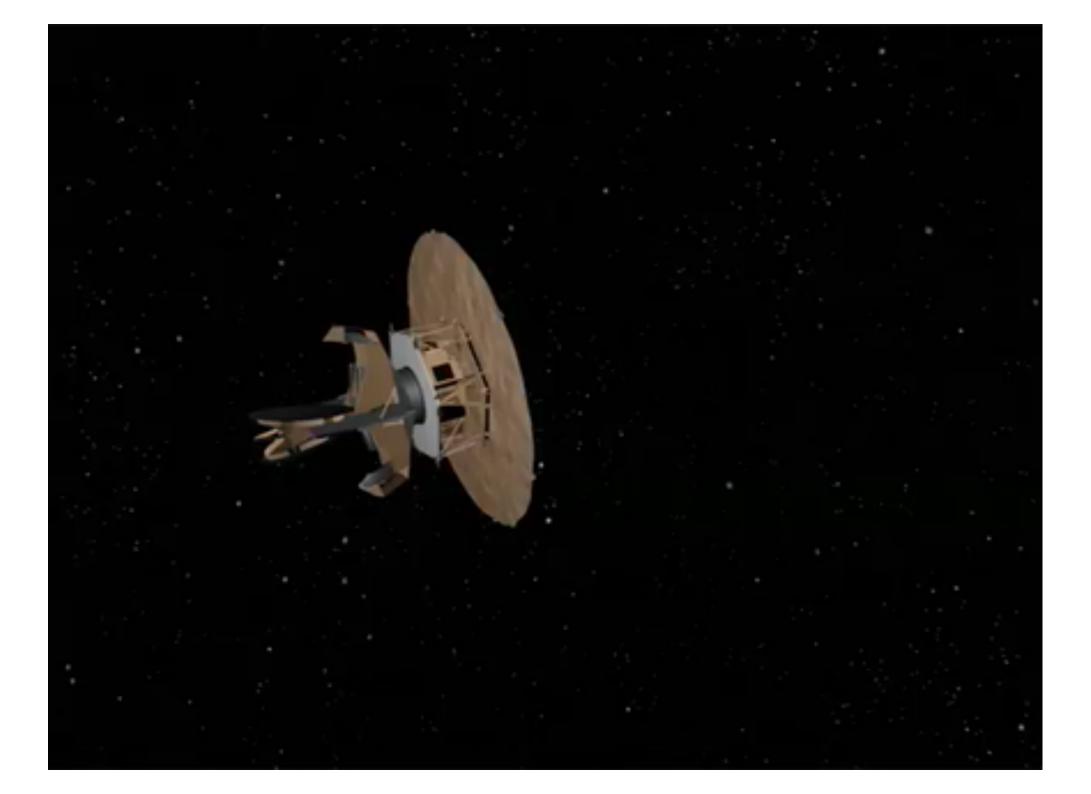


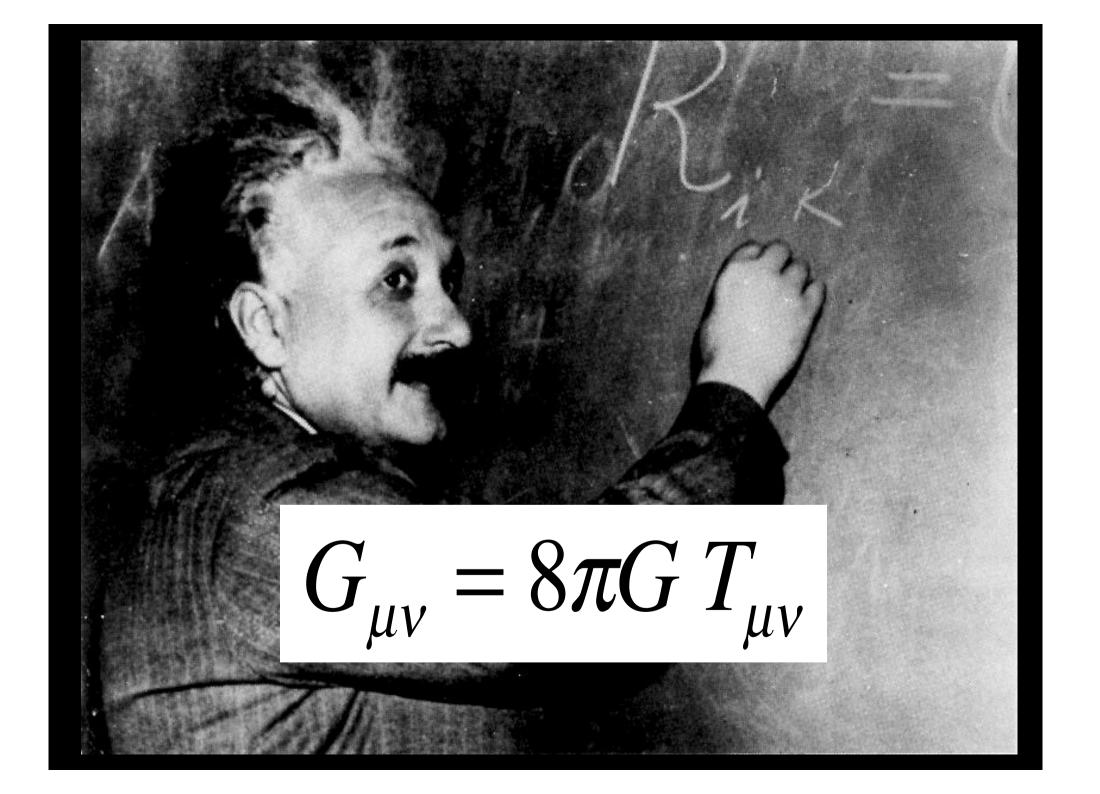
Juan García-Bellido Inst. Física Teórica UAM 7th March 2013

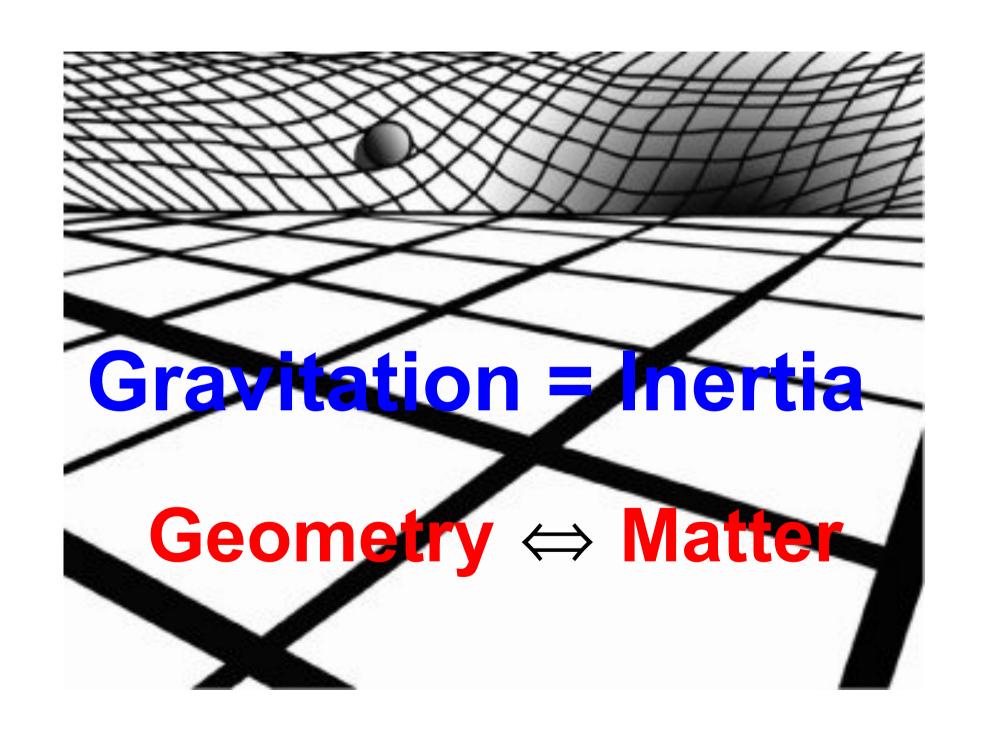
Overview

- I. The accelerating Universe Dark Energy
- II. Structure Formation

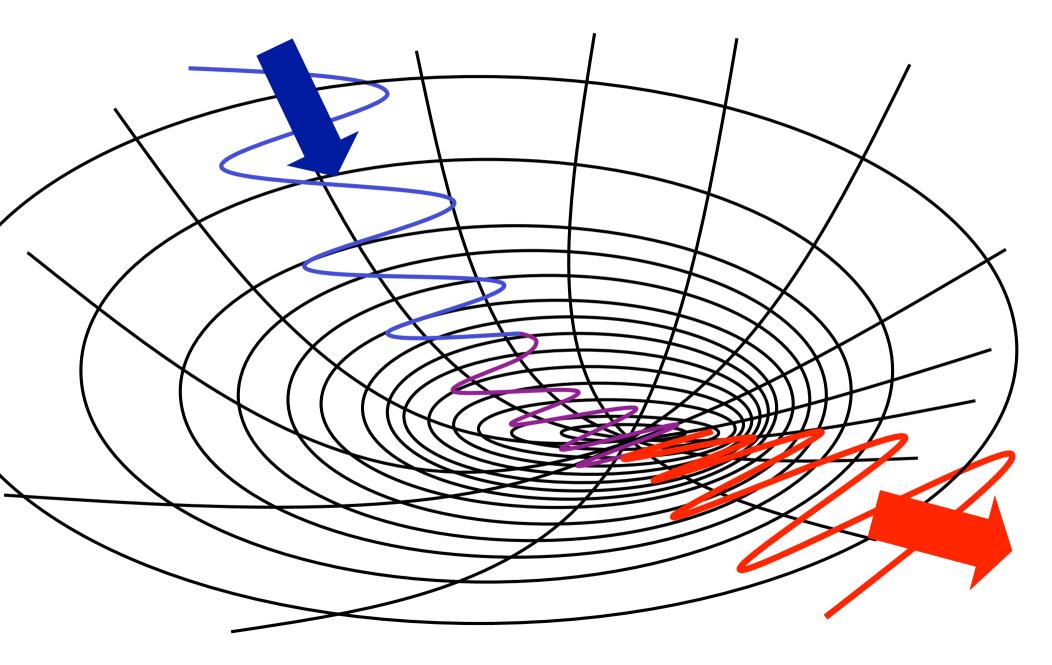
 Dark Matter
- III. CMB Anisotropies
 Inflation



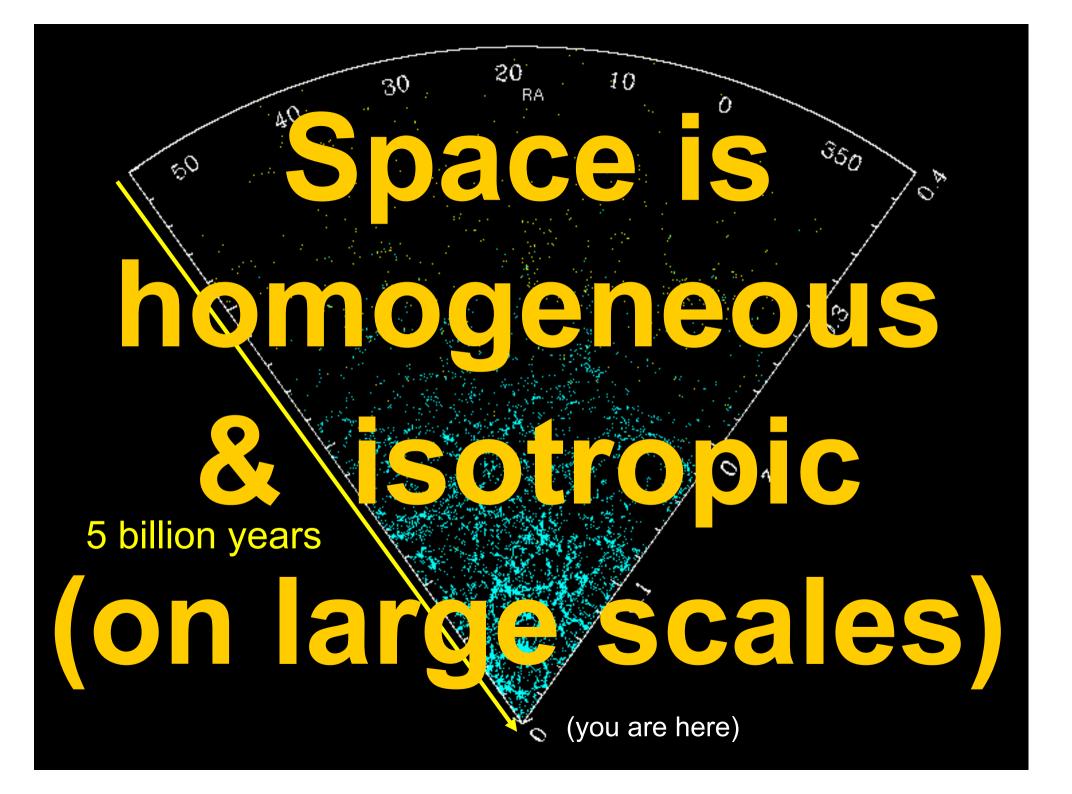




Gravitacional blueshift & redshift



What is the Geometry of Universe?



General Relativity Universe expansion

Hot Big Bang

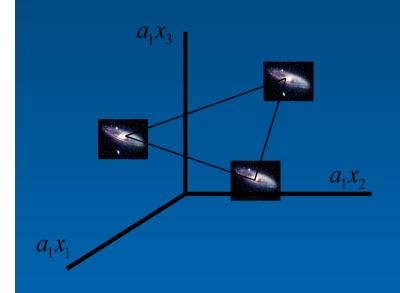
General Relativity

$$G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Homogeneity and Isotropy

$$ds^{2} = -dt^{2} + \underline{a^{2}(t)} \left[\frac{dr^{2}}{1 - Kr^{2}} + r^{2} d\Omega \right] FRW$$

$$ds^{2} = -dt^{2} + a^{2}(t)(dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2})$$

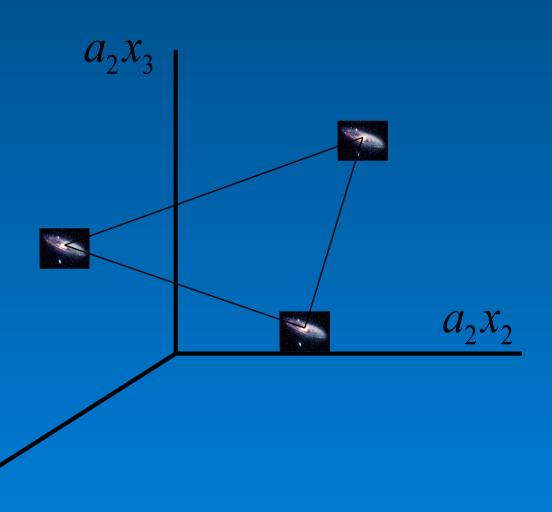


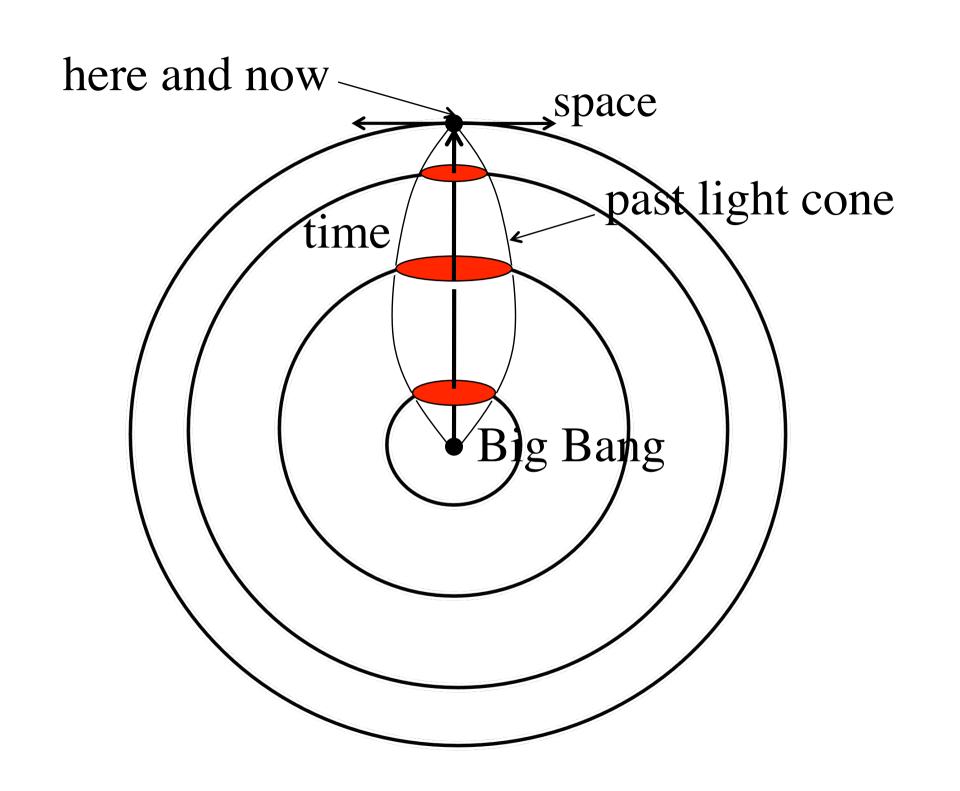
scale factor

$$a(t_2) > a(t_1)$$

 $a_2 x_1$

flat space





Spatial Curvature

$$^{(3)}R = \frac{6K}{a^2(t)}$$

Closed

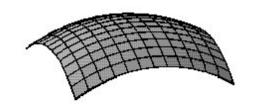
Flat

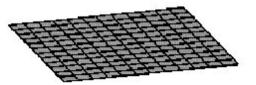
Open

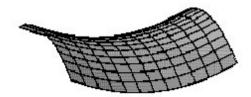
$$K = +1$$
 $K = 0$

$$K = 0$$

$$K = -1$$







Matter Content: Perfect Fluid

$$T_{\mu\nu} = p g_{\mu\nu} + (\rho + p) U_{\mu} U_{\nu}$$

Isotropic in its rest frame:

$$T^{\mu}_{v} = diag(-\rho(t), p(t), p(t), p(t))$$

Energy density conservation:

$$D_{\mu}T^{\mu}{}_{\nu} = 0 \Rightarrow \dot{\rho}(t) + 3\frac{\dot{a}}{a}(\rho(t) + p(t)) = 0$$

Einstein-Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \qquad ij + 00$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{00}$$

Equation of state of matter

$$p(t) = w \rho(t)$$
 barotropic fluid

Friedmann equation

$$\frac{1}{2}m\dot{a}^2 - \frac{GMm}{a} = -\frac{mK}{2}$$

$$T + V = E$$

$$M = \frac{4\pi}{3} \rho \ a^3$$

K = 0 escape velocity

K > 0 recollapse

K < 0 expand forever

Universe dynamics (K=0)

Radiation:
$$p = \rho/3$$

$$\rho_R \propto a^{-4}$$

$$a_R \propto t^{1/2}$$

Matter:

$$p \ll \rho$$

$$\rho_M \propto a^{-3}$$

$$a_M \propto t^{2/3}$$

Vacuum:

$$p = -\rho$$

$$\rho_{V} \propto a^{0}$$

$$a_V \propto e^{Ht}$$

Cosmological Parameters

Rate of Expansion (Hubble)

$$H_0 = \frac{\dot{a}}{a}(t_0) = 100 h \text{ km/s/Mpc}$$

$$H_0^{-1} = 9.773 h^{-1} \text{ Gyr}$$

$$cH_0^{-1} = 3000 \, h^{-1} \, \text{Mpc}$$

$$1 \text{ pc} = 3.262 \text{ ly} = 3.086 \times 10^{16} \text{ m}$$

Critical density (K=0)

$$\rho_c(t_0) = \frac{3H_0^2}{8\pi G}$$

$$=1.88 h^2 10^{-29} \text{ g/cm}^3$$

=
$$2.77 h^{-1} 10^{11} M_{\odot} / (h^{-1} Mpc)^{3}$$

= 11.26
$$h^2$$
 protons/m³

Density parameter

$$\Omega_0 = \frac{8\pi G}{3H^2} \rho(t_0) = \frac{\rho}{\rho_c} (t_0)$$

$$\Omega_0 = \Omega_R + \Omega_M + \Omega_\Lambda$$

$$\Omega_R = \frac{\rho_R}{\rho_c}(t_0) \qquad \Omega_M = \frac{\rho_M}{\rho_c}(t_0)$$

$$\Omega_{\Lambda} = \frac{\Lambda}{3H_0^2} \qquad \Omega_K = \frac{-K}{a_0^2 H_0^2}$$

Spatial Curvature

$$K = +1$$

$$K = 0$$

$$K = -1$$

Closed

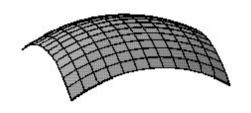
Flat

Open

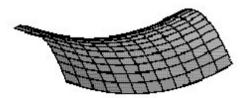
$$\Omega_0 > 1$$

$$\Omega_0 = 1$$

$$\Omega_0 < 1$$







Cosmic Sum Rule

Friedmann equation

$$H^{2} = \frac{8\pi G}{3} (\rho_{R} + \rho_{M}) + \frac{\Lambda}{3} - \frac{K}{a^{2}}$$

Today:
$$1 = \Omega_R + \Omega_M + \Omega_\Lambda + \Omega_K$$

No vacuum:
$$\Omega_{\Lambda} = 0 \implies \Omega_{K} = 1 - \Omega_{M}$$

Flat space:
$$\Omega_K = 0 \implies \Omega_{\Lambda} = 1 - \Omega_M$$

Deceleration parameter

$$q_0 = -\frac{a\ddot{a}}{\dot{a}^2}(t_0) = \frac{4\pi G}{3H_0^2}(\rho + 3p)$$

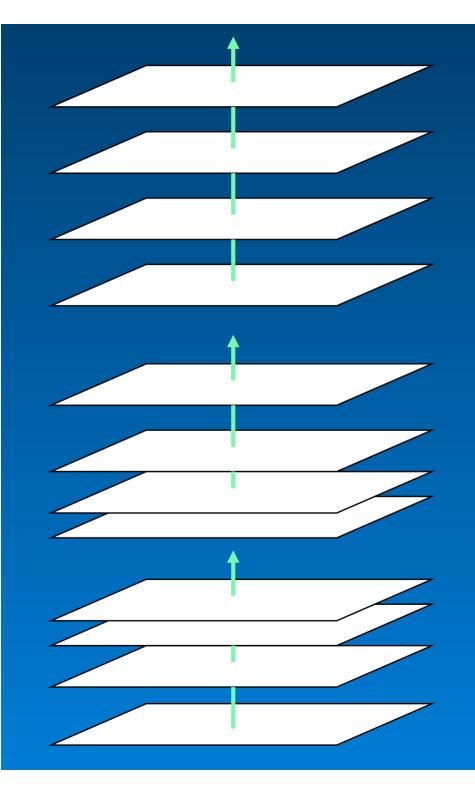
$$q_0 = \Omega_R + \frac{1}{2}\Omega_M - \Omega_{\Lambda} + \frac{1}{2}\sum_x (1 + 3w_x)\Omega_x$$

Matter domination:

$$q_0 > 0$$

Vacuum domination:

$$q_0 < 0$$



Uniform expansion

$$q_0 = 0$$

$$\Omega_M = 2\Omega_{\Lambda}$$

Accelerated expansion

$$q_0 < 0$$

$$\Omega_M < 2\Omega_\Lambda$$

Decelerated expansion

$$q_0 > 0$$

$$\Omega_M > 2\Omega_{\Lambda}$$

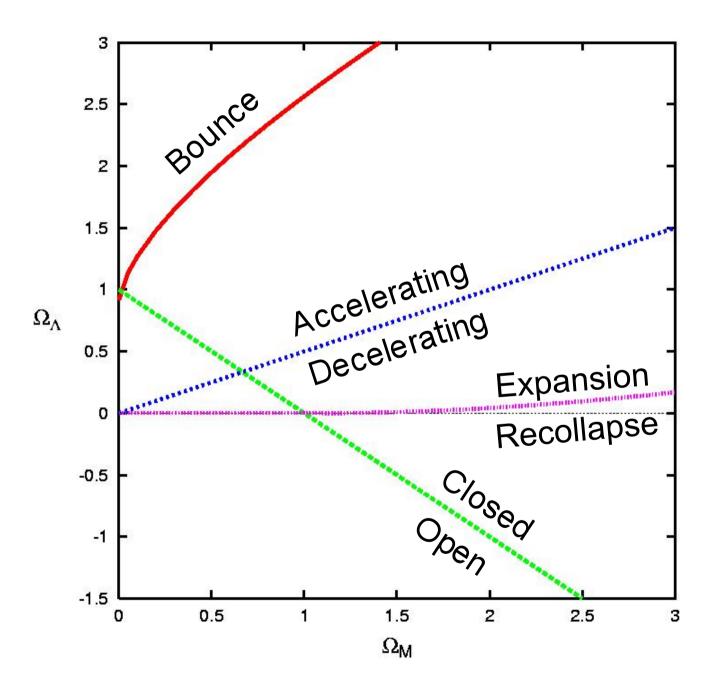
Bounce
$$H_0 t_0 = \int_0^1 \frac{da}{\sqrt{1 + \Omega_M (\frac{1}{a} - 1) + \Omega_\Lambda (a^2 - 1)}} = \infty$$

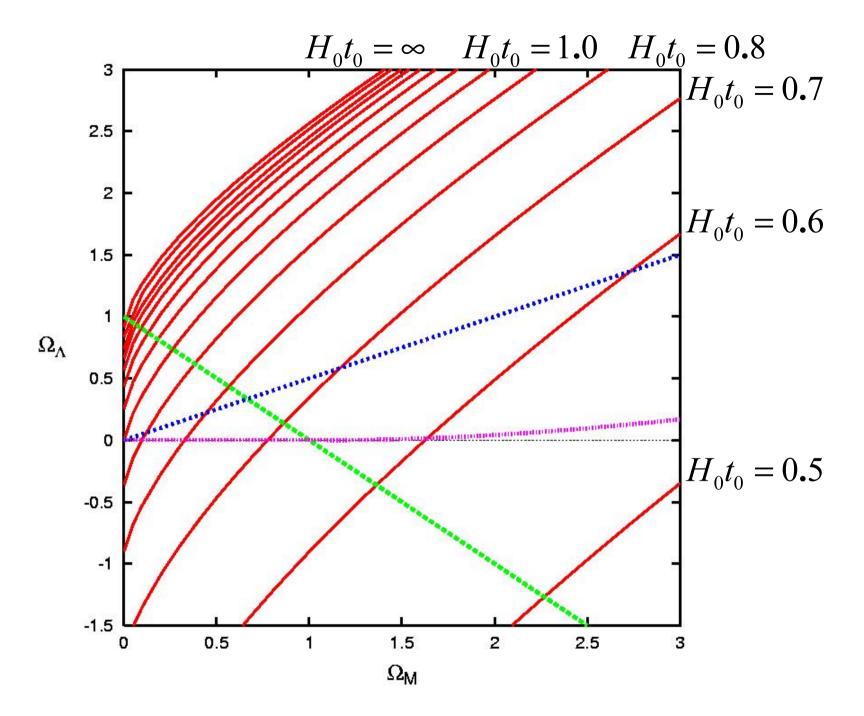
Uniform exp.
$$q_0 = 0 \implies \Omega_{\Lambda} = \frac{1}{2}\Omega_{M}$$

Critical univ.

$$\Omega_{\Lambda} = \begin{cases} 0 & \Omega_{M} \leq 1 \\ 4\Omega_{M} \sin^{3}\left[\frac{1}{3} \arcsin\left(\frac{\Omega_{M}-1}{\Omega_{M}}\right)\right] & \Omega_{M} > 1 \end{cases}$$

Flat space
$$\Omega_K = 0 \implies \Omega_{\Lambda} = 1 - \Omega_M$$





Cosmological Parameters

 H_0 Rate of expansion

 t_0 Age of the Universe

*q*₀ Acceleration Parameter

 Ω_K Spatial Curvature

 $\Omega_{\scriptscriptstyle M}$ Dark Matter

 Ω_{Λ} Cosmological Constant

 Ω_B Baryon Density

 $\Omega_{
m v}$ Neutrino Density

The Expanding Universe

Geodesic motion

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}_{v\lambda} u^{\nu} u^{\lambda} = 0; \qquad u^{\mu} = (\gamma, \gamma v^{i})$$

$$\Gamma_{ij}^0 = \frac{\dot{a}}{a} g_{ij} \quad \Rightarrow \quad |\vec{u}| \propto \frac{1}{a} \quad \Rightarrow \quad |\vec{p}| \propto \frac{1}{a}$$

Photon redshift

$$p = \frac{h}{\lambda}$$

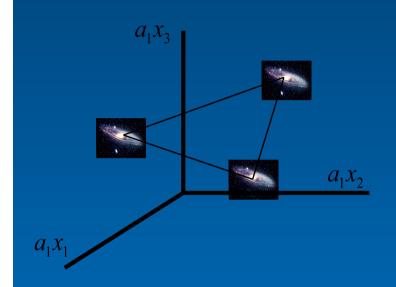
$$\frac{\lambda_1}{\lambda_0} = \frac{a(t_1)}{a(t_0)} \implies z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a_0}{a_1} - 1$$



redshift

$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a_0}{a_1} = 1 + z$$

$$ds^{2} = -dt^{2} + a^{2}(t)(dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2})$$

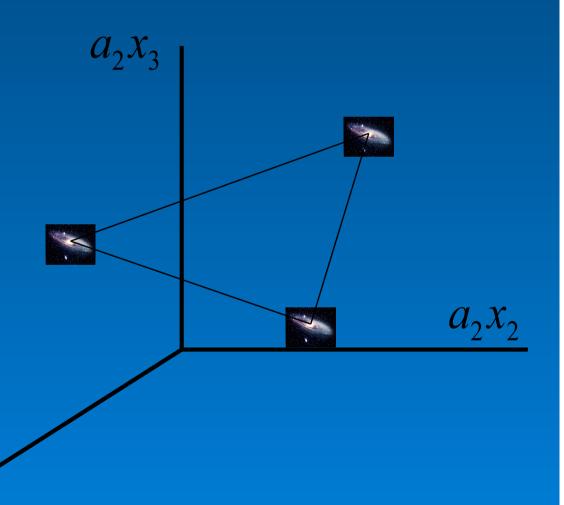


scale factor

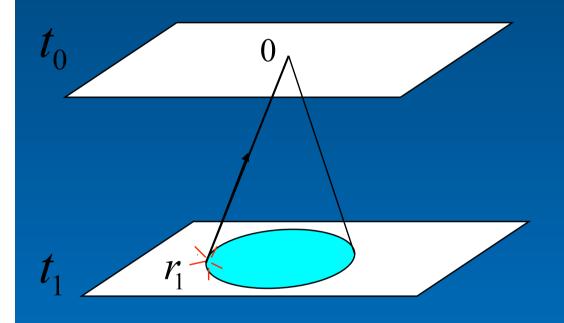
$$\frac{a(t_2)}{a(t_1)} = \frac{1+z_1}{1+z_2}$$

$$a_2 x_1$$

flat space



FRW kinematics



Physical distance

$$d = a_0 r_1 \quad (1^{st} order)$$

Light cone:

$$0 = -dt^2 + a^2(t) \frac{dr^2}{1 - Kr^2}$$

$$\int_{t_{1}}^{t_{0}} \frac{dt}{a(t)} = \int_{0}^{r_{1}} \frac{dr}{\sqrt{1 - Kr^{2}}} = f(r_{1}) = \begin{cases} \mathbf{arcsin} \, r_{1} & K = 1 \\ r_{1} & K = 0 \\ \mathbf{arcsinh} \, r_{1} & K = -1 \end{cases}$$

Taylor expansion

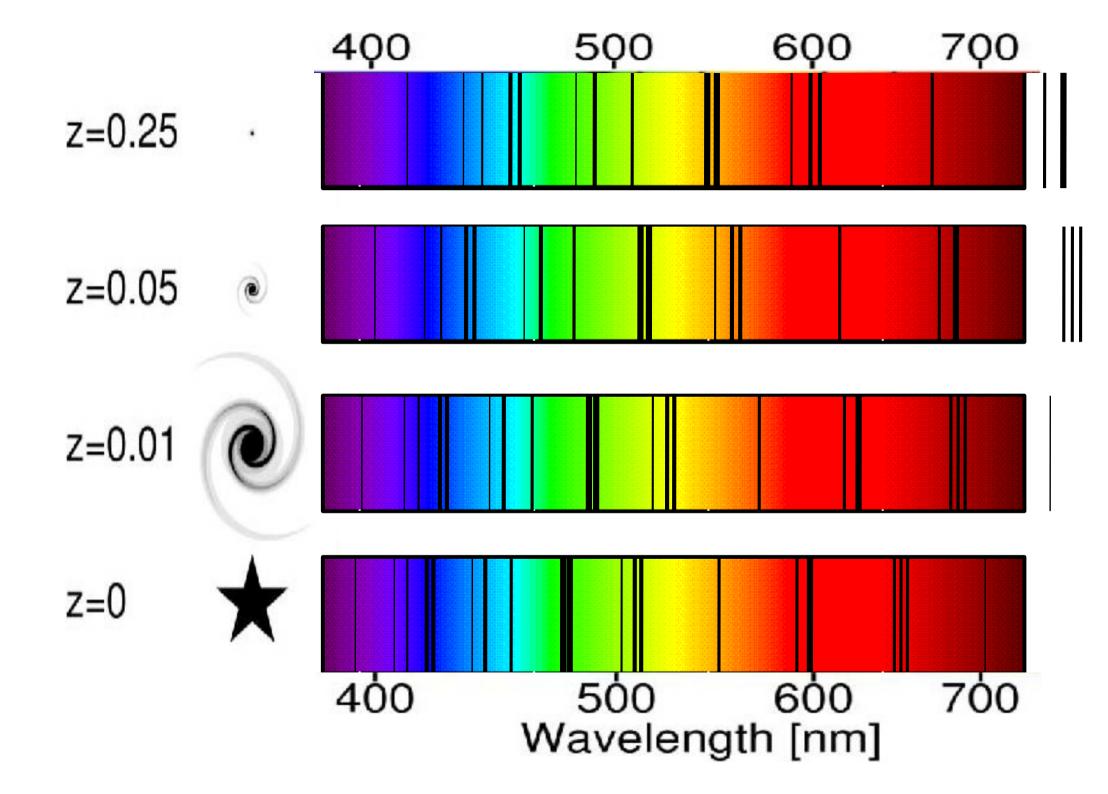
$$\frac{1}{1+z} = \frac{a(t)}{a_0} = 1 + H_0(t-t_0) + O(t-t_0)^2$$

To first approximation

$$r_1 \approx f(r_1) = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0} (t_1 - t_0) + \dots = \frac{z}{a_0 H_0} + \dots$$

Hubble law

$$H_0 d = H_0 a_0 r_1 = z \approx vc$$



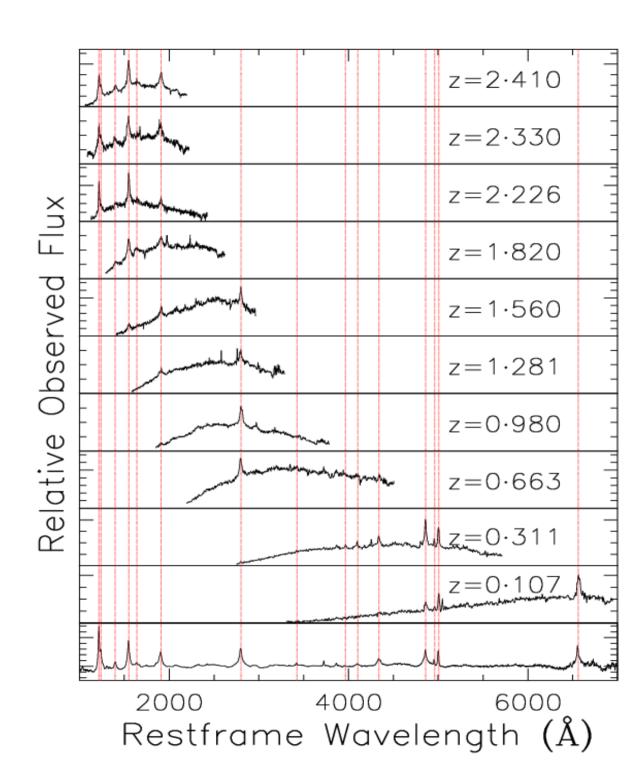
Edwin P. Hubble

Mount Wilson

Mount Palomar







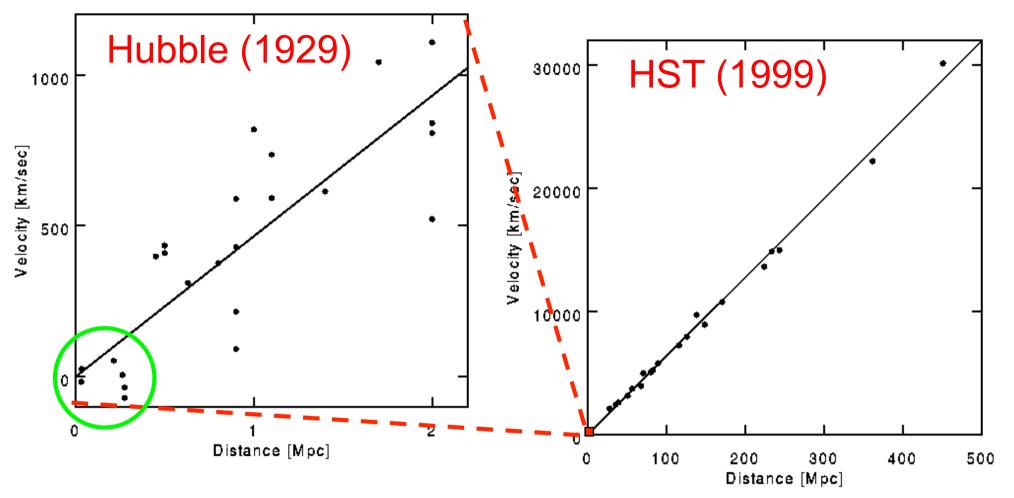
Redshifts to galaxies

Hubble law

$$H_0 d = z \approx vc$$

Hubble Space Telescope





 $H_0 = 500 \, \text{km/s/Mpc}$

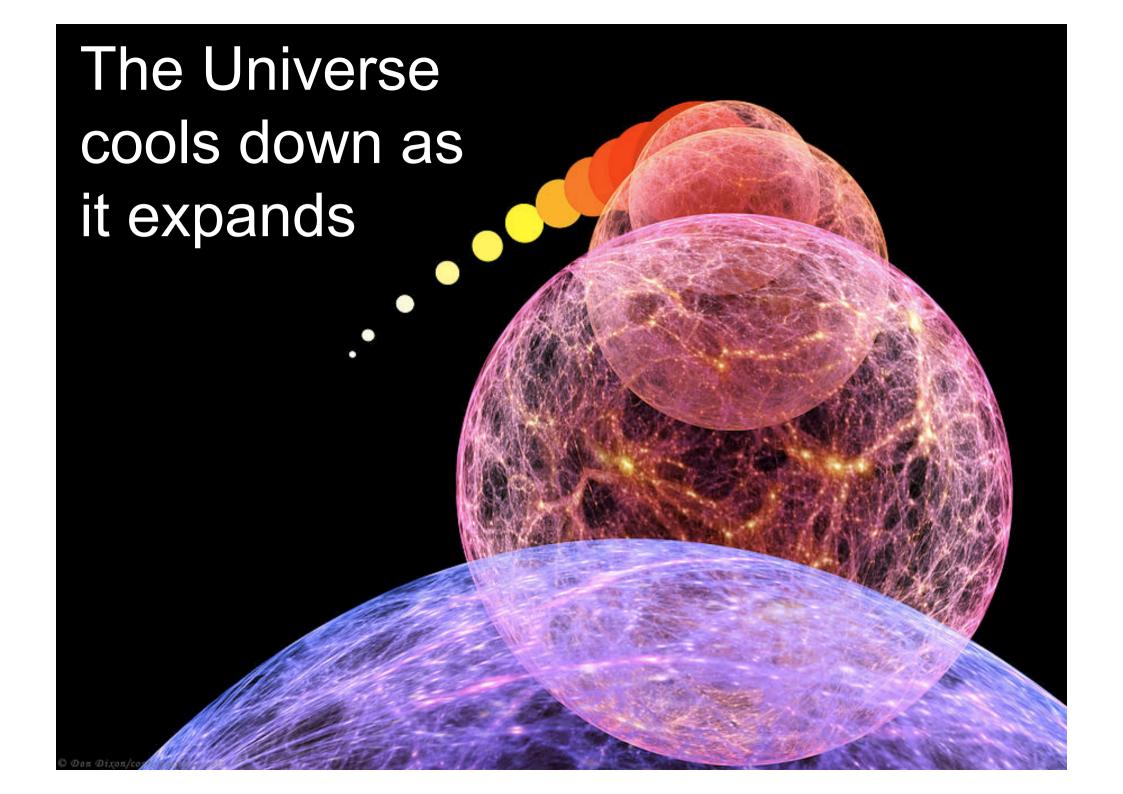
Dominated by systematic errors!

$$H_0 = 70 \text{ km/s/Mpc}$$
 $z \le 0.1$

ne AGIMO Universe

If the universe is expanding, necessarily it must have been denser and hotter in the past

Tracing the history of the universe, we reach the realm of high energy physics and particle accelerators



Fluids in thermal and chemical equilibrium

$$n = \frac{g}{2\pi^2} \int_m^\infty dE \, \frac{E(E^2 - m^2)^{1/2}}{e^{(E-\mu)/T} \pm 1} \,,$$

$$\rho = \frac{g}{2\pi^2} \int_{m}^{\infty} dE \, \frac{E^2 (E^2 - m^2)^{1/2}}{e^{(E-\mu)/T} \pm 1} \,,$$

$$p = \frac{g}{6\pi^2} \int_{m}^{\infty} dE \, \frac{(E^2 - m^2)^{3/2}}{e^{(E-\mu)/T} \pm 1} \, .$$

Relativistic particles

Gas of relativistic $(m \ll T)$, non degenerate $(\mu \ll T)$ particles

$$n = \frac{g}{2\pi^2} \int_0^\infty \frac{E^2 dE}{e^{E/T} \pm 1} = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{Bosons} \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3 & \text{Fermions} \end{cases},$$

$$\rho = \frac{g}{2\pi^2} \int_0^\infty \frac{E^3 dE}{e^{E/T} \pm 1} = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{Bosons} \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 & \text{Fermions} \end{cases},$$

$$p=\frac{1}{3}\rho\,, \qquad \langle E\rangle\equiv\frac{\rho}{n}=\left\{\begin{array}{ll} \frac{\pi^4}{30\zeta(3)}\,T\simeq 2.701\,T & \text{Bosons}\\ \frac{7\pi^4}{180\zeta(3)}\,T\simeq 3.151\,T & \text{Fermions} \end{array}\right.$$

Non-relativistic particles

Gas of non-relativistic $(m\gg T)$ particles, with chemical pot. μ

$$n = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T},$$

$$\rho = m n, \qquad \langle E \rangle \equiv \frac{\rho}{n} = m + \frac{3}{2}T$$

$$p = n T \ll \rho.$$

Its contribution to the total energy of the universe is always subdominant (Boltzman suppressed) w.r.t. relativistic species.

Energy density and entropy of the Universe

$$\rho_{\rm R} = \frac{\pi^2}{30} g_* T^4, \qquad p_{\rm R} = \frac{1}{3} \rho_{\rm R},$$

$$g_*(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4,$$

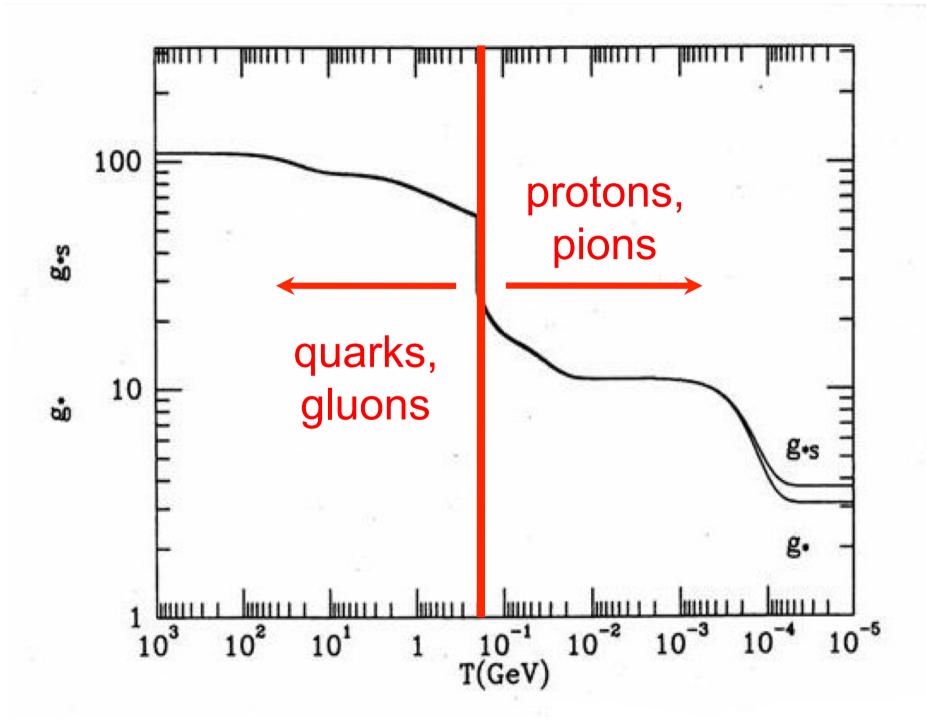
Relation between time and temperature

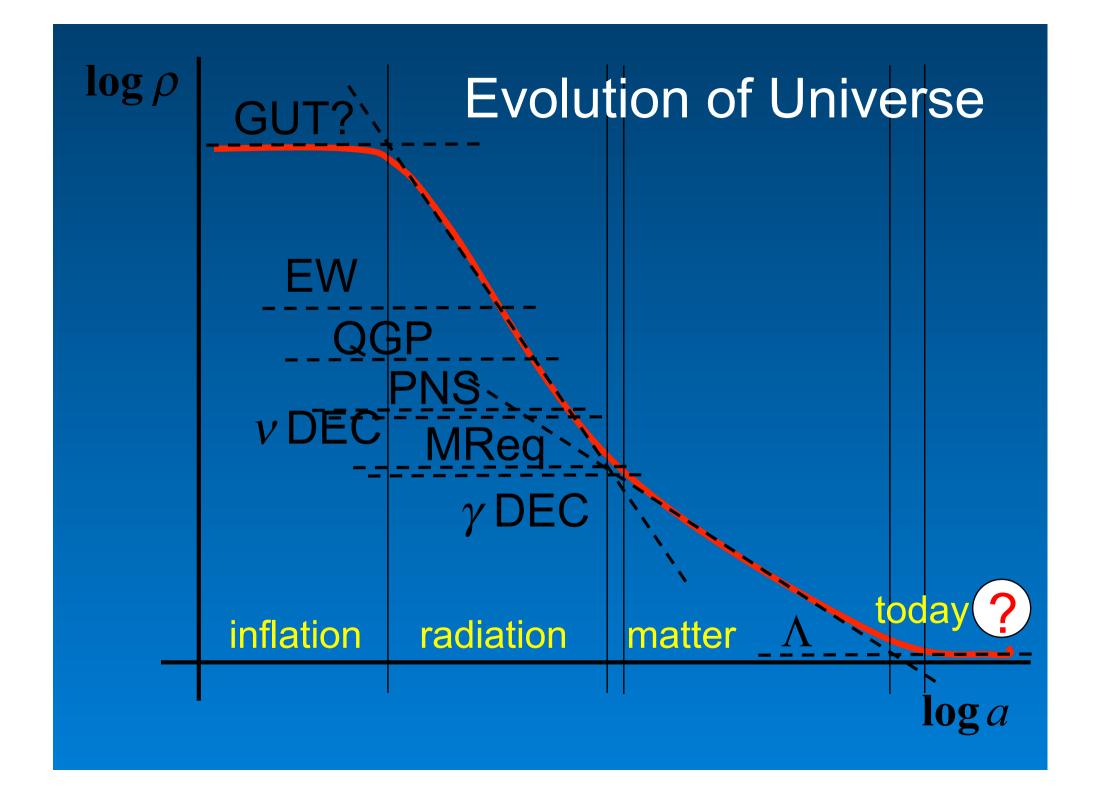
$$H = 1.66 g_*^{1/2} \frac{T^2}{M_P} = \frac{1}{2t} \implies t = 0.301 g_*^{-1/2} \frac{M_P}{T^2} = 2.42 g_*^{-1/2} \left(\frac{\text{MeV}}{T}\right)^2 \text{ s}$$

Entropy is conserved, while the Universe expands adiabatically

$$S = \frac{2\pi^2}{45} g_{*S} (aT)^3 = \text{const.},$$

$$g_{*S}(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

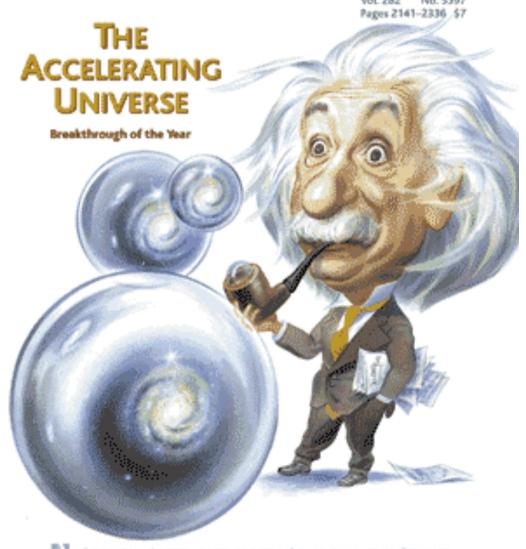




ne Accelerating Universe

18 December 1998

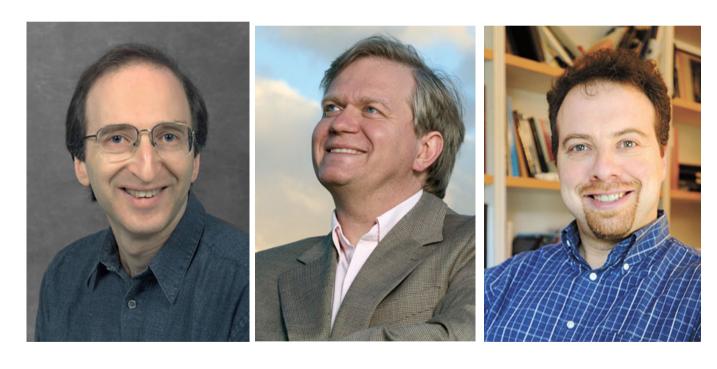
Vol. 282 No. 5397 Pages 2141–2336 \$7





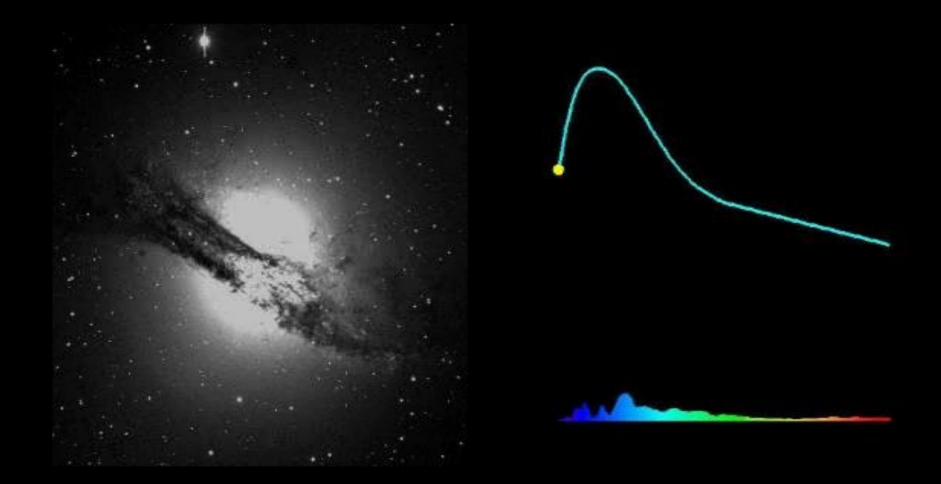
The Nobel Prize in Physics 2011

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

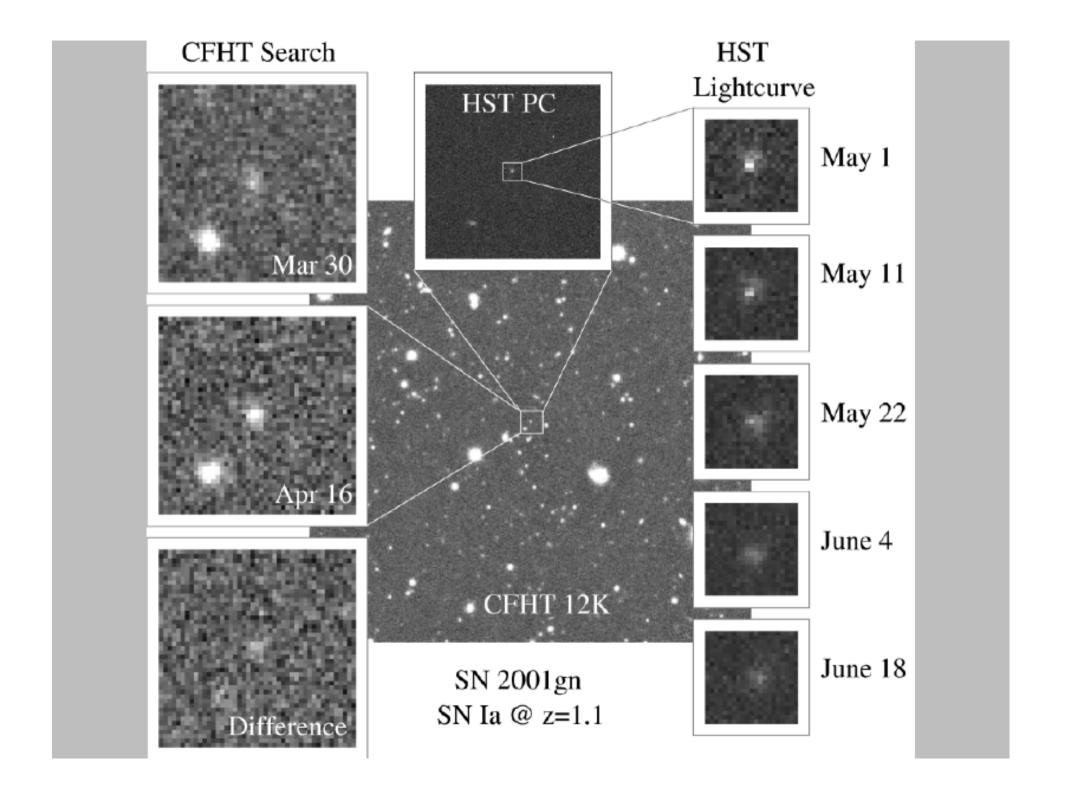


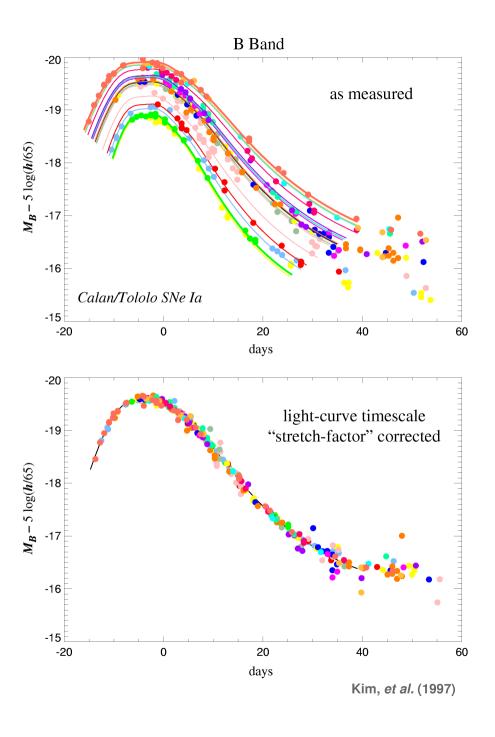
Saul Perlmutter Brian P. Schmidt Adam G. Riess





Epoch 1 Epoch 2 Epoch 2 - Epoch 1





Supernovae la Lightcurves & Stretch-factor

SNIa as Standard Candles

Luminosity distance

L Absolute Luminosity of source (dE/dt)

F Measured Flux at detector (dE/dt dA)

$$F = \frac{L}{4\pi (1+z)^2 a_0^2 r^2(z)} = \frac{L}{4\pi d_L^2(z)}$$

$$H_0 d_L(z) = (1+z) |\Omega_K|^{-1/2} \sinh \left[\int_0^z \frac{|\Omega_K|^{1/2} dz'}{H(z')} \right]$$

Effective magnitude

$$m(z) \equiv M + 5\log_{10}\left(\frac{d_L(z)}{\text{Mpc}}\right) + 25$$
$$= \overline{M} + 5\log_{10}[H_0d_L(z)]$$

Taylor expansion to third order

$$\frac{a(t)}{a_0} = 1 + H_0(t - t_0) - \frac{q_0}{2!}H_0^2(t - t_0)^2 + \frac{j_0}{3!}H_0^3(t - t_0)^3 + \dots$$

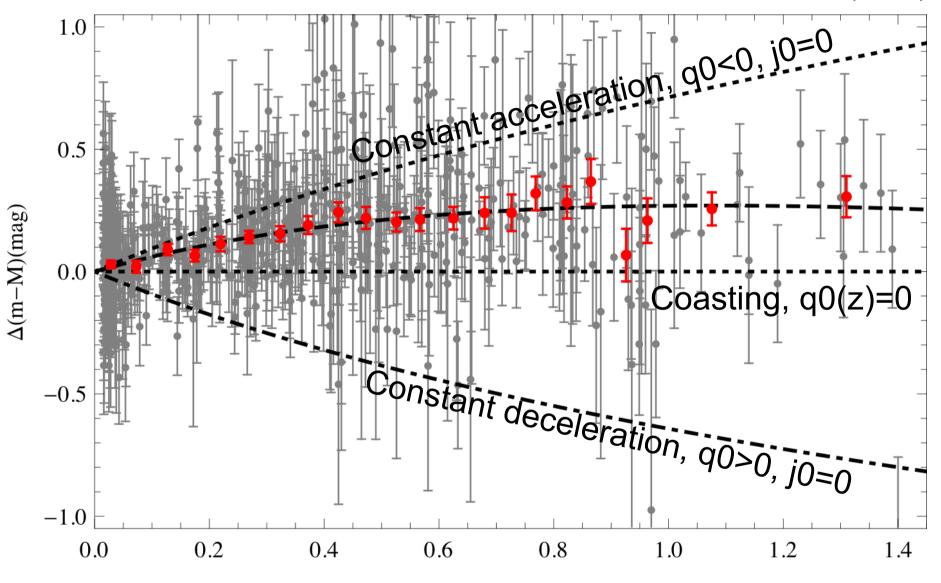
$$H(t) = \frac{\dot{a}}{a} \qquad q(t) = -\frac{\ddot{a}}{aH^2} \qquad j(t) = \frac{\ddot{a}}{aH^3}$$

To very good approximation, to $O(z^3)$

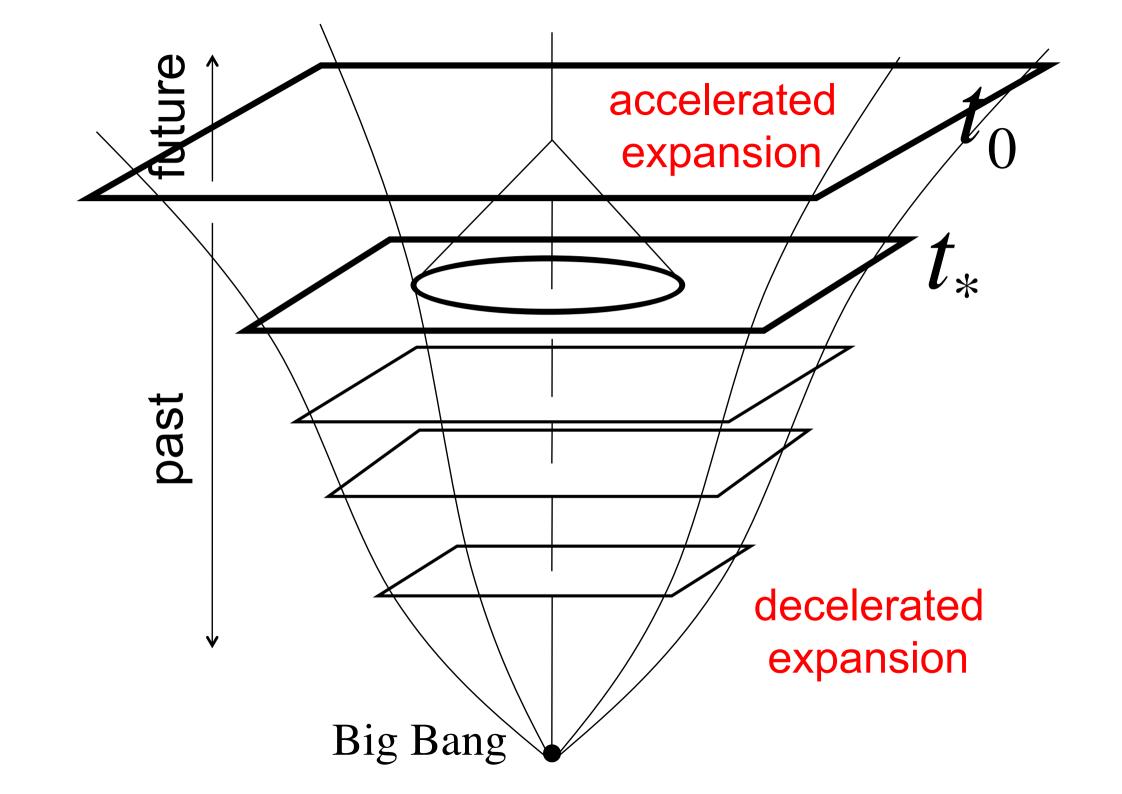
$$d_L(z) = \frac{cz}{H_0} \left\{ 1 + \frac{1}{2} (1 - q_0)z - \frac{1}{6} (1 - q_0 - 3q_0^2 + j_0)z^2 + \ldots \right\}$$

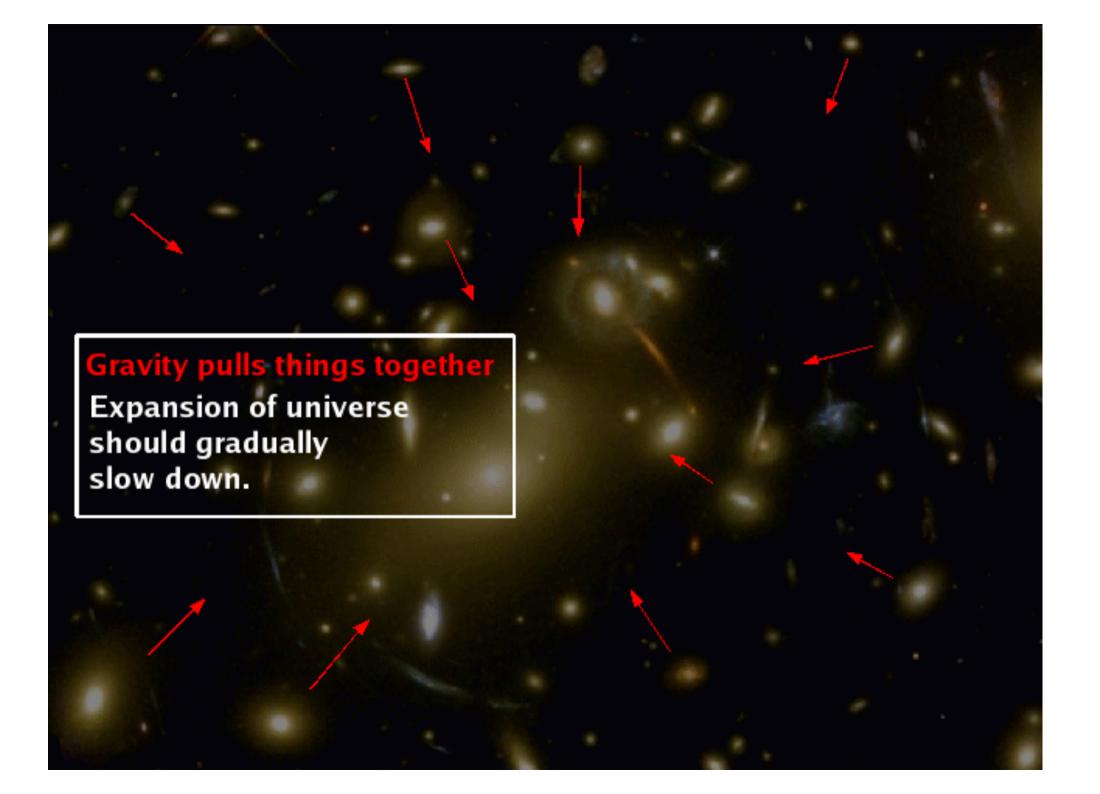
Union-2 SNe

Amanullah et al. (2010)



 \mathbf{Z}



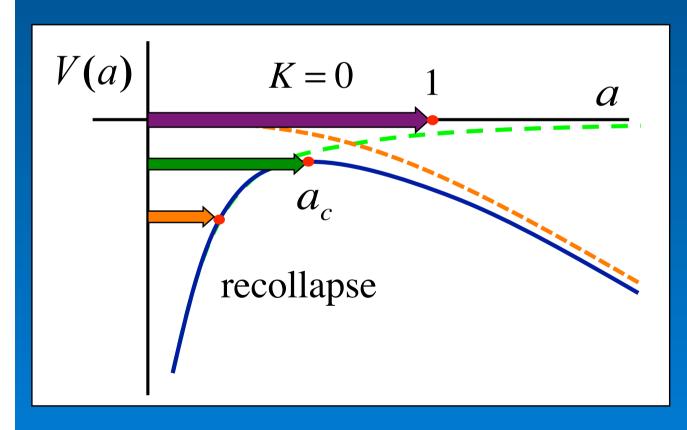




Einstein-de Sitter model (∧>0)

$$\frac{1}{2}\dot{a}^2 - \frac{GM}{a} - \frac{\Lambda}{6}a^2 = -\frac{K}{2}$$

$$T + V = E$$



$$a_c = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$

coasting
point
(unstable)

What is the acceleration of the universe today?

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}$$
 Friedmann

$$\ddot{a}_0 = \left(-\frac{\Omega_M}{2} + \Omega_{\Lambda}\right) a_0 H_0^2$$

$$= 0.5863 \ a_0 t_0^{-2}$$

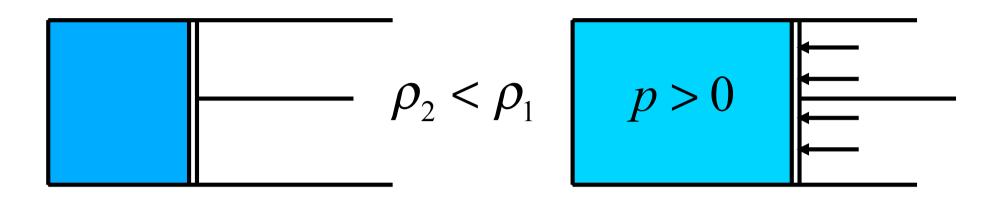
$$= 9.2 \times 10^{-10} \ \text{ms}^{-2}$$



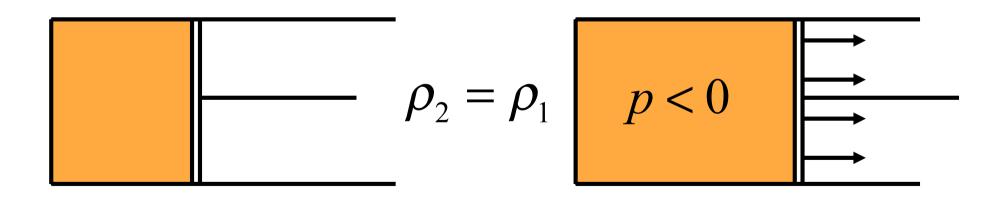
"THE UNIVERSE IS EXPANDING FASTER THAN EVER, AND I DON'T EVEN FEEL A BREEZE."

HOW GO We interpret this DE?

Normal Matter



$$d(\rho V) + pdV = TdS = 0$$

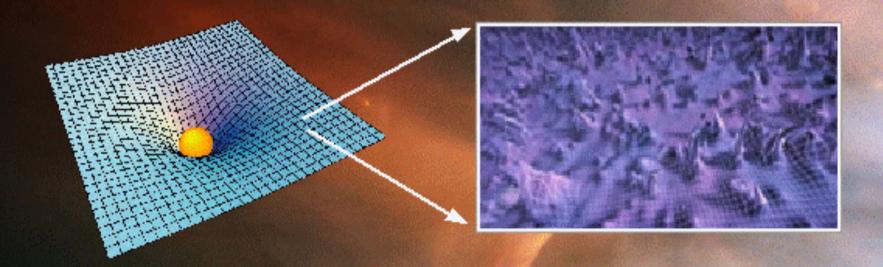


Vacuum Energy

The Physics of Nothing

How can *nothing* be most of *everything* in the universe?

The answer (maybe) is quantum uncertainty: "empty space" is a sea of virtual particles winking in and out of existence:



Nothing is something!

Cosm. Const. = Vacuum Energy

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu} = p_{\upsilon} g_{\mu\nu} = -\rho_{\upsilon} g_{\mu\nu} \implies \Lambda = 8\pi G \rho_{\upsilon}$$

$$\rho_{\upsilon} = \sum_{i} \int_{0}^{\Lambda_{UV}} \frac{d^{3}k}{(2\pi)^{3}} \frac{\hbar \omega_{i}(k)}{2} = \frac{\hbar \Lambda_{UV}^{4}}{16\pi^{2}} \sum_{i} N_{i} + O(m^{2}\Lambda^{2})$$

$$\Lambda_{UV} \approx M_{Pl} \implies \rho_{\upsilon}^{th} \approx 10^{120} \rho_{\upsilon}^{obs} = 10^{120} (10^{-3} eV)^{4}$$

$$\Lambda_{UV} \approx M_{Higgs} \implies \rho_{\upsilon}^{th} \approx 10^{65} \rho_{\upsilon}^{obs}$$

Nature of Dark Energy?

$$\Lambda = 8\pi G \, \rho_v = \text{const.} \quad \Rightarrow \quad w_v = \frac{p_v}{\rho_v} = -1$$

$$\rho_x \neq \text{const.} \quad \Rightarrow \quad w_x \neq -1$$

$$H^{2}(z) = H_{0}^{2} \left[\Omega_{M} (1+z)^{3} + \Omega_{x} e^{\int_{0}^{z} (1+w_{x}(u)) \frac{3du}{1+u}} + \Omega_{K} (1+z)^{2} \right]$$

$$q(z) = -1 + \frac{d \ln H(z)}{d \ln (1+z)}$$
$$= \frac{1}{2}\Omega_0 + \frac{3}{2}w_x(z)\Omega_x(z)$$

Coasting Point

Assuming $w_x = w = \text{const.} < 0$

$$q(z) = \frac{1}{2} \left[\frac{\Omega_M + (1+3w)\Omega_x (1+z)^{3w}}{\Omega_M + \Omega_x (1+z)^{3w} + \Omega_K (1+z)^{-1}} \right] = 0$$

$$\Rightarrow z_c = \left(\frac{(3|w| - 1)\Omega_x}{\Omega_M} \right)^{\frac{1}{3|w|}} - 1$$

 $z > z_c$ universe decelerating $z < z_c$ universe accelerating

e.g.
$$w = -1 \implies z_c = \left(\frac{2\Omega_{\Lambda}}{\Omega_M}\right)^{\frac{1}{3}} - 1 \approx 0.7$$

Model Building

SCDM
$$H(z) = H_0(1+z)^{3/2}$$
 ruled out!

$$\Lambda \text{CDM} \quad H(z) = H_0 [\Omega_M (1+z)^3 + 1 - \Omega_M]^{1/2}$$

$$\Rightarrow \quad \Omega_M = 0.27 \pm 0.03$$

$$\Lambda w CDM \quad H(z) = H_0 [\Omega_M (1+z)^3 + \Omega_\Lambda (1+z)^{3(1+w)}]^{1/2}$$

$$\Rightarrow \Omega_M = 1 - \Omega_{\Lambda} = 0.3$$
, $w = -1.02 \pm 0.10$

 $\Lambda CDM - w(z)$

$$H(z) = H_0[\Omega_M (1+z)^3 + \Omega_\Lambda \exp[3\int_0^z (1+w(u))\frac{du}{1+z}]]^{1/2}$$

New Observational Probes

Modern Cosmological Probes

- We have a whole array of tools at our disposal for studying the Universe with increasing detail.
- It allows us to disentangle the astrophysics from the cosmology and both from fundamental physics.
- We are now learning how to deal with systematic errors inherent to observations.

Four main observational probes:

- Gravitational lensing
- Supernovae
- Clusters of galaxies
- Baryon Acoustic Oscillations

Gravitational lensing

Purely geometric phenomenon, only depends on the distribution of matter between the source and us.

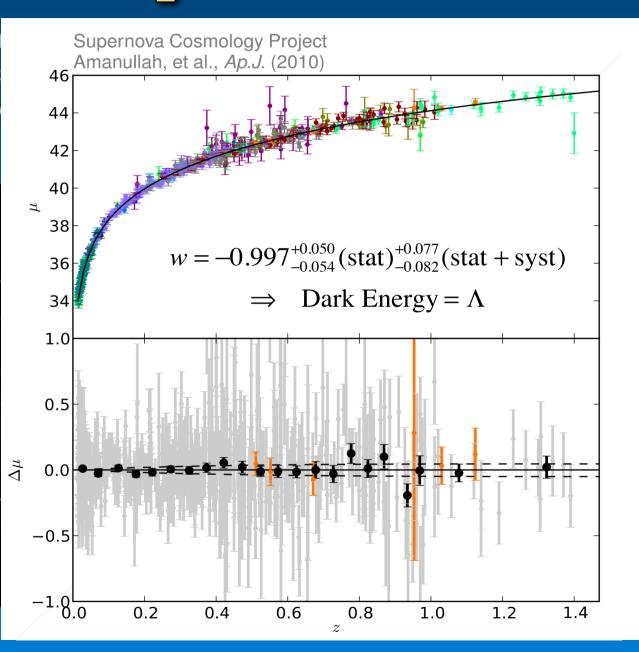
Allows us to model the mass distributions and measure their content.

It is a clean and reliable probe.



Supernovae

Stars th great di standar



een at d as

Clusters of galaxies

The largest virialized structures in the Universe.

Their X-ray emission allow us to estimate their mass.

Help determine the Halo Mass Function

Their number density in the Universe is very sensitive to cosmological parameters.

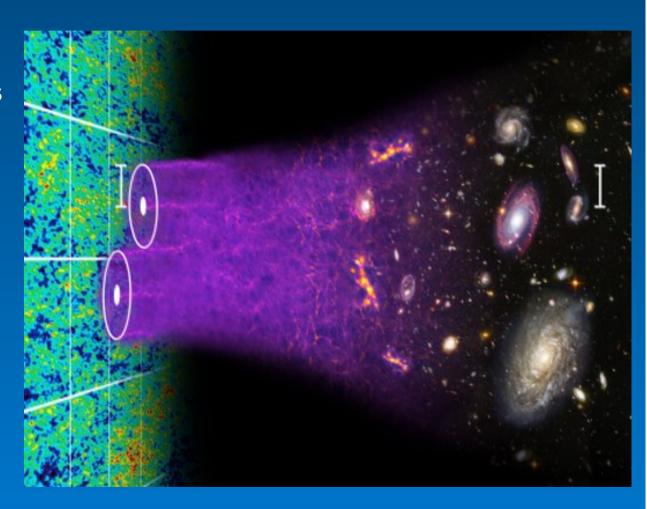


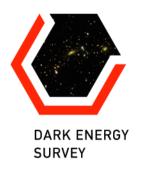
Baryon Acoustic Oscillations

The plasma before photon decoupling has fluctuations that propagate like sound waves.

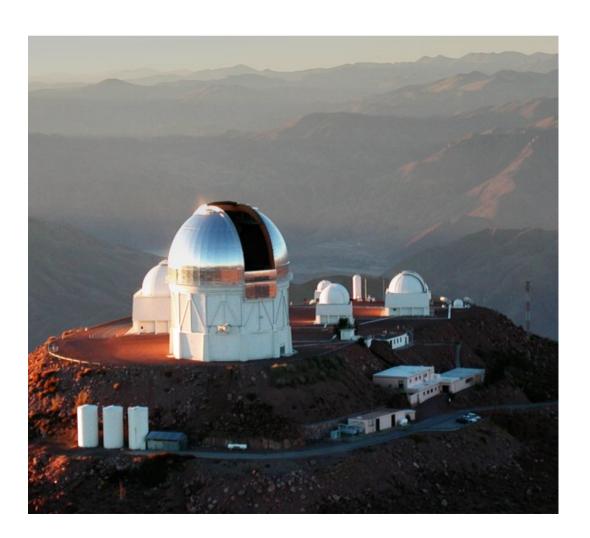
At decoupling there is a characteristic scale, the sonic horizon, that can be used as a standard ruler.

Its evolution with redshift since then is an excellent cosmic probe.





Blanco 4m telescope Cerro Tololo, Chile



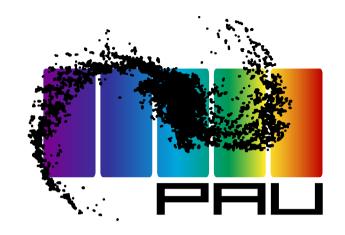
Dark Energy Survey

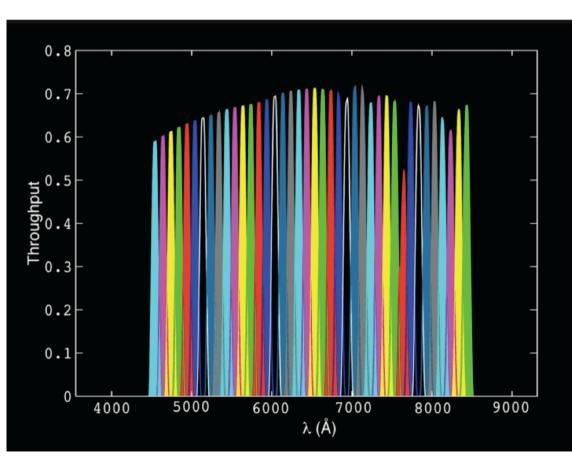
500 million galaxies 5000 deg sq. Δz_{photo} = 0.05 (1+z) 20 bins z range [0.2,1.5]

Cost: 100M\$



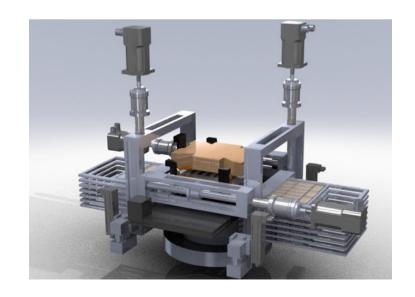


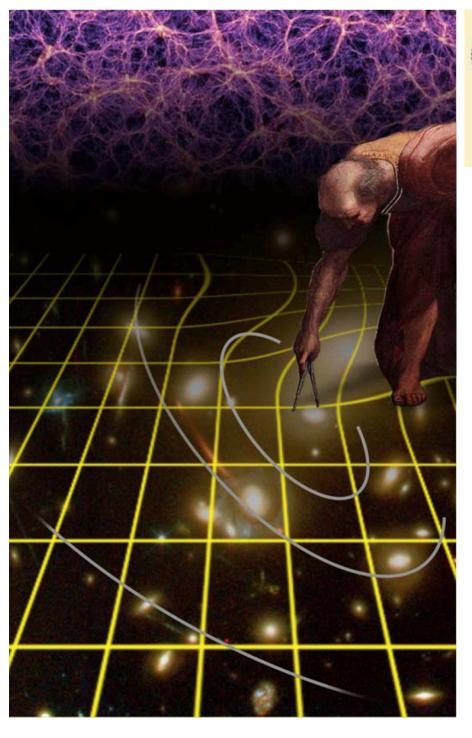




100 million galaxies 200 – 1000 deg sq. Δz_{photo} = 0.0035 (1+z) 100 bins z range [0.2,1.5] "Tomography"

Cost: 10M\$







EUCLID

Spectroscopic survey

100 million galaxies 15,000 sq. deg Δz_{spec} = 0.001 (1+z) 8 bins z range [0.5,2.1]

Cost: 1B\$

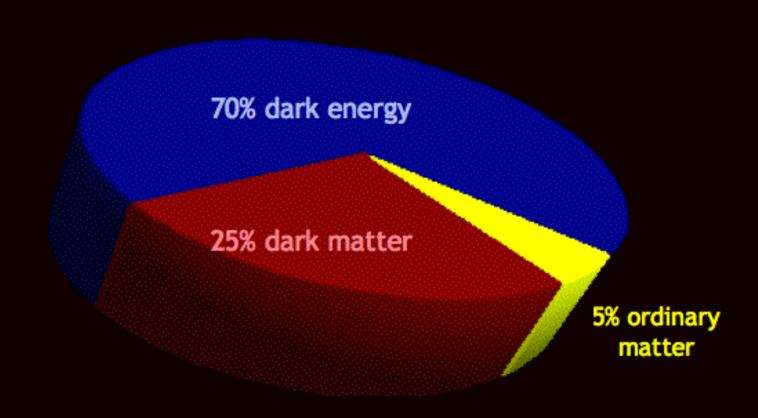
Imaging survey

1000 million galaxies 15,000 deg sq.

Δz_{photo} = 0.05 (1+z) 5 bins z range [0.5,3.0]

OWards the EUTURE

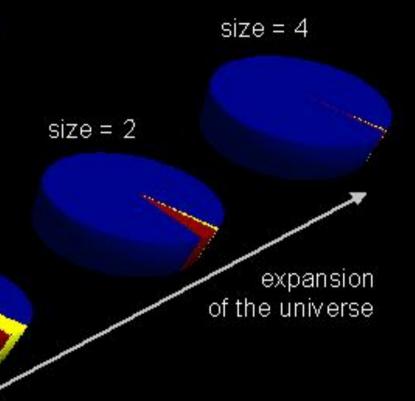
We have a complete inventory of the universe.



One puzzle: all matter <u>dilutes away</u>, but dark energy remains constant. So why are they (very roughly) comparable today?

size = $\frac{1}{2}$

size = 1



size = 1/4

The past was dominated by matter, the future will be dominated by dark energy. What makes the present day so special?