# Status and outlook on global $B \to X_s \gamma$ & $|V_{ub}|$ fits

**SIMBA** Collaboration:

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# Talk overview

I. Introduction
II. Analysis and Theory Uncertainties for $B \to X_s \gamma$
III. SuperB Factories demonstration fit for $|V_{ub}|$
IV. Summary and Outlook

# I.a Introduction

Inclusive $|V_{ub}|$ in a nutshell: ( better overview → my talk yesterday)

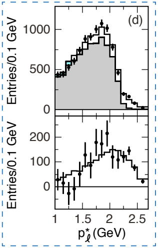
1 Measure the partial branching fraction $\Delta\mathcal{B}(B \to X_u \, \ell \, \bar{\nu}_\ell)$

  * Select phase-space regions more-or-less enriched with $B \to X_u \, \ell \, \bar{\nu}_\ell$

$$\to \quad |V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}}{\tau_B \Delta\Gamma_{\text{theory}}}}$$

2 External input needed for $\Delta\Gamma_{\text{theory}}$

  * $m_b$ from $B \to X_c \, \ell \, \bar{\nu}_\ell$ or elsewhere
  * Shape function model (tested against $B \to X_s \, \gamma$)




[Phys.Rev.D86,032004]

Inclusive $|V_{cb}|$ in a nutshell:

* Global fit to kinematic moments measured in $B \to X_c \, \ell \, \bar{\nu}_\ell$ to extract $|V_{cb}|$, $m_b$, and non-perturbative parameters.

Goal of SIMBA: Employ strategy that proved successful for $|V_{cb}|$ to $|V_{ub}|$.

# I.b SIMBA

Goal of SIMBA: Employ strategy that proved successful for $|V_{cb}|$ to $|V_{ub}|$.

- Determine $|V_{ub}|$, $m_b$, and shape function (SF) simultaneously.
- Combine different decay modes, measurements, and experiments:
  1. Various $B \to X_s \gamma$ spectra
     Information about the SF, $m_b$, and $C_7$
  2. Various $B \to X_u \ell \bar{\nu}_\ell$ partial BFs (or spectra)
     Information about $|V_{ub}|$, the SF, and $m_b$. Differential spectra would be more powerful in constraining the SF
  3. External constraints on $m_b$, and shape function moments (from $B \to X_c \ell \bar{\nu}_\ell$ or other sources)

Benefits of a global fit: Minimizing uncertainties, by making maximal use of all available information; consistent treatment of all correlated uncertainties

(experimental, theoretical, and from input parameters)

Where do we stand?

1. $B \to X_s \gamma$: OK $\to$ progress on theory uncertainties, will show latest fits
2. $B \to X_u \ell \bar{\nu}_\ell$: More work needed $\to$ show toy fit using theory at NLO
3. $B \to X_c \ell \bar{\nu}_\ell$ constraints: (OK) $\to$ will not show fits.

[Anna-Sophia Lacker]

# I.c Master formulae

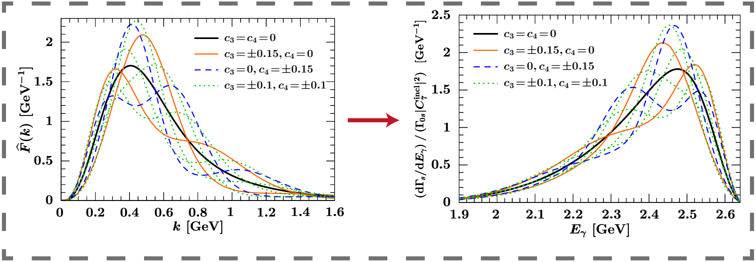
The Master formulae for differential spectra:

$$\mathrm{d}\Gamma_u \propto |V_{ub}|^2 \left\{ \widehat{W}_u \otimes \widehat{F} + \ldots \right\}$$

$$\mathrm{d}\Gamma_s \propto |V_{tb}\, V_{ts}^*|^2\, m_b^2 \left\{ \left|C_7^{\mathrm{incl}}\right|^2 \left[ \left( \widehat{W}_{77}^{\mathrm{sing}} + \widehat{W}_{77}^{\mathrm{nons}} \right) \otimes \widehat{F} + \sum_n W_{77,n}\, F_n^{subl} \right] + \sum_{i,j\neq 7} \left[ \Re(C_7^{incl})\, 2C_i\, \widehat{W}_{7i}^{nons} + C_i C_j \widehat{W}_{ij}^{nons} \right] \otimes \widehat{F} + \ldots \right\}$$

shape function $\widehat{F}$ $\leftrightarrow$ Differential shape ; $|V_{ub}|^2$ and $|V_{tb}\, V_{ts}^*|^2$ $\leftrightarrow$ Normalization of spectra

$\rightarrow$ Different SFs lead to different differential spectra:



# Part II

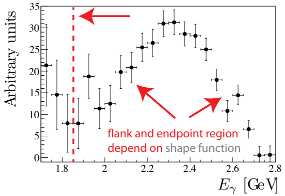
Analysis of $B \to X_s \gamma$ and Theory Uncertainties

# II.a Introduction

- $B \to X_s \gamma$ very promising to probe Flavor sector for new physics
- Most precise measurements at high $E_\gamma$

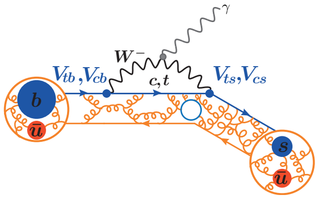
  $\Updownarrow$

  Theory most precise with low $E_\gamma$ cut

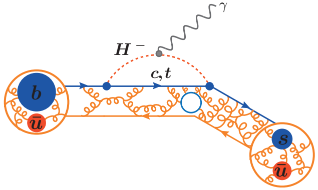



[PRL:103241801]

Standard Model $B \to X_s \gamma$:




2HDM contribution:




- Rising $E_\gamma$ cut $\leftrightarrow$ dependence on parton distribution function of $b$-quark ($\hat{=}$ Shape function)
- **HFAG** extrapolates $\Delta\mathcal{B}$ to a lower cut $E_\gamma > 1.6$

| Reference | $\Delta\mathcal{B}(E_\gamma > 1.6\text{ GeV})$ |
|---|---|
| **HFAG** [arXiv:1010.1589] | $(\mathbf{3.55 \pm 0.24 \pm 0.09}) \times \mathbf{10^{-4}}$ |
| Misiak et al. [PRL:98:022002] | $(\mathbf{3.15 \pm 0.23}) \times \mathbf{10^{-4}}$ |

$\Rightarrow$ **SIMBA** tests Standard Model without need of extrapolations

# II.b Shape function and Master fomulae

- Treat unknown shape function $\widehat{F}(k)$ as expansion of set of basis functions:

$$\widehat{F}(k) = \frac{1}{\lambda}\left[\sum_n c_n\, f_n(k)\right]^2 \qquad \text{with} \quad \int_0^\infty \mathrm{d}k\, \widehat{F}(k) = \sum_n c_n^2 = 1$$

$\lambda$ is a parameter of the basis, $f_n$ are the basis functs. with coeff. $c_n$.

- Non-perturbative physics in coefficients $c_n$ $\rightarrow$ determine from measured differential $E_\gamma$ spectra
- finite exp. input $\leftrightarrow$ series must be truncated

Aim negligible model dependence w.r.t. exp. uncert.

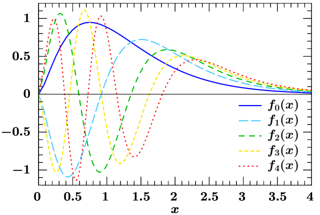
- Master formula for differential decay rate:

$$\mathrm{d}\Gamma_s \propto |V_{tb}\, V_{ts}^*|^2\, m_b^2 \Big\{ \left|C_7^{\text{incl}}\right|^2 \Big[ \Big(\widehat{W}_{77}^{\text{sing}} + \widehat{W}_{77}^{\text{nons}}\Big) \otimes \widehat{F} + \sum_n W_{77,n}\, F_n^{subl} \Big] + \sum_{i,j\neq 7} \Big[ \Re(C_7^{incl})\, 2C_i\, \widehat{W}_{7i}^{nons} + C_i C_j \widehat{W}_{ij}^{nons} \Big] \otimes \widehat{F} + \dots \Big\}$$

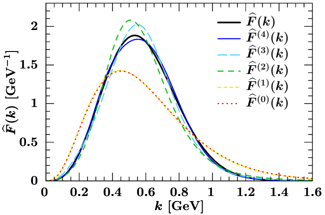
$C_7^{\text{incl}}$ sums all contributions creating same effective $b \to s\gamma$ vertex prop. to $C_7$. Included at full **NNLL+NNLO**. $C_{i\neq 7}$ fixed at SM values.

- Absorb sub-leading $1/m_b$ corrections: $\widehat{\mathcal{F}}(k) = \widehat{F}(k) + \frac{1}{m_b}\sum_n F_n^{subl}$

Used basis functions:




Expansion of model function:




[PRD:78:114014]

# II.c Truncation uncertainty and basis

- finite exp. input $\leftrightarrow$ series must be truncated

$$\widehat{F}(k) = \frac{1}{\lambda}\left[\sum_n c_n\, f_n(k)\right]^2$$

$\Rightarrow$ Induces residual basis (= model) dependence

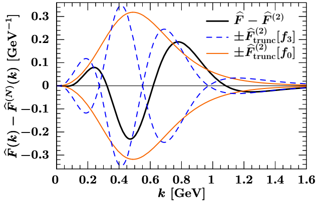
- Truncation error scales with truncation order $N$

$$1 - \sum_{n=0}^{N} c_n$$

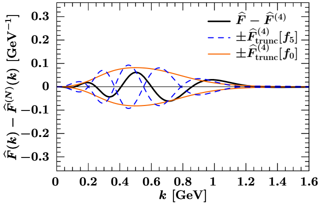
- Optimal $N$ and $\lambda$ (= basis) determined from data
  - $\Rightarrow$ Choose $\lambda$ so series converges quickly
  - $\Rightarrow$ Choose $N$ so truncation error is small w.r.t. exp. uncert.
  - $\Rightarrow$ Add more terms with more precise data

$\Rightarrow$ Must be careful not to 'overtune' things

Truncation error with $N = 2$:




Truncation error with $N = 4$:




[PRD:78:114014]

# II.d Experimental input and Fit

- Analyze four $E_\gamma$ spectra from *BABAR* and *Belle*

  a *Belle* inclusive (in $\Upsilon(4S)$ frame): Inclusive $E_\gamma$ spectrum using **605 fb$^{-1}$** and a leptonic tag.
  $\Rightarrow$ Use eff. corrected spectrum and smear theory

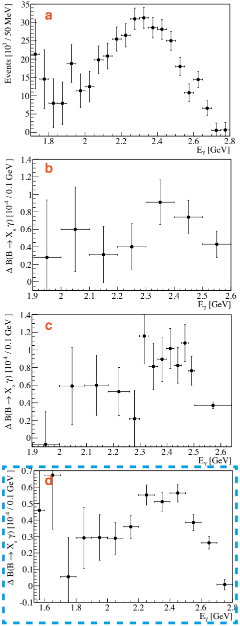
  b *BABAR* with hadronic tags (in $B$ frame): $E_\gamma$ spectrum using **210 fb$^{-1}$** and a hadronic tag.

  c *BABAR* sum-over-exclusive modes (in $B$ frame): $E_\gamma$ spectrum is recon. using the had. mass $m_X$ using **82 fb$^{-1}$**.

  d *BABAR* inclusive (in $\Upsilon(4S)$ frame): Inclusive $E_\gamma$ spectrum using **347 fb$^{-1}$**
  $\Rightarrow$ Use eff. corrected and resol. unfolded spectrum

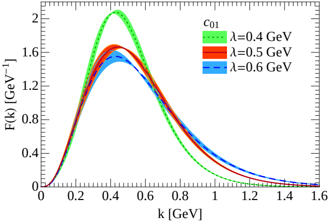
Fit Procedure: Use a $\chi^2$ fit

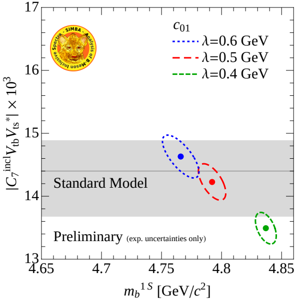
- Float $C_7^{\text{incl}}$ and number of $c_n$ coefficients
  ($C_{i\neq7}$ fixed at SM values)
- Evaluate model dependence for several bases:
  Different Bases $\leftrightarrow$ $\lambda = 0.4 - 0.6$ GeV
- Pick an expansion with negligible model dependence w.r.t. experimental uncertainty



# II.e Basis independence

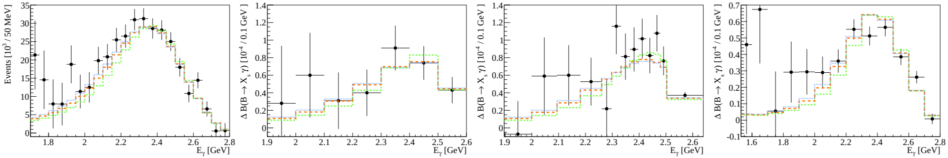
- Fit with two basis functions ($c_{01}$):
  - → Equivalent to fixed model with fitted 1st moment
  - → All fits with good $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50



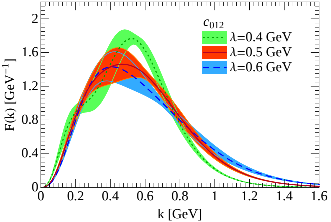

The fitted $|C_7^{\rm incl}\, V_{tb}\, V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\, V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

⇒ Exp. uncertainties underestimate model dependence

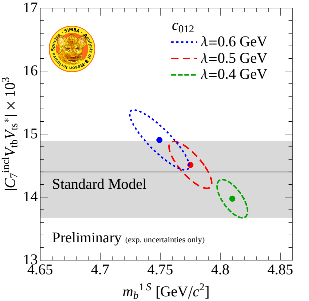


# III.e Basis independence

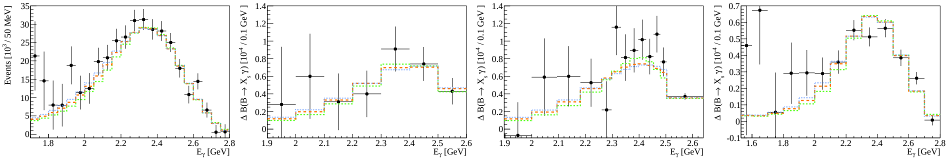
- Fit with three basis functions ($c_{012}$):




- $\chi^2$/ndf: 46.5/49; 42.5/49; 41.6/49

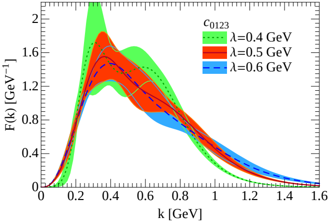


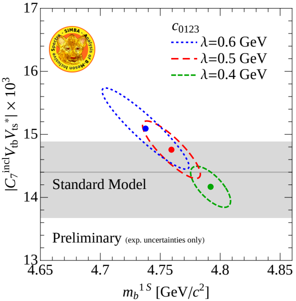

The fitted $|C_7^{incl}\, V_{tb}\, V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\, V_{ts}^*| = 40.68^{+0.4}_{-0.5}$



## II.e Basis independence

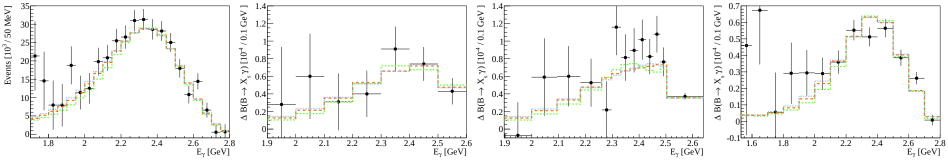
- Fit with four basis functions ($c_{0123}$):



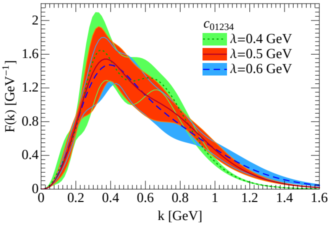

The fitted $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

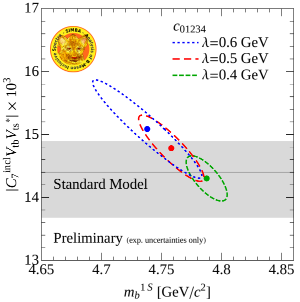
- $\chi^2/\text{ndf}$: 43.7/48; 41.7/48; 41.4/48



# II.e Basis independence

- Fit with five basis functions ($c_{01234}$):

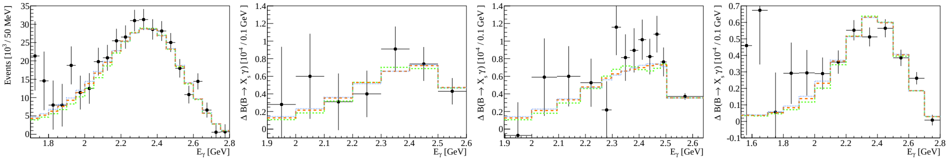



- $\chi^2$/ndf: 43.0/47; 41.6/47; 41.4/47

⇒ With enough coeff., results agree within uncert. and become basis (= model) independent

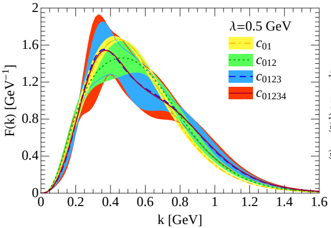
The fitted $|C_7^{incl}\ V_{tb}\ V_{ts}^*|$ values are compared with the NLO Standard Model prediction using $|V_{tb}\ V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

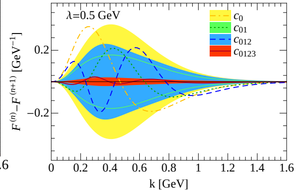


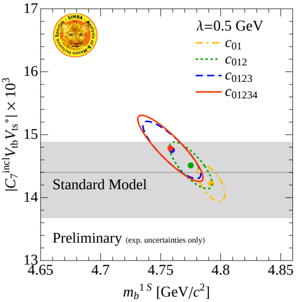
# II.f Fit result for $\lambda = 0.5$ GeV

- Fits with 2,3,4 & 5 basis functions: ($c_{01}, c_{012}, c_{0123}, c_{01234}$)
- Shape function and estimated basis dependence

determined from $n+1$ coefficient and envelop from first basis function



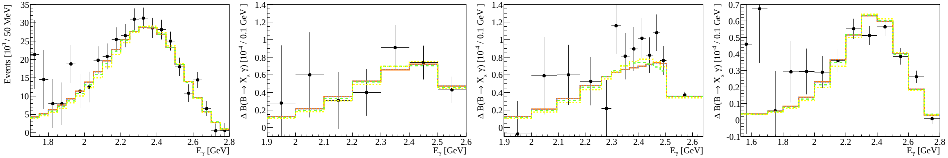

⇒ Uncertainties underestimated with too few coeff.

→ would need to include additional uncertainty due to truncation

⇒ Very little change by including 5*th* coefficient ($c_4$)

→ truncation uncertainty negligible compared to other uncertainties

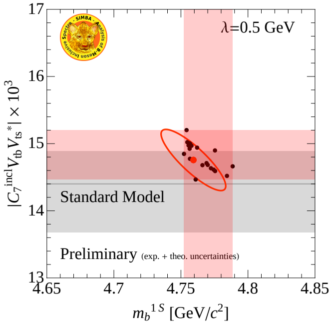
Fitted values of $|C_7^{\text{incl}} V_{tb} V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb} V_{ts}^*| = 40.68^{+0.4}_{-0.5}$



# II.g Theory Uncertainties

- Largest theory uncert. from higher order pert. theory.
- Evaluated by varying SCET scales: $\mu_h$; $\mu_j$; $\mu_s$; $\mu_{NS}$
- Probe contour with 22 variations and repeat fits:

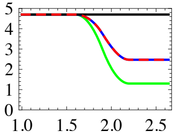
Use fit with $\lambda = 0.5$ GeV and $c_{0123}$



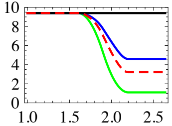

The red shaded region shows the largest extend of the probed variations

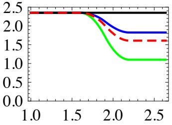
⇒ Shift central value scales to middle of contour results in symmetric theory uncert. interval.

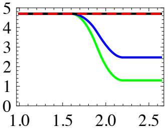
Curent central value scales:



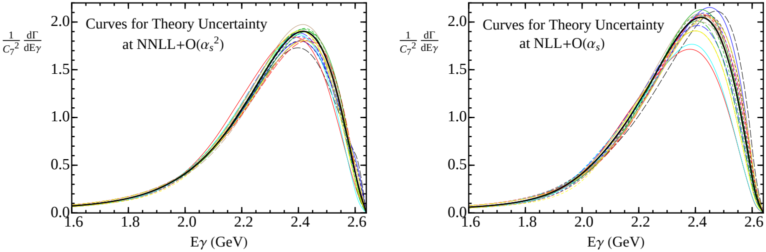
3 example variations:



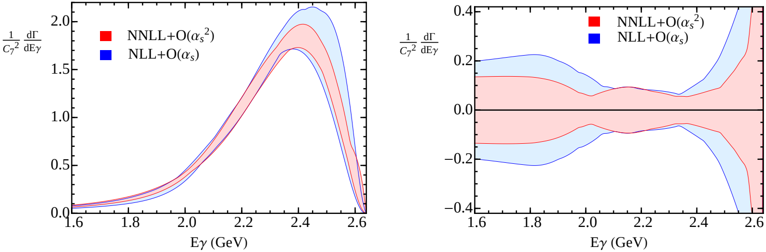




# II.h Differential theory uncertainty



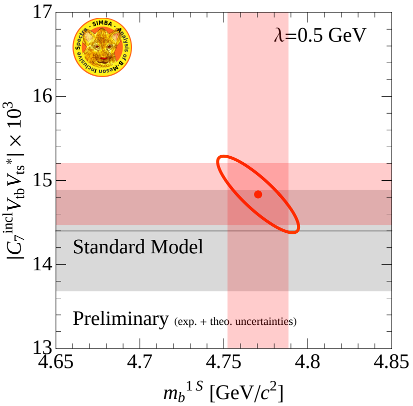

top: the impact on the scale variations on the differential spectra at NLL and NNLL are shown

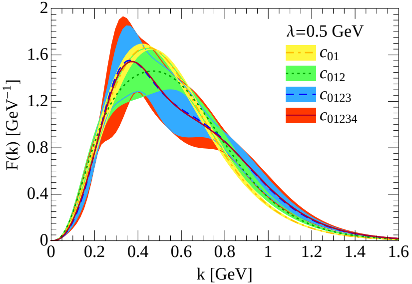



bottom: the resulting envelope and normalized envelopes at NLL and NNL are shown

# II.i Summary for $B \to X_s\gamma$

- Obtained value of $C_7^{\rm incl}$ which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data




⇒ Test of Standard Model with negligible model uncertainties from non-perturbative QCD effects

# Part III

SuperB Factory demonstration fit for $|V_{ub}|$

# III.a SuperB Factory demonstration fit

Why global fits at SuperB Factories? Global fit approach can be very powerful with high statistics

* Measure spectra in addition to (partial) BFs to maximize the available shape information, especially in $B \to X_u \ell \bar{\nu}_\ell$

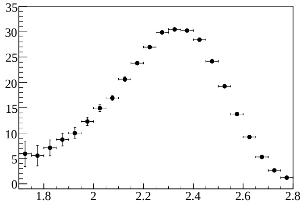
  Shape information is the key to constraining subleading corrections

* Large datasets can be taken advantage to aggressively reject background at the cost of efficiency and to maximize resolution

  (Super clean B-tagging idea as aired by K. Tackmann at SuperB workshop)

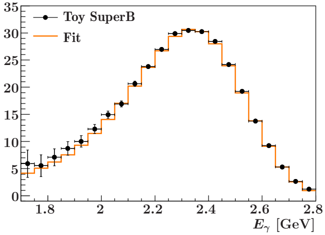
## Toy $B \to X_s \gamma$ for $75\,\text{ab}^{-1}$

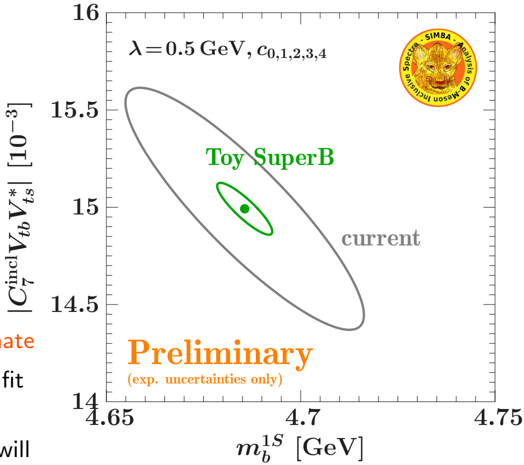
- Spectrum generated with $\lambda = 0.6$ GeV, $c_0 = 1$
- Uncertainties and correlations obtained from incl. *Belle* spectrum:
  * Stat. uncertainties scaled by luminosity
  * Syst. uncertainties scaled by 1/3
  * Correlations and detector res. assumed to be the same (likely a bit on the optimistic side)



# III.b SuperB Factory $B \to X_s\gamma$ fit

Fit result with 5 coefficients and $\lambda = 0.5$ GeV:



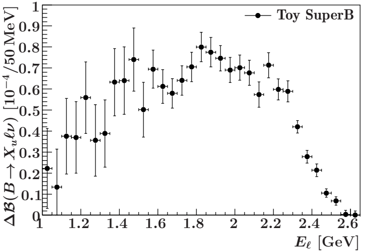



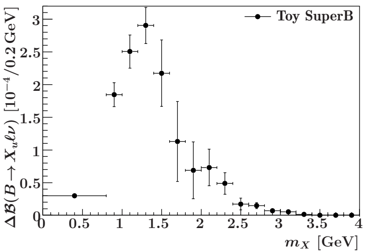

- Theoretical uncertainties will dominate
- High precision data can be used to fit more $c_n$ and for subleasing effects
- Everything at NLL+NLO since we will also include $B \to X_u \, \ell \, \bar{\nu}_\ell$, for simplicity ignore subleading SF

# III.c SuperB Factory $B \to X_s\gamma$ fit

## Toy $B \to X_u\,\ell\,\bar{\nu}_\ell$ for 75 $\mathrm{ab}^{-1}$

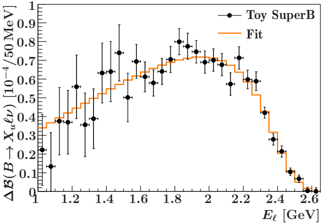
- $m_X$ and $E_\ell$ spectra generated with $\lambda = 0.6$ GeV, $c_0 = 1$
- Uncertainties and corral. inspired by BaBar [Phys.Rev.D86,032004]
  - * Assuming main uncertainties and corr. due to $B \to X_u\,\ell\,\bar{\nu}_\ell$ background
  - * Aiming to be conservative, but clear caveat: no resolution effects considered.

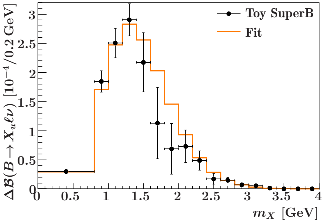


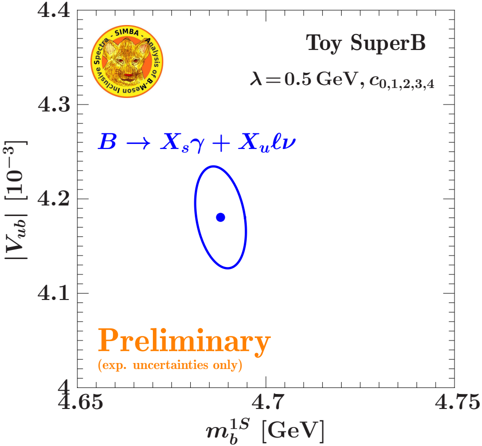



# III.d SuperB Factory $B \to X_u \ell \bar{\nu}_\ell + B \to X_s \gamma$ fit

Fit result with 5 coefficients and $\lambda = 0.5$ GeV:



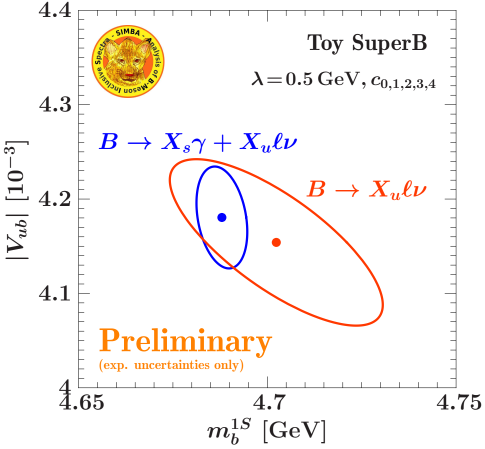

- Large amount of data can be used to push analyses to the limits

  on the experimental as well as on the theory side

- Subleading effects between $B \to X_s\gamma$ and $B \to X_u \ell \bar{\nu}_\ell$ can be addressed.

# III.e SuperB Factory $B \to X_u \ell \bar{\nu}_\ell + B \to X_s \gamma$ fit

Fit result with 5 coefficients and $\lambda = 0.5$ GeV:




- Fitting only $B \to X_u \ell \bar{\nu}_\ell$ eliminates sensitivity to subleading effects.

# IV Summary and Outlook

- Global fits for $|V_{ub}|$ can be a powerful tool at SuperB Factories.
- Presented a status update on **SIMBA**

**Where do we stand?**

1. $B \to X_s \gamma$: OK $\to$ progress on theory uncertainties, will show latest fits
2. $B \to X_u \ell \bar{\nu}_\ell$: More work needed $\to$ show toy fit using theory at NLO
3. $B \to X_c \ell \bar{\nu}_\ell$ constraints: (OK) $\to$ will not show fits.

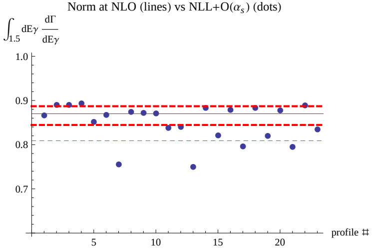
$\to$ Working on wrapping up $B \to X_s \gamma$, and shift attention to $B \to X_u \ell \bar{\nu}_\ell$.

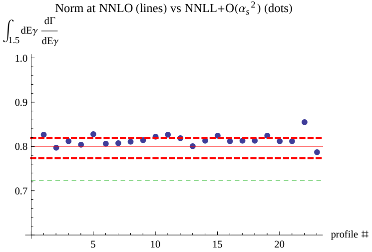
- Thinking about how to merge theoretical and experimental uncertainties into one CI.
- Main challenge for $B \to X_u \ell \bar{\nu}_\ell$: different subleasing $1/m_b$ corrections to shape function.
- $b \to c$ constraints + $B \to X_s \gamma$: work fine in fits. Glad to learn from C. Schwanda that there are new $1S$ values!

Thank you!

# Backup

# A.b Differential theory uncertainty

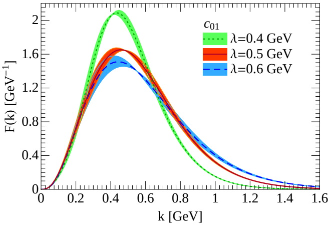


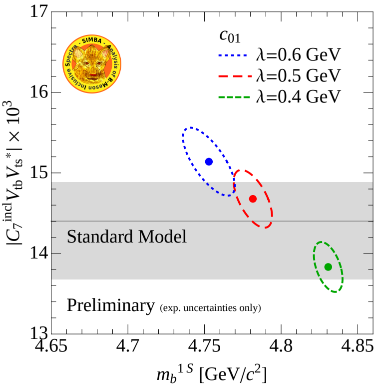

left and right: The fixed order theory uncertainty (at NLO and NNLO) is compared with the estimated uncertainty of the resumed NNLL/NNLO calculation used in this work: the red solid line corresponds to the fixed order result with a scale of $\mu = 4.7$, the red upper and lower dashed lines correspond to a variation of $\mu = 9.4$ and $\mu = 2.35$, respectively. The green line corresponds to the chosen scale of Misiak et al. [PRL:98:022002] (which uses a different definition of $C_7$ than this work). The blue dotes correspond to the chosen scale variations of the resumed NNLL/NNLO calculation. Our profiles have reasonable agreement with the fixed order results and also taking the range of dots as an uncertainty in this integral, our NNLL and NLL norms agree within uncertainties.

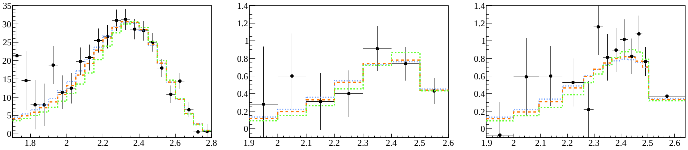
# B. Result without BaBar incl. spectrum

- Fit with two basis functions ($c_{01}$):



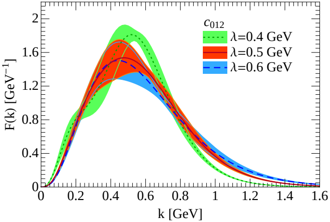

- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50



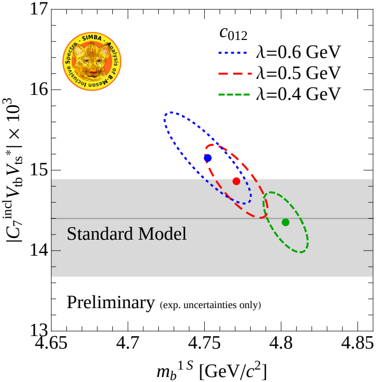



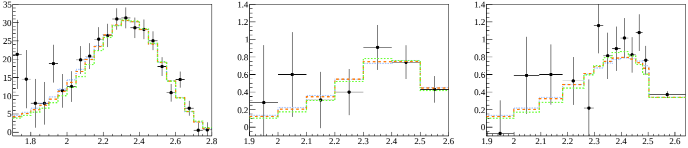
# B.a Basis independence

- Fit with three basis functions ($c_{012}$):



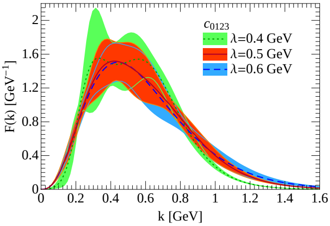

- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50



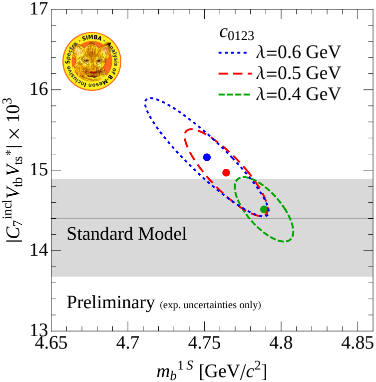



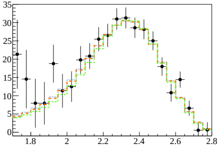
# B.a Basis independence

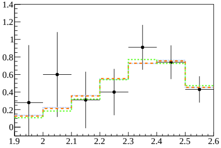
- Fit with four basis functions ($c_{0123}$):

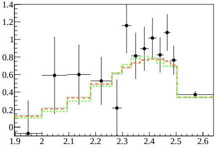



- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50



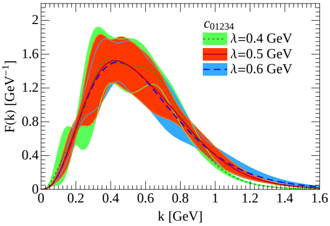





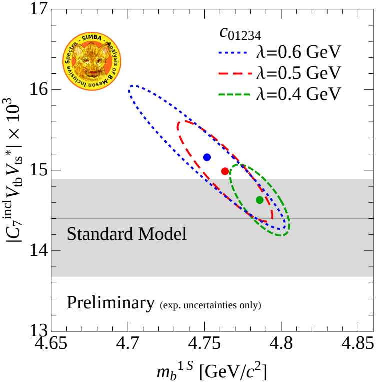


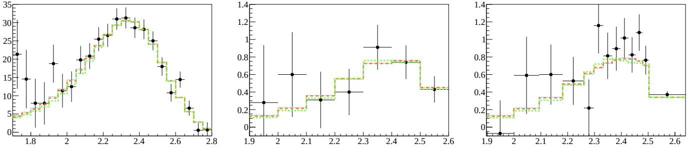
# B.a Basis independence

- Fit with five basis functions ($c_{01234}$):




- $\chi^2$/ndf: 53.8/50; 44.0/50; 42.3/50

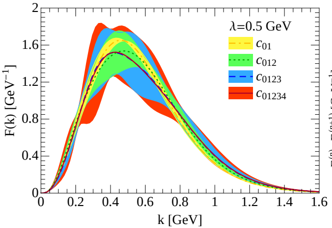


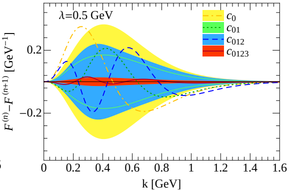



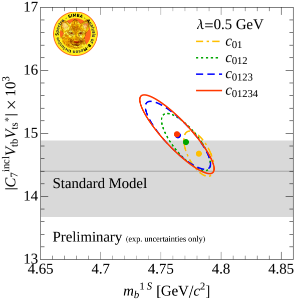
# B.b Fit result for $\lambda = 0.5$ GeV

- Fits with 2,3,4 & 5 basis functions: ($c_{01}, c_{012}, c_{0123}, c_{01234}$)
- Shape function and estimated basis dependence

  determined from $n+1$ coefficient and envelop from first basis function



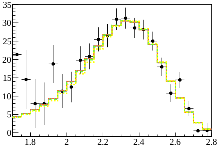

Fitted values of $|C_7^{\text{incl}}\, V_{tb}\, V_{ts}^*|$ are compared with the NLO Standard Model prediction using $|V_{tb}\, V_{ts}^*| = 40.68^{+0.4}_{-0.5}$

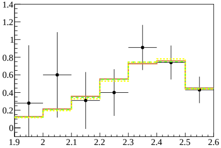
$\Rightarrow$ Uncertainties underestimated with too few coeff.

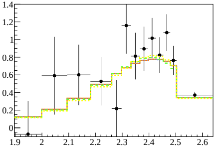
$\rightarrow$ would need to include additional uncertainty due to truncation

$\Rightarrow$ Very little change by including 5*th* coefficient ($c_4$)

$\rightarrow$ truncation uncertainty negligible compared to other uncertainties

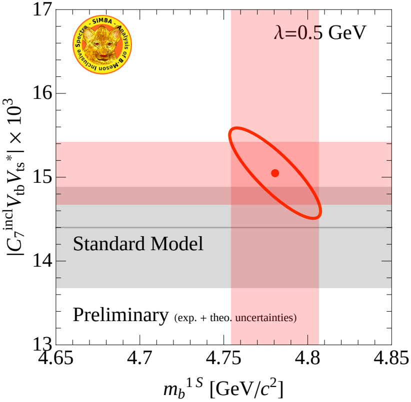


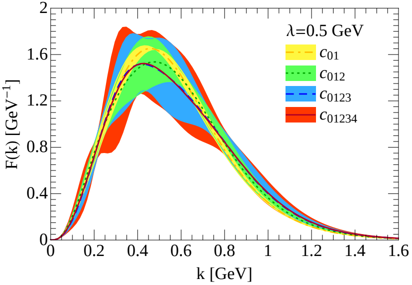




# B.c Theory uncertainty and results

- Obtained value of $C_7^{\text{incl}}$ which is very good agreement with Standard Model
- Non-perturbative shape function (with abs. $1/m_b$ corrections) determined by data



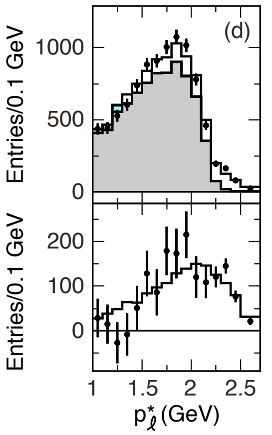
# C.a Regions of phase space

Recap on regions of phase space for $B \to X_u \, \ell \, \bar{\nu}_\ell$ and $B \to X_s \, \gamma$:

- SF region at large $E_\ell$ (endpoint) and $E_\gamma$ (peak region):
  experimentally clean(er) $\leftrightarrow$ theoretically more difficult
- OPE region at small $E_\ell$, large $q^2$ and small $E_\gamma$:
  large backgrounds $\leftrightarrow$ theoretically easier
- In between region $m_X \backsim m_D$; moderately large $E_\ell$ and $E_\gamma$

$\Rightarrow$ No 'golden' regions

$\Rightarrow$ Including a wide region needs a combination of optimal theory description for each region




[Phys.Rev.D86,032004]