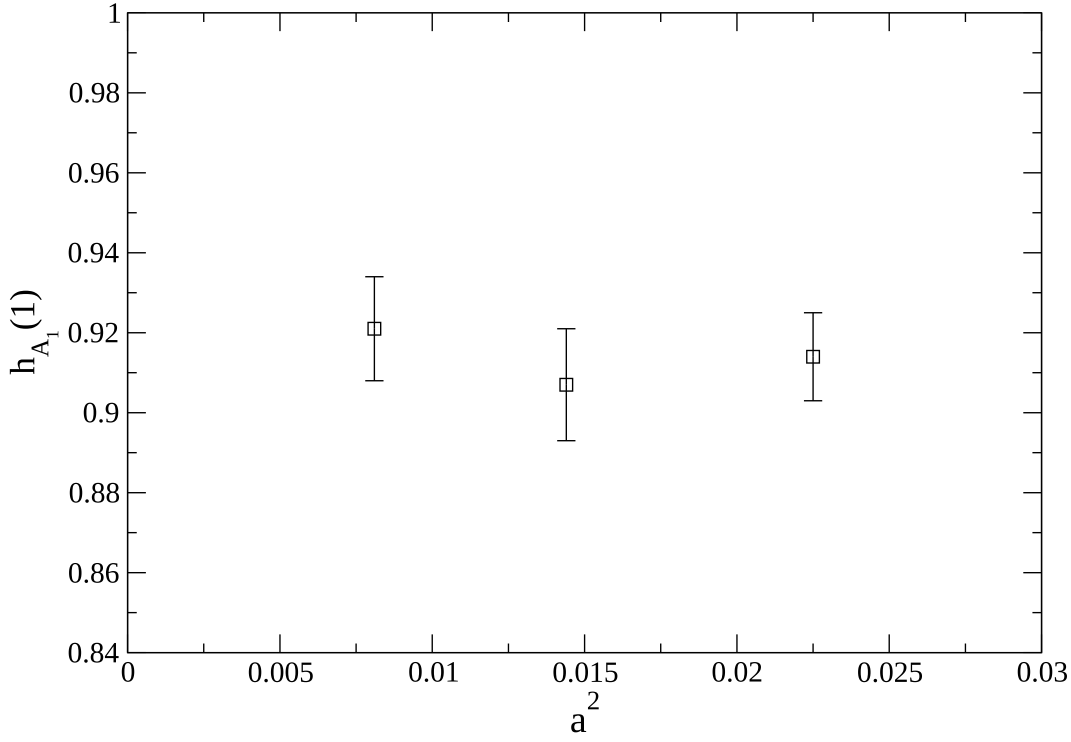
Improved data set



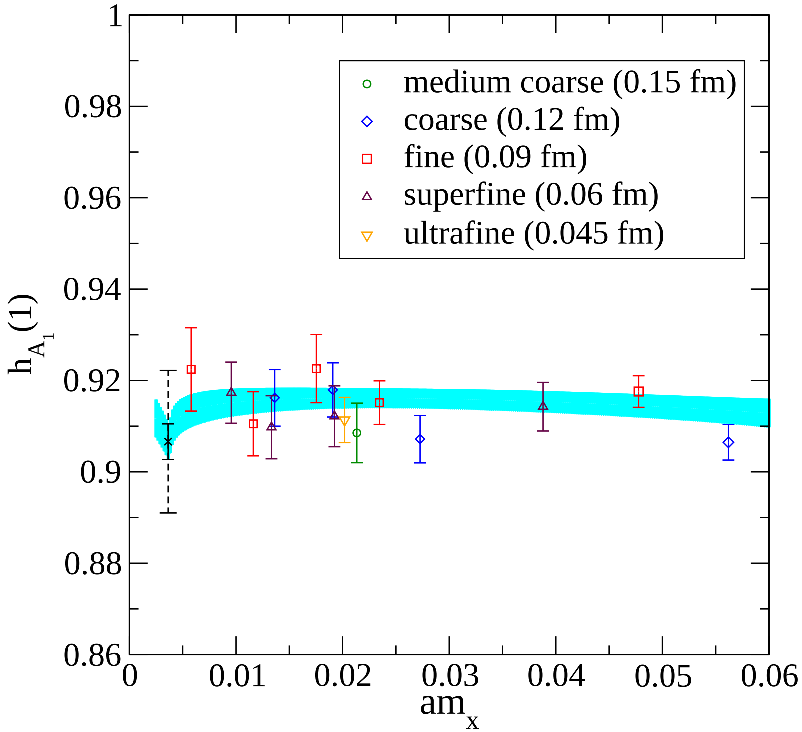
Range of lattice spacing and light quark mass used here (colored disks) and in 2008 (black circles).
The area is proportional to the size of the ensemble.

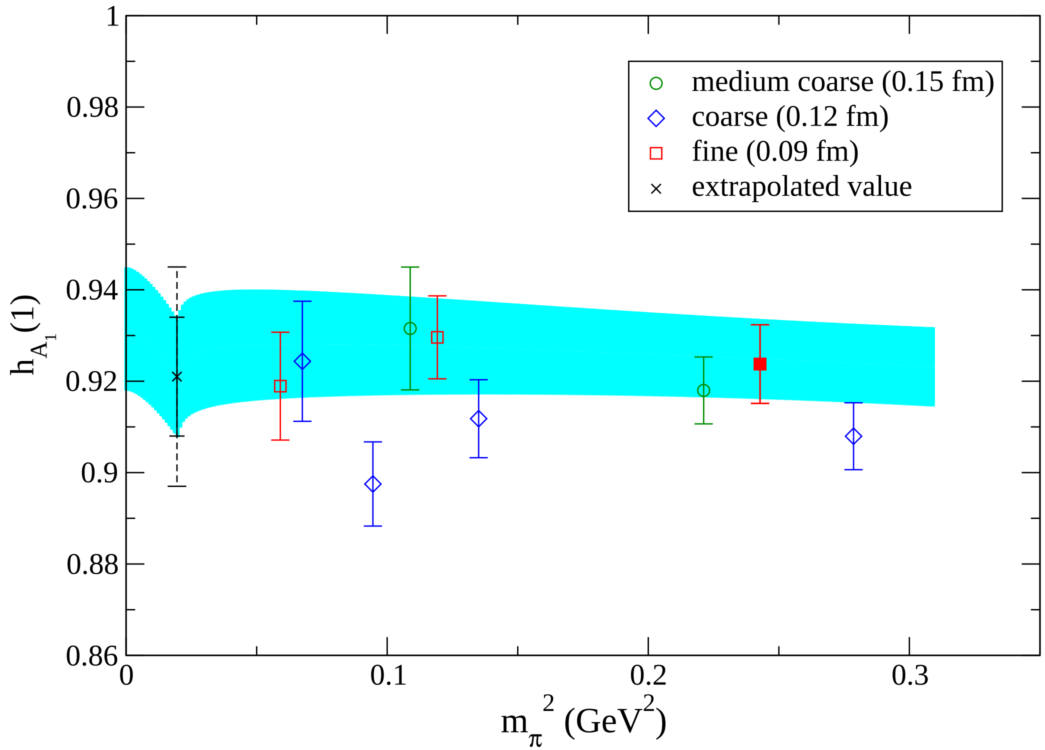
Discretization effects (2012 vs 2008)





Chiral Extrapolation (2012 vs 2008)





2012 fit: $\chi^2/\text{dof} = 3.15/5$, p-value = 0.87.

Error budget (2008 vs 2012)

uncertainty	old $h_{A_1}(1)$	new $h_{A_1}(1)$
statistical	1.4%	0.4%
$g_{\pi DD^*}$	0.9%	0.9%
chiral/continuum extrap	0.9%	0.7%
HQ discretization errors	1.5%	1.0%
kappa tuning	0.7%	-
perturbation theory	0.3%	0.3%
u_0 tuning	0.4%	-
Total	2.6%	1.6%

κ tuning error is now part of the statistical error. There is no longer a u_0 mistuning, so this error goes away.

$B \to D^* \ell \nu$ Status Summary

Slight tweaks in b quark mass determination so analysis is not yet finished.

Previous FNAL/MILC publication quoted 2.6% error on $|V_{cb}|$ and CKM 2010 update quoted 1.8%.

Present full data set analysis quotes a 1.6% error, with a central value that will not move very far from the previous update. Still 2σ or more away from inclusive determination.

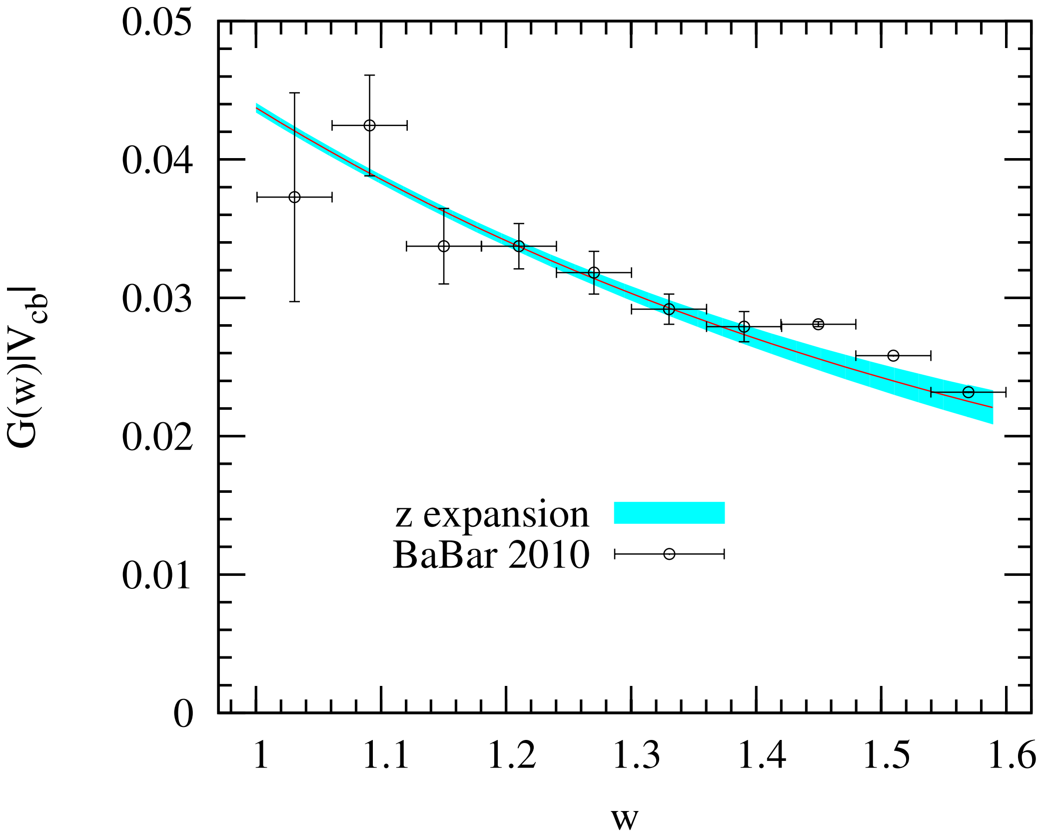
$|V_{cb}|$ from $B \to D\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} \times |V_{cb}|^2 |\mathcal{G}_{B\to D}(w)|^2 \quad (2)$$

where $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v'), and where

$$\mathcal{G}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \quad (3)$$

$B \to D\ell\nu$ at non-zero recoil



A comparison of the form factor shape using lattice calculations and the z expansion to BaBar data assuming $|V_{cb}| = 41.4 \times 10^{-3}$. More data and analysis in progress for precision determination of $|V_{cb}|$.

$R(D)$

BaBar has measured:

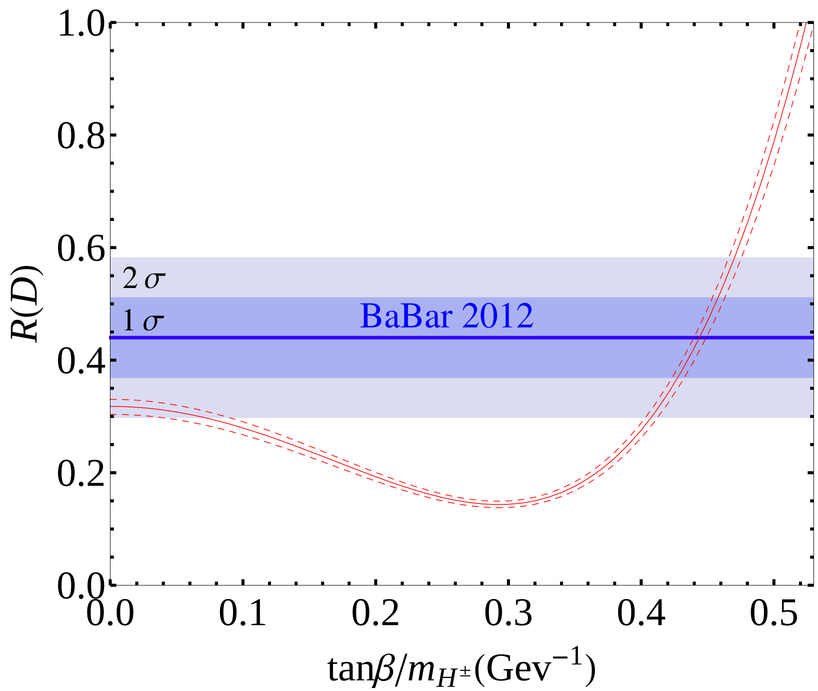
$$R(D^*) = \frac{\mathcal{B}(B \to D^* \tau \nu)}{\mathcal{B}(B \to D^* \ell \nu)} = 0.332 \pm 0.030, \quad (4)$$

$$R(D) = \frac{\mathcal{B}(B \to D \tau \nu)}{\mathcal{B}(B \to D \ell \nu)} = 0.440 \pm 0.072. \quad (5)$$

Standard Model predictions are $R(D^*) = 0.252(3)$ and $R(D) = 0.296(16)$ [Fajfer, et. al., arXiv:1203.2654] using kinematic and dispersive constraints on the shape and HQET to relate the unmeasured f_0 to the measured f_+. (Note $|V_{cb}|$ cancels in the ratio in the SM.)

Can do (first) unquenched lattice calculation of $R(D)$ as spin-off of previous work.

$R(D)$



$R(D)$ in a two-Higgs doublet model. Plot from Daping Du for the Fermilab Lattice and MILC Collaborations.
Tension in Standard Model from combination of $R(D)$ and $R(D^*)$ is reduced from 3.4 σ to 3.2 σ.
Useful to compute $R(D^*)$ using Lattice QCD. Future project!

Conclusions

Discrepancy between exclusive and inclusive $|V_{cb}|$ survives further scrutiny.

Must continue to improve calculation, and go to non-zero recoil for $B \to D^* \ell\nu$.

Calculations of $B \to D\ell\nu$ at zero-recoil underway.