QCD under Extreme Conditions

(second lecture)

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Second lecture:

★ Effective model building

★ Z(N), the Polyakov loop, and confinement

★ Chiral symmetry breaking

★ Two examples on relevance and difficulties in exploring the phase diagram

- Chiral magnetic effect and the strong CP problem
- Drawing the phase diagram

★ Final comments

To go beyond in our study of the phases of QCD, we need to know its symmetries, and how they are broken spontaneously or explicitly. But QCD is very complicated:

- First, it is a non-abelian SU(N_c) gauge theory, with gluons living in the adjoint representation
- Then, there are N_f dynamical quarks (who live in the fund. rep.)
- On top of that, all these quarks have masses which are all different! Very annoying from the point of view of symmetries!

So, in studying the phases of QCD, we do it by parts, and consider many "cousin theories" which are very similar to QCD but simpler (more symmetric). We also study the dependence of physics on parameters which are fixed in nature.

Basic hierarchy for effective model building:

pure glue SU(N):

- Z(N) symmetry (SSB)
- order parameter: Polyakov loop L
- deconfining trans.: N=2 (2nd order), N=3 (weakly 1st order)

+ massless quarks:

- chiral symmetry (SSB)
- order parameter: chiral condensate σ
- Z(N) explicitly broken, but rise of L ⇨ deconf.
- chiral trans.: N=3,2 (N_f=2) – 2nd order

+ massive quarks:

Z(N) and chiral explicitly broken

Yet vary remarkably and L ↔ σ

SU(N_c), Z(N_c) and the Polyakov loop

For the QCD Lagrangian (massless quarks)

$$\mathcal{L} = \frac{1}{2} tr G^2_{\mu\nu} + \bar{q} i \gamma^\mu D_\mu q$$

$$D_\mu = \partial_\mu - igA_\mu \quad , \quad G_{\mu\nu} = \frac{1}{-ig}[D_\mu, D_\nu]$$

we have invariance under local SU(N_c). In particular, we have invariance under elements of the center Z(N_c)

$$\Omega_c = e^{i\frac{2n\pi}{N_c}} \mathbf{1}$$

At finite temperature, one has also to impose the following boundary conditions:

$$A_\mu(\vec{x}, \beta) = +A_\mu(\vec{x}, 0) \quad , \quad q(\vec{x}, \beta) = -q(\vec{x}, 0)$$

Any gauge transf. that is periodic in t will do it. However, 't Hooft noticed that the class of possible tranfs. is more general!

They are such that

$$\Omega(\vec{x}, \beta) = \Omega_c \quad , \quad \Omega(\vec{x}, 0) = 1$$

$$A^{\Omega}(\vec{x}, \beta) = \Omega_c^{\dagger} A_{\mu}(\vec{x}, \beta) \Omega_c = +A_{\mu}(\vec{x}, 0)$$

$$q^{\Omega}(\vec{x}, \beta) = \Omega_c^{\dagger} q(\vec{x}, \beta) = e^{-i\phi} q(\vec{x}, \beta) \neq -q(\vec{x}, 0)$$

keeping the gauge fields invariant but not the quarks!

For pure glue, this $Z(N_c)$ symmetry is exact, and we can define an order parameter from the trace of the Polyakov loop:

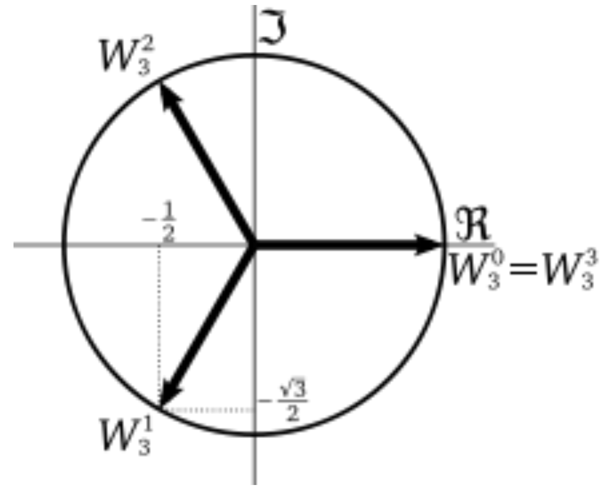
$$\ell(\vec{x}) = \frac{1}{N_c} \text{ Tr } L(\vec{x}) = \frac{1}{N_c} \text{ Tr } \mathcal{P} \exp \left[ig \int_0^{\beta} d\tau \; \tau^a A_0^a(\vec{x}, \tau) \right]$$

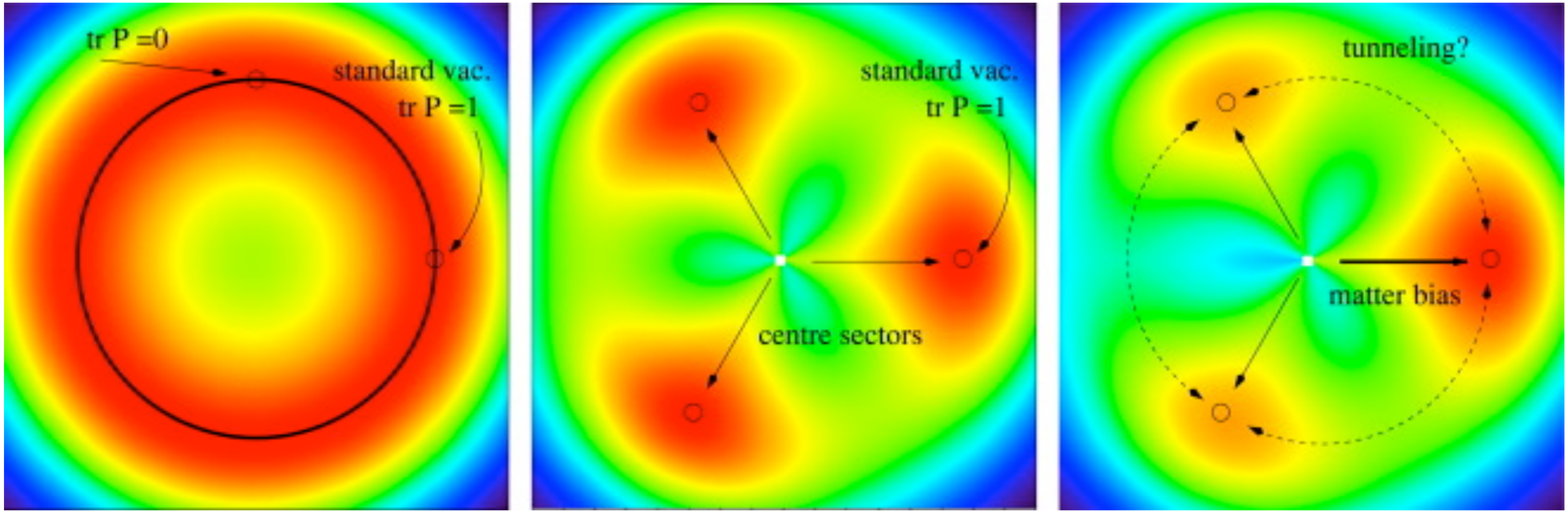
$$L(\vec{x}) \mapsto \Omega_c \; L(\vec{x}) \; \mathbf{1} = e^{i \frac{2n\pi}{N_c}} L(\vec{x})$$

Exercise: show it in detail!

N.B: ℓ is gauge invariant.

Roots of unity:





[Langfeld & Wipf (2012)]

At very high T, g ~ 0, and β -> 0, so that

$$\langle \ell \rangle = e^{i\frac{2n\pi}{N_c}} \ell_0 \,, \;\; \ell_0 \sim 1$$

and we have a N-fold degenerate vacuum, signaling SSB of global $Z(N_c)$. At T = 0, confinement implies that $\ell_0 = 0$ ['t Hooft (1978)].

Then, ℓ_0 can be used as an order parameter for the deconfining transition:

$$\ell_0 = 0 \;\;,\;\; T < T_c \;\;;\;\; \ell_0 > 0 \;\;,\;\; T > T_c$$

Usually the Polyakov loop is related to the free energy of an infinitely heavy test quark via [McLerran & Svetitsky (1981)]

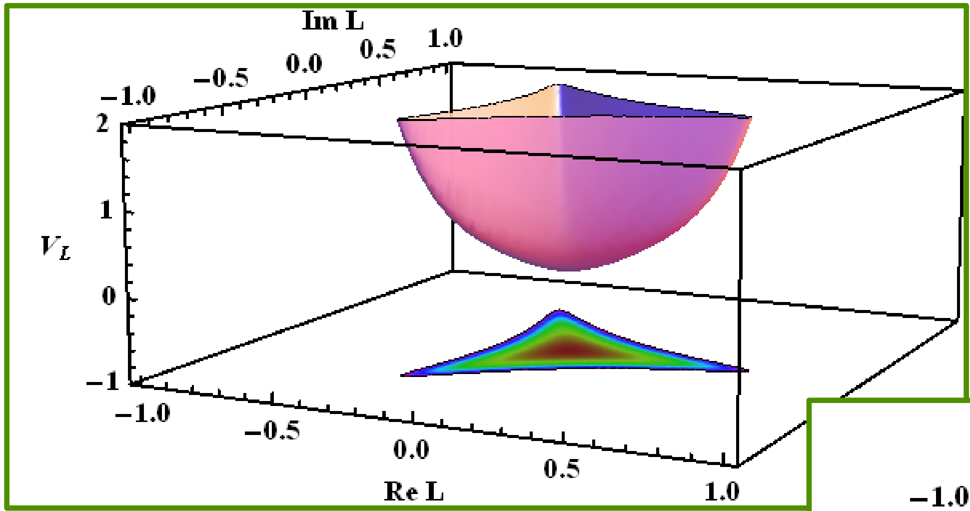
$$\langle \ell \rangle = e^{-F_{test}/T}$$

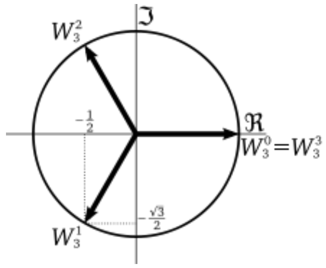
(confinement: no free quark)

Exercise: do you see any possible problem in the equation above? (Cf. 1st eqn in this slide!) If so, could we, instead, relate < ℓ > to the propagator for a test quark?

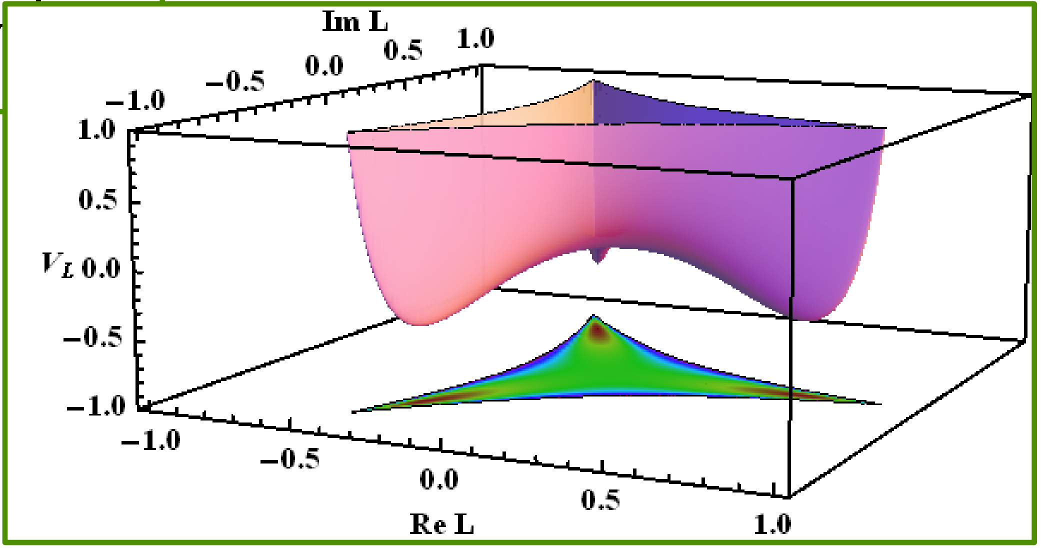
Form of the effective potential for the Polyakov loop

$T \ll T_c$





$T \gg T_c$



Adding massless quarks (chiral symmetry)

In the limit of massless quarks, QCD is invariant under global chiral rotations $U(N_f)_L \times U(N_f)_R$ of the quark fields.

One can rewrite this symmetry in terms of vector (V = R + L) and axial (A = R - L) rotations

$$U(N_f)_L \times U(N_f)_R \sim U(N_f)_V \times U(N_f)_A$$

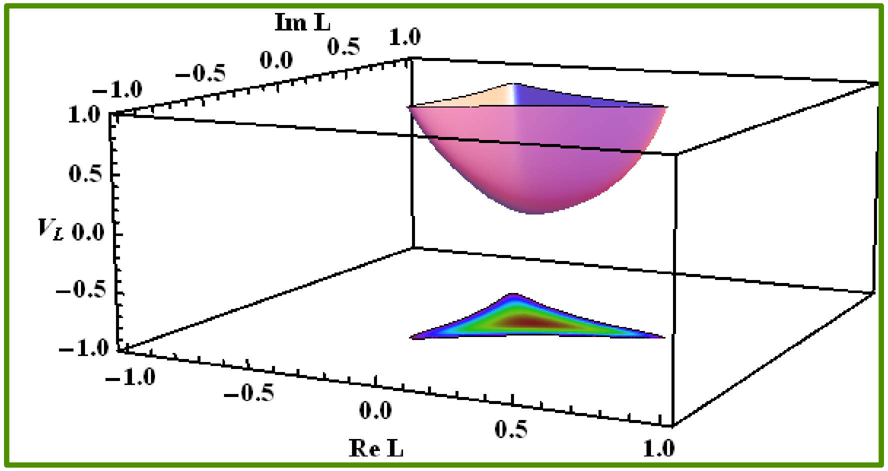
As U(N) ~ SU(N) x U(1), one finds

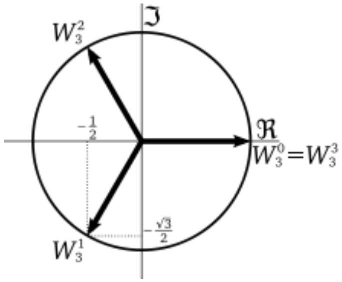
$$U(N_f)_L \times U(N_f)_R \sim SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

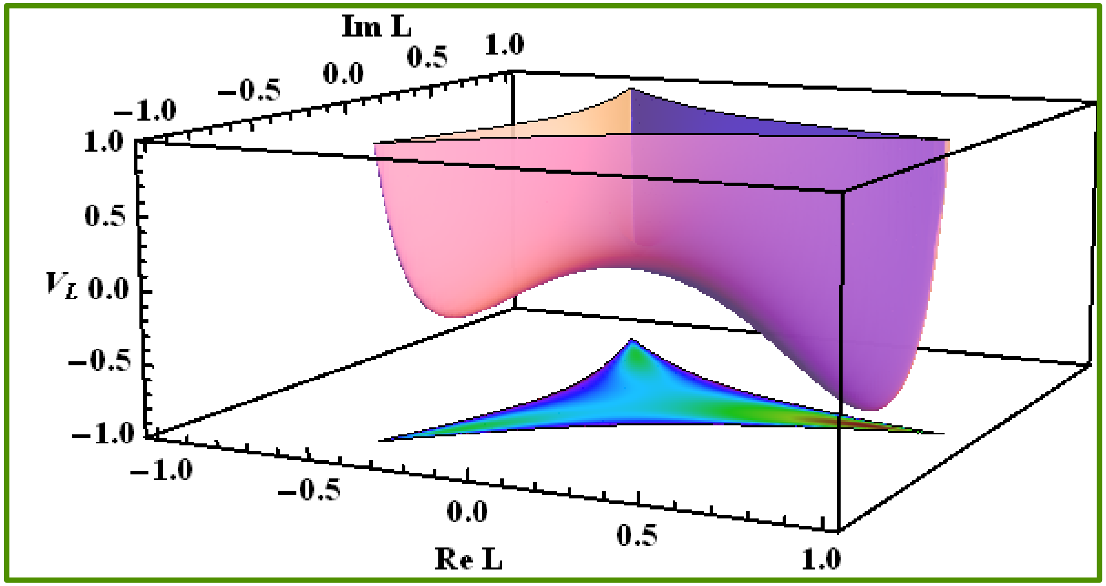
where we see the $U(1)_V$ from quark number conservation and the $U(1)_A$ broken by instantons.

Exercise: can we restore $U(1)_A$ in a hot and dense medium? Think of possible consequences and observables.

Quarks (even massless) break explicitly Z(3)!







In QCD, the remaining $SU(N_f)_L \times SU(N_f)_R$ is explicitly broken by a nonzero mass term. Take, for simplicity, $N_f=2$:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}^f_L\gamma^\mu D_\mu\psi_L + \bar{\psi}^f_R\gamma^\mu D_\mu\psi^f_R - m_u\left(\bar{u}_L u_R + \bar{u}_R u_L\right) - m_d\left(\bar{d}_L d_R + \bar{d}_R d_L\right)$$

so that, for non-vanishing $m_u = m_d$, the only symmetry that remains is the vector isospin $SU(2)_V$. In the light quark sector of QCD, chiral symmetry is just approximate.

Then, for massless QCD, one should find parity doublets in the vacuum, which is not confirmed in the hadronic spectrum. Thus, chiral symmetry must be broken in the vacuum by the presence of a quark chiral condensate, so that

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

and the broken generators allow for the existence of pions, kaons, ...

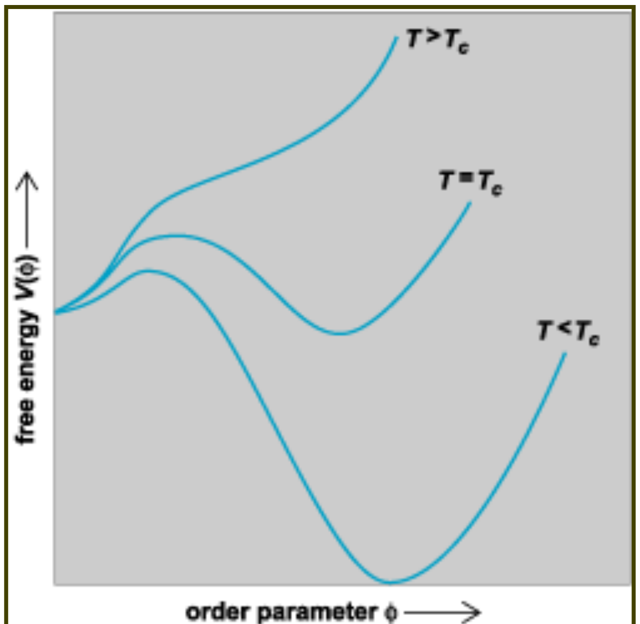
Hence, for massless QCD, we can define an order parameter for the SSB of chiral symmetry in the vacuum – **the chiral condensate:**

$$\langle 0|\bar{\psi}\psi|0\rangle = \langle 0|\bar{\psi}_L\psi_R|0\rangle + \langle 0|\bar{\psi}_R\psi_L|0\rangle$$

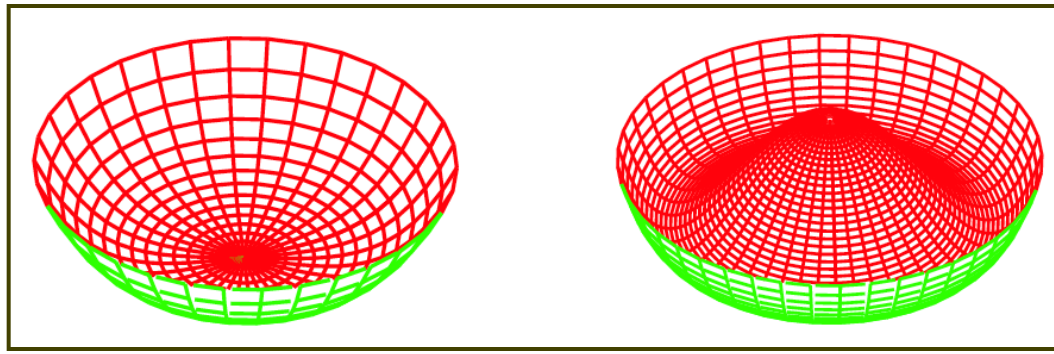
so that this vacuum expectation value couples together the L & R sectors, unless in the case it vanishes.

For very high temperatures or densities (low α_s), one expects to restore chiral symmetry, melting the condensate that is a function of T and m and plays the role of an order parameter for the chiral transition in QCD.

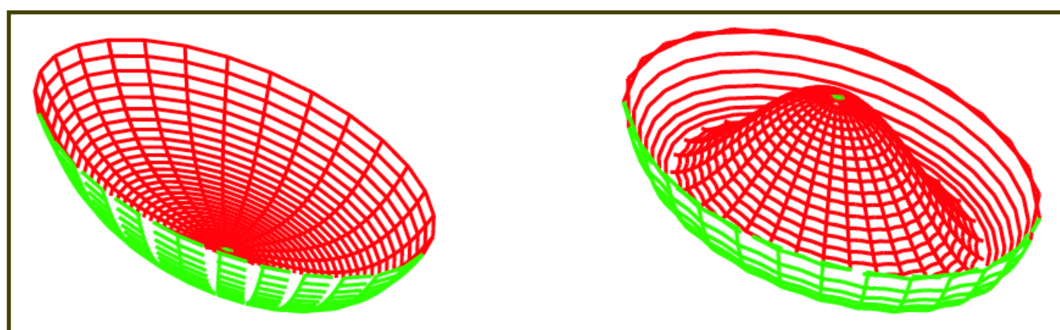
Form of the effective potential for the chiral condensate



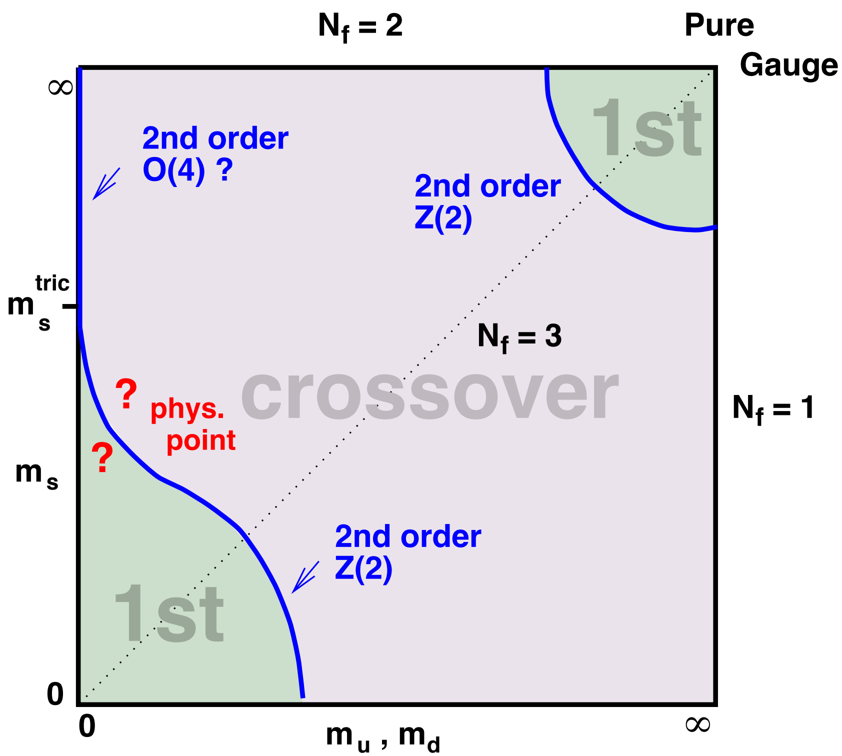
$T \gg T_c$: $T \ll T_c$:

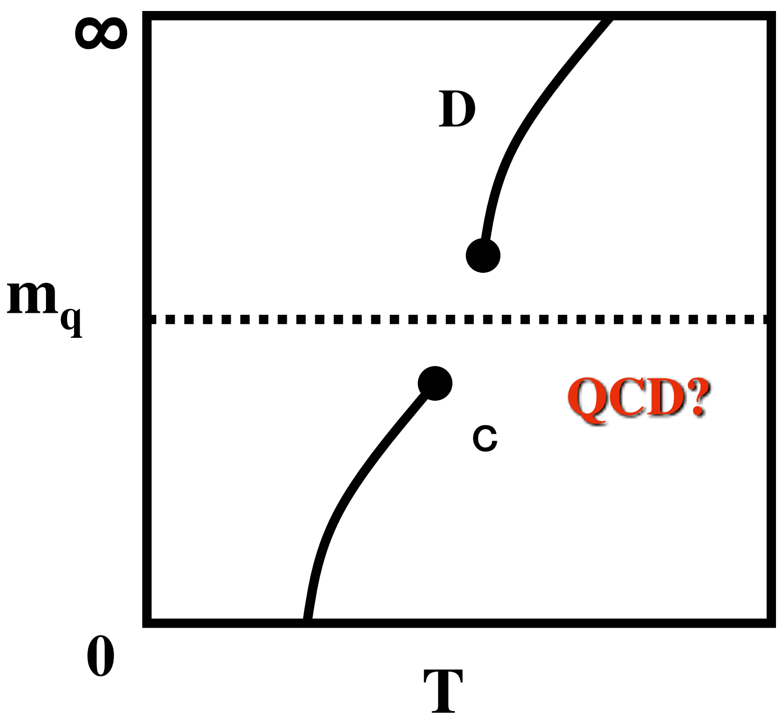


+ explicit symmetry breaking:



From pure gauge deconfinement to the chiral limit: Playing with quark masses in the "QCD transition(s)"

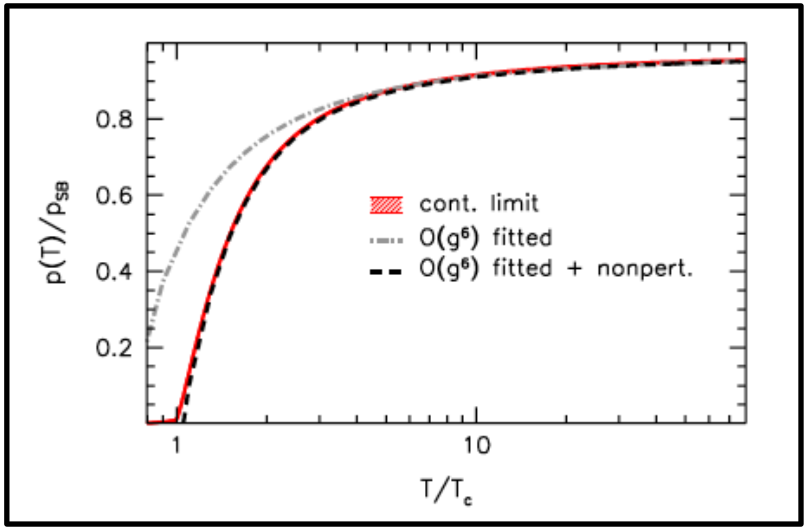




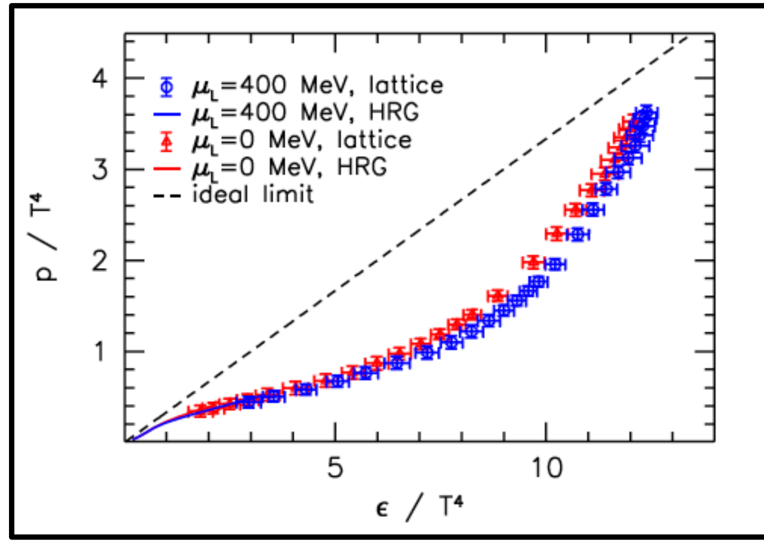
[Gavin, Gocksch & Pisarski (1994)]

Some results from the lattice [Borsányi et al (2012), from 3 papers]

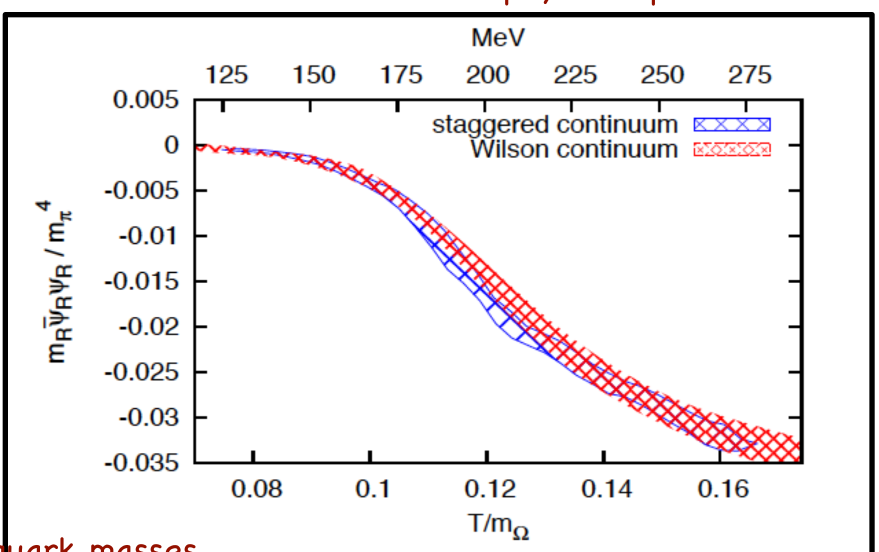
pure gauge and pQCD



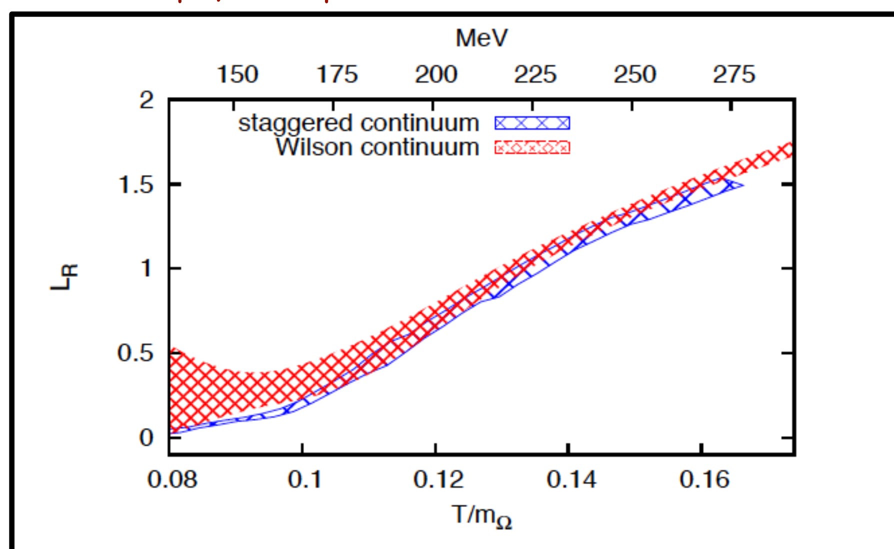
QCD with physical quark masses



QCD with physical quark masses



QCD with physical quark masses



quark masses

- Two relevant phase transitions in QCD associated with SSB mechanisms for different symmetries of the action

- Approximate $Z(N_c)$ symmetry and deconfinement [exact for pure gauge $SU(N_c)$]. Order parameter: (trace of the) Polyakov loop

- Approximate chiral symmetry and chiral transition [exact for massless quarks]. Order parameter: chiral condensate.

- Some good estimates within a very simple framework: the bag model. Very crude, disagrees with lattice QCD on the nature of the transition, but still used in several calculations (EoS for compact stars, hydro evolution of the QGP, etc.)

- Going beyond: effective models (based on symmetries of S_{QCD})

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Effective models: general idea

- Keep relevant symmetries

- Try to include in the effective action all terms allowed by the chosen symmetries

- Mimic QCD at low energy using a simpler field theory

- Analytic results: estimates, qualitative behavior, etc.

- Examples: linear σ model, NJL model, Polyakov loop model, ...

- Just part of the story - combined with lattice QCD and/or data from experiments may provide good insight

NB: chiral perturbation theory – "the" effective model

Examples:

Linear σ model:

$$\mathcal{L} = \bar{q}\left[i\gamma^\mu\partial_\mu - g\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)\right]q + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right) - U(\sigma,\vec{\pi})$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - H\sigma$$

NJL model:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - m_0\right)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

Polyakov loop model:

$$\mathcal{V}(\ell) = \left(-\frac{b_2}{2}|\ell|^2 - \frac{b_3}{6}(\ell^3 + (\ell^*)^3) + \frac{1}{4}(|\ell|^2)^2\right) b_4 T^4$$

Example: LSM & the chiral transition

- Symmetry: for massless QCD, the action is invariant under $SU(N_f)_L \times SU(N_f)_R$
- "Fast" degrees of freedom: quarks
  "Slow" degrees of freedom: mesons
- Typical energy scale: hundreds of MeV
- For $SU(N_f=2)$, for simplicity, we have pions and the sigma
- Framework: coarse-grained Landau-Ginzburg effective potential
- SU(2) x SU(2) spontaneously broken in the vacuum
- Can also accommodate explicit breaking by massive quarks

Building the effective lagrangian

[with scalars allowed by χ symmetry]

- Kinetic terms:

$$\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}$$

$$\bar{q}(i\partial\!\!\!/)q$$

- Fermion-meson interaction:

$$g(i\bar{q}\gamma_5\vec{\tau}q)\cdot\vec{\pi}$$

$$g(\bar{q}q)\sigma$$

- Chiral self-interaction:

$$V = V\left(\pi^2+\sigma^2\right) = \frac{\lambda}{4}\left(\left(\pi^2+\sigma^2\right) - f_\pi^2\right)^2$$

- Explicit chiral symmetry breaking term:

$$\delta\mathcal{L}_{SB} = h\sigma$$

Exercise: prove that the terms above are the ones allowed by chiral symmetry, except for the last which breaks it explicitly.

Parameters should be fixed such that:

- $SU(2)_L \times SU(2)_R$ is spontaneously broken in the vacuum, with $\langle\sigma\rangle = f_\pi$, $\langle\pi\rangle = 0$
- h should be related to the nonzero pion mass (plays a role analogous to an external magnetic field for a spin system)
- f_π = 93 MeV is the pion decay constant, determined experimentally. It comes about when one computes the weak decay of the pion, which is proportional to the amplitude

$$\langle 0|J^{\mu a}_{axial}(x)|\pi^b(q)\rangle = -if_\pi q^\mu \delta^{ab} e^{-iq\cdot x}$$

a,b: isospin

- The small but nonzero pion mass breaks "softly" the axial current:

$$\langle 0|\partial_\mu J^{\mu a}_{axial}(x)|\pi^b(q)\rangle = -f_\pi q^2 \delta^{ab} e^{-iq\cdot x} = f_\pi m_\pi^2 \delta^{ab} e^{-iq\cdot x}$$

- PCAC: "partial conservation of the axial current"

- Including a term ~ $h\sigma$ brings the following consequences:

  - The true vacuum (in the σ direction) is shifted

[we can redefine f_π such that it coincides with the experimental value]

- The σ mass is modified

$$m_\sigma^2 = \left(\frac{\partial^2 V}{\partial \sigma^2}\right)_{\sigma_0} = 2\lambda f_\pi^2 + \frac{h}{f_\pi}$$

- Pions acquire a nonzero mass

$$m_\pi^2 = \left(\frac{\partial^2 V}{\partial \pi^2}\right)_{\sigma_0} = \frac{h}{f_\pi} > 0$$

which fixes h to be:

$$h = f_\pi m_\pi^2$$

Then, all parameters can be chosen to reproduce the vacuum features of mesons.

- The connection with the quark mass is given by the Gell-Mann--Oakes--Renner (GOR) relation:

$$\langle 0|h\sigma|0\rangle = \langle 0|(-m\bar{\psi}\psi)|0\rangle \quad \Longrightarrow \quad m_\pi^2 f_\pi^2 = -\frac{m_u + m_d}{2} \langle 0|\bar{u}u + \bar{d}d|0\rangle$$

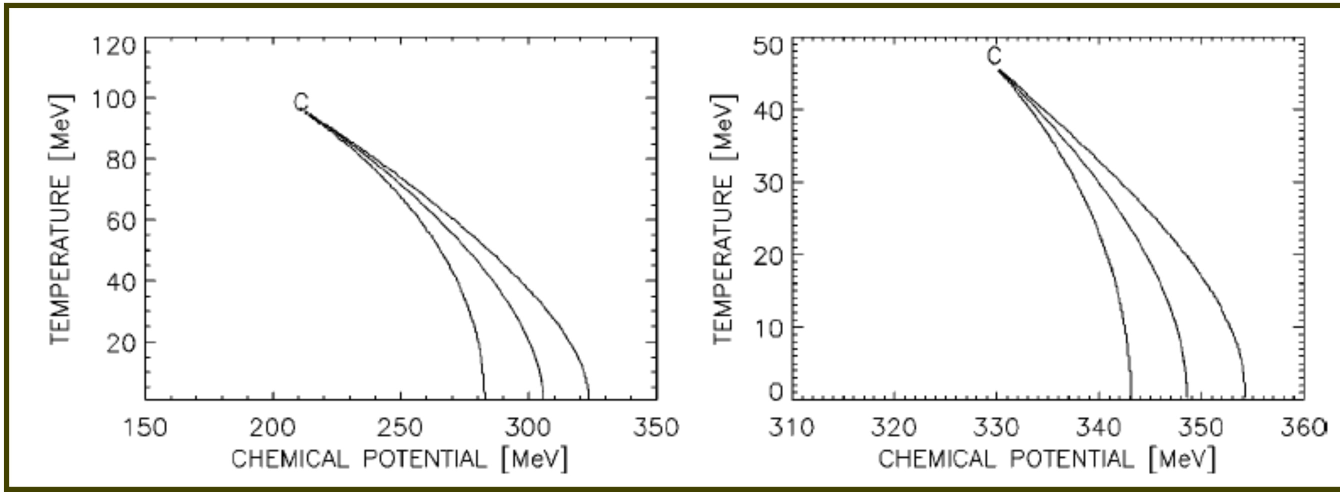
↕ "by construction", since one wants this term to mimic the QCD explicit breaking of chiral symmetry

↕ Connection not only between m_π and m_q, but also between the σ field condensate and the chiral condensate

- In a medium, one can use <σ>(T) in the effective model to describe the melting of the chiral condensate at high T.

Phase diagram and effective potential

(linear σ)



(NJL)

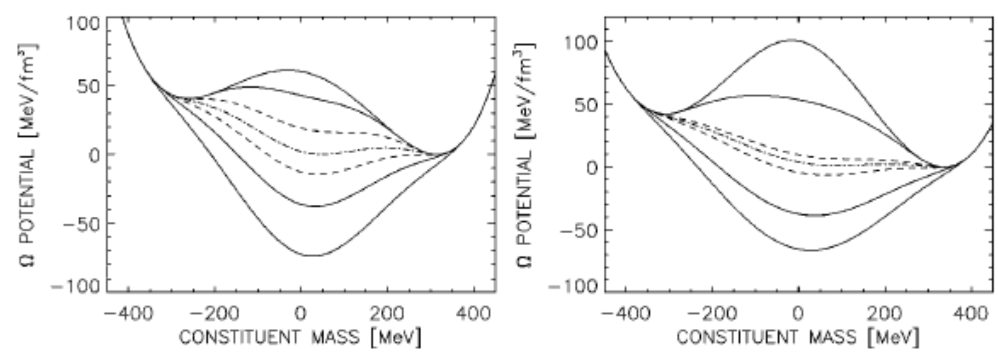


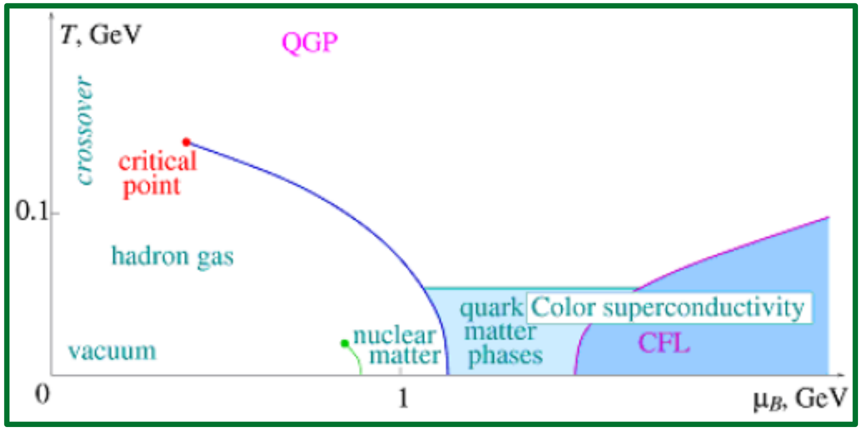
FIG. 3. The thermodynamical potentials Ω for the sigma model (left) and the NJL model (right). For both models $T=0$. The levels correspond to (starting from the top) $\mu=[0,225,279,306,322,345,375]$ MeV in the sigma model and $\mu=[0,288,343,348,35,378,396]$ MeV for the NJL model.

[Scavenius et al. (2001)]

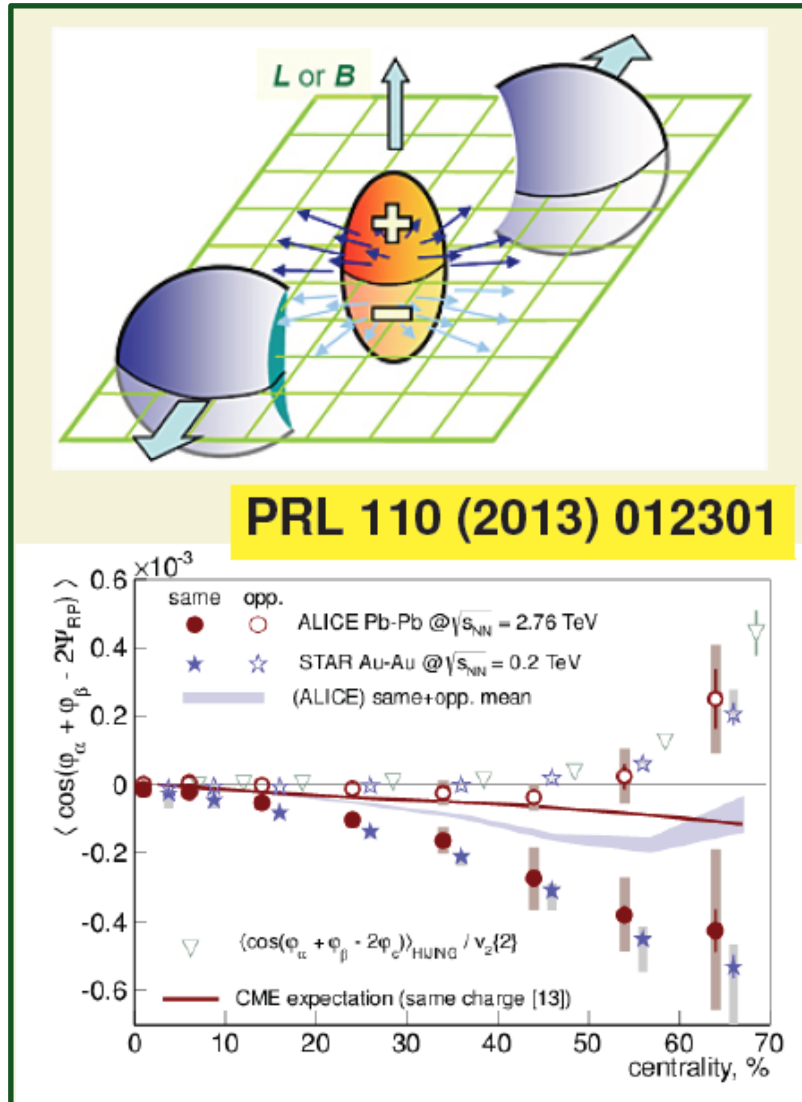
Two examples on relevance and difficulties in exploring the phase diagram:

strong CP problem & CME

"drawing" the phase diagram



Plots from Gonzalez's talk!



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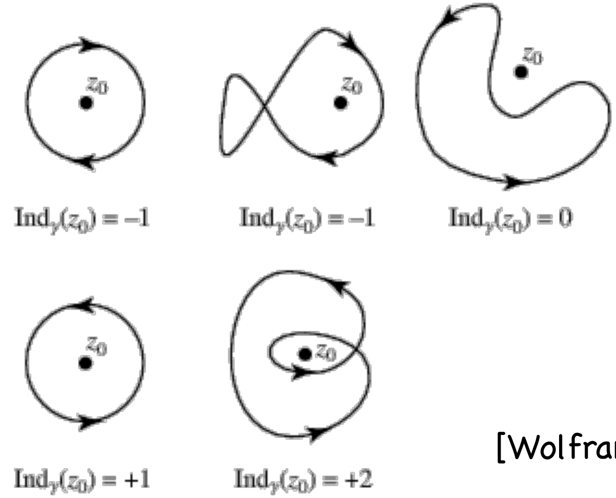
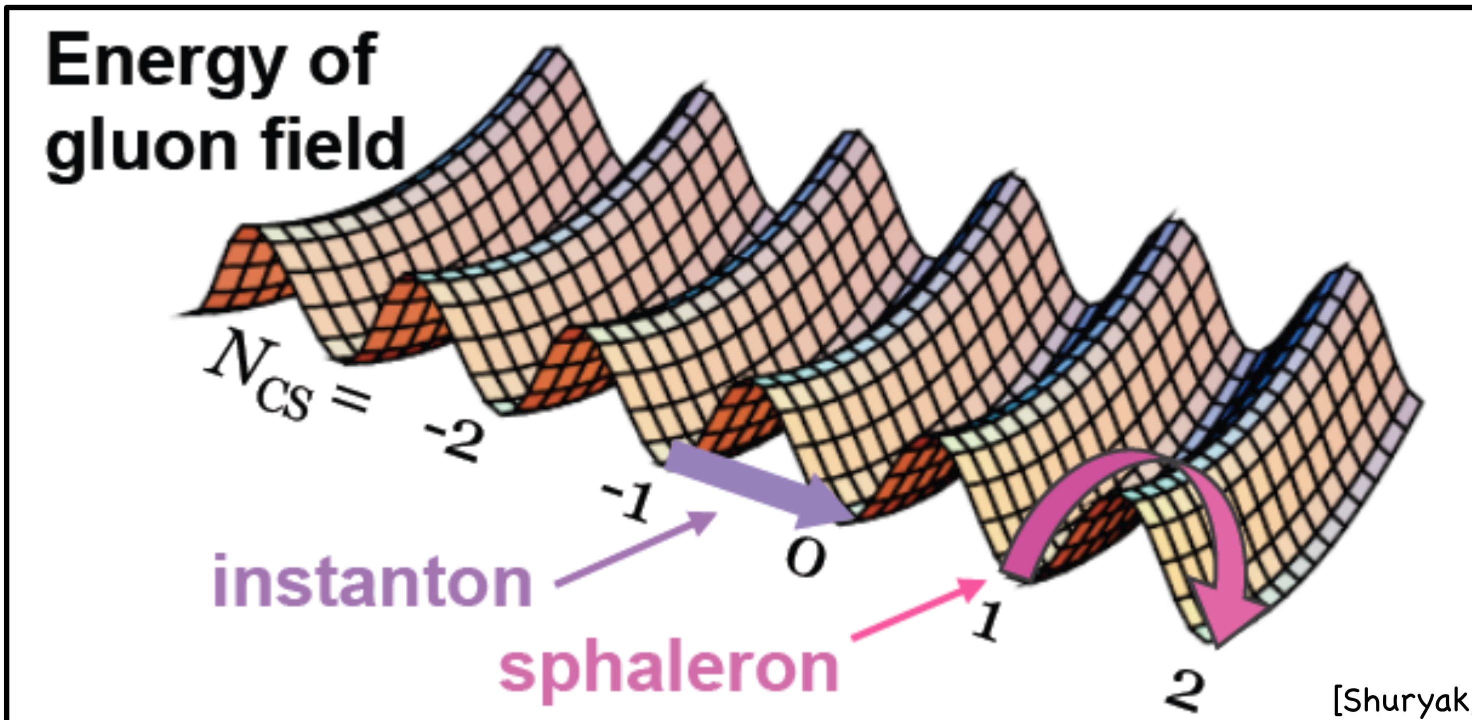
Strong CP problem

(a real fundamental problem!!)



The vacuum of QCD is topologically nontrivial!

[Wolfram]



[Shuryak, 2009]

Strong CP problem & HIC

- Topologically nontrivial configurations of the gauge fields allow for a CP-violating term in the Lagrangian of QCD

$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 F^{\mu\nu a} \tilde{F}^a_{\mu\nu}$$

However, experiments indicate $\theta < 10^{-10}$.

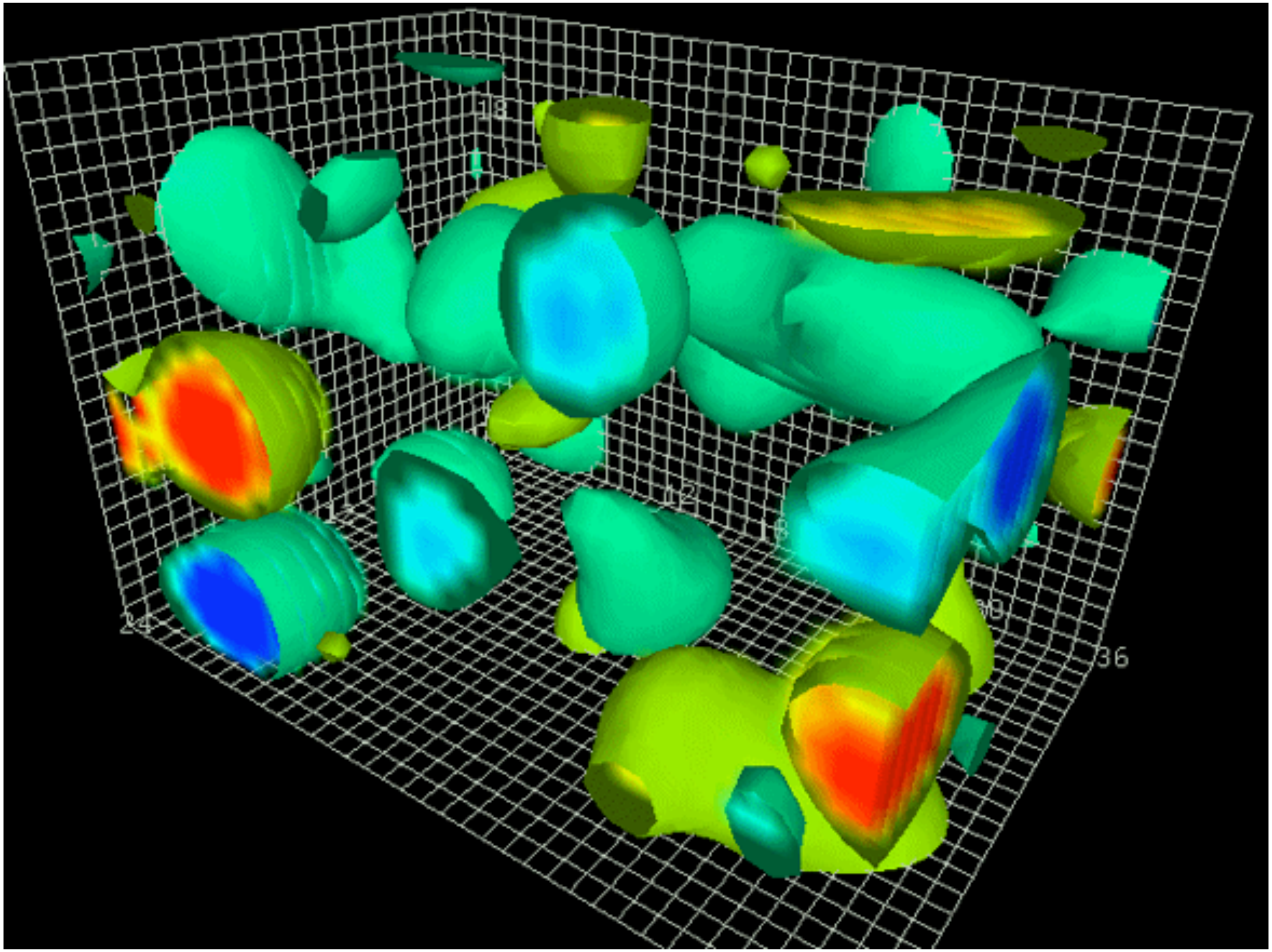
- Spontaneous breaking of P and CP are forbidden in the true vacuum of QCD for $\theta = 0$ [Vafa & Witten (1984)]. However, this does not hold at finite temperature [Bronoff & Korthals Altes; Azcoiti & Galante; Cohen,…] and metastable states are allowed -> chance to probe the topological structure of QCD !

- Metastable P- and CP-odd domains could be produced in heavy ion collisions [Kharzeev, Pisarski & Tytgat (1998)]

- Signature? Mechanism based on the separation of charge -> the chiral magnetic effect [Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008); Fukushima, Kharzeev & Warringa (2008)] under very strong magnetic fields in non-central collisions; sensitive experimental observable [Voloshin (2000,2004)]

[Leinweber]

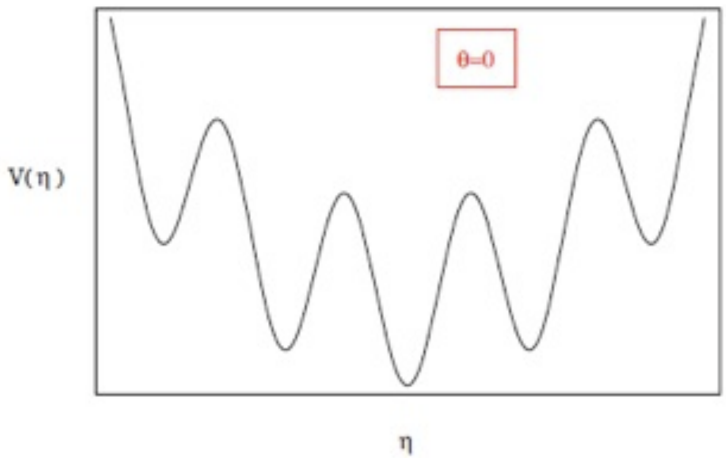
Topological number fluctuations in QCD vacuum



From Kharzeev, QM 2009:

Chern-Simons number in hot QCD

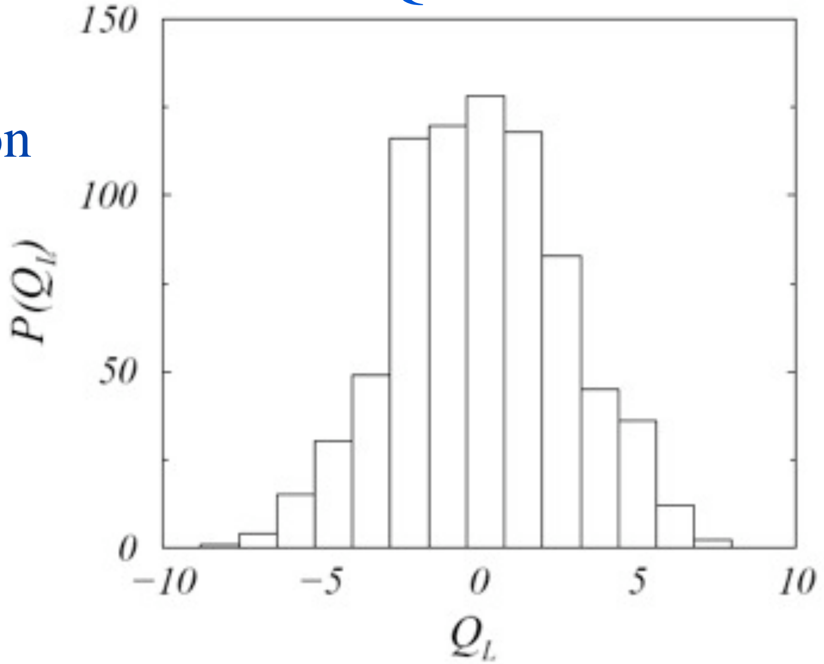
The existence of P and CP odd domains expected close to the deconfinement phase transition



DK, R. Pisarski, M. Tytgat,

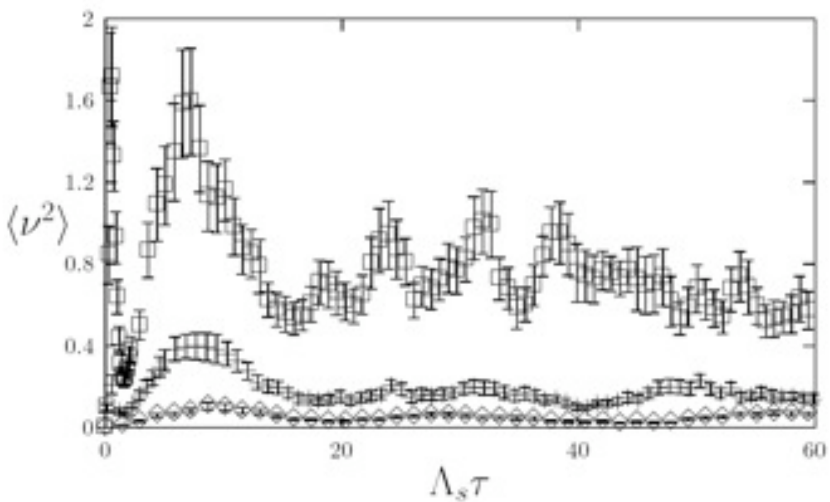
Phys.Rev.Lett.81:512-515,1998

Lattice QCD results:

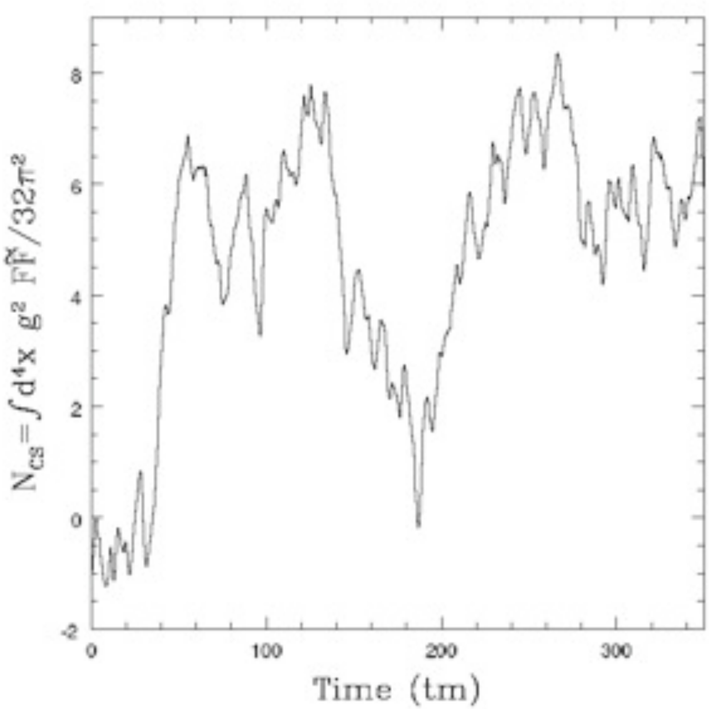


B.Alles, M.D'Elia and A.DiGiacomo, hep-lat/0004020

Diffusion of Chern-Simons number in QCD: real time lattice simulations



DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002

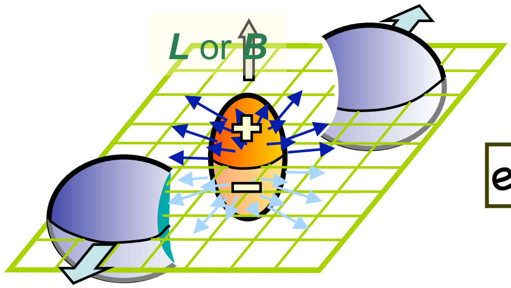


P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

High temperature and low density

high magnetic fields in non-central RHIC collisions

[Kharzeev, McLerran & Warringa (2008)]



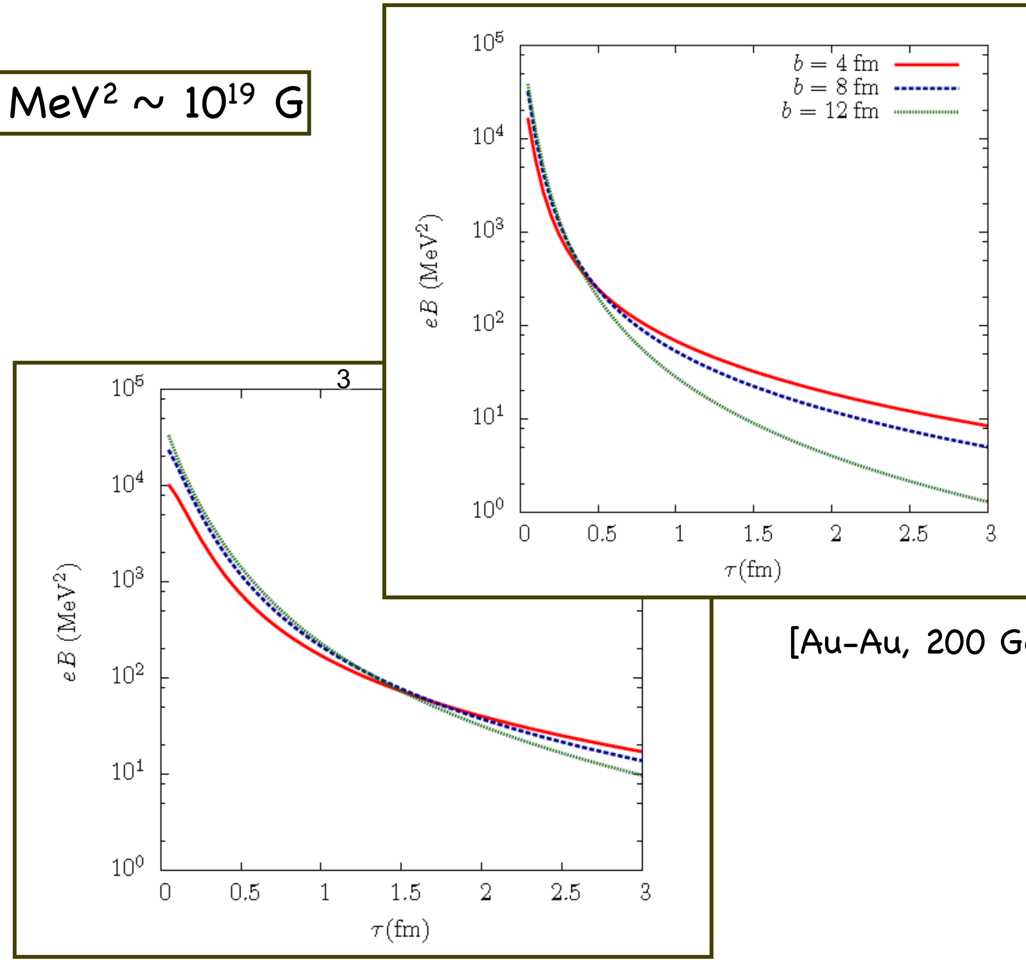
[Voloshin, QM2009]

$eB \sim 10^4$-10^5 MeV^2 $\sim 10^{19}$ G

For comparison:

- "Magnetars": B ~ 10^{14}-10^{15} G at the surface, higher in the core [Duncan & Thompson (1992/1993)]
- Early universe (relevant for nucleosynthesis): B~ 10^{24} G for the EWPT epoch [Grasso & Rubinstein (2001)]

Plus: mechanism based on separation of charge for the detection of the Chiral Magnetic Effect and P-odd effects [Voloshin (2000,2004), Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008); Fukushima, Kharzeev & Warringa (2008)]



[Au-Au, 200 GeV]

[Au-Au, 62 GeV]

❖ CP violation in heavy ion collisions

• Rate of *instanton* transitions at zero temperature (tunneling) ['t Hooft (1976)]:

(no quarks)
$$\frac{dN_t^{\pm}}{d^3x dt d\rho} = 0.0015 \left(\frac{2\pi}{\alpha_s}\right)^6 e^{-2\pi/\alpha_s} \frac{1}{\rho^5}$$
$$Q_w = \pm 1$$

• High T tends to decrease this rate [Pisarski & Yaffe (1980], but allows for *sphaleron* transitions (rate <u>increases</u> with T). For Yang-Mills [Moore et al. (1998); Bodeker et al. (2000)]:

$$\frac{dN_t^{\pm}}{d^3x dt} \sim 25.4 \, \alpha_w^5 T^4$$

• Estimate for QCD via N_c scaling [Kharzeev et al (2008)]:

$$\frac{dN_t^{\pm}}{d^3x dt} \sim 192.8 \, \alpha_s^5 T^4$$

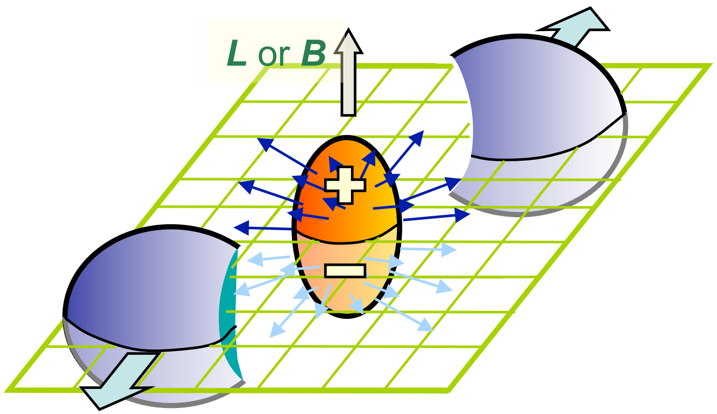
• Due to the anomaly the Ward identities are modified, and the charges Q_L and Q_R obey the following relations (for $N_+=N_-$ at $t \to -\infty$):

$$(N_L - N_R)_{t\to\infty} = 2N_f Q_w$$
$$N_L - N_R = \int d^4x \, (Q_L - Q_R) = -\int d^4x \, Q_5$$

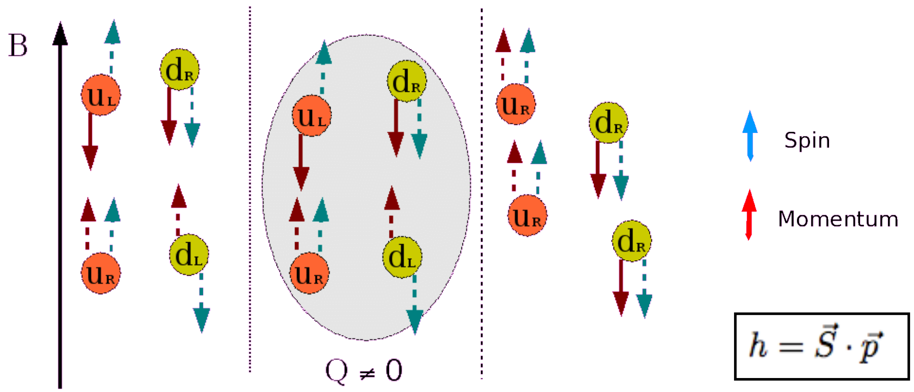
so that fermions interacting with non-trivial gauge fields ($Q_w \neq 0$) have their chirality changed!

❖ Chiral magnetic effect

In non-central heavy ion collisions a strong magnetic field is generated in the orbital angular momentum direction (perpendicular to the reaction plane) and there can be regions with $Q_w \neq 0$ (inducing *sphaleron* transitions):



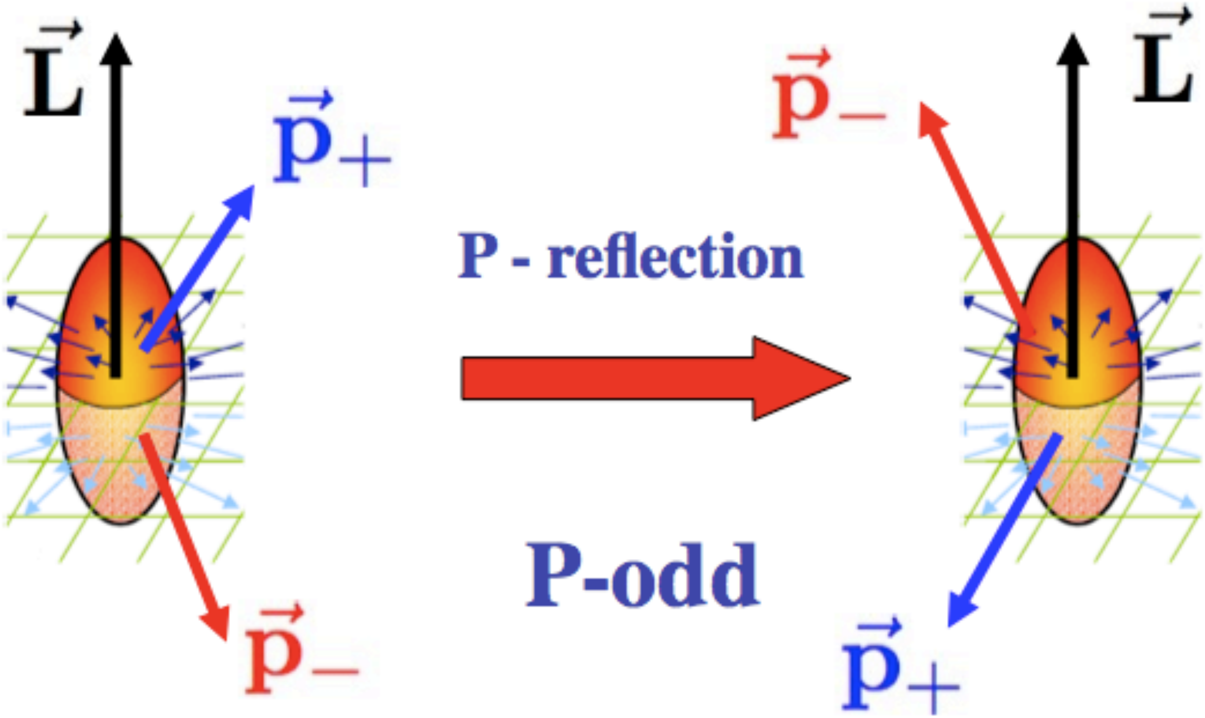
[Voloshin, QM2009]



[Kharzeev, McLerran & Warringa (2008)]

- The strong B field restricts quarks (all in the lowest Landau level, aligned with B) to move along its direction
- Q_w=-1, e.g., converts L -> R: inversion of the direction of momentum
- Net current and charge difference created along the B direction

Charge separation = parity violation:



$$\mathbf{P}: \quad \vec{p} \rightarrow -\vec{p}; \quad \vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L}$$

[Kharzeev, QM2009]

Magnetic QCD

Several theoretical/phenomenological questions arise:

How does the QCD phase diagram look like including a nonzero uniform B ? (another interesting "control parameter" ?)

Where are the possible metastable CP-odd states and how "stable" they are? What are their lifetimes ?

Are there modifications in the nature of the phase transitions ?

Are the relevant time scales for phase conversion affected ?

Are there other new phenomena (besides the chiral magnetic effect) ?

What is affected in the plasma formed in heavy ion collisions ?

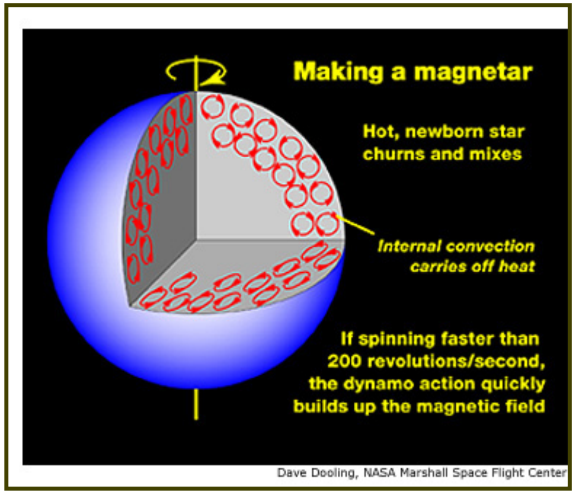
Which are the good observables to look at ? Can we investigate it experimentally ? Can we simulate it on the lattice ?

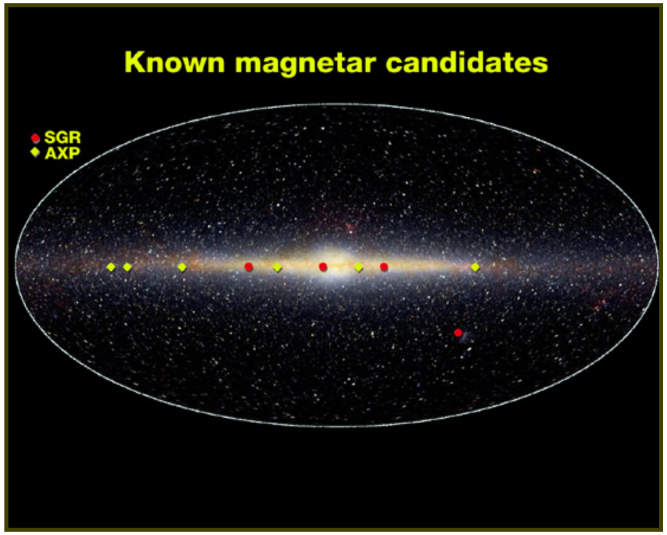
Besides…

Strong interactions under intense magnetic fields can be found, in principle, in a variety of systems:

❖ High density and low temperature

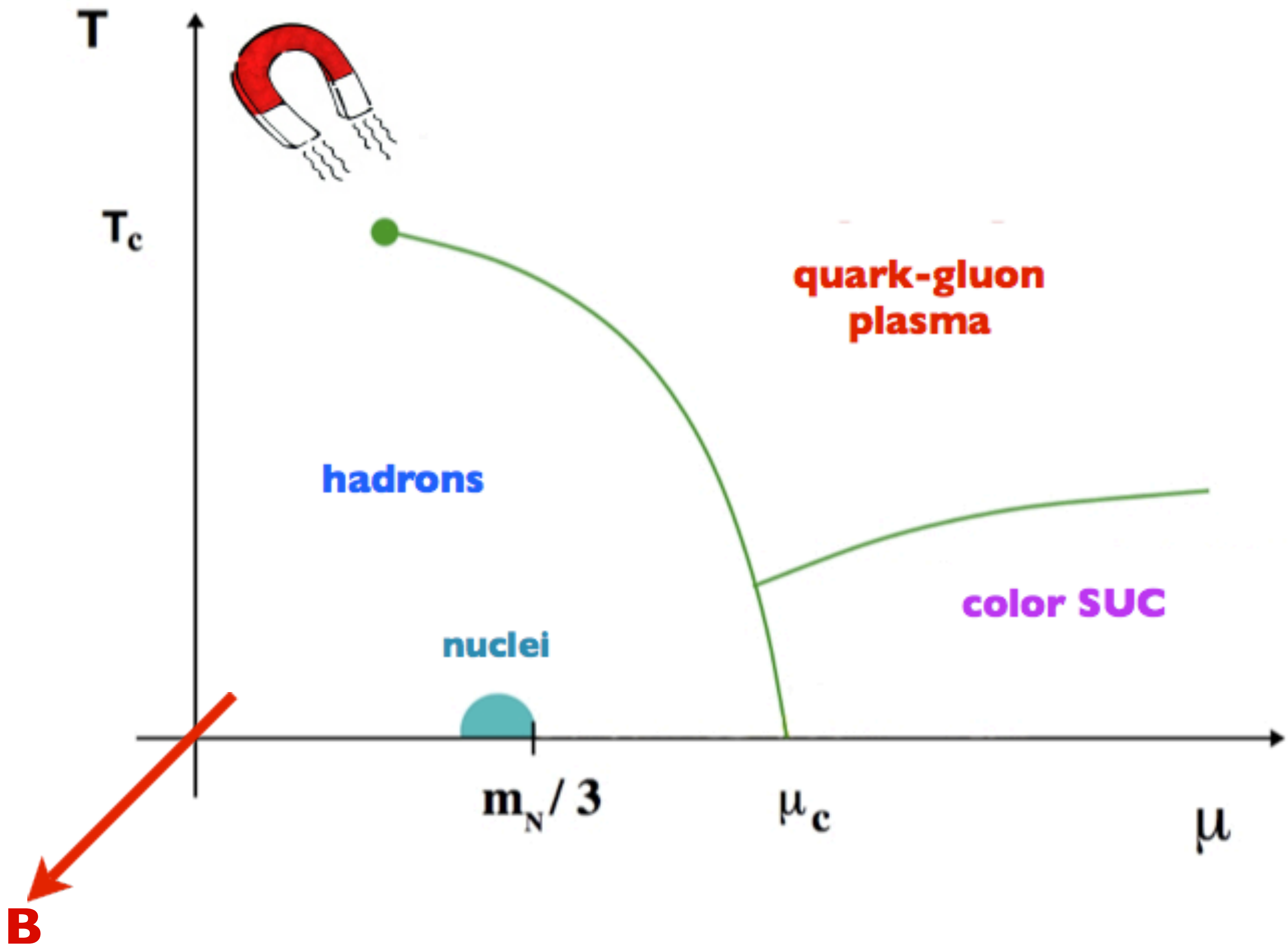
- "Magnetars": $B \sim 10^{14}$–10^{15} G at the surface, much higher in the core [Duncan & Thompson (1992/1993)]





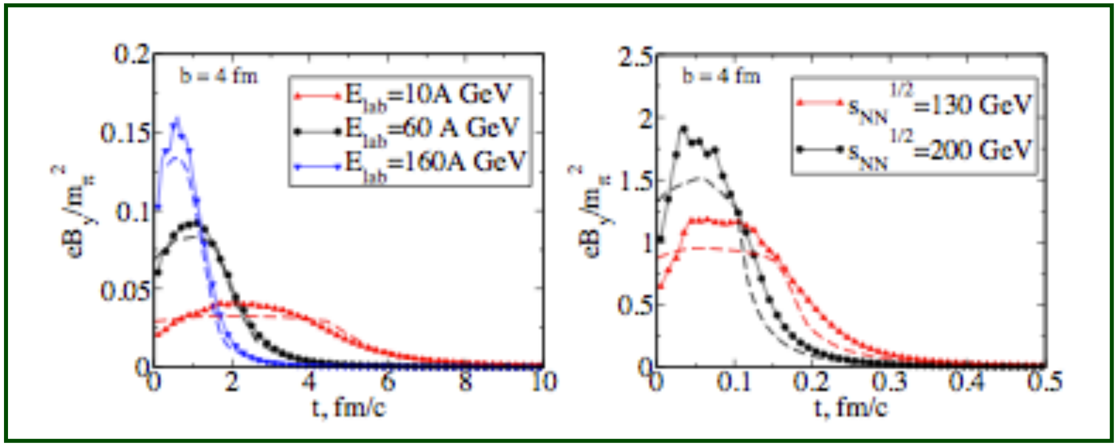
- Stable stacks of π^0 domain walls or axial scalars (η,η') domain walls in nuclear matter: $B \sim 10^{17}$–10^{19} G [Son & Stephanov (2008)

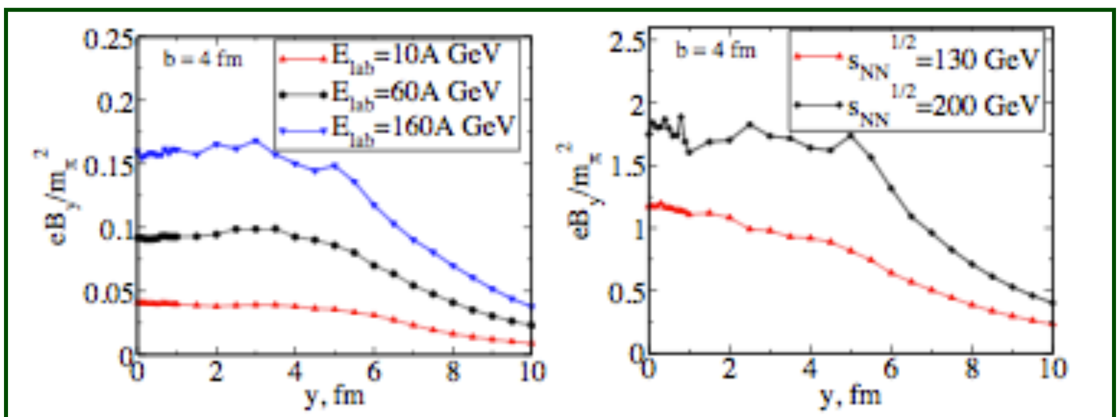
Pictorially:



HIC for FAIR
Helmholtz International Center
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

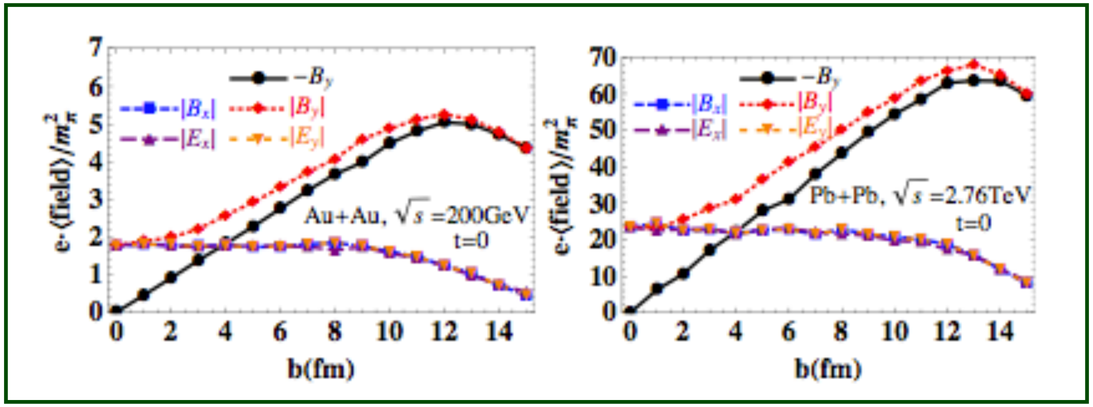
High magnetic fields in heavy-ion collisions have been computed...





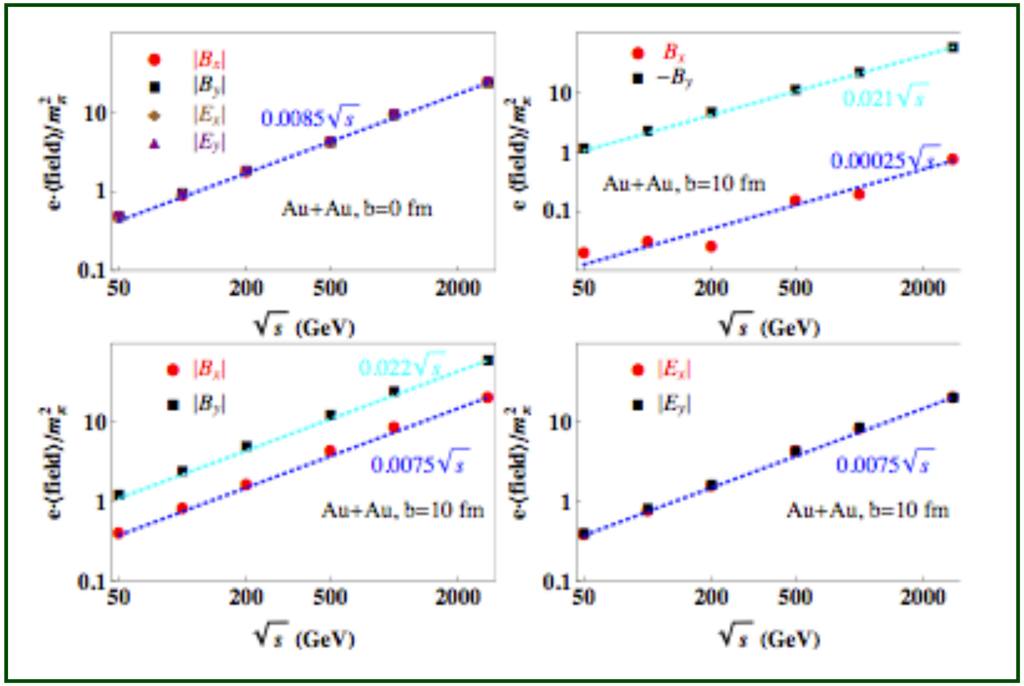
- RHIC energies, higher fields at the LHC
- Fields very flat in the central region (system may be deconfined/chiral)
- semi-analytic estimates & UrQMD agree well

[Skokov, Illariunov & Toneev (2009)]



- HIJING computation
- RHIC vs LHC energies

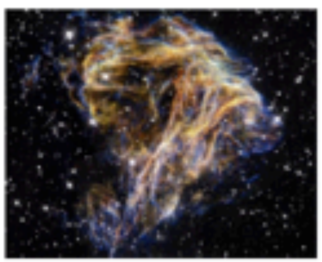
- Huge fields for ultra-peripheral collisions due to event-by-event fluctuations
- Possible vacuum SUC via ρ meson condensation [Chernodub (2010)]
- Possible building of spin-charge correlation for quarks



[Deng & Huang (2012)]

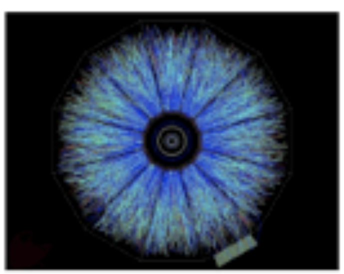
Comparison of magnetic fields





The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5 x 10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\ \mathrm{Gauss}$$

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Strong motivation: in-medium strong interactions under extreme magnetic fields are:

- of experimental relevance
  - HICs, early universe, magnetars
- rich in new phenomenology
  - Chiral magnetic effect, new QCD phase diagram, vacuum SUC
- amenable to lattice simulations: new open channel for comparison!
  - model constraining, tests for pQCD and nonpert. methods, ...

Incorporating a magnetic background in loop integrals

Let us assume the system is in the presence of a strong magnetic field background that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{choice of gauge}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

- charged mesons **(new dispersion relation)**:

$$\begin{aligned}(\partial^2 + m^2)\phi = 0 \\ \partial_\mu \to \partial_\mu + iqA_\mu\end{aligned} \quad \longrightarrow \quad \varphi''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]\varphi(y) = 0$$

Landau levels:

$$\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B \qquad \omega_B \equiv \frac{|q|B}{m}$$

$$\Longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

- quarks (new dispersion relation):

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu$$

$$\longrightarrow \quad u''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2 + qB\sigma}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]u(y) = 0$$

$$\Rightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n + 1 - \sigma)|q|B \qquad \sigma = \pm 1$$

- integration measure:

T = 0:

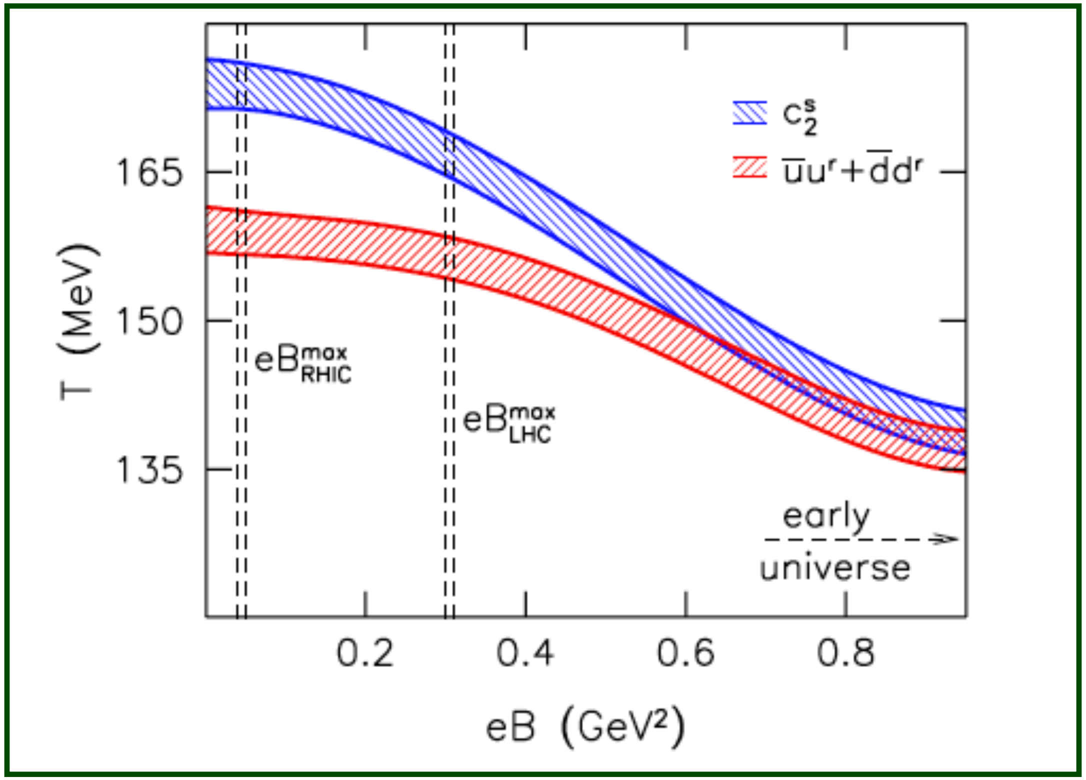
$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi}\sum_{n=0}^{\infty}\int \frac{dk_0}{2\pi}\frac{dk_z}{2\pi}$$

T > 0:

$$T\sum_{\ell}\int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi}\sum_{\ell}\sum_{n=0}^{\infty}\int \frac{dk_z}{2\pi}$$

l: Matsubara index
n: Landau level index

From the lattice:

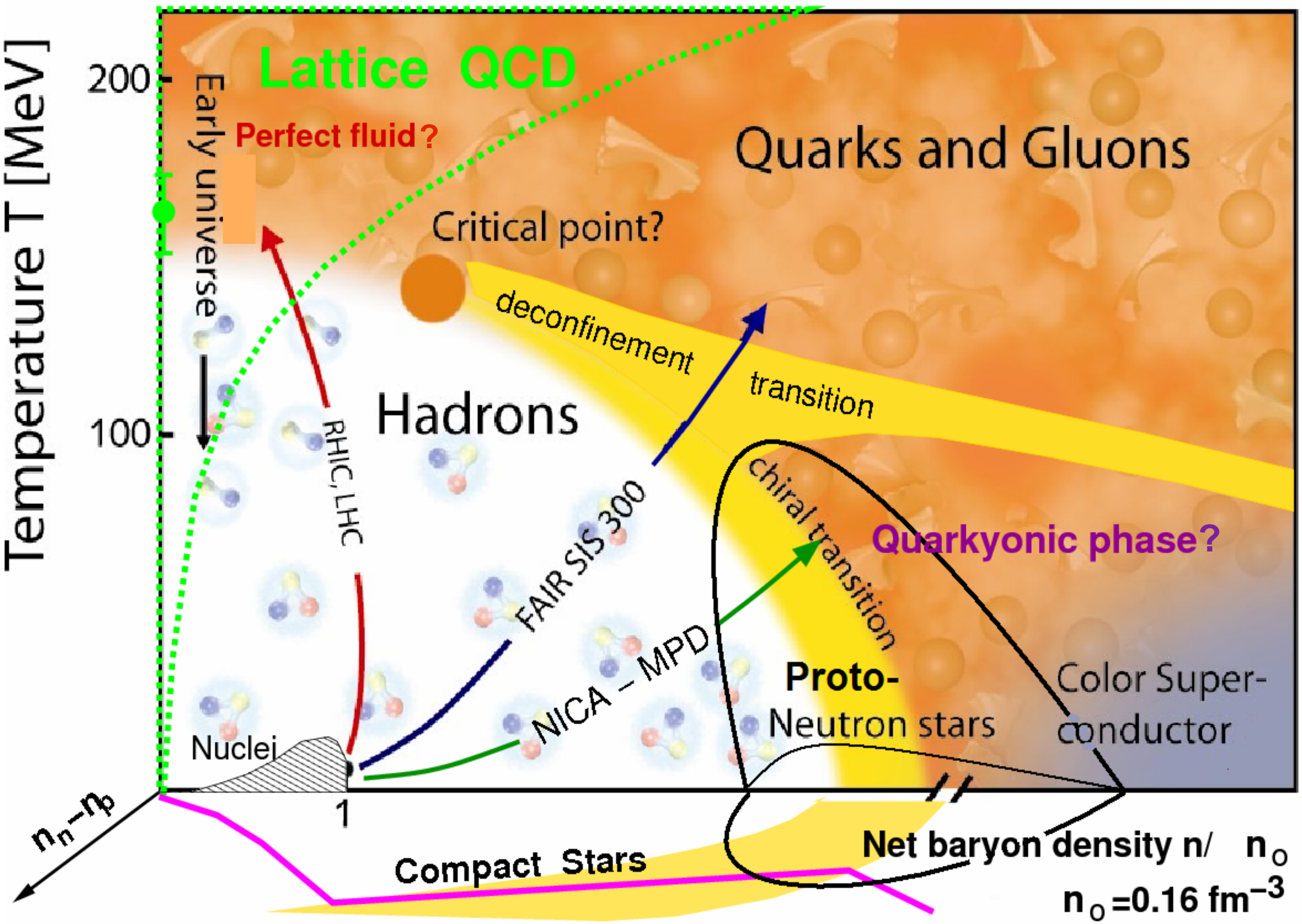


[Bali et al (2012)]

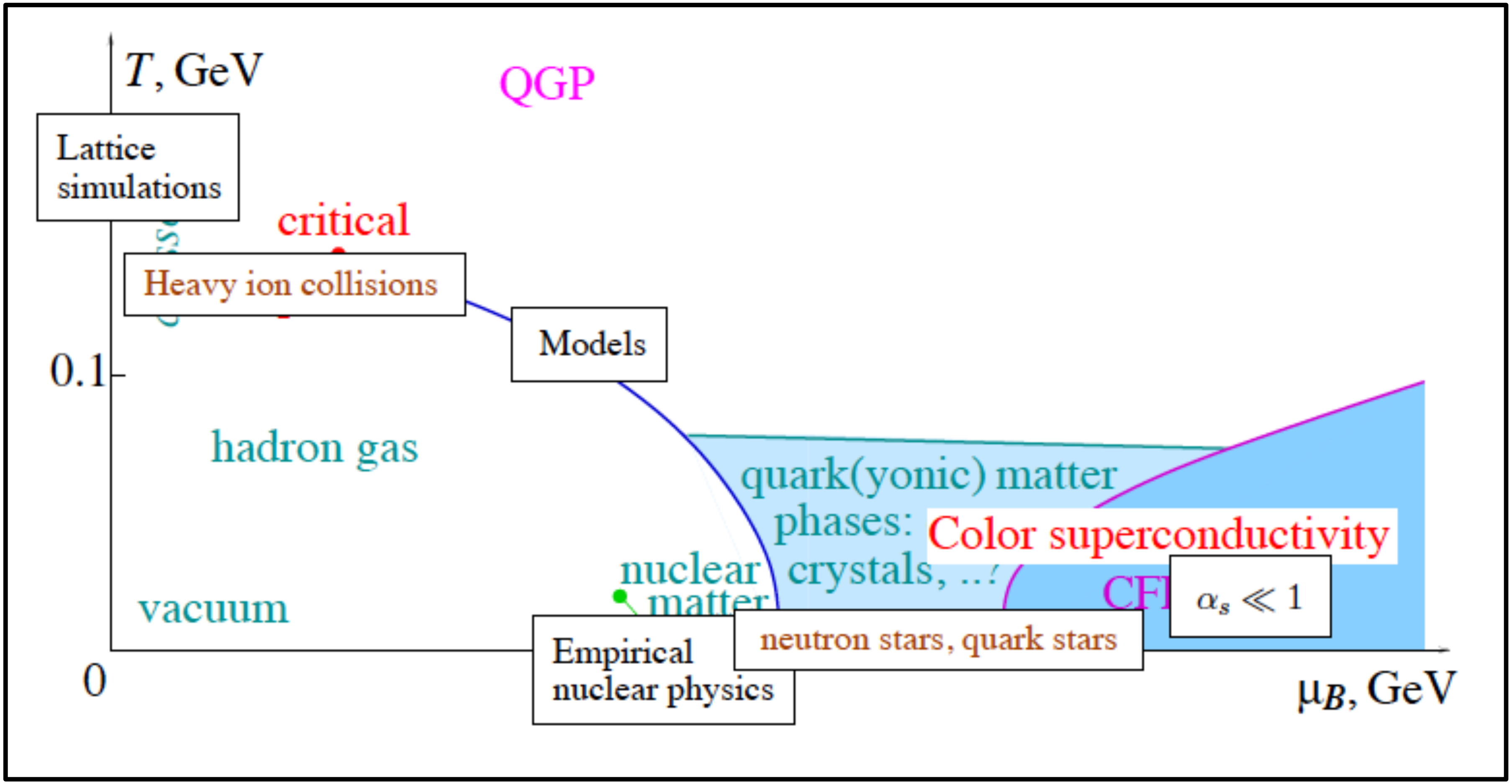
A result that is not obtained in the majority of models!

The field is wide open yet... → More details in the discussion sessions!!

"Drawing" the phase diagram

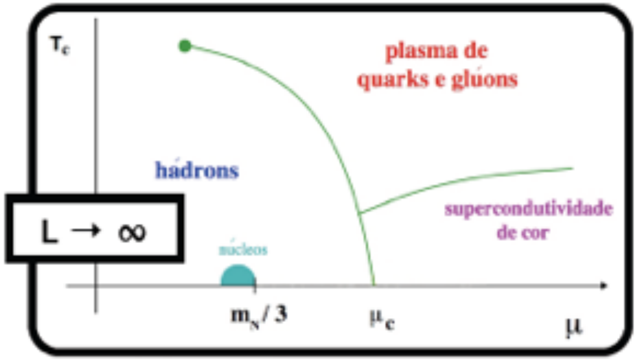


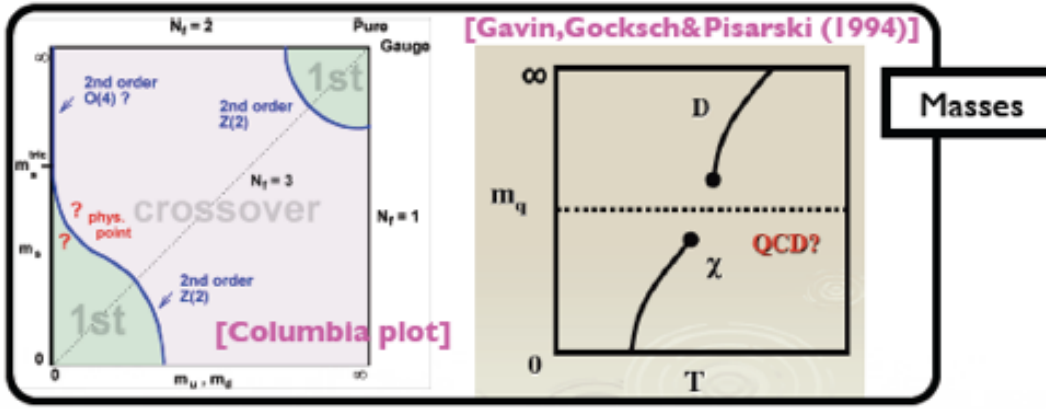
Using all we can!

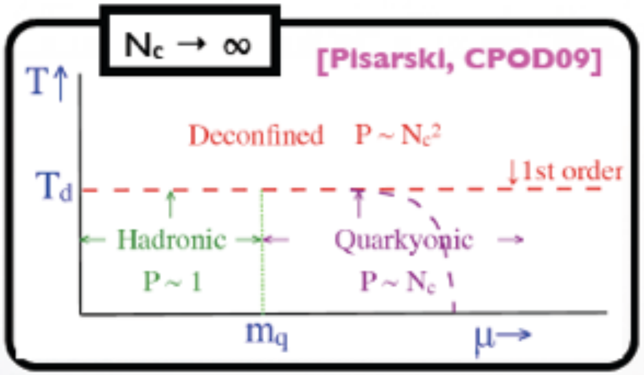


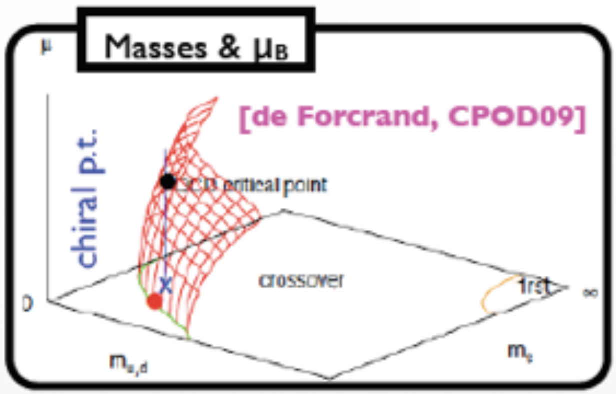
[Stephanov (2010)]

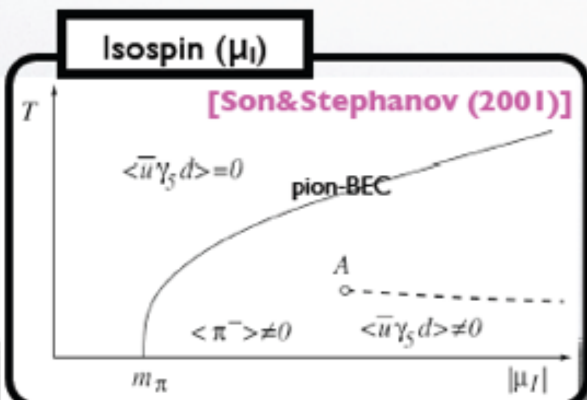
Many phase diagrams... A few examples:

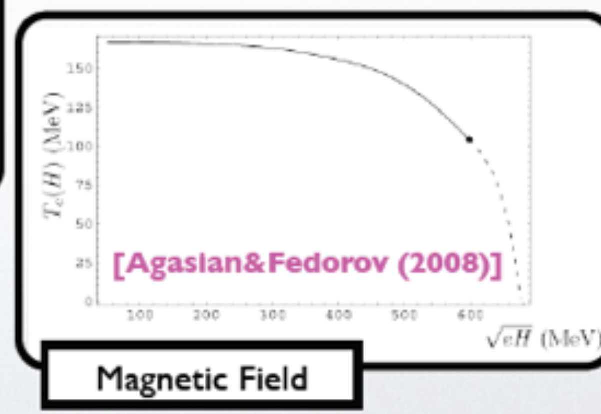








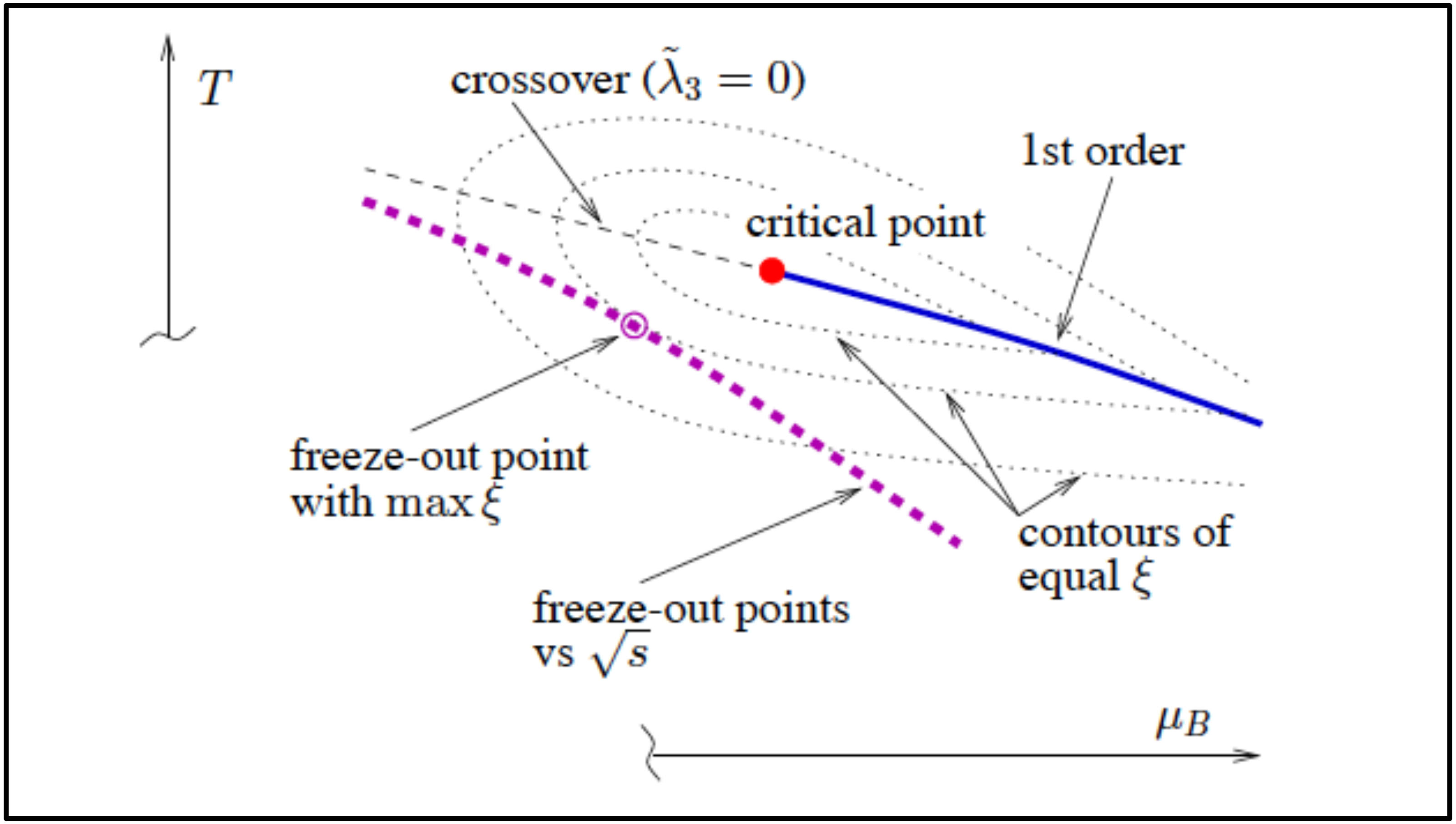




In the critical region ⟶ large fluctuations expected!

$$\xi_\infty \sim t^{-\nu} \quad ; \quad t \equiv \frac{T - T_{\rm CEP}}{T_{\rm CEP}}$$

$$\langle \sigma^n \rangle \sim \xi^{p_n} \, f_n(\xi/L)$$



[Stephanov (2010)]

★ Experimentalists can measure multiplicities that fluctuate event-by-event

★ These fluctuations increase in magnitude near the critical point as a power of the correlation length (contribution of the order parameter)

$$\langle \sigma^n \rangle \sim \xi^{p_n} f_n(\xi/L)$$

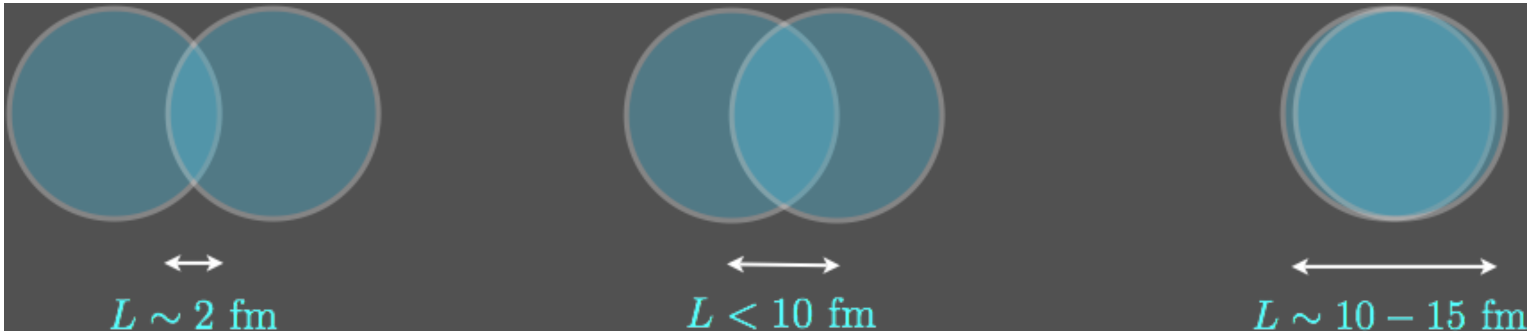
$$\boxed{\langle (\delta N)^3 \rangle \sim \xi^{4.5}, \qquad \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \sim \xi^7}$$ [Stephanov (2009)]

★ Higher cumulants grow faster with ξ, BUT are harder to compute and much harder to measure (large background/signal!)

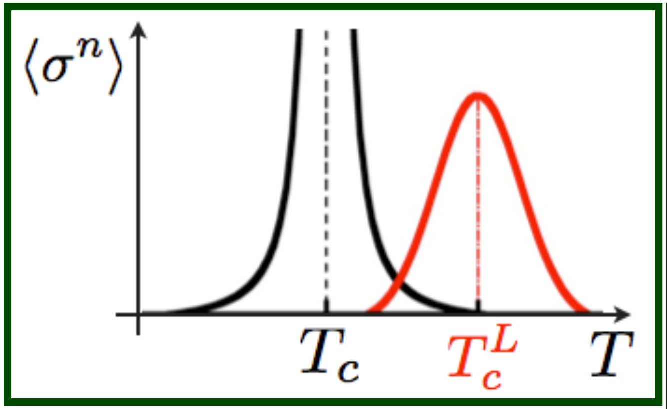
★ Combinations of moments may help... [Athanasiou, Rajagopal & Stephanov (2010)]

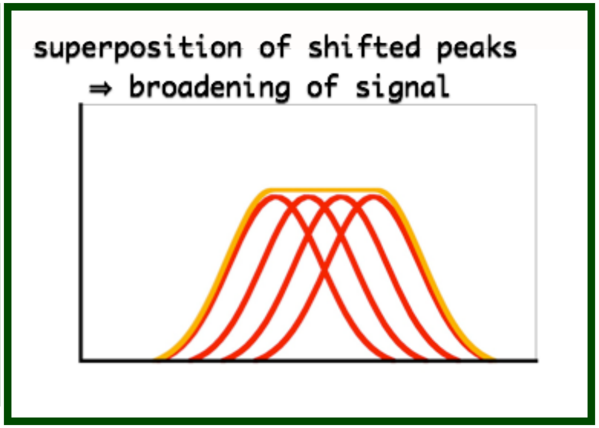
So, there is the background to be surpassed, there is critical slowing down, and...

- In heavy ion collisions the system size will depend on centrality:



- Measurements will generally probe *pseudocritical,* smoothened, shifted thermodynamic quantities. Ex. – cumulants:





- Most (≈all) signatures based on non-monotonic behavior of observables [e.g.: Stephanov (2009)].

- partially hidden by background. shifts and smoothening.

More details in the discussion sessions!!

Instead of conclusions... just a final comment

To make progress in understanding (or at least in collecting facts about) (de)confinement and chiral symmetry, we need it all:

- Experiments and observations
- Lattice simulations
- Theory developments
- Effective models

And also combinations whenever possible.

It is crucial to have theorists and experimentalists working and discussing together!

A last piece of advice: don't trust theorists that much...