QCD under Extreme Conditions

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Outline

First lecture:

★ First question you should ask: WHY??

- Motivation and some disturbing facts

✳ Collective effects x "fundamental" physics

★ Second question you should ask: WHERE??

- Accelerator experiments, astro, early universe

✳ My focus: the EoS

★ Third question you should ask: HOW??

- Theory, models, etc

Second lecture:

★ Effective model building

★ Z(N), the Polyakov loop, and confinement

★ Chiral symmetry breaking

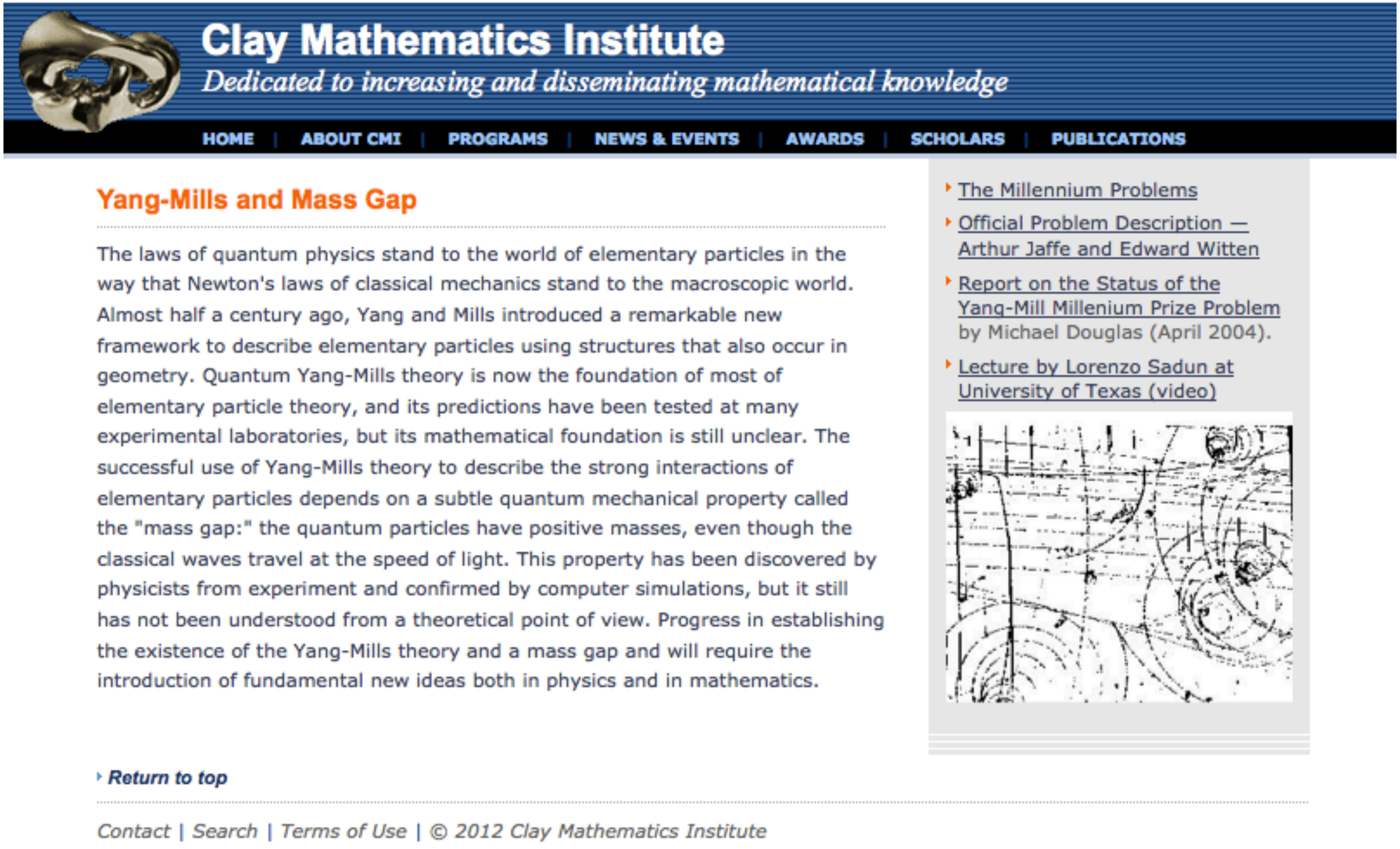
★ Two examples on relevance and difficulties in exploring the phase diagram

- Chiral magnetic effect and the strong CP problem
- Drawing the phase diagram

★ Final comments

First question you should ask:
WHY??

Confinement: one of the 6 unsolved Millennium Problems...



Yang-Mills and Mass Gap

The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap:" the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap and will require the introduction of fundamental new ideas both in physics and in mathematics.

- The Millennium Problems
- Official Problem Description — Arthur Jaffe and Edward Witten
- Report on the Status of the Yang-Mill Millenium Prize Problem by Michael Douglas (April 2004).
- Lecture by Lorenzo Sadun at University of Texas (video)

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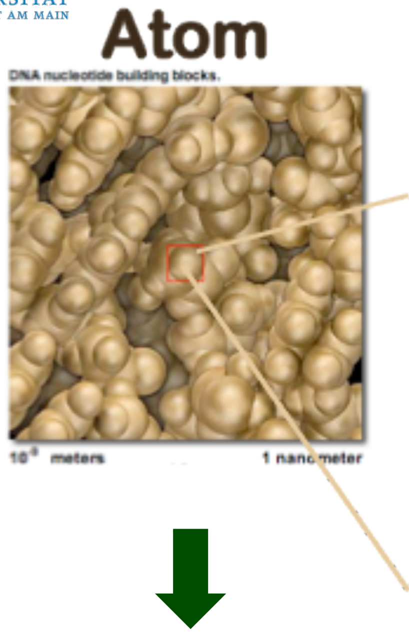
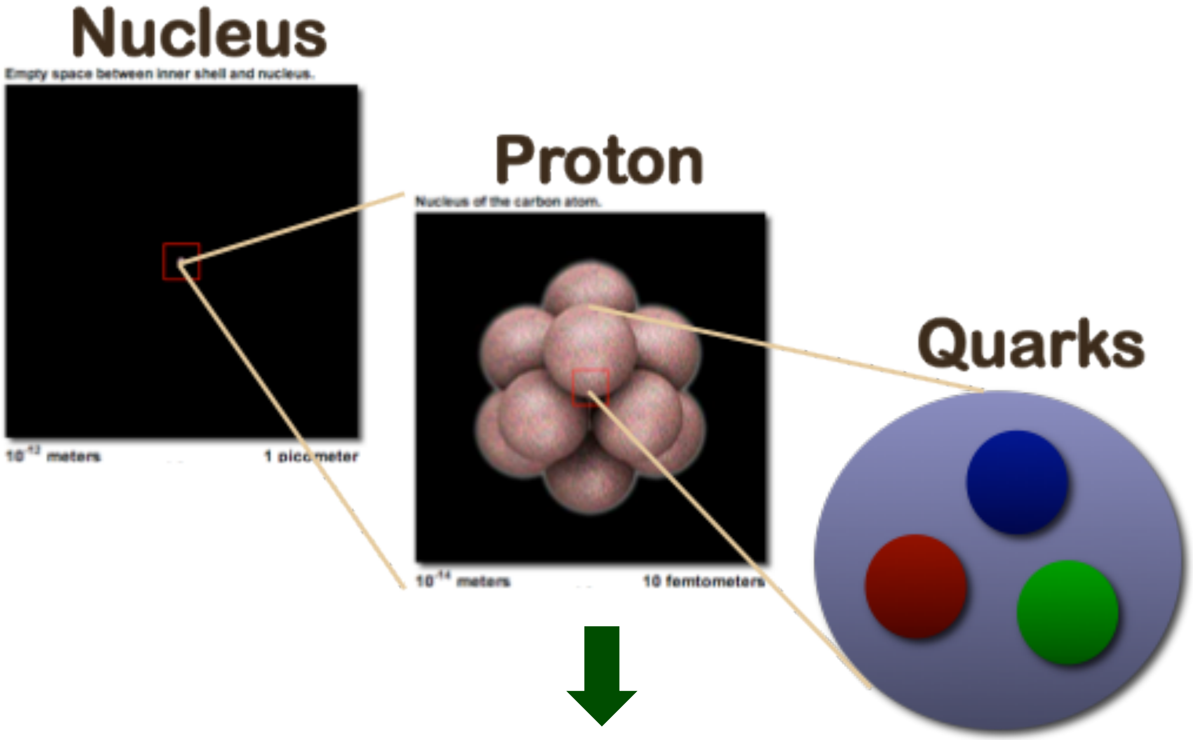
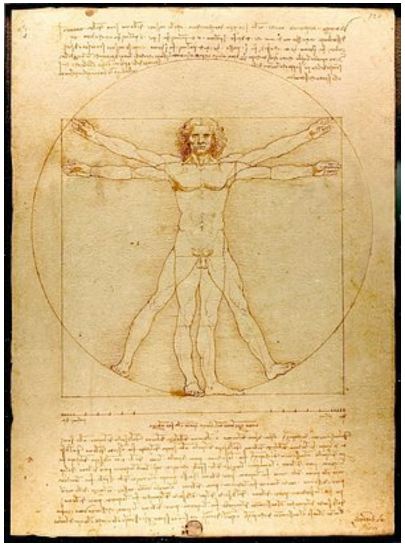
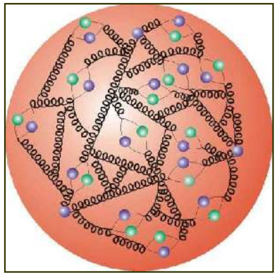
Not only a cute (and very tough!) math problem…



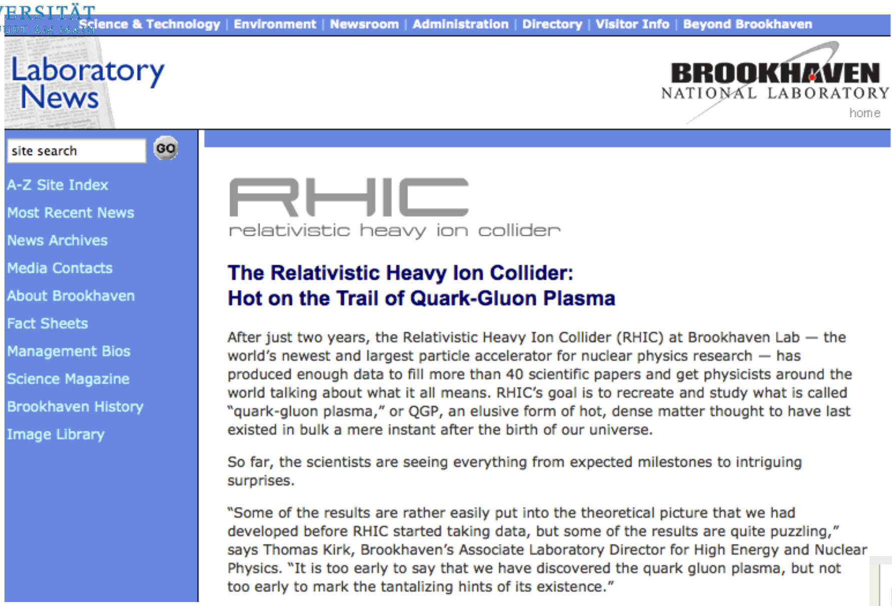






(Univ. Oxford)

Real origin of mass…
Sorry, Higgs!



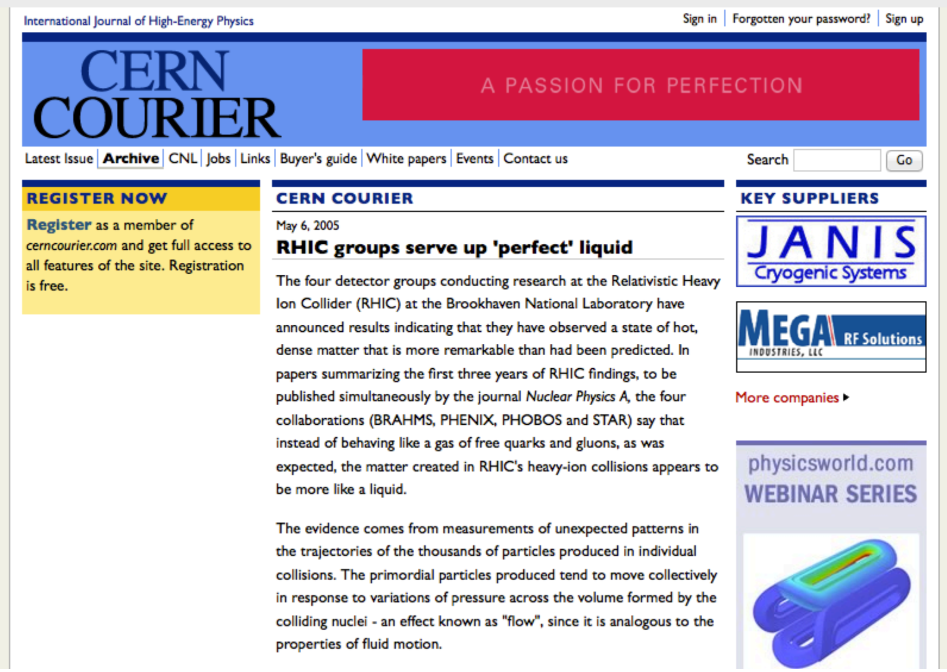
The Relativistic Heavy Ion Collider:
Hot on the Trail of Quark-Gluon Plasma

After just two years, the Relativistic Heavy Ion Collider (RHIC) at Brookhaven Lab — the world's newest and largest particle accelerator for nuclear physics research — has produced enough data to fill more than 40 scientific papers and get physicists around the world talking about what it all means. RHIC's goal is to recreate and study what is called "quark-gluon plasma," or QGP, an elusive form of hot, dense matter thought to have last existed in bulk a mere instant after the birth of our universe.

So far, the scientists are seeing everything from expected milestones to intriguing surprises.

"Some of the results are rather easily put into the theoretical picture that we had developed before RHIC started taking data, but some of the results are quite puzzling," says Thomas Kirk, Brookhaven's Associate Laboratory Director for High Energy and Nuclear Physics. "It is too early to say that we have discovered the quark gluon plasma, but not too early to mark the tantalizing hints of its existence."

Deconfinement: matter put under extreme conditions...



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May 6, 2005

RHIC groups serve up 'perfect' liquid

The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory have announced results indicating that they have observed a state of hot, dense matter that is more remarkable than had been predicted. In papers summarizing the first three years of RHIC findings, to be published simultaneously by the journal *Nuclear Physics A*, the four collaborations (BRAHMS, PHENIX, PHOBOS and STAR) say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy-ion collisions appears to be more like a liquid.

The evidence comes from measurements of unexpected patterns in the trajectories of the thousands of particles produced in individual collisions. The primordial particles produced tend to move collectively in response to variations of pressure across the volume formed by the colliding nuclei - an effect known as "flow", since it is analogous to the properties of fluid motion.

... can behave in unexpected ways.

In spite of the fantastic success of the Standard Model, we don't understand a few essential mechanisms...

Another why? (this one is, of course, enough!)

The behavior of QCD at high temperature is of obvious interest. It provides the answer to a childlike question:
What happens if you keep making things hotter and hotter?

The behavior of QCD at large net baryon density (and low temperature) is also of obvious interest. It answers yet another childlike question: What will happen when you keep squeezing things harder and harder?

(Frank Wilczek, Phys. Today, August 2000)

The Nobel Prize in Physics 2004:
"for the discovery of asymptotic freedom in
the theory of the strong interaction"

Physicist's question:

What is the inner structure of matter and the nature of strong interactions under extreme conditions of temperature and density?

- **Experiments:** "squeeze", "heat", "break"
- **Theory:** in-medium quantum field theory, i.e.
  - finite-temperature QCD
  - finite-density QCD
  - effective models

- Some disturbing "facts" about confinement:

- Confinement seems to be a property of hadrons.
- Confinement seems to be present in QCD.
- Lattice: evidence of confinement in pure gauge and QCD.
- There seems to be a confinement scale (masses, f_π, Λ_{QCD}).

- Some annoying problems:

- Theory is nonperturbative at relevant scales.
- Lattice is great but not perfect, and not Nature.
- "seems" is pretty annoying in all the "facts" above...

- We don't know that much, so we need a lot of input...

To make progress in understanding (or at least in collecting facts), we need it all:

- Experiments and observations
- Lattice
- Theory
- Effective models

And also combinations whenever possible. Because...

- Experiments are really expensive, hard and "dirty".

- Lattice has still several limitations (not only hardware...).

- Theory (QCD) can be (barely) solved in very specific situations (usually away from the interesting region).

- Effective models capture just part of the game.

In terms of thermodynamics, or many-body or statistical mechanics, the basic ideas are:

✧ Perturbing the (confined) vacuum to study confinement by

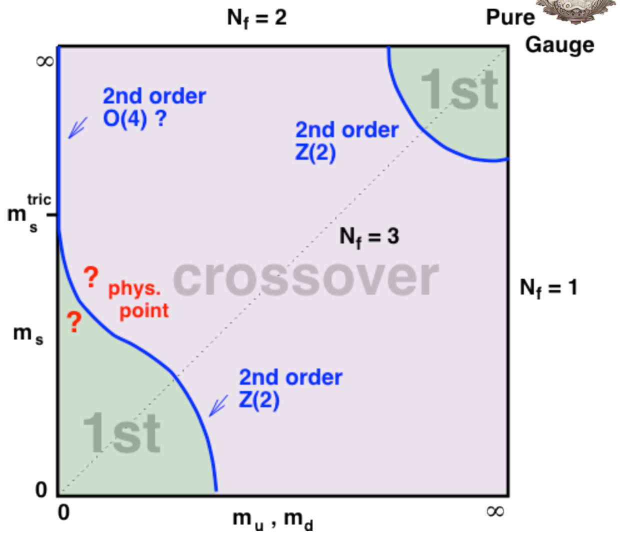
- heating (temperature)
- squeezing or unbalancing species (chemical potentials: baryon number, isospin, strangeness, ...)
- using classical external fields (magnetic, ...)

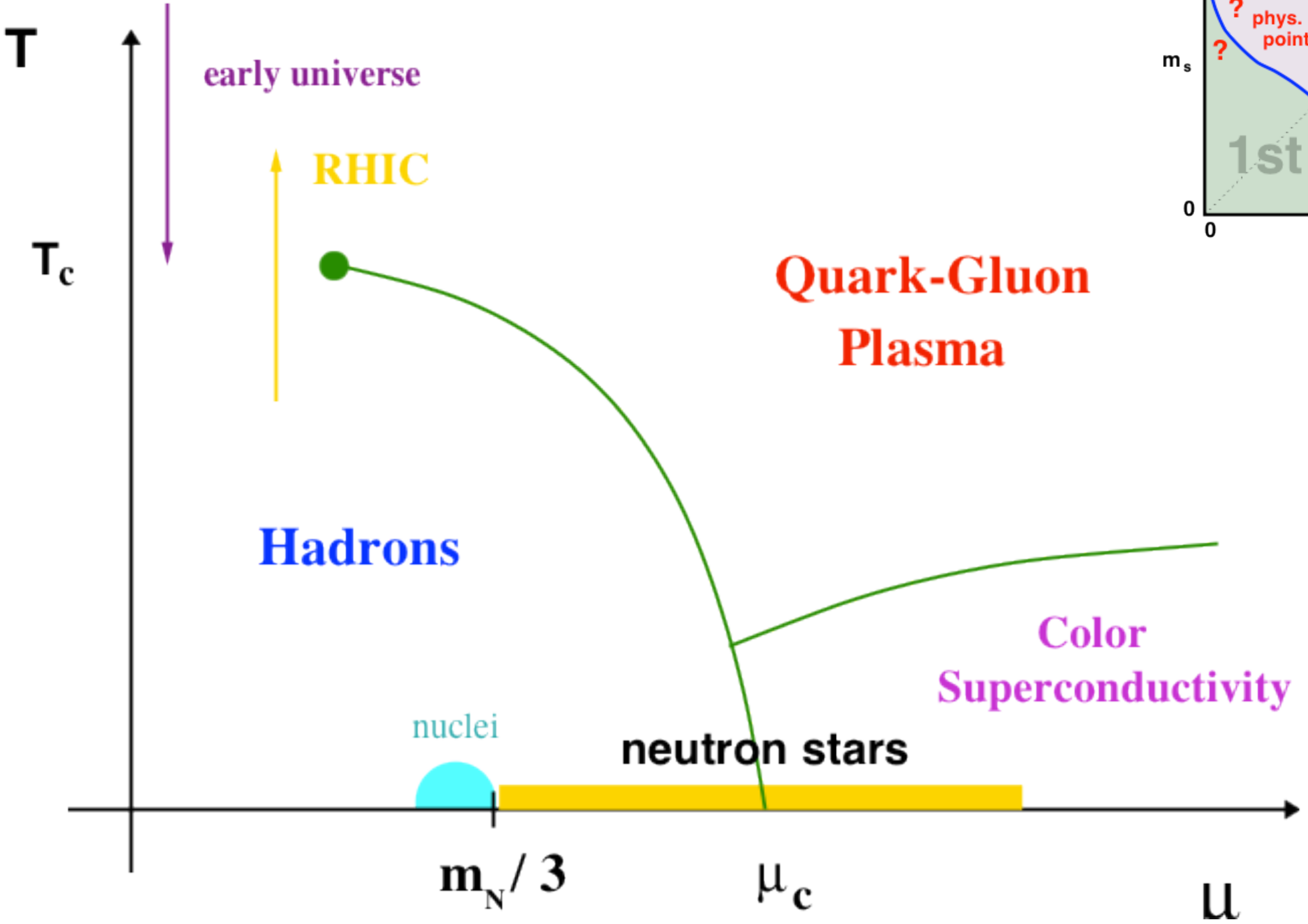
and taking the system away from confinement and back.

✧ Relating (or not) confinement to other key properties of strong interactions and QCD (chiral symmetry, etc).

✧ Trying to draw all possible phase diagrams of QCD and "cousin theories" to learn basic facts.

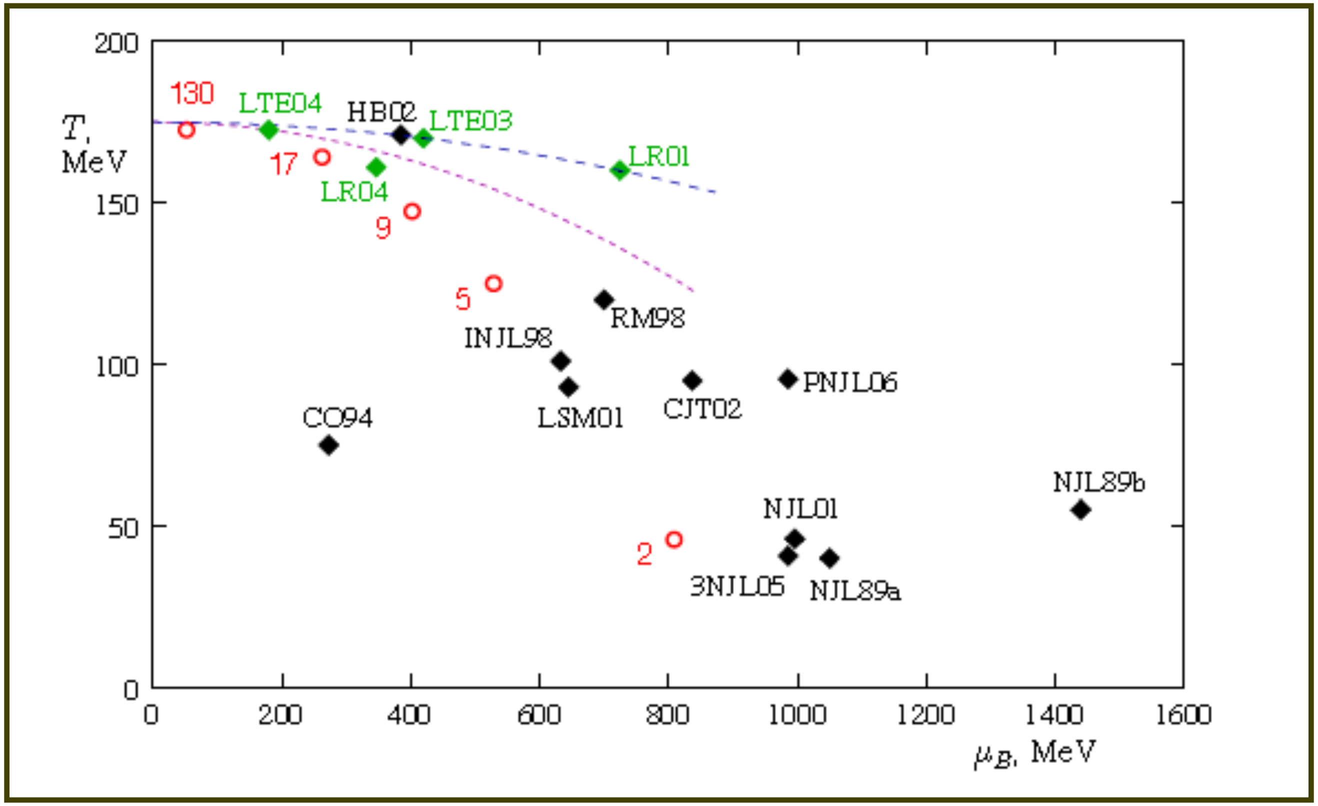
Example – cartoon of a phase diagram:





Example of "we don't know that much":

position of the critical endpoint of QCD:



[Stephanov (2006)]

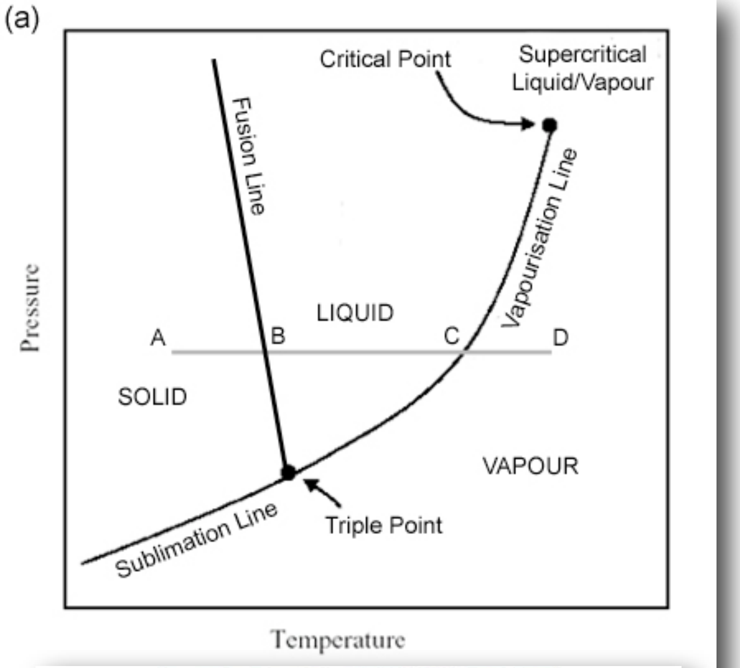
- Model predictions, lattice, freeze-out points in HIC

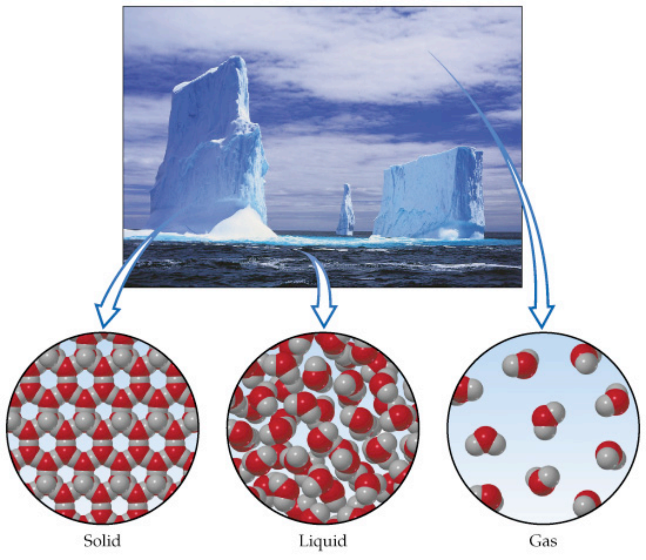
Second question you should ask: WHERE??

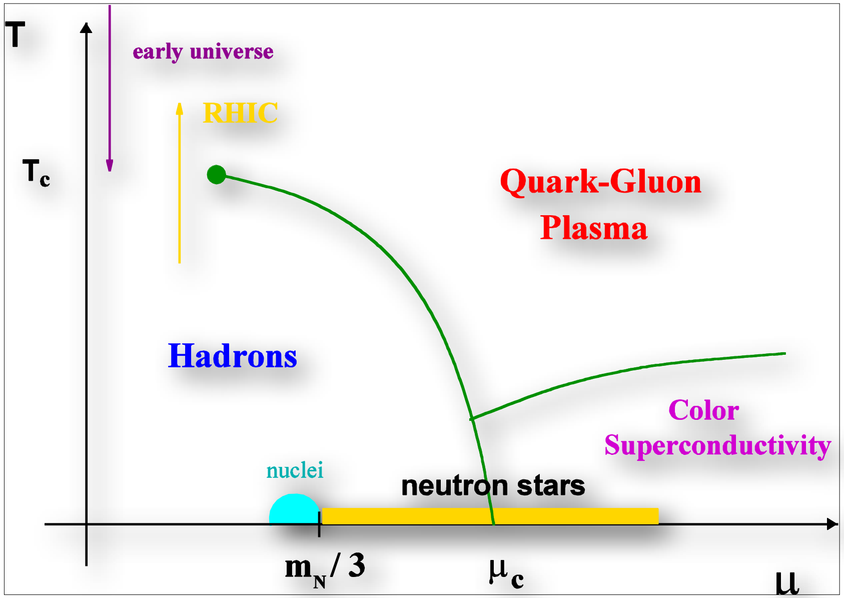
But before that, let's discuss an important point…

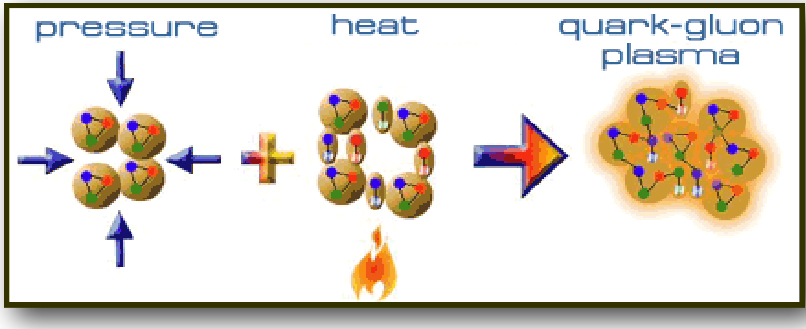
Collective effects x microscopic ("fundamental") physics

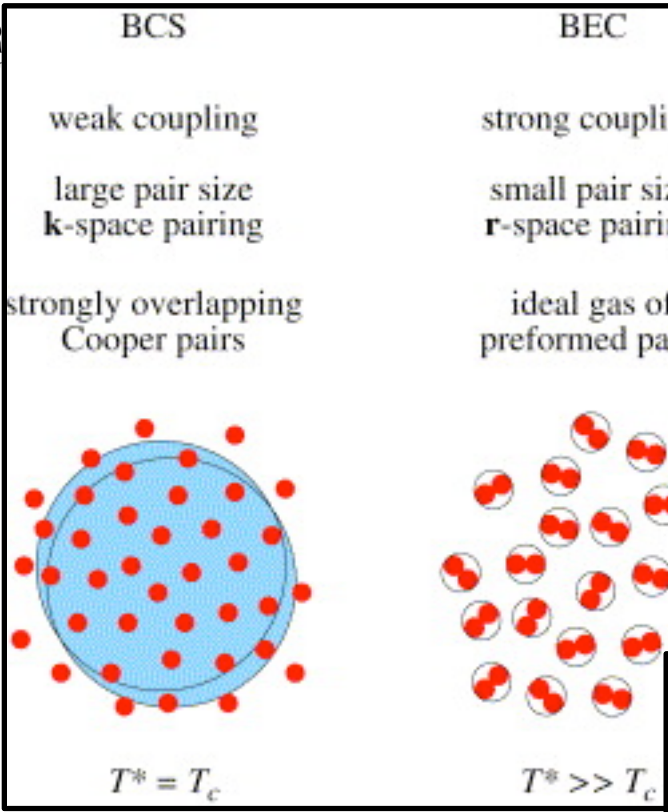
Collective effects x microscopic ("fundamental") physics





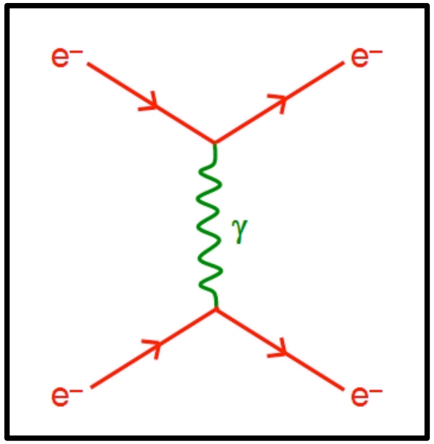


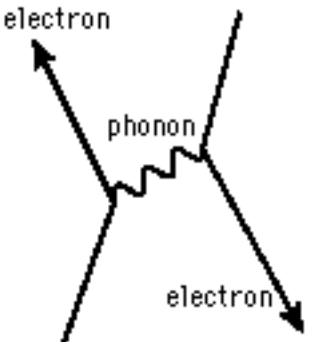




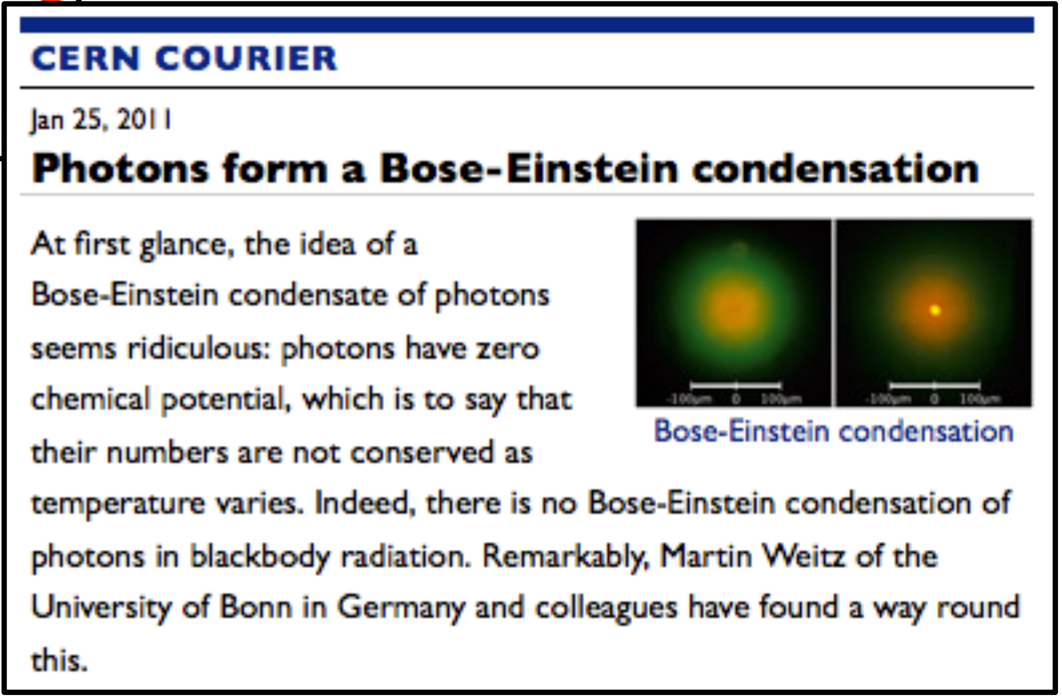
BCS	BEC
weak coupling	strong coupling
large pair size **k**-space pairing	small pair size **r**-space pairing
strongly overlapping Cooper pairs	ideal gas of preformed pairs
$T^* = T_c$	$T^* >> T_c$

[Chen et al (2005)]





[GITAM]



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Jan 25, 2011

Photons form a Bose-Einstein condensation

At first glance, the idea of a Bose-Einstein condensate of photons seems ridiculous: photons have zero chemical potential, which is to say that their numbers are not conserved as temperature varies. Indeed, there is no Bose-Einstein condensation of photons in blackbody radiation. Remarkably, Martin Weitz of the University of Bonn in Germany and colleagues have found a way round this.

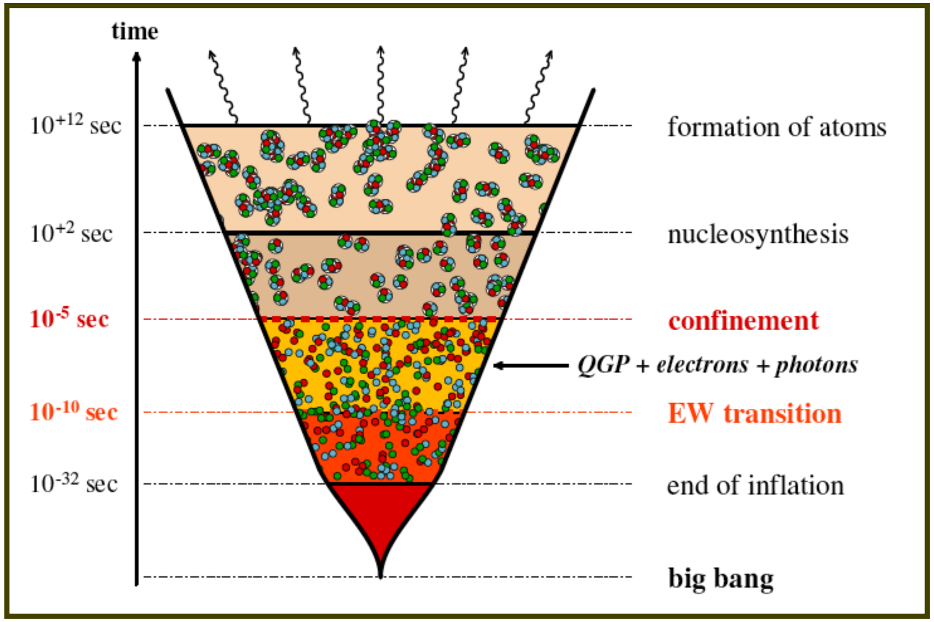
Bose-Einstein condensation

Now, ok, back to the...

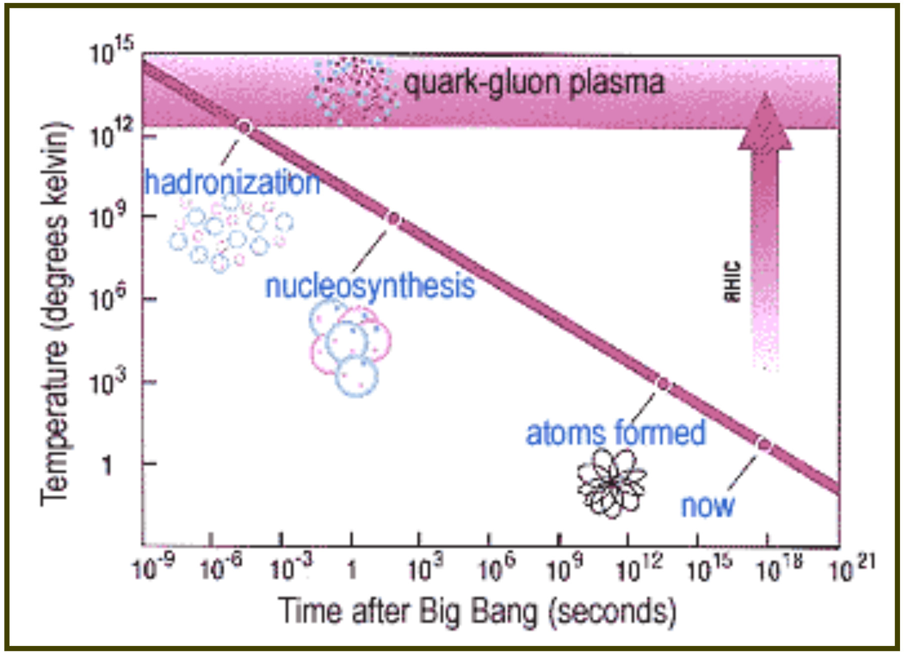
Second question you should ask:
WHERE??

Experiments and observations

First time: the early universe



(RHIC)

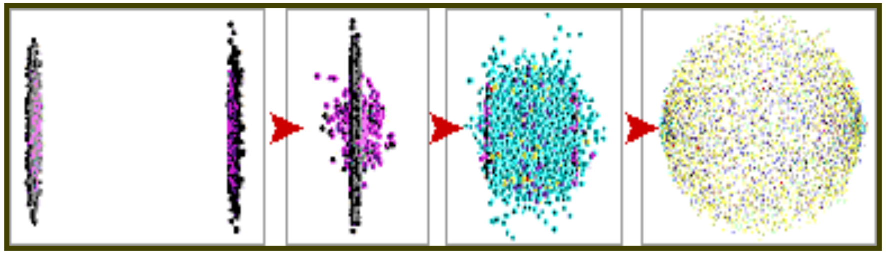


(RHIC)

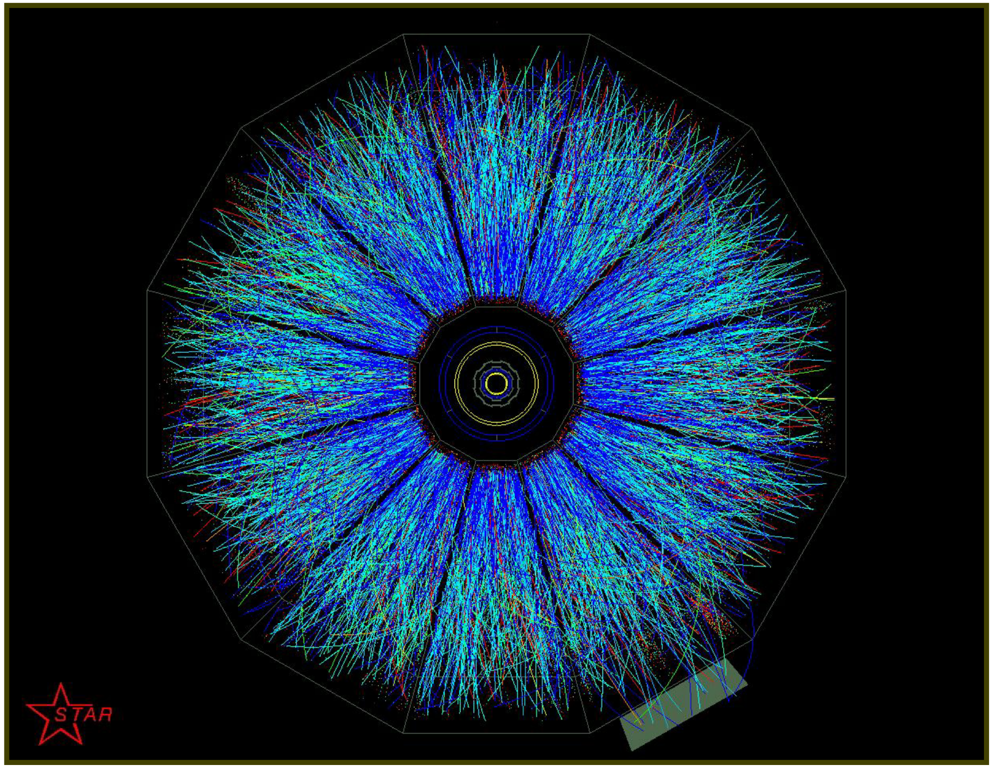
Temperature-driven transitions (very low μ)

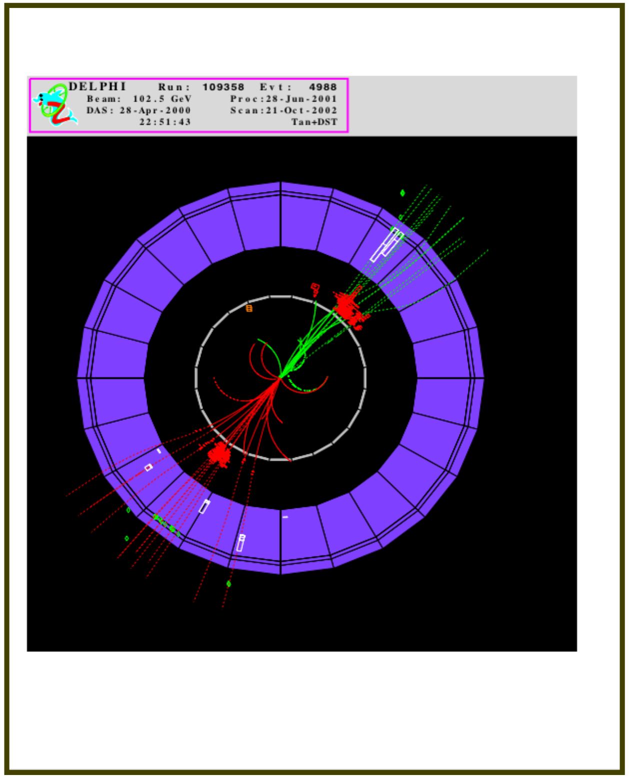
Observables: relics from that time?

In the lab – heavy ion collisions (“Little Bang”)



LEP: $e^+ + e^- \to q\ \bar{q}$ (≈200 GeV)





Big Bang vs. Little Bang

Using a simple approximation for the EoS, we can estimate the typical sizes:

$$3p \approx \epsilon \approx \frac{\pi^2}{30} N(T) T^4$$

Early universe (Big Bang):

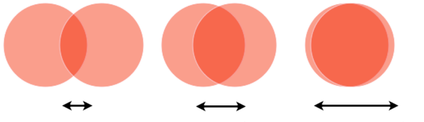
The radius of the universe, as given by the particle horizon in a Robertson-Walker spacetime, where the scale factor grows as a(t)~t^n, is given by (n=1/2, N~50 for QCD)

$$L_{univ}(T) \approx \frac{1}{4\pi}\left(\frac{1}{1-n}\right)\left(\frac{45}{\pi N(T)}\right)^{1/2}\frac{M_{Pl}}{T^2} = \frac{1.45\times 10^{18}}{(T/\mathrm{GeV})^2\sqrt{N}}\ \mathrm{fm}$$

The system is essentially in the thermodynamic limit!

Heavy ion collisions (Little Bang): $L_{QGP} \leq 10 - 15\ \mathrm{fm}$

The system created in HIC's is FINITE, and its size is CENTRALITY-DEPENDENT, $L(N_{\mathrm{part}})$:



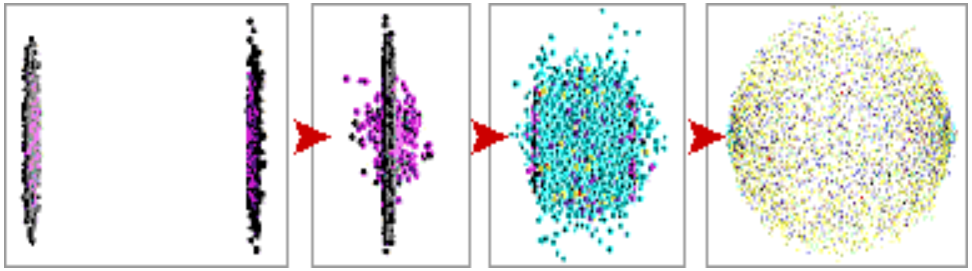
The system can be very small!

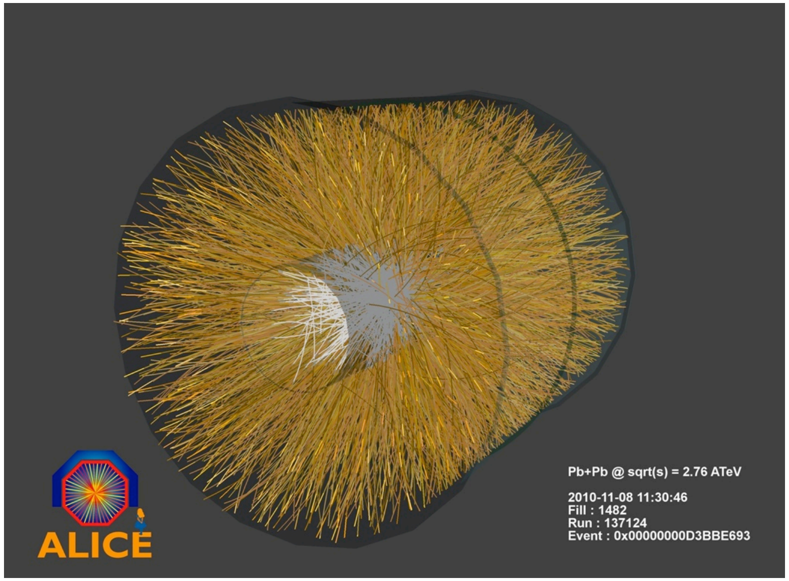
Huge differences in (time and length scales) between Big and Little Bangs…

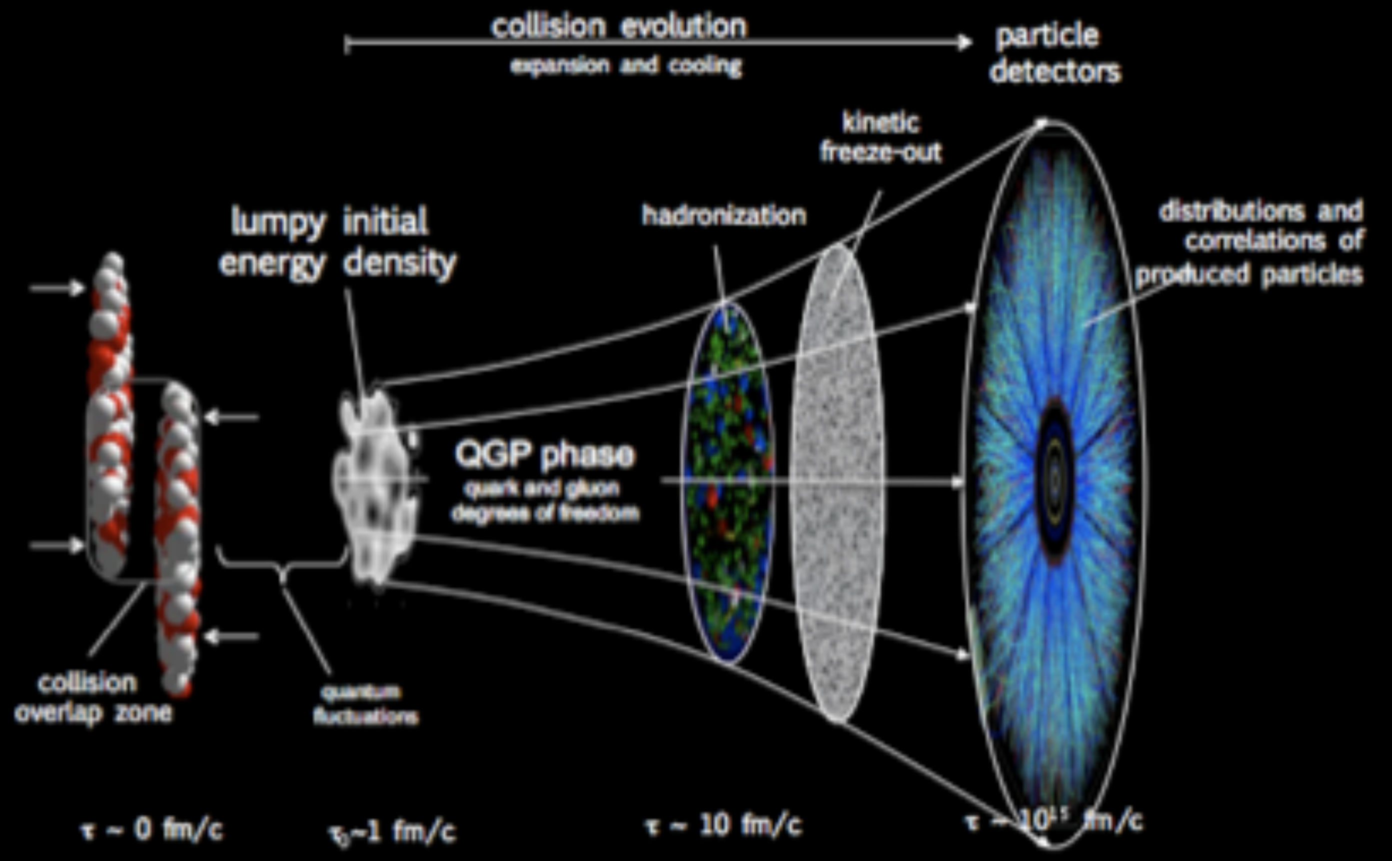
Some scales:

- Electroweak transition ~ 100 GeV -> way too high…
- Chiral & deconfinement transitions ~ 150 MeV

QCD phase transitions in the lab!
(very low μ, temperature-driven)





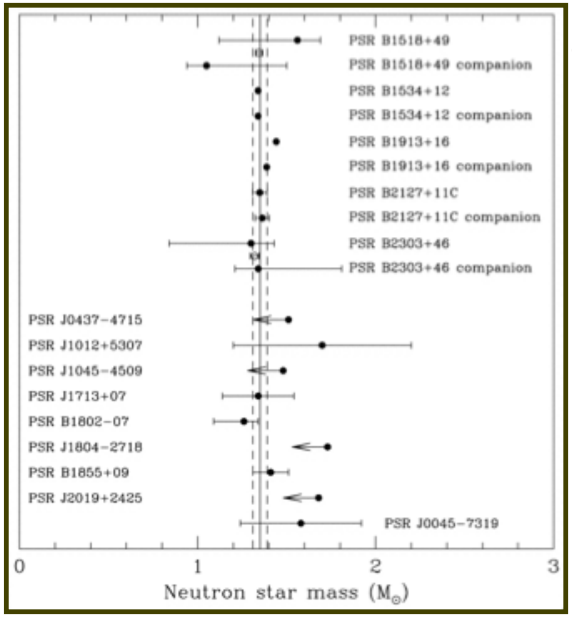


Elliptic Flow: A Study of Space-Momentum Correlations In Relativistic Nuclear Collisions.
P. Sorensen, Quark Gluon Plasma, Vol. 4, World Scientific, arXiv:0905.0174 [nucl-ex]

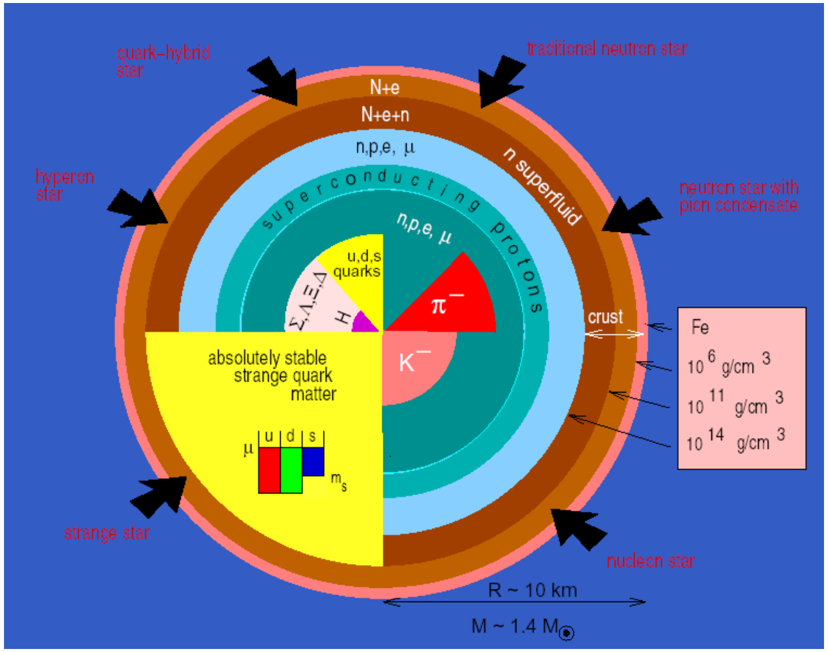
Compact stars (structure, supernovae, etc)

- New phases, condensates, color superconductivity, etc in the core.

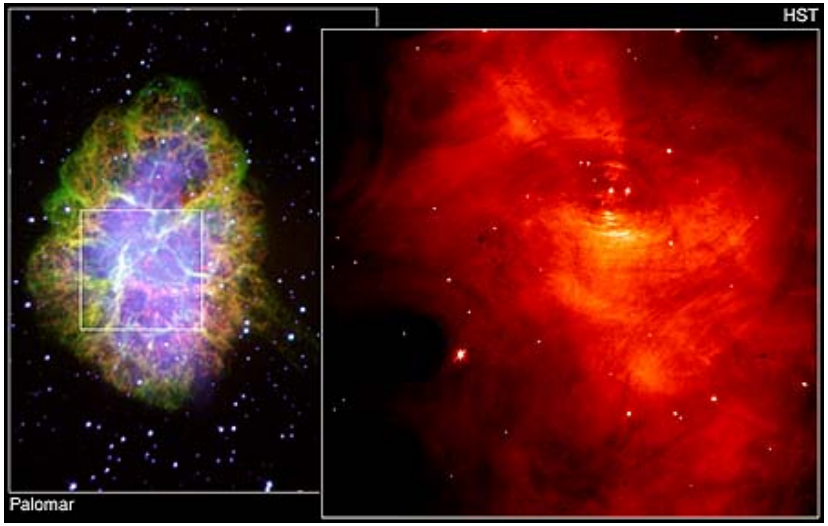
- Deconfinement and χ symmetry might affect explosion mechanism via EoS.



(Thorsett & Chakrabarty, 1999)



(F. Weber, 2000)



(NASA)

Some extreme numbers:

RHIC:

$v_{beam} \approx 0.99995\ c$

$T_c \approx 200$ MeV $\approx 2 \times 10^{12}$ K

⇒ QFT at "high" temperature

Compact stars:

$n_0 = 3 \times 10^{14}\ \mathrm{g/cm^3} = 0.16\ \mathrm{fm^{-3}}$

$n_{core} \approx (4 - 15)\ n_0$ $[\langle n_{Earth} \rangle \approx 5.5\ \mathrm{g/cm^3}]$

M ≈ 1 – 2 solar masses $[M_S \approx 2 \times 10^{33}\ \mathrm{g}]$

R ≈ 6 – 16 Km $[R_S \approx 7 \times 10^5\ \mathrm{km}]$

⇒ QFT at "high" density + General Relativity

Third question you should ask:
HOW??
(theory, models, etc)

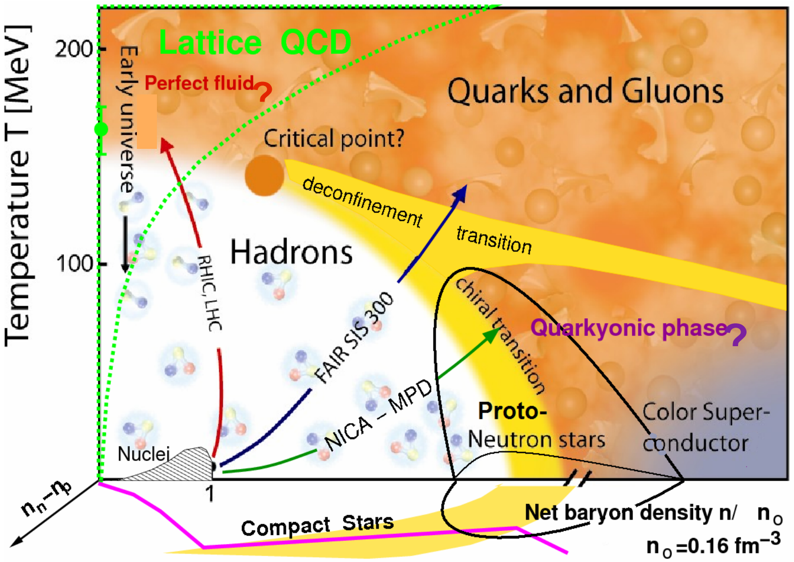
But before that, let me state my focus here, and why so...

Focus: the equation of state (EoS)

Why? Because, besides carrying all the thermodynamic equilibrium info we may want, it is also the basic crucial ingredient for dynamics, structure, etc.

1. Phase diagram structure (every detail):

pressure(T, μ ,B,etc)



2. Structure of compact stars:

Tolman-Oppenheimer-Volkov equations:

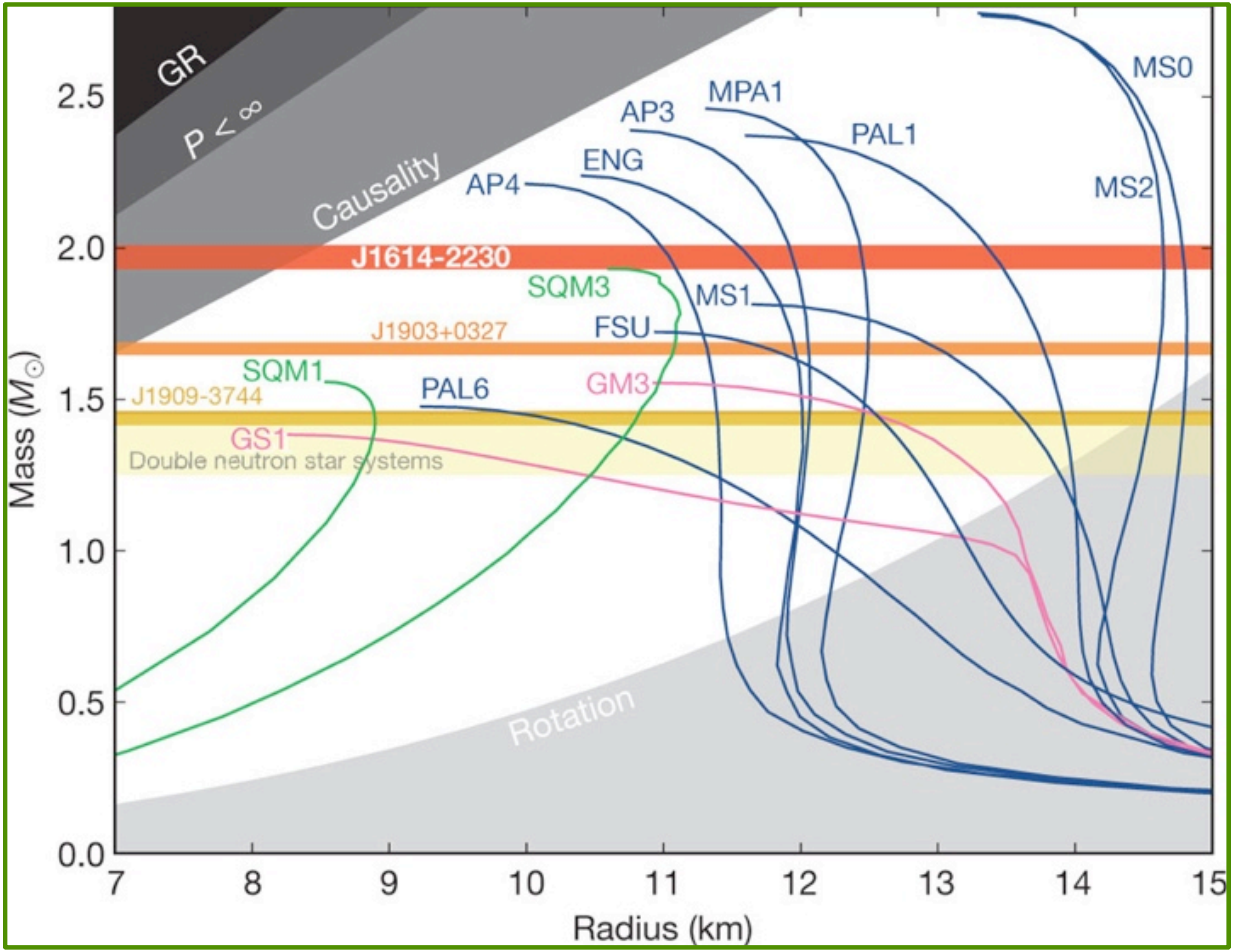
- Einstein's GR field equations
- Spherical symmetry
- Hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1-\frac{2G\mathcal{M}(r)}{r}\right]}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\epsilon(r) \ ; \quad \mathcal{M}(R) = M$$

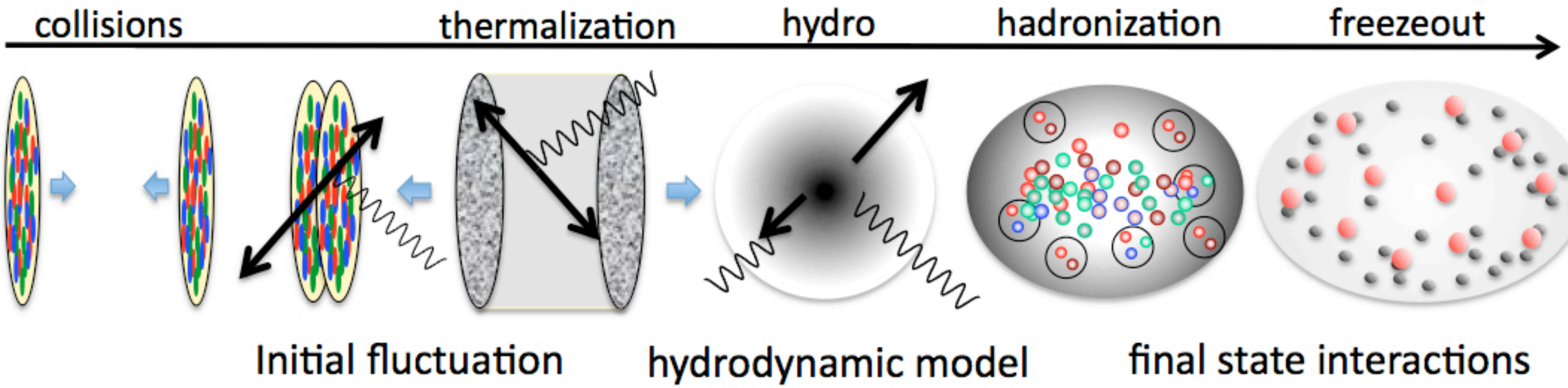
- Given the EoS p = p(e), one can integrate the TOV equations from the origin until the pressure vanishes p(R) = 0

- Different EoS's define different types of stars: white dwarfs, neutron stars, quark stars, strange stars, ...



[Demorest et al (2010)]

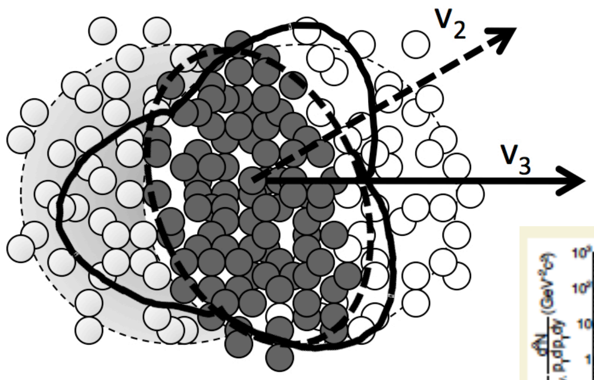
3. Hydro in heavy ion collisions:

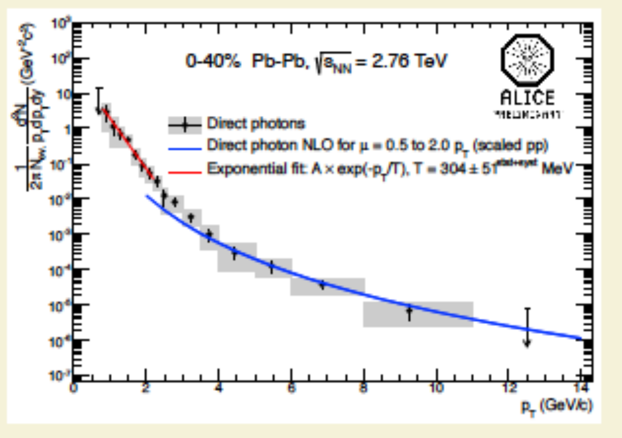


collisions thermalization hydro hadronization freezeout

Initial fluctuation hydrodynamic model final state interactions

[Nonaka et al (2012)]





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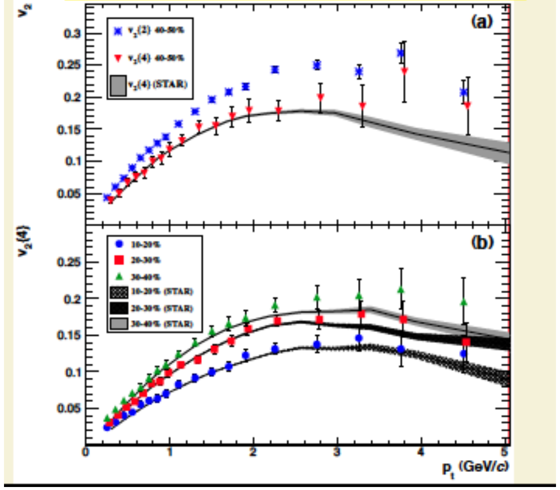


[from Gonzalez's talk]

Ideal hydrodynamics

- The fundamental equation of hydrodynamics is simply the conservation of energy-momentum,

  $$\partial_\mu T^{\mu\nu} = 0$$

  and of the baryon number,

  $$\partial_\mu (n_{_B} v^\mu) = 0$$

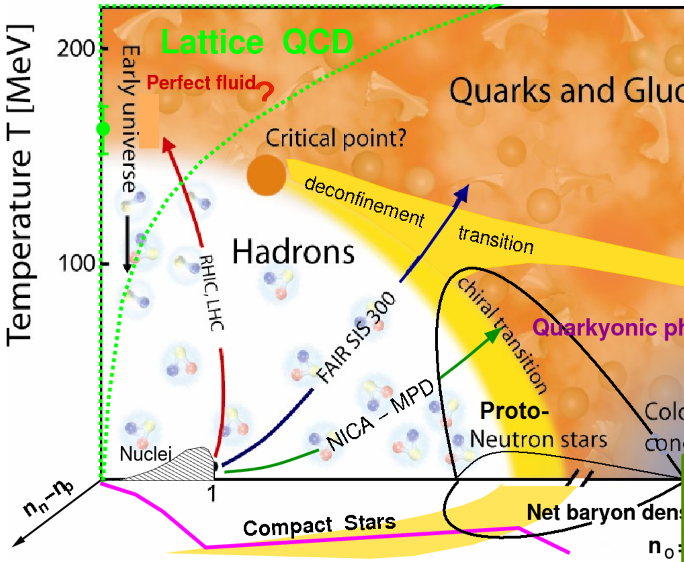
- The unknown functions are :
  - $p(t, \vec{x}), \epsilon(t, \vec{x}), n_{_B}(t, \vec{x})$
  - $v^\mu(t, \vec{x})$ (3 unknowns only, since $v_\mu v^\mu = 1$)

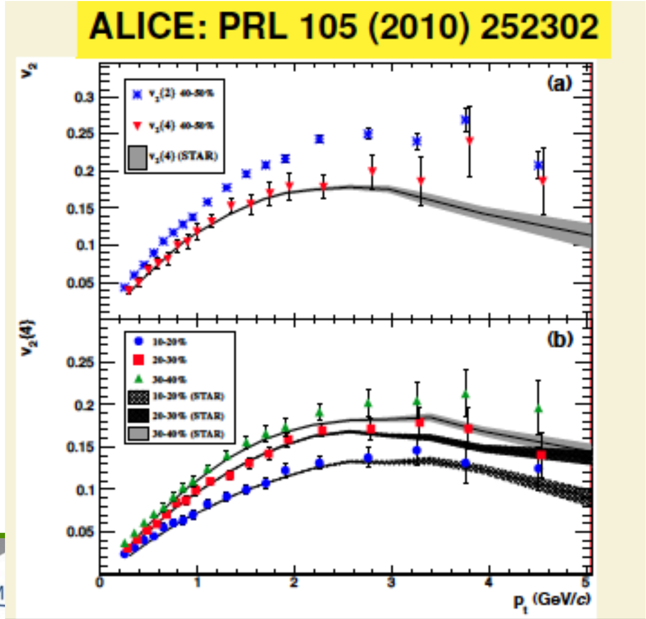
- $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu (n_{_B} v^\mu) = 0$ give only 5 equations

- An additional constraint comes from the equation of state of the matter under consideration, as a relation between the local pressure p and energy density ϵ

[Gelis (2012)]

So, it is clear that we REALLY need the EoS:





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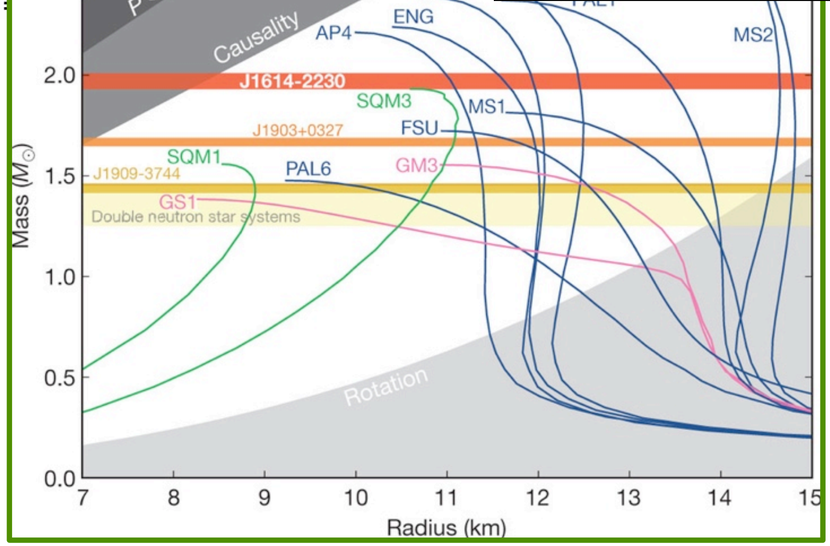


Now, ok, back to the…

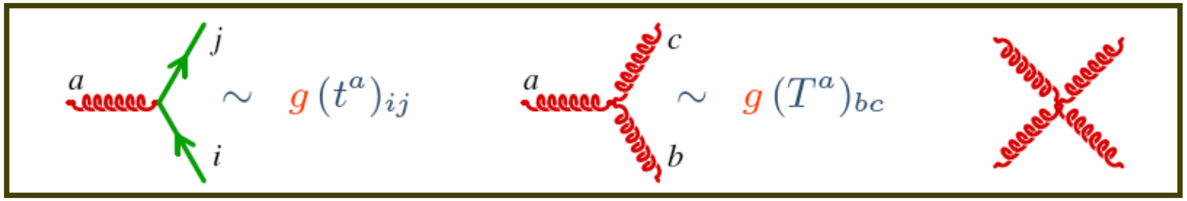
Third question you should ask:
HOW??
(theory, models, etc)

Theory (QCD)

The theory is, in principle, given:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_f \overline{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A^a_\mu \tau^a)\psi_f$$

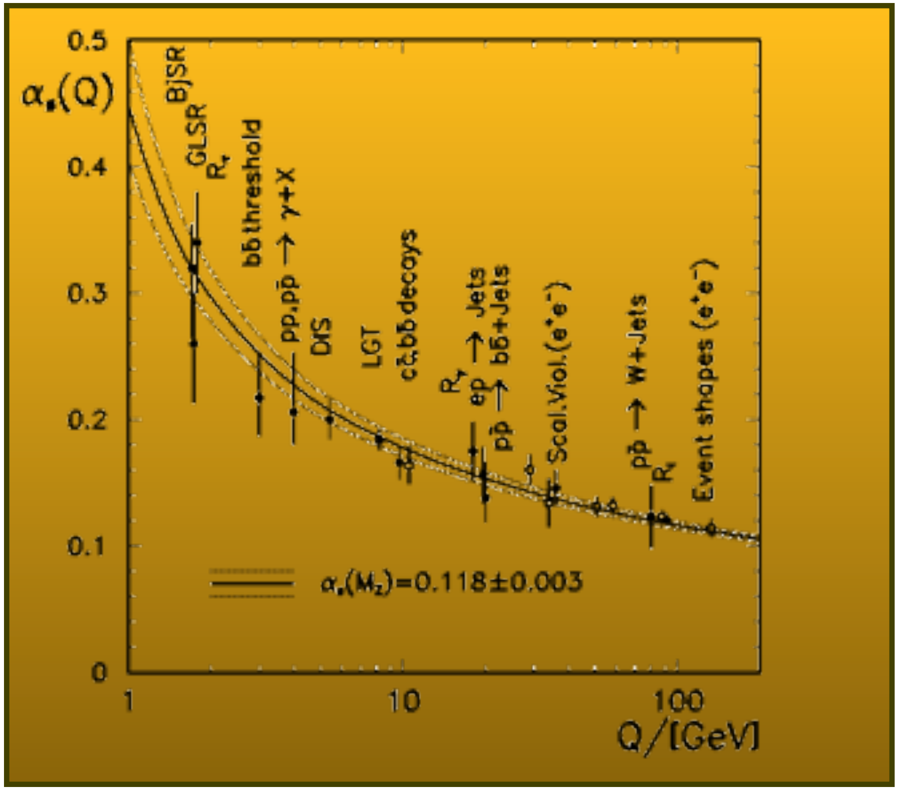
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

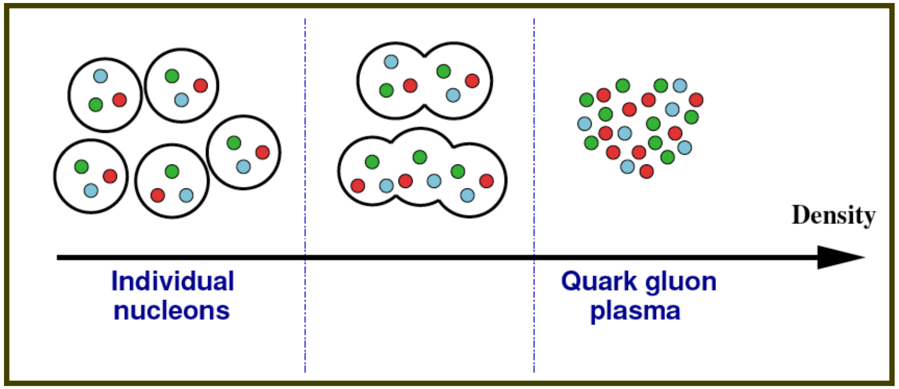


Then: compute the thermodynamic potential, blah blah...

Well, it's not so simple...

Asymptotic freedom & the vacuum of QCD





- Matter becomes simpler at very high temperatures and densities (T and μ as energy scales in a plasma), but very complicated in the opposite limit...

- T and μ are not high enough in the interesting cases...

- Finite T pQCD is very sick...

Finite T pQCD is IR sick...

Nonlinear effects become important for: $\langle(\partial A)^2\rangle \sim g^2\langle A^4\rangle \sim g^2\langle A^2\rangle^2$

Long-wavelength fluctuations: $\langle A^2\rangle_\kappa \approx \int^\kappa \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{k}\frac{1}{\mathrm{e}^{k/T}-1} \approx \int^\kappa \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{T}{k^2} \approx \kappa T$

Scale for PT breakdown: $\kappa \approx g^2 T$

- **Hard degrees of freedom: the plasma particles**

$$k \sim T \qquad \langle A^2\rangle_T \sim T^2 \qquad \langle(\partial A)^2\rangle_T \sim T^4 \qquad g^2\langle A^2\rangle_T^2 \sim g^2T^4$$

- **Soft degrees of freedom, collective modes**

$$k \sim gT \qquad \langle A^2\rangle_{gT} \sim gT^2 \qquad \langle(\partial A)^2\rangle_{gT} \sim g^3T^4 \qquad g^2\langle A^2\rangle_{gT}^2 \sim g^4T^4$$

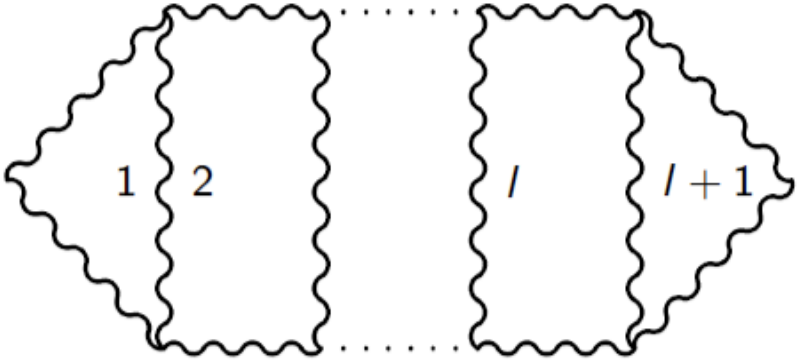
- **Ultrasoft degrees of freedom, unscreened magnetic fluctuations**

$$k \sim g^2T \qquad \langle A^2\rangle_{g^2T} \sim g^2T^2 \qquad \langle(\partial A)^2\rangle_{g^2T} \sim g^6T^4 \qquad g^2\langle A^2\rangle_{g^2T}^2 \sim g^6T^4$$

[Blaizot (2004)]

Even worse: Linde's problem [from Philipsen, Trento 2009]

- $(l+1)$-loop diagram: **contribution to pressure**



contribution from Matsubara 0-mode:

$$\boxed{P \sim g^{2l}\left(T\int d^3p\right)^{l+1} p^{2l}(p^2+m^2)^{-3l}}$$

$$\begin{array}{rll} g^{2l} & \text{for} & l=1,2 \\ g^6T^4\ln(T/m) & \text{for} & l=3 \\ g^6T^4(g^2T/m)^{l-3} & \text{for} & l>3 \end{array}$$

magnetic mass $m_{mag} \sim g^2T \Rightarrow$ **all loops** $(l>3)$ contribute to g^6

even for weak coupling!

Equation of state - naïve field map

hadrons
low temp.
and density

……………………………………………

???
$T \sim T_c$; $n \sim n_c$

quarks
high temp.
and density

hadronic models,
nuclear field theory
(us!)

pQCD at $T>0$ & $\mu>0$
asymptotic freedom

where all the things that matter happen…

there is no appropriate formalism yet…

Ok, let's not desperate… as we said, there are many ways out!

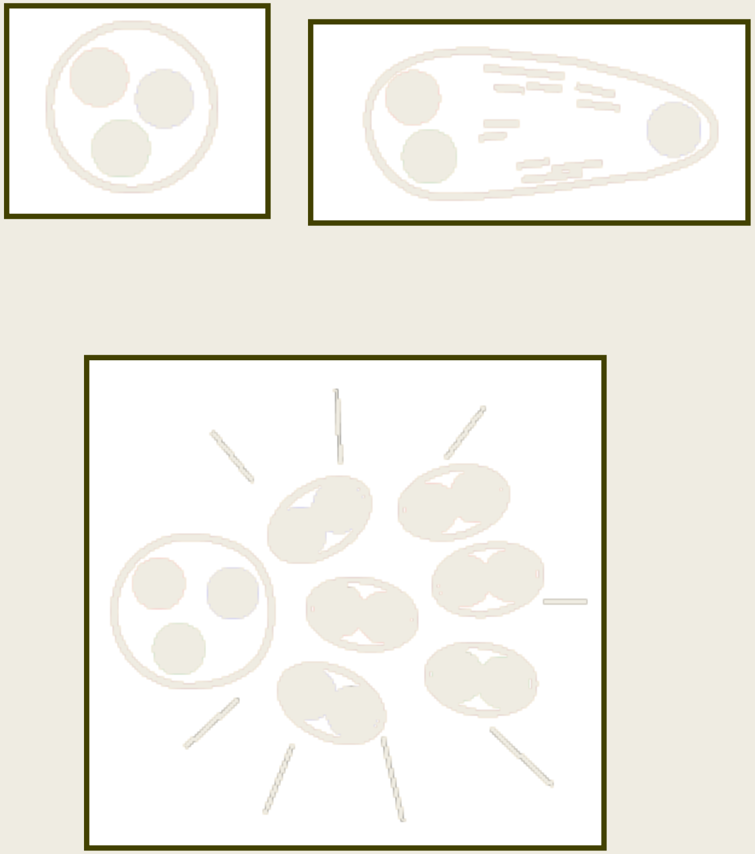
Some popular examples:

- Very intelligent and sophisticated brute force: lattice QCD
- Intensive use of symmetries: effective field theory models
- Redefining degrees of freedom: quasiparticle models
- "Moving down" from high-energy pQCD
- "Moving up" from hadronic low-energy (nuclear) models

And we can and should also combine a bunch of them.

But, before all that, let's do something REALLY simple...

The MIT bag model (70's)



Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium.
+ eventual corrections ~ α_s

- Asymptotic freedom: free quarks and gluons inside color singlet bags

- Confinement: vector current vanishes on the boundary

- Confinement achieved by assuming a constant energy density for the vacuum (negative pressure) -> bag constant (B)

- B: phenomenological parameter, extracted from fits to masses

  (difference in energy density between the QCD and the pert. vacua)

- Hadron mass (spherical bag):

E_h = "vacuum" + kinetic ~ $\boxed{\frac{4}{3}\pi R^3 B + \frac{const}{R}}$ ⇨ $\boxed{M_h = \frac{16}{3}\pi R_h^3 B}$

- Hadron pressure: $\boxed{p_h = -\frac{\partial E_h}{\partial V} = -B + \frac{const}{4\pi R^4} = 0}$ (at equilibrium)

- Assuming a deconfining transition, the pressure in the QGP phase within this model is given by

$$\boxed{p = \left(\nu_b + \frac{7}{4}\nu_f\right)\frac{\pi^2 T^4}{90} - B}$$

whereas the pressure in the hadronic phase (pion gas) is

$$\boxed{p_\pi = \nu_\pi \frac{\pi^2 T^4}{90}}$$

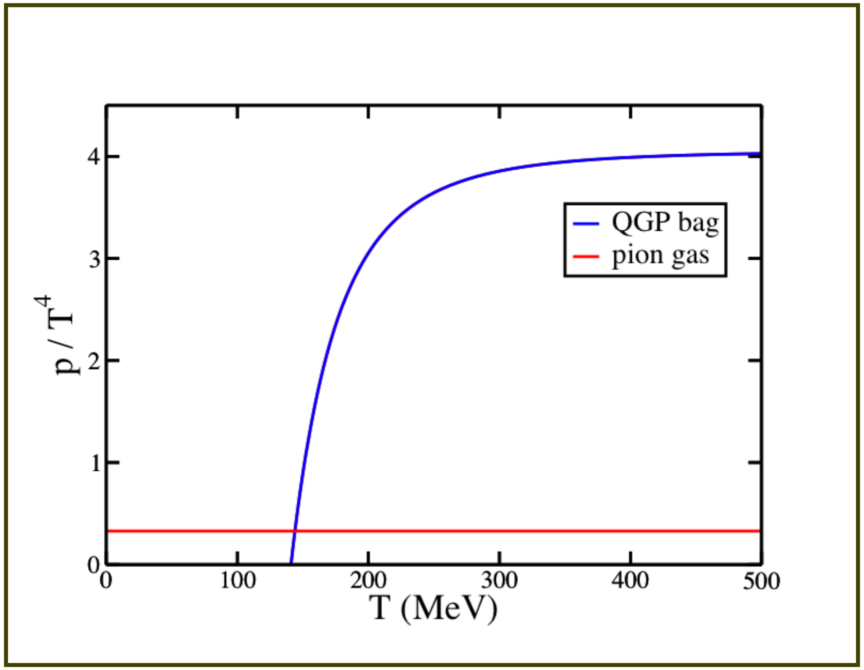
neglecting masses for simplicity. Here, we have the following numbers of d.o.f.'s: $\nu_\pi = 3$, $\nu_b = 2\,(N_c^2 - 1)$ and $\nu_f = 2\,N_c N_f$

For instance, for $N_c = 3$, $N_f = 2$ and $B^{1/4} = 200$ MeV:

$$\boxed{T_c = \left(\frac{45B}{17\pi^2}\right)^{1/4} \approx 144\ MeV}$$

[not so bad as compared to lattice QCD results]

and a 1st order transition [differs from lattice QCD results]



To go beyond in our study of the phases of QCD, we need to know its symmetries, and how they are broken spontaneously or explicitly. But QCD is very complicated:

- First, it is a non-abelian SU(N_c) gauge theory, with gluons living in the adjoint representation
- Then, there are N_f dynamical quarks (who live in the fund. rep.)
- On top of that, all these quarks have masses which are all different! Very annoying from the point of view of symmetries!

So, in studying the phases of QCD, we do it by parts, and consider many "cousin theories" which are very similar to QCD but simpler (more symmetric). We also study the dependence of physics on parameters which are fixed in nature.

Basic hierarchy for effective model building:

pure glue SU(N):

- **Z(N) symmetry (SSB)**
- order parameter: Polyakov loop **L**
- deconfining trans.: N=2 (2nd order), N=3 (weakly 1st order)

+ massless quarks:

- **chiral symmetry (SSB)**
- order parameter: chiral condensate **σ**
- Z(N) explicitly broken, but rise of **L** ➪ deconf.
- chiral trans.: N=3,2 ($N_f=2$) – 2nd order

+ massive quarks:

Z(N) and chiral explicitly broken

Yet vary remarkably and **L** ↔ **σ**

Next lecture:

★ Effective model building

★ Z(N), the Polyakov loop, and confinement

★ Chiral symmetry breaking

★ Two examples on relevance and difficulties in exploring the phase diagram
  - Chiral magnetic effect and the strong CP problem
  - Drawing the phase diagram

★ Final comments