Modeling of Metallic Micro-Protrusions on the surface of High Gradient Structures

A study of coupled thermal and electrical effects giving rise to dark current and thermal breakdown.

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Motivation

- Micron sized structures on the surface are a source of dark current.
- Dark current causes performance loss and breakdown.
- Local heating of these structures may affect the emission leading to phase transitions and runaway scenarios

- Connection to current accelerator science is already pointed out by previous speakers

Outline

- Point Charge Model and geometry, field enhancement calculations
- General Thermal Field Equation and comparison with Fowler Nordheim emission
- A simple Space Charge model
- Inside the metal E-field calculation
- Solution of heat equation
- Possible other effects and discussion

Assumptions

- A single cylindrically symmetric, micron sized object with a sharp tip will be simulated.
- 'The protrusion' is made of the same material as the surface. A metal like Cu.
- Electrical relaxation times ($\sim 10^{-14}$s) are very short compared to pulse duration ($> 10^{-7}$s). Therefore background is essentially static. The surface field is typically around 100-300 MV/m
- We will skip the analysis of the evolution or root cause of these protrusions.

The Point Charge Model

- We need to describe the geometry in order to simulate
- An image charge model called PCM is a simple and efficient analytical tool to produce arbitrary protrusion geometries.
- Experimental measurements of dark current can be reproduced by using this image charge model.
- Image charges stacked upon each other with decreasing strength will be considered.

PCM

- Protrusion models can be constructed using the PCM
- We build a potential function of the form:

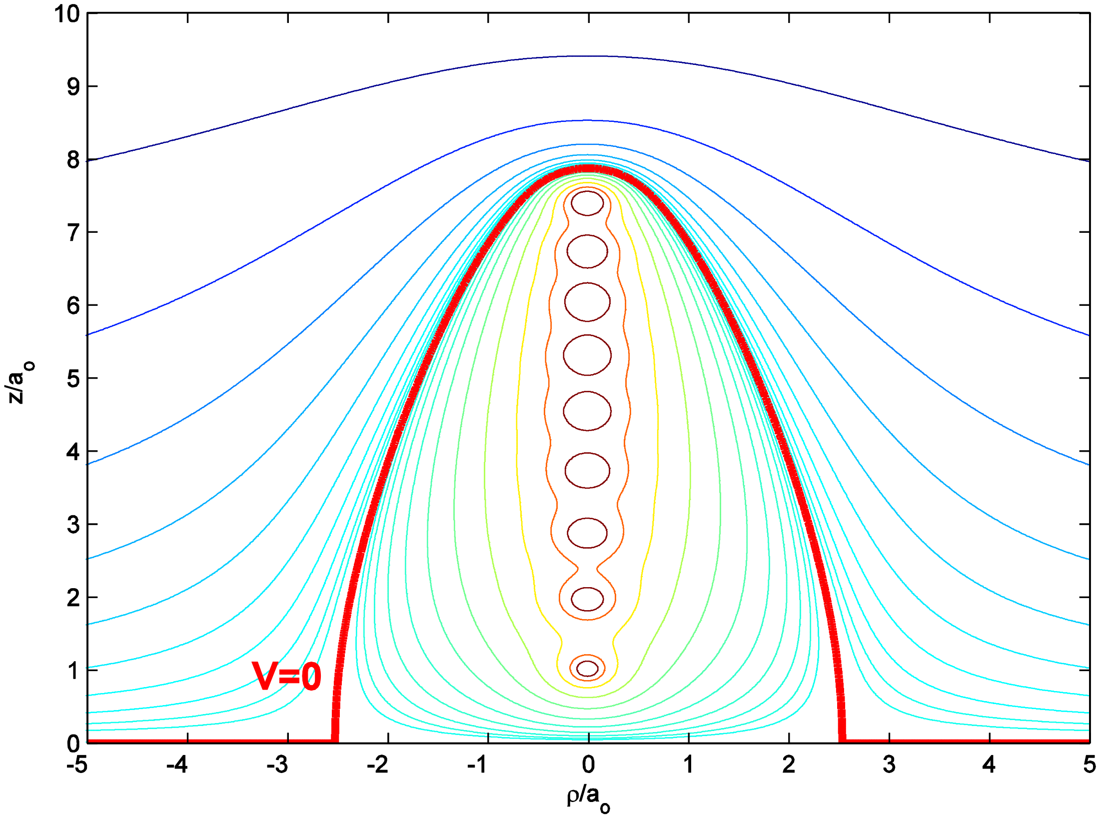
$$V_n(\rho, z) = F_0 a_0 \left\{ -\frac{z}{a_0} + a_0 \sum_{j=0}^{n} \frac{\lambda_j}{\sqrt{\rho^2 + (z - z_j)^2}} - a_0 \sum_{j=0}^{n} \frac{\lambda_j}{\sqrt{\rho^2 + (z + z_j)^2}} \right\}$$

- As an image charge potential this describes the potential outside.
- $V_n(\rho, z) = 0$ identifies the protrusion profile constructed by using an n charge PCM model.
- The sharp tip amplifies the background electric field by a factor of β ranging from 10 to 100.

Jensen and Lau and O'shea ***Electron emission contributions to dark current and its relation to microscopic field enhancement and heating in accelerator structures.*** PHYS.REV. ST - ACCELERATORS AND BEAMS **11**, 081001 (2008)

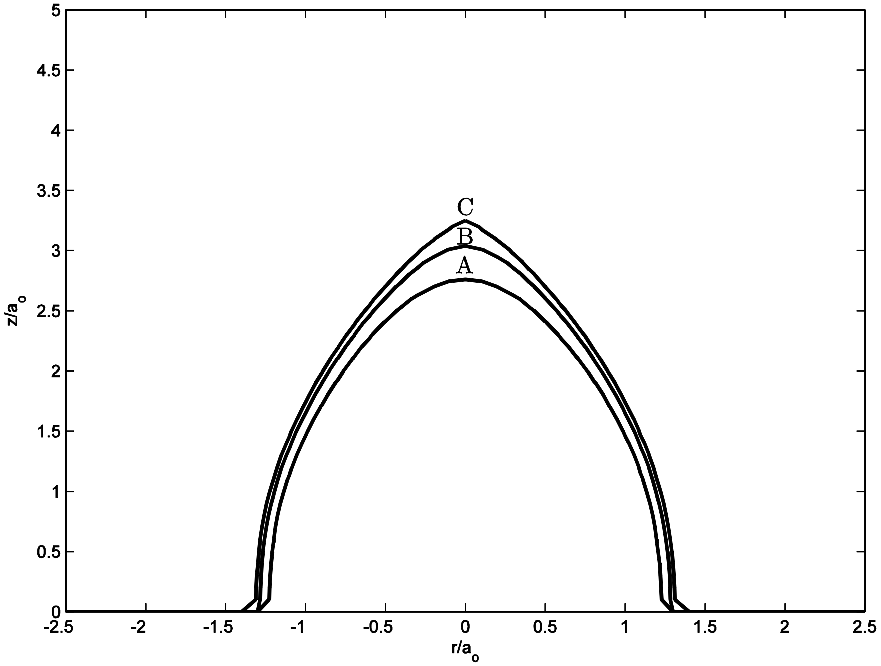
The Protrusion Geometry

- For $n = 9$ and $r = 0.95$ we have:



The Protrusion Geometry

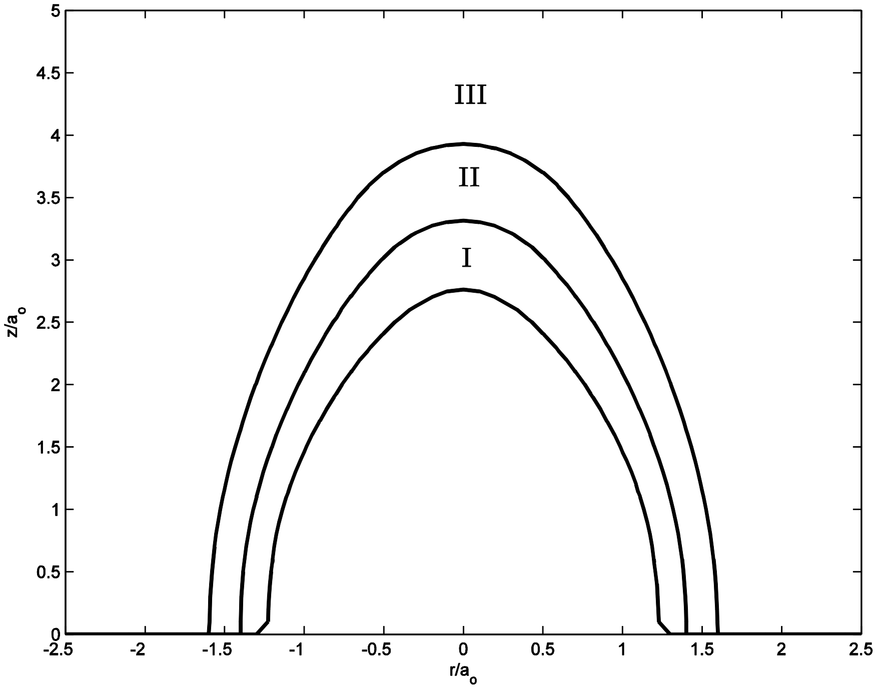
- As the number of charges increase sharper protrusions are obtained.
- We will show that there exists n_{max} such that $n > n_{max}$ are destroyed due to extreme E-field and temperature conditions at the tip.



A: n=4; B: n=6 C: n=12;

The Protrusion Geometry

- The ratio of two successive charges is the other input parameter in this model.



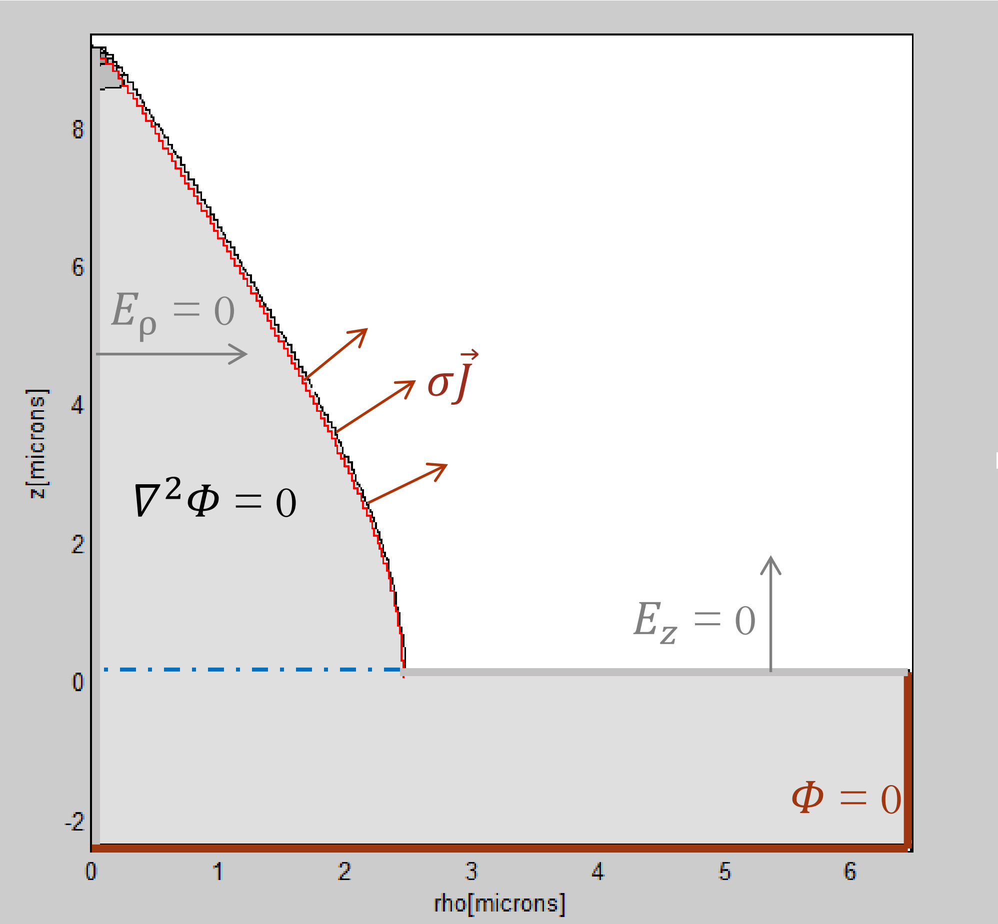
I: r=0.69; II: r=0.8; III:r=0.9

The Boundary Conditions

- We solve the Laplace equation in the shaded region with the specified boundary conditions.

  The boundary condition for the surface of the protrusion is determined by the current density flowing through the boundary.

- The region includes a chunk from the rest of the surface for a more realistic calculation



The Surface Current

- The current in question is due firstly, to the intense electric field amplified at the sharp tip of the structure. This is called Field electron emission and quantified by the Fowler-Nordheim formula.
- Secondly, to the high temperatures occurring via a combination of Joule and resistive heating. This is called thermionic emission and described by the Richardson-Laue-Dushman formula.
- Although their domains of validity do not overlap, both of these formulas are deduced from the same expression:

$$J(F,T) = q/h \int_{-\infty}^{\infty} D(E,F) f(E,T) dE$$

The Surface Current

- Here f is the supply function:

$$N(E,T) = 4\pi mkTh^{-3} \ln\left(1 + \exp\left(-\frac{E-\zeta}{kT}\right)\right)$$

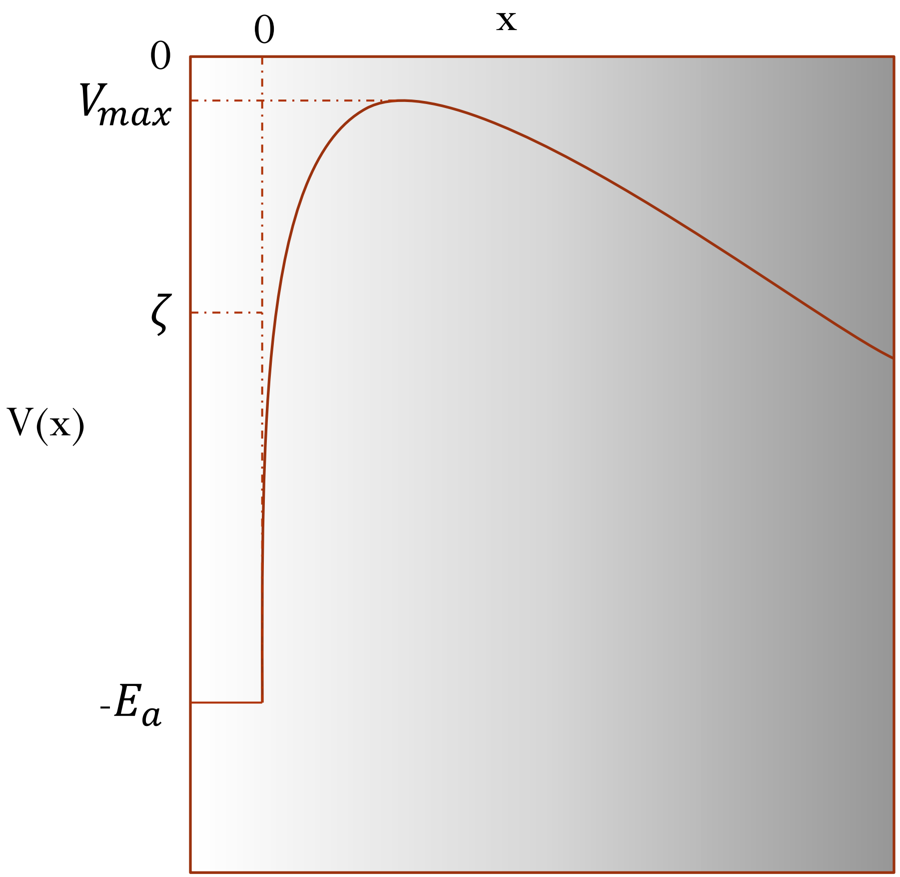
- And D is the transition probability. A WKB type approximation provides a formula:

$$D(F,E) = \frac{1}{1 + exp\left\{-4\pi i/h \int_{x_1}^{x_2} p(\zeta)\, d\zeta\right\}}$$

Where p is the complex momentum of bound state and $x_{1,2}$ are the zeros of p^2.

The Barrier Potential

$$V(x) = -\frac{e^2}{4x} - eFx \qquad for\ x > 0$$



Range of E field

- We can calculate V_{max} as $-\sqrt{e^3 F}$
- So an electron at Fermi level sees a barrier of

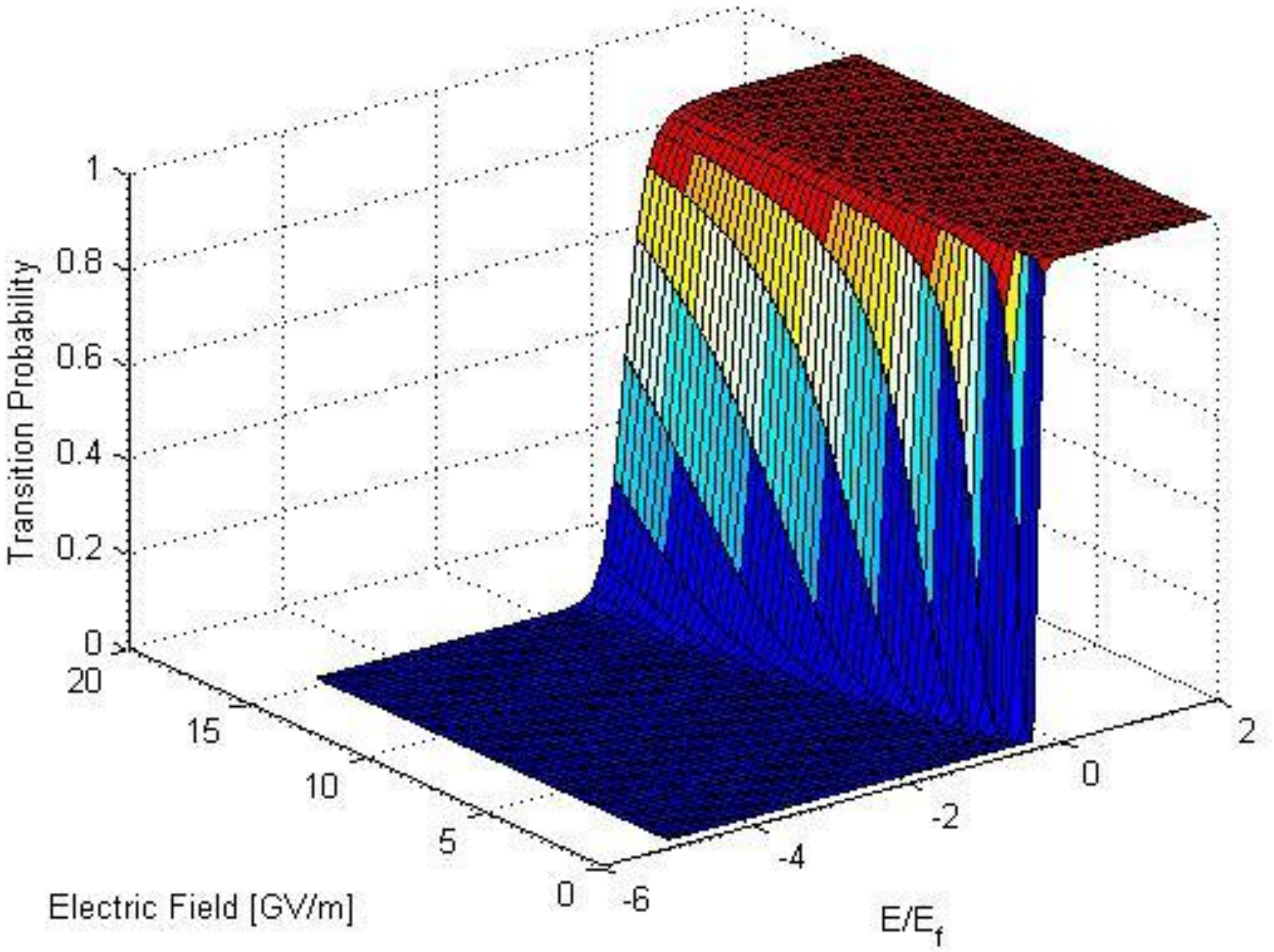
height $V_{max} - \zeta$.

- V_{max} was lower than the Fermi level, there would be no bound electron which is not physical.
- So we should avoid this case, by restricting the E field.
- For a work function of 4.7 eV, the field should be less then

15 GV/m.

- For a background surface field of 300 MV/m, the amplification factor β must be less than 50.

Transition Probability



Current Density Comparison

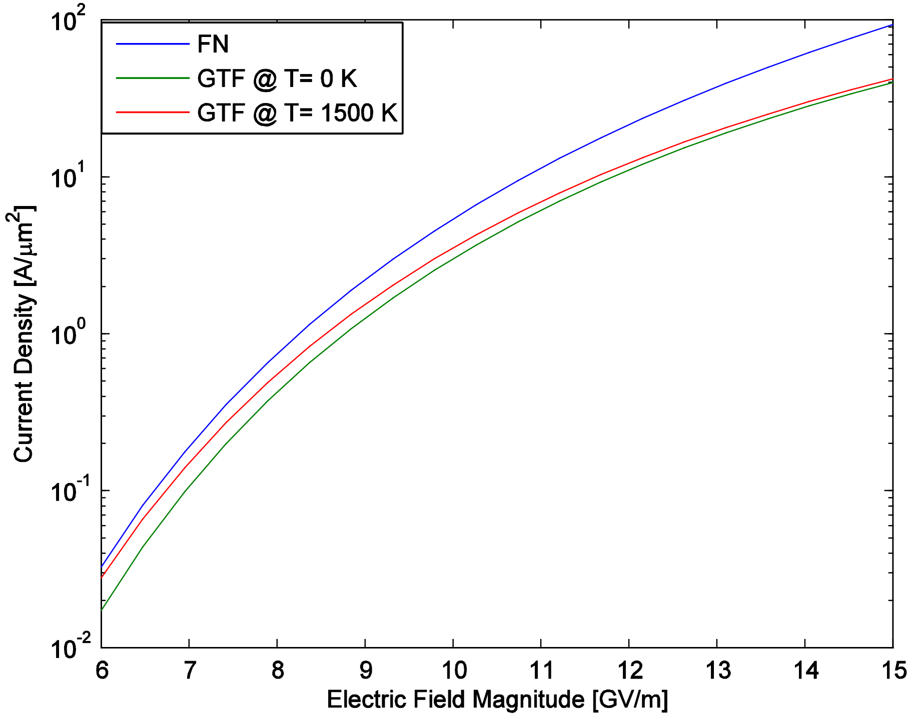
- Current is mostly concentrated at the tip, sharper tips emit exponentially higher current.
- Our aim is to calculate E-field inside. At the tip we expect:

$\sigma_{metal} \cdot E_{inside} = J_{GTF}(\rho = 0)$

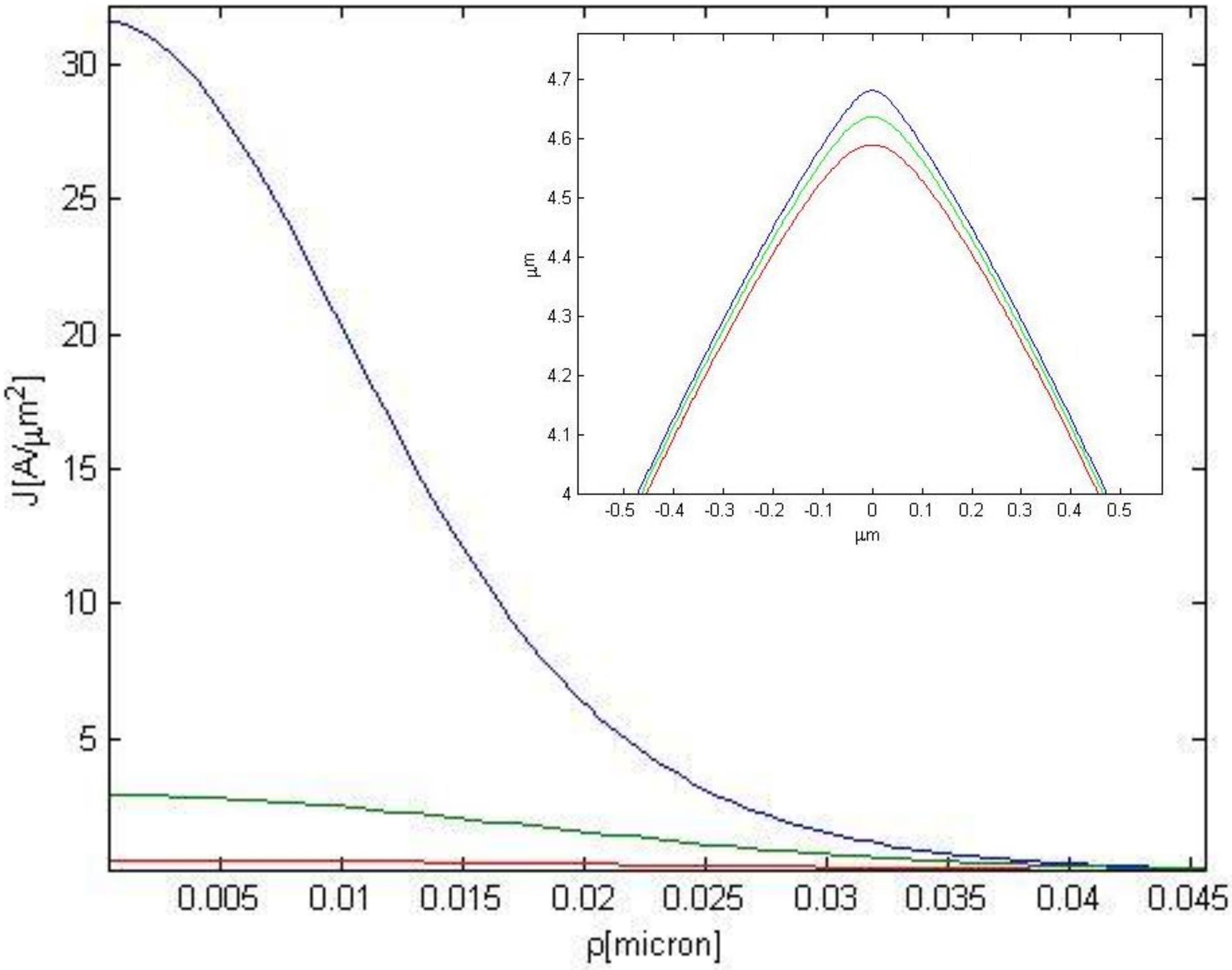
which is $\approx 34 \frac{A}{\mu m^2}$ for $\beta = 50$ and $F_o = 300\ MV/m$.

where $\sigma_{Cu} = 59.6\ S/\ \mu m$

Hence E_{inside} should be around $0.57\ V/\ \mu m$ at the tip.



Current Density Profile



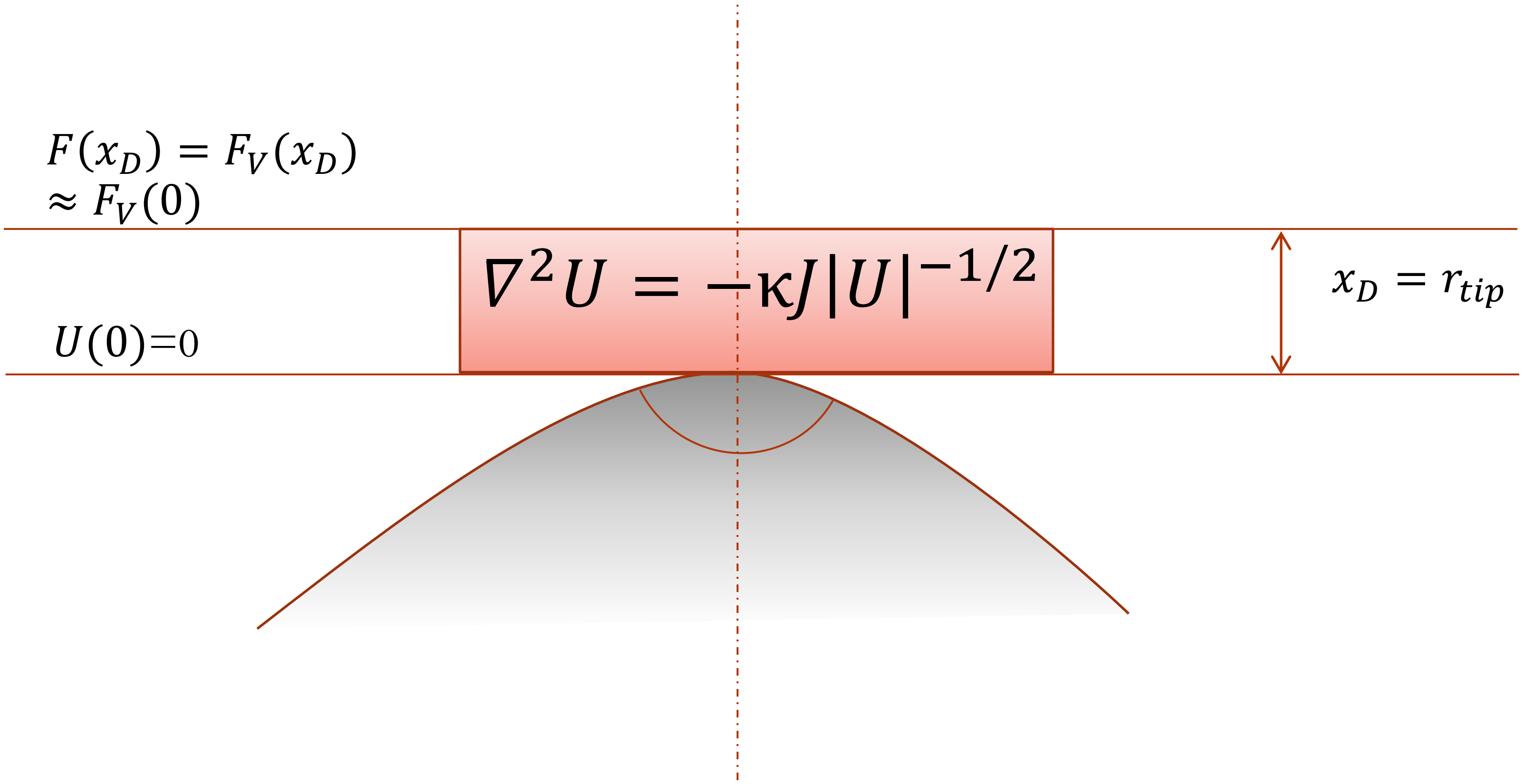
Space Charge Effects

- Barbour *et*. *al*. predicts space charge effects should be relevant above 0.4 A/μm^2.
- We need to model space charge effects and incorporate it to our results.
- Exact modeling requires PIC codes which is beyond the scope of this study
- Feng *et*. *al*. studied the transition from FN to space charge limited emission in an 2006 paper.
- We will describe our own simpler model.

Modified Child-Langmuir Model.

- Two major concerns:
  - There is no definite anode unlike in the CL model.
  - There is a 3D emitter with non-trivial geometry
- The current is concentrated at the tip
- As me move away from the emitting tip, electron density drops very fast due to the 3D geometry.
- Therefore the tip radius serves as a good scale over which the space charges are spread.
- The tip is assumed to be a planar cathode
- Typical range is 20 nm to 300 nm.
- Instead of an anode we will use a Neumann Boundary condition.

Space Charge Model



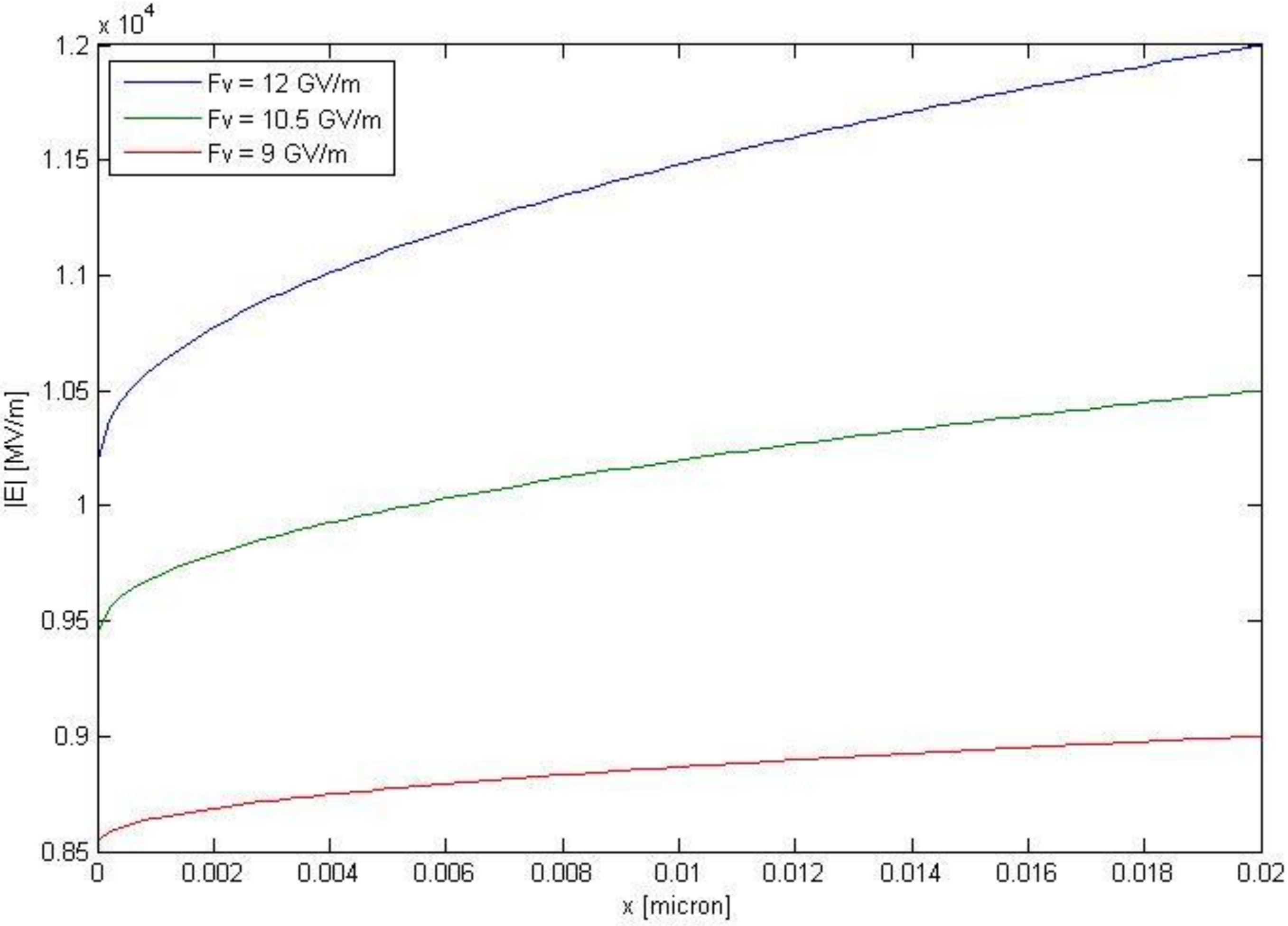
$$F(x_D) = F_V(x_D)$$
$$\approx F_V(0)$$

$$\nabla^2 U = -\kappa J|U|^{-1/2}$$

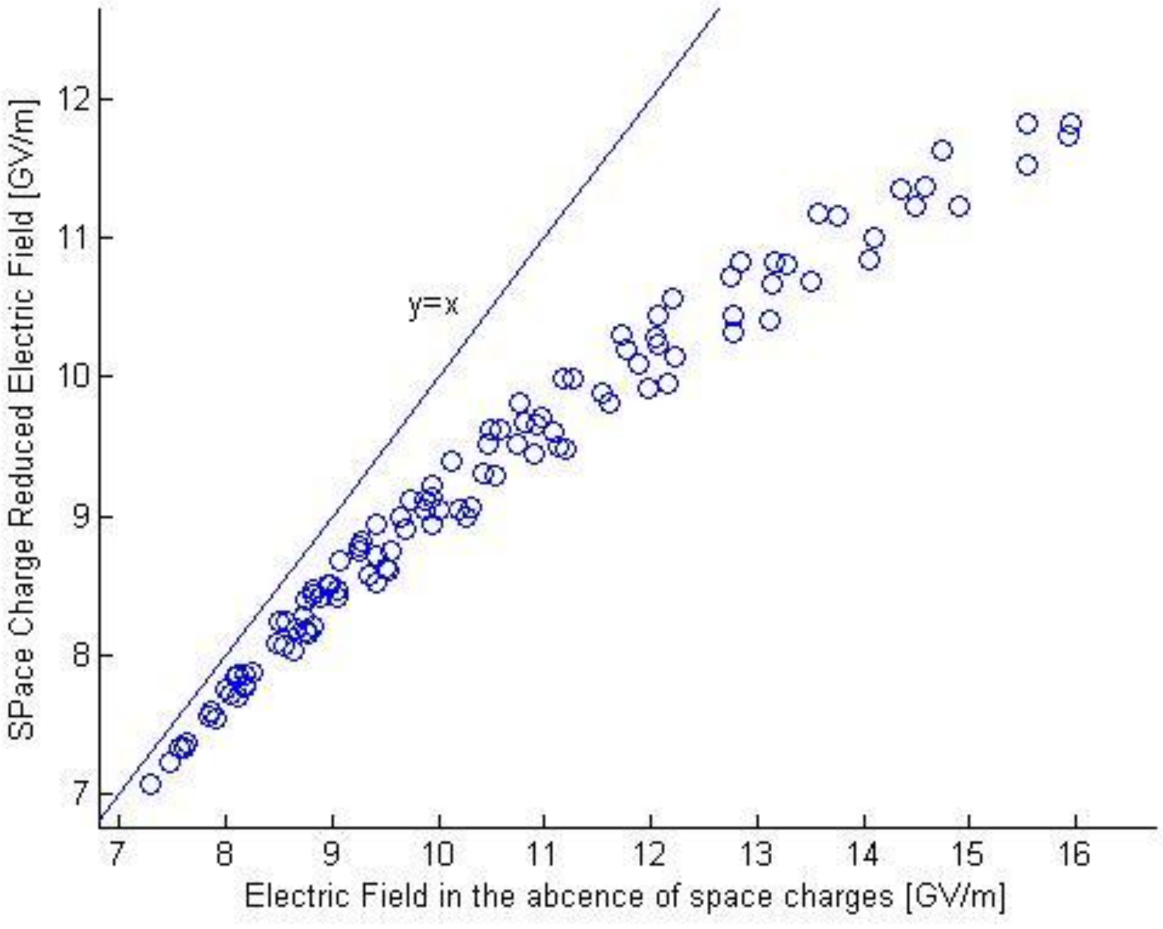
$$x_D = r_{tip}$$

$$U(0)=0$$

Space Charge reduced Electric Field

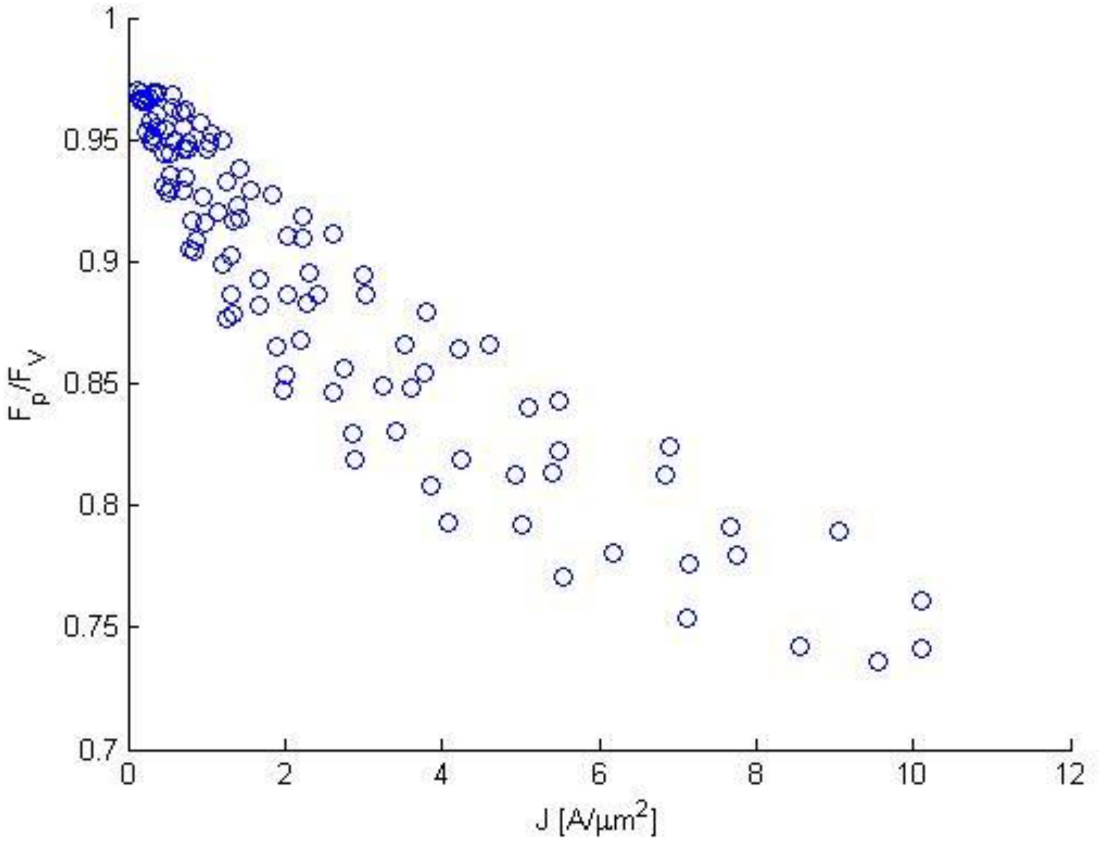


Space Charge Reduced Electric Field

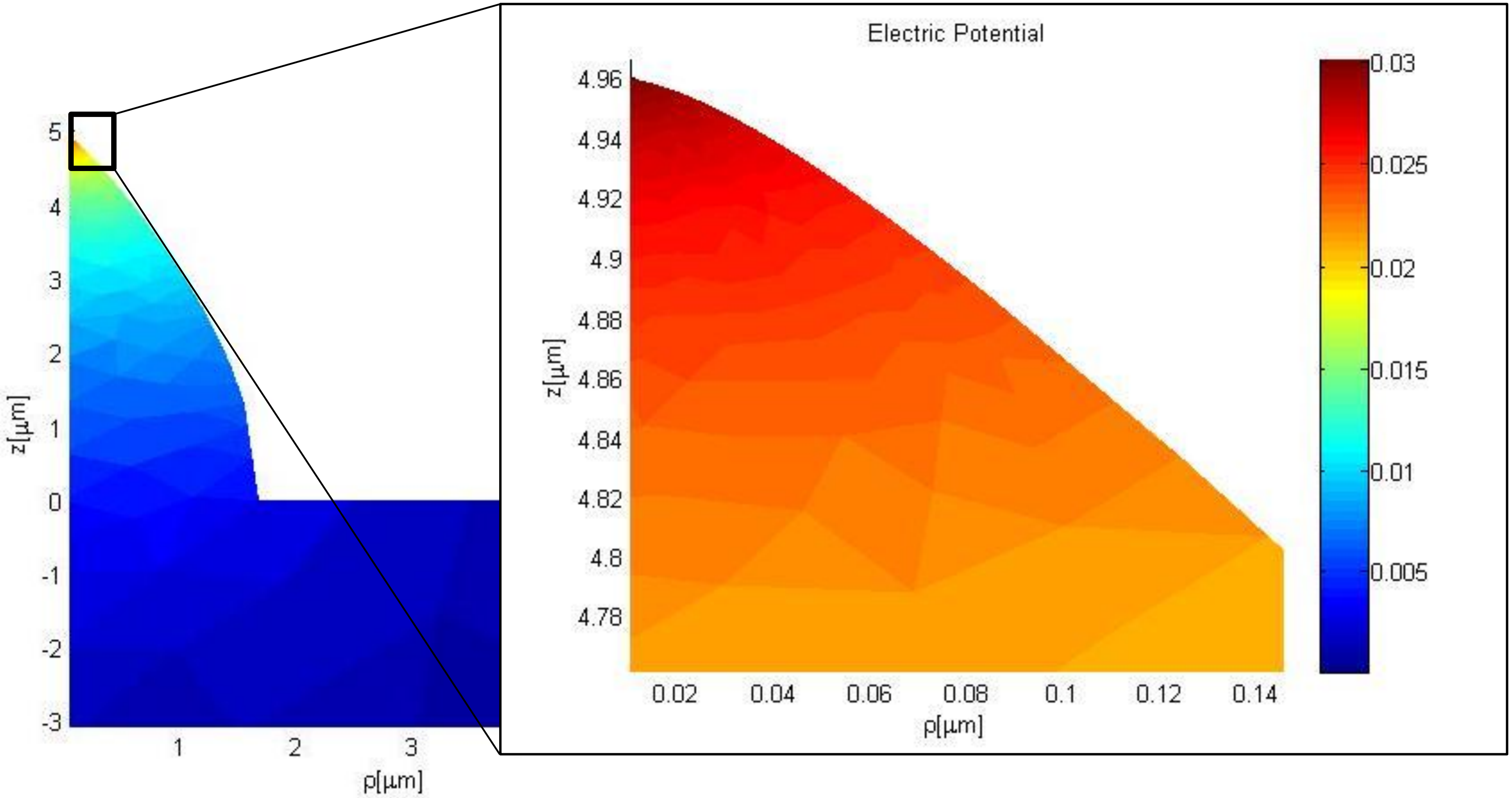


Onset of Space Charge Effect as Current Density grows

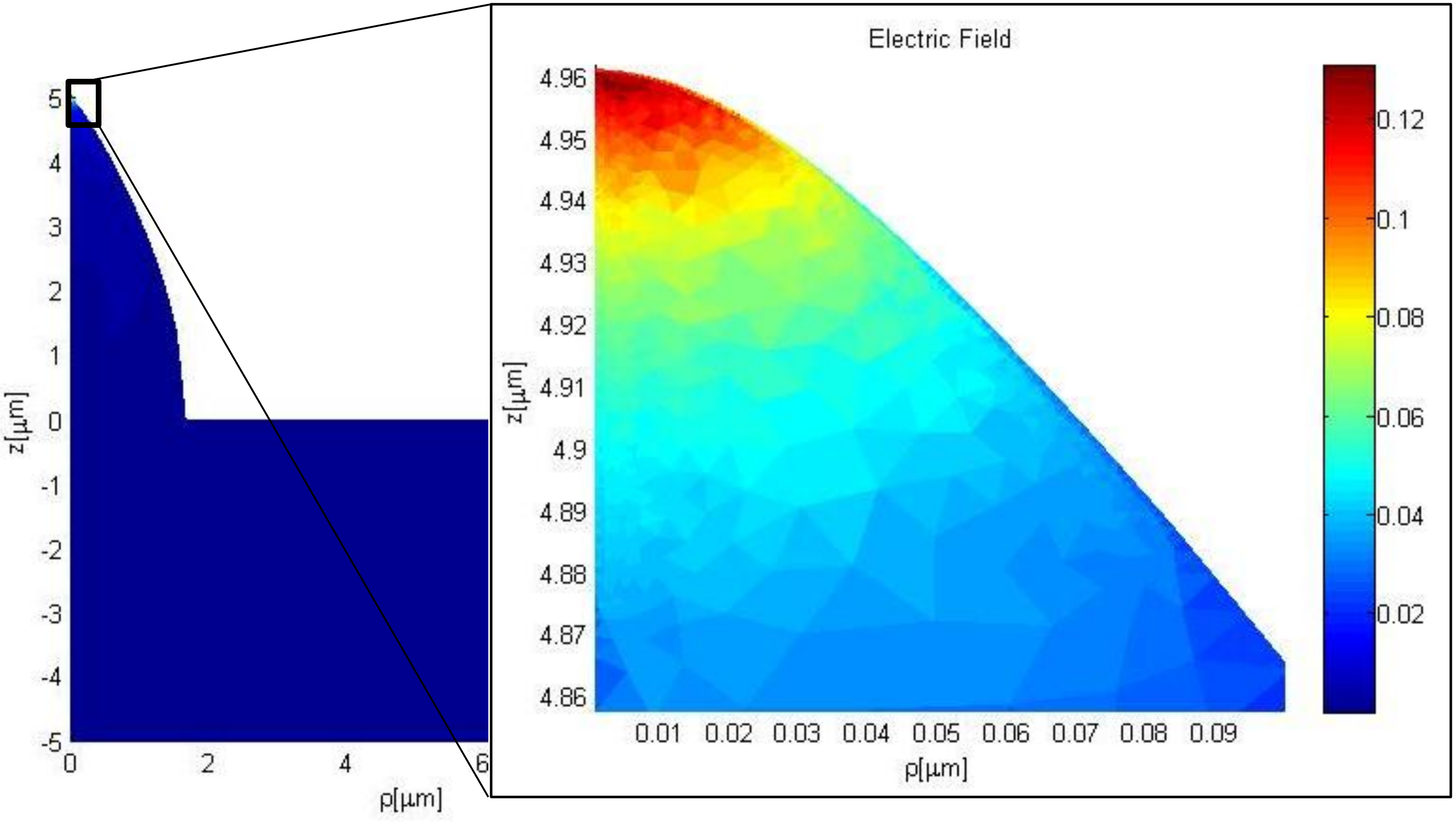
- Although the space charge effect is not prominent till very high current densities; consistent solution of space Charge equations reduced the current emission by up to 3 to 7 fold



Simulation Results



Simulation Results



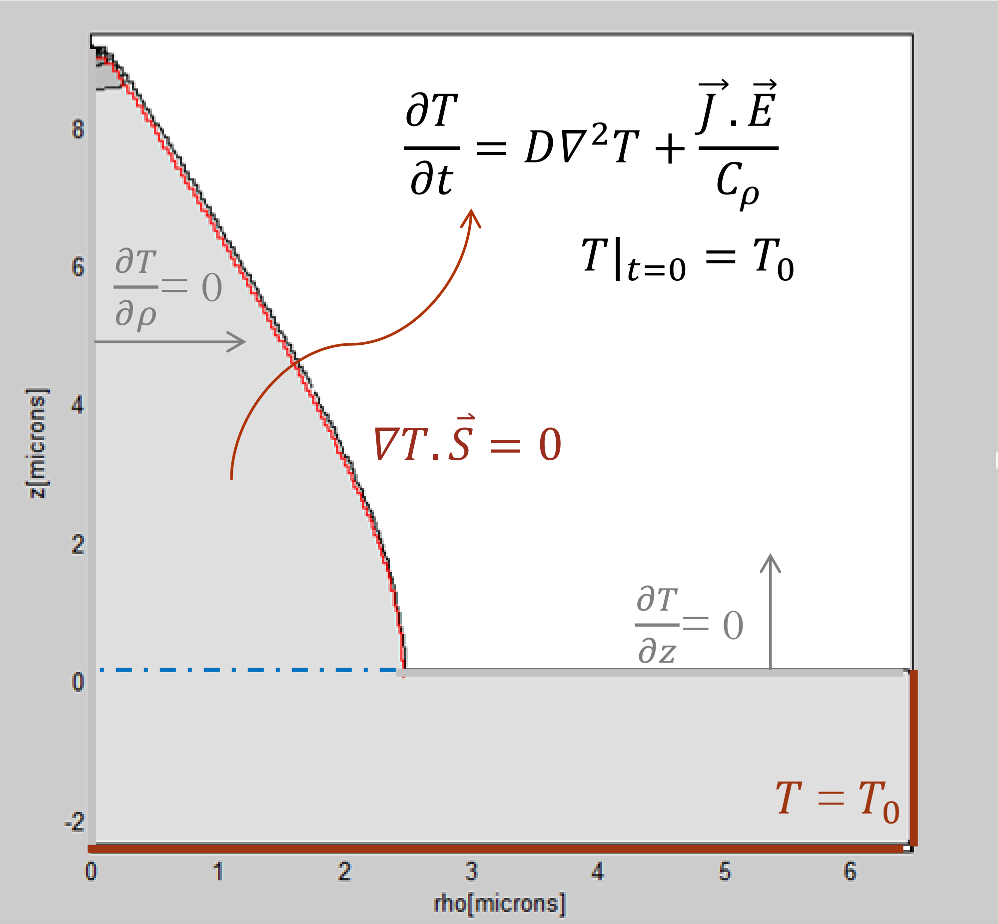
Heating

- The Heat Equation is solved subject to the following initial and boundary conditions.
- For Cu:

$$D = 0.121\,{}^{\mu m^2}/_{ns}$$

$$C_\rho = 3.56\,{}^{mW.ns}/_{K.\mu m^3}$$

- As it seems, heat dissipation takes place predominantly at the tip and ultimately depends on the sharpness of the protrusion i.e. number of charges in PCM.



$$\frac{\partial T}{\partial t} = D\nabla^2 T + \frac{\vec{J}.\vec{E}}{C_\rho}$$

$$T|_{t=0} = T_0$$

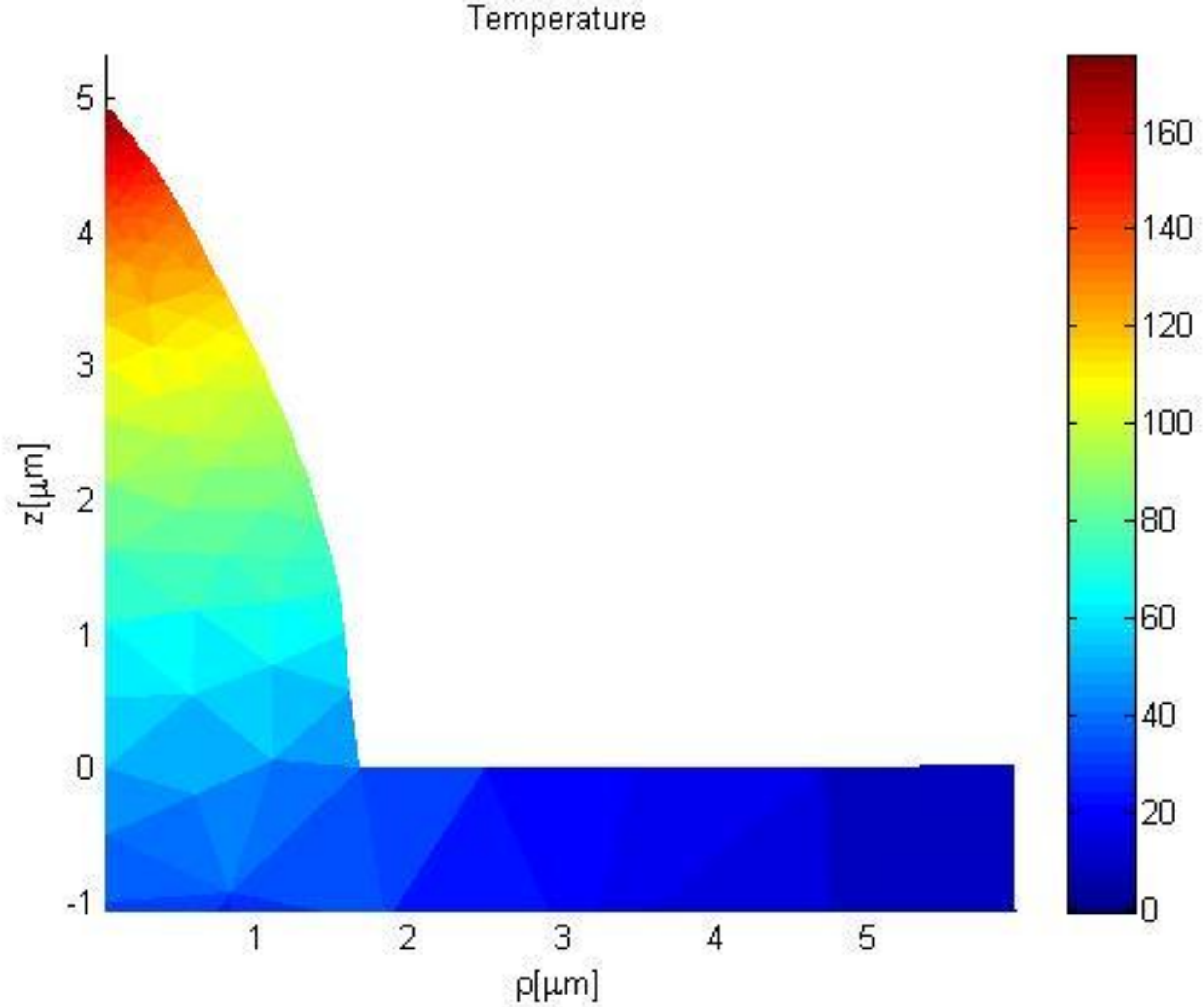
$$\frac{\partial T}{\partial \rho} = 0$$

$$\nabla T.\vec{S} = 0$$

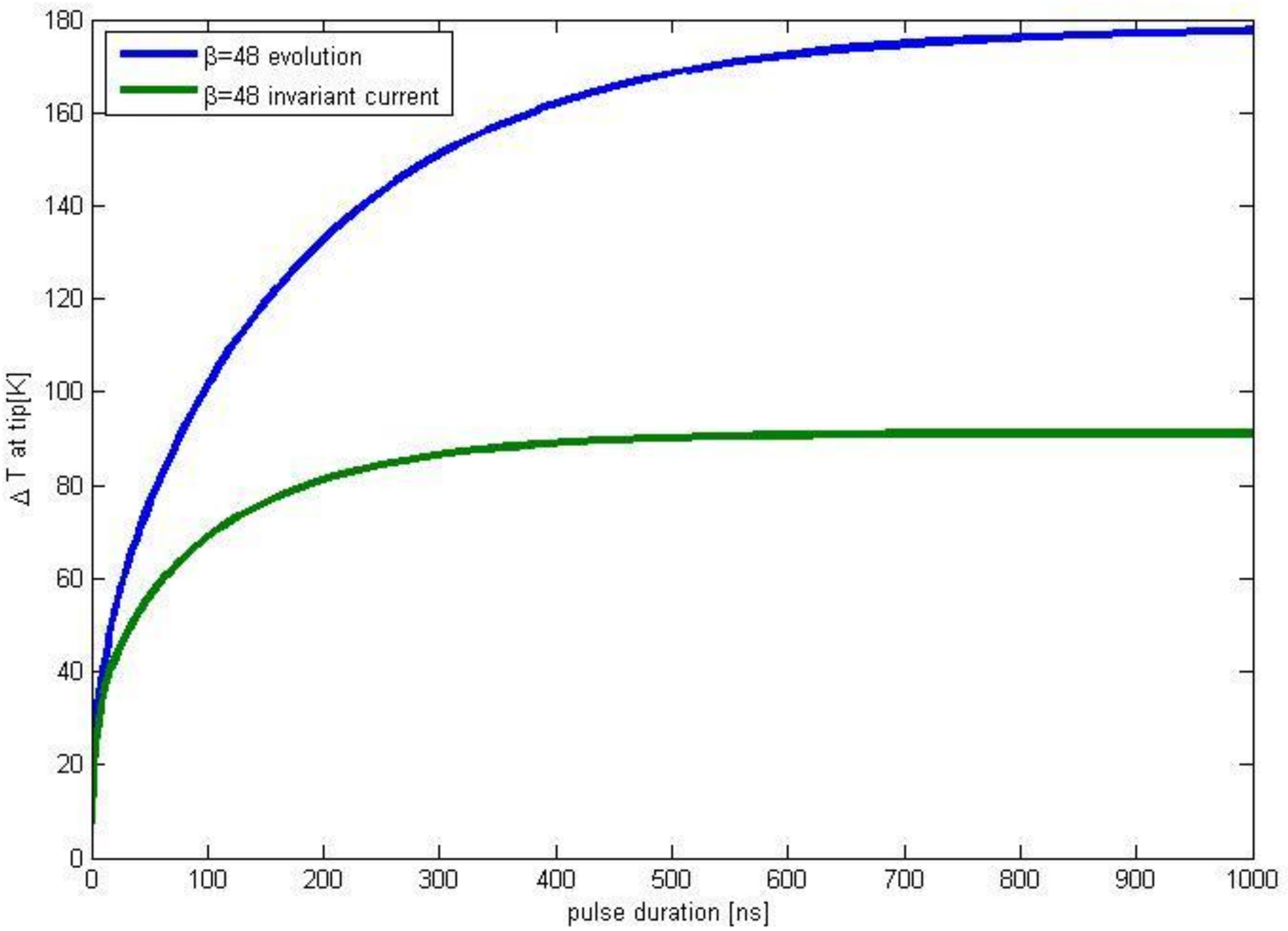
$$\frac{\partial T}{\partial z} = 0$$

$$T = T_0$$

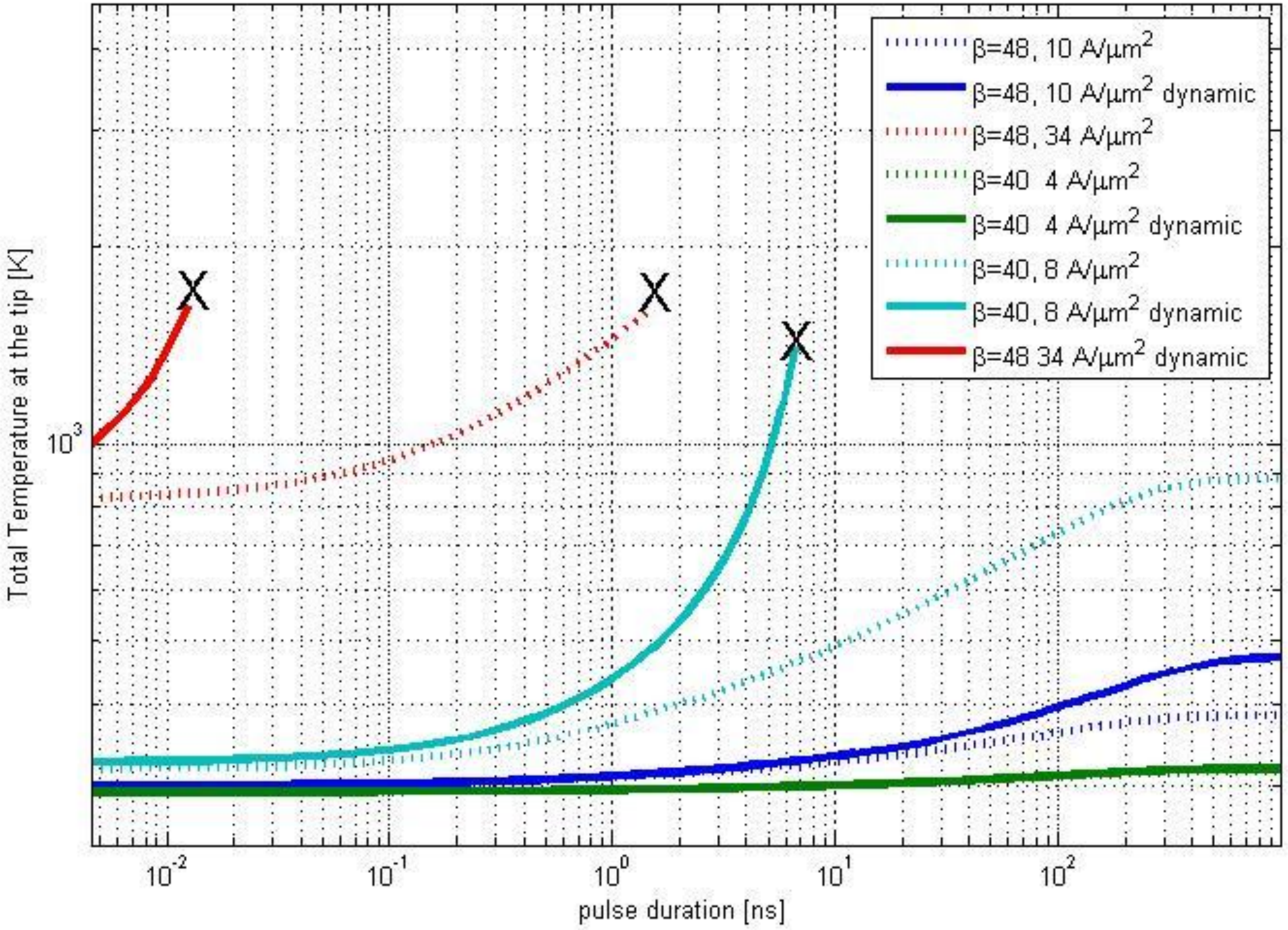
Simulation Results
Tip β=48, Pulse duration $1\mu s$



Evolution of Temperature at the tip



Stability and phase transition



Summary

- We have considered a coupled thermal and electrical simulation, including the following effects:
  - Space Charge
  - General Thermal Field Emission
  - Hans Grüneisen effect on the conductivity.
  - Joule Heating
  - Thermopower is negligible due to small temperature gradient
- More on the way:
  - Nottingham Heating/cooling
  - Better models for space charge and plasma sheath phenomena

Future Work

- Phase transition and mixed phase simulations
  - Taylor cones, EM Stress analysis
- Thermo mechanical stress is ruled out due to almost harmonic temperature field, but worth revisiting.
- Explosive Emission, destroyed tip and micron sized particles in the tube.
- Finally tie up everything to observed break down stats.

Acknowledgements:

This work is supported by the DoE