Higgs production in gluon fusion beyond NNLO

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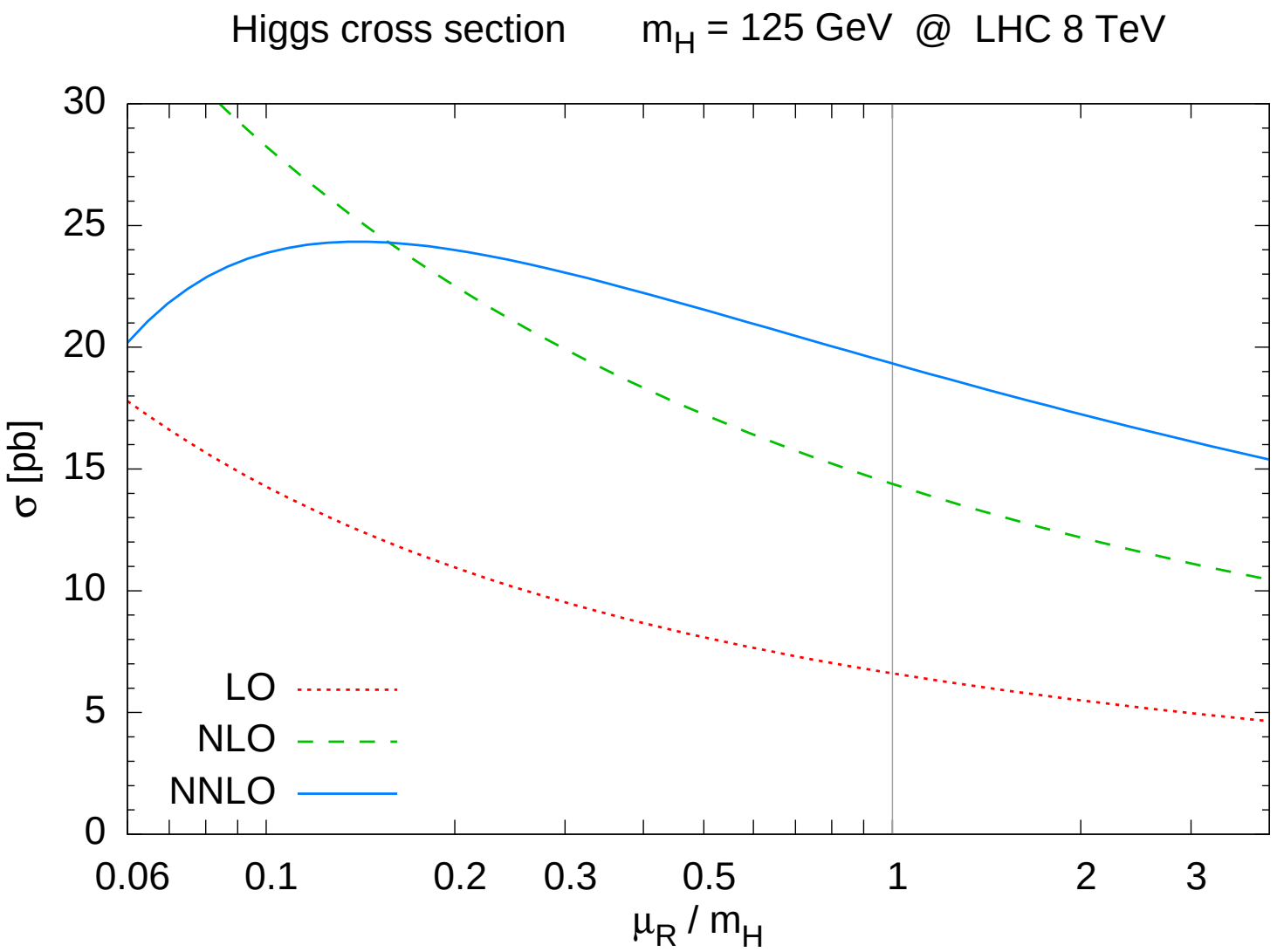
DESY Hamburg

LHCP, Barcelona, May 14, 2013

Work in collaboration with:
Richard Ball, Stefano Forte, Simone Marzani, Giovanni Ridolfi

arXiv:1303.3590

Higgs production in gluon fusion to NNLO



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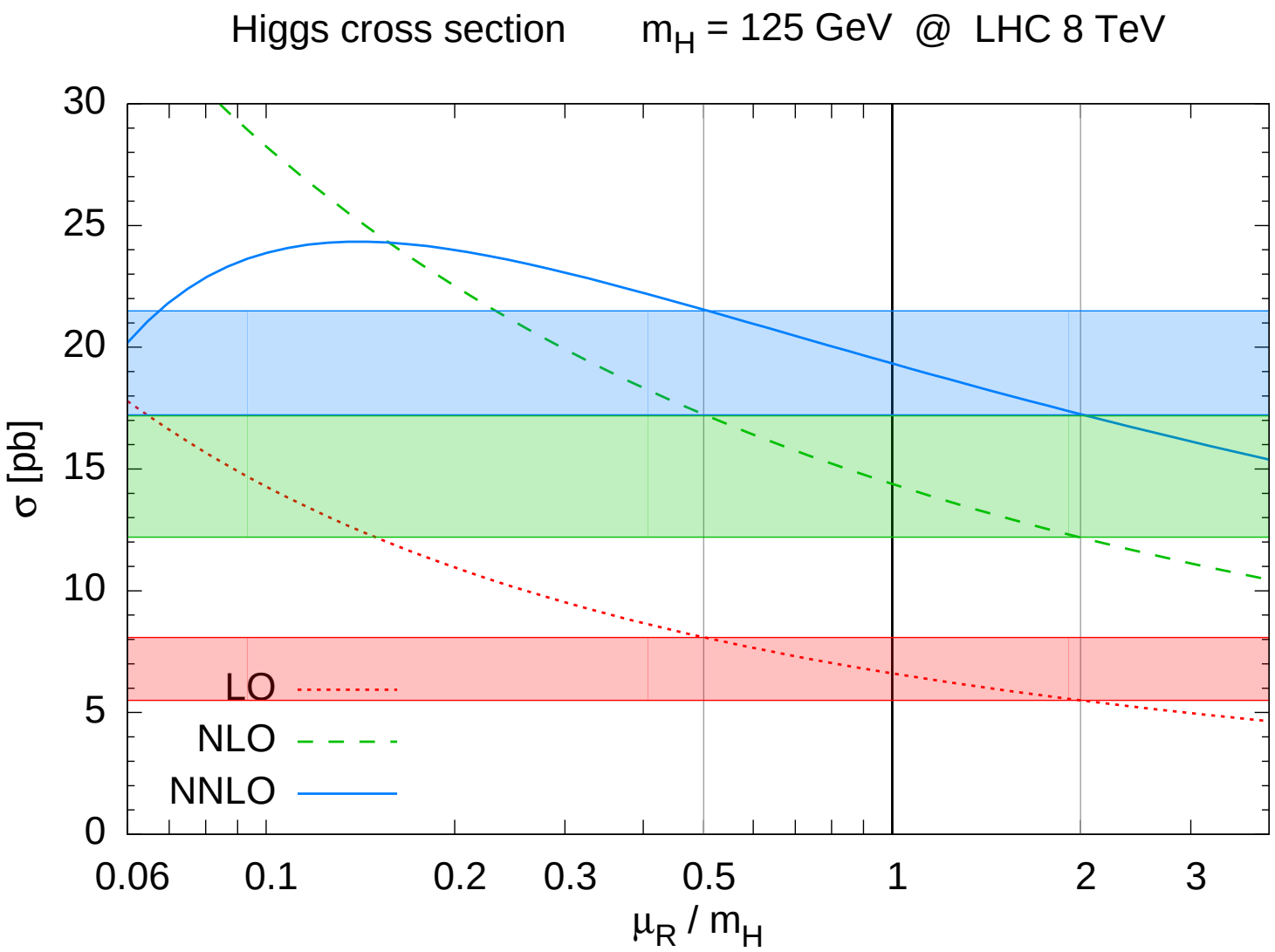
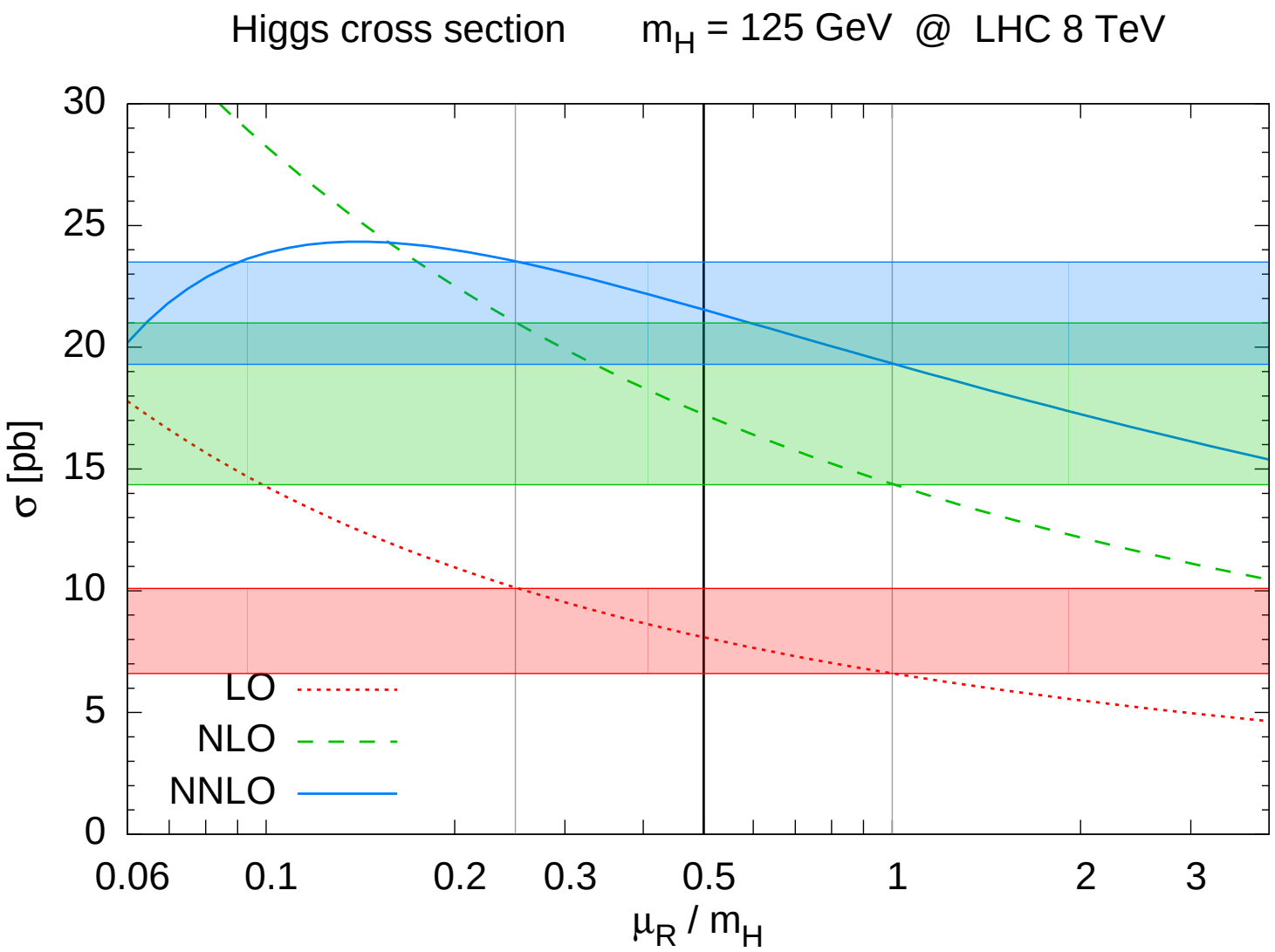


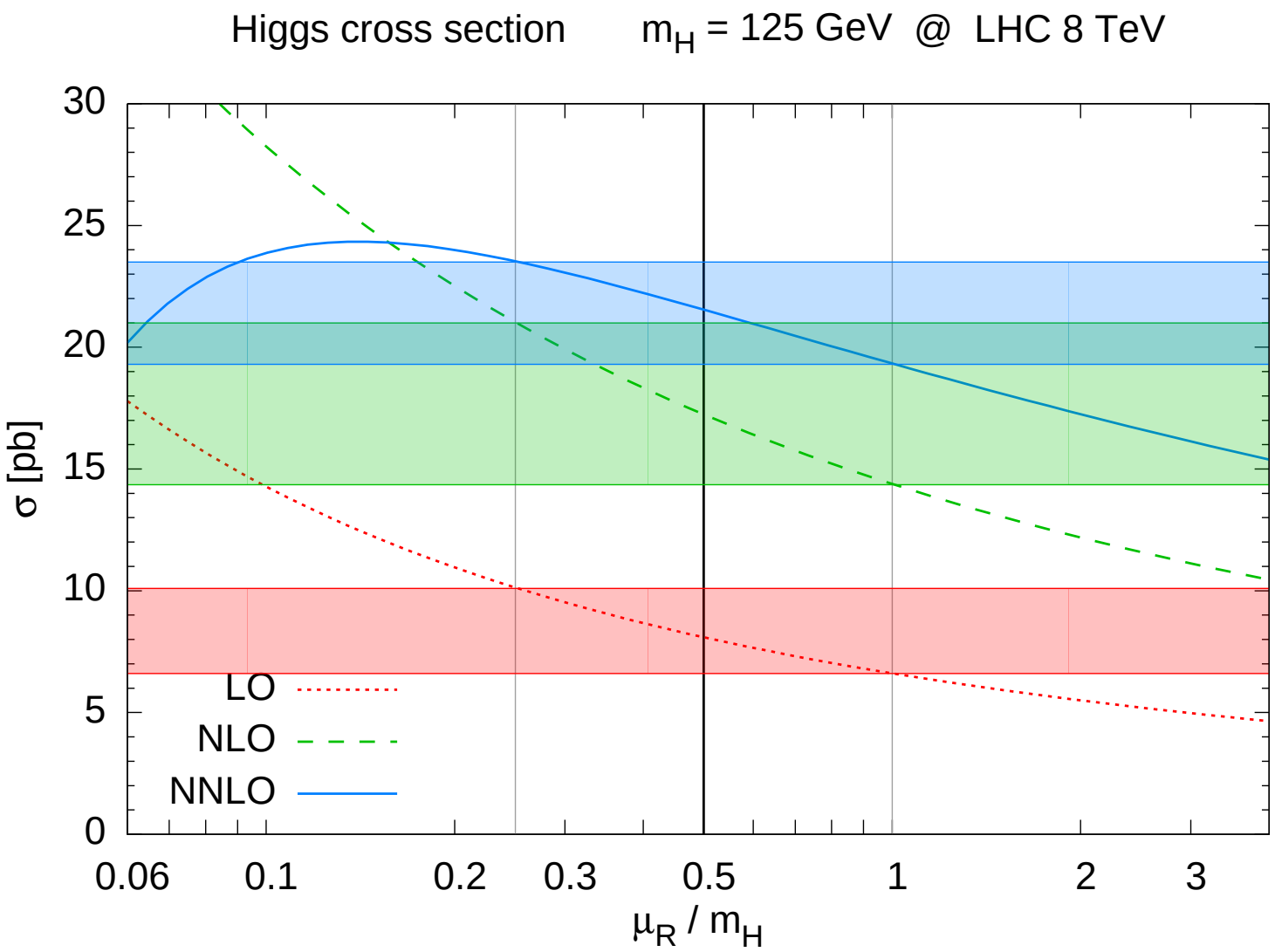
$$1/2 < \mu/m_H < 2$$

Higgs production in gluon fusion to NNLO



$$1/4 < \mu/m_H < 1$$

Higgs production in gluon fusion to NNLO



Convergence is slow! NNLO not definitive.

Higgs production: *inclusive* cross section

$$\sigma(\tau) = \tau\,\sigma_0 \sum_{ij} \int_\tau^1 \frac{dz}{z}\, \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) C_{ij}(z,\alpha_s), \qquad \tau = \frac{m_H^2}{s}, \qquad z = \frac{m_H^2}{\hat{s}}$$

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Ingredients of our N^{3}LO prediction

gg channel only:

$$C_{gg}(z,\alpha_s) \simeq C_{\text{soft}}(z,\alpha_s) + C_{\text{high-energy}}(z,\alpha_s)$$

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Outline:

- construction of $C_{\text{soft}}(z,\alpha_s)$ and $C_{\text{high-energy}}(z,\alpha_s)$
- comparison against known NLO and NNLO
- results at NNNLO

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$$N \to \infty \qquad\qquad N \to 1$$

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- comparison against known NLO and NNLO
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Mellin space: $$C_{gg}(N,\alpha_s) = \int_0^1 dz\, z^{N-1}\, C_{gg}(z,\alpha_s)$$

High-energy part: $C_{\text{high-energy}}$

Leading Log poles in $C_{gg}(N, \alpha_s)$: $\frac{\alpha_s^k}{(N-1)^k}$ $\left(\alpha_s^k \frac{\log^{k-1} z}{z}\right)$

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$\gamma_+(N)$: DGLAP anomalous dimension (largest eigenvalue)

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- valid at LL (with running coupling effects)

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- we use an expansion of $\gamma_+(N)$ to NLL
- momentum conservation $C_{\text{high-energy}}(N = 2, \alpha_s) = 0$

Soft part: C_{soft}

From soft-gluon (threshold) resummation

$$C_{gg}(N,\alpha_s) \overset{N\to\infty}{=} g_0\left(\alpha_s, \frac{m_H}{m_t}\right) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right]$$

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in z-space linear combination of $\left(\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}\right)_+$

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<u>Our improvements:</u>

- we use the correct logs (from kinematics): $\left(\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}\right)_+$

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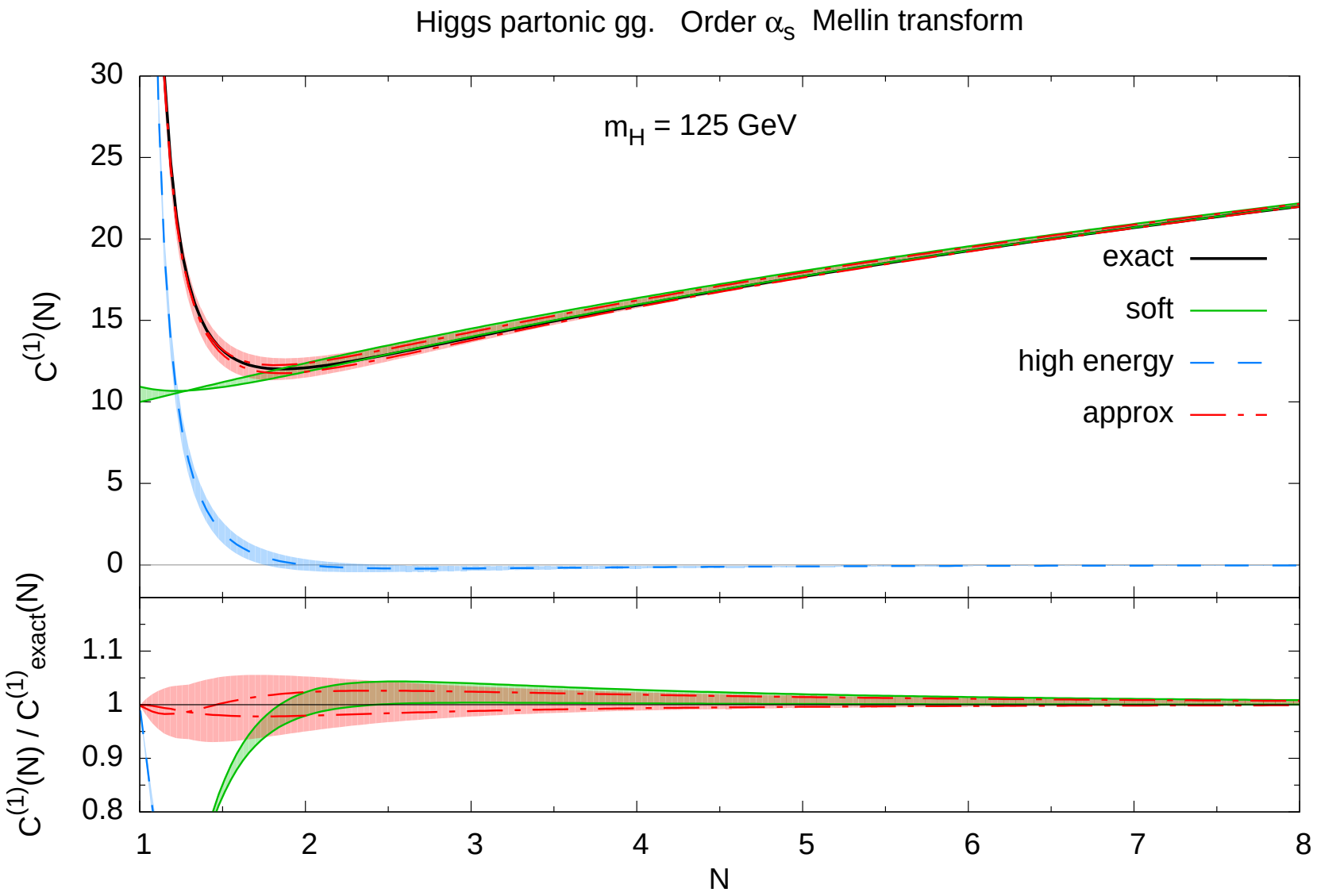
- we supply each emission with the Altarelli-Parisi splitting

$$P_{gg} = \frac{C_A}{\pi} \frac{1 - 2z + 3z^2 - 2z^3 + z^4}{z\,(1-z)} \qquad (z < 1)$$

(avoiding double counting) [Krämer, Laenen, Spira 1997]

Partonic comparison

$$C_{gg}(N,\alpha_s) = 1 + \alpha_s\, C^{(1)}(N) + \alpha_s^2\, C^{(2)}(N) + \alpha_s^3\, C^{(3)}(N) + \ldots$$



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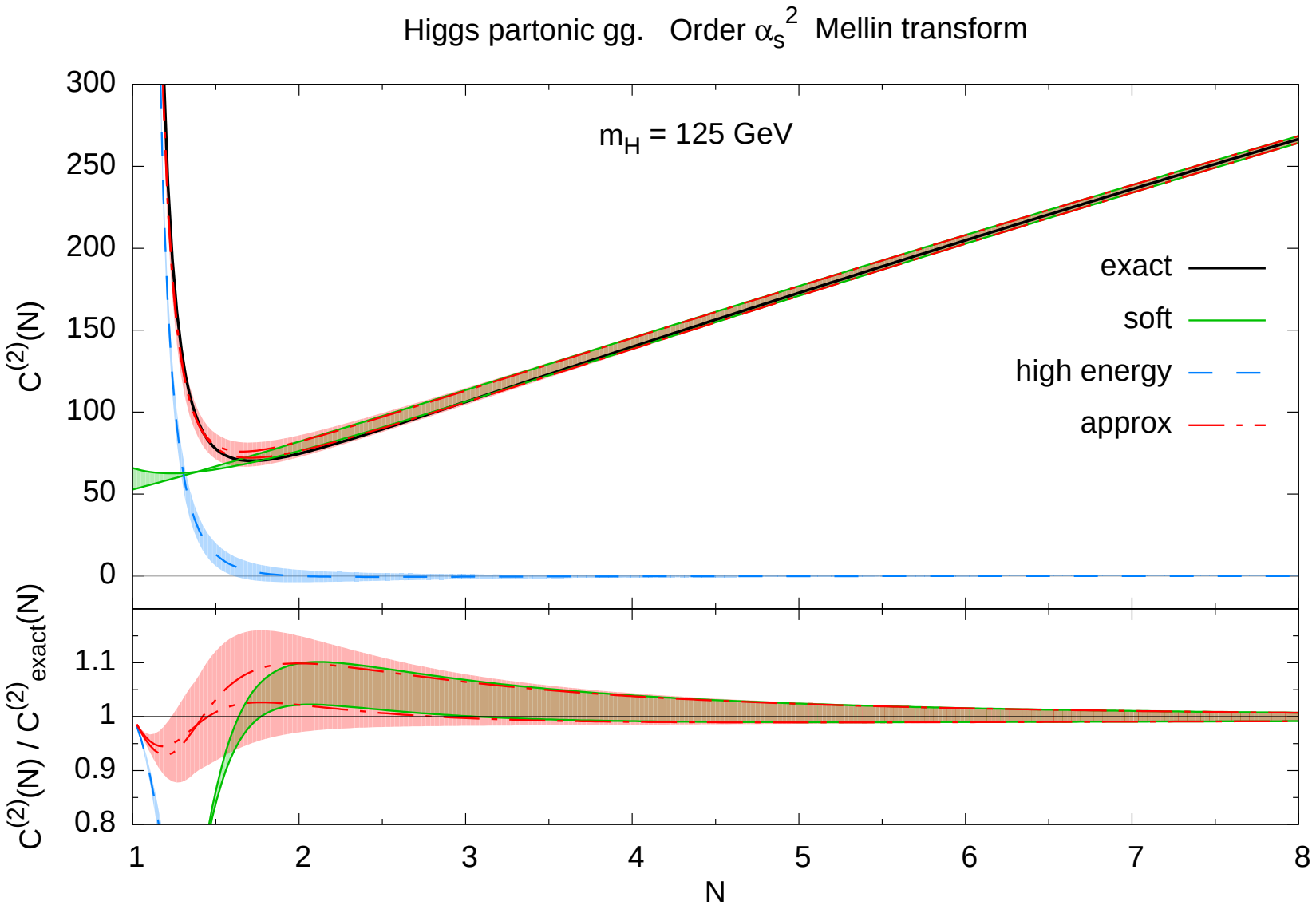

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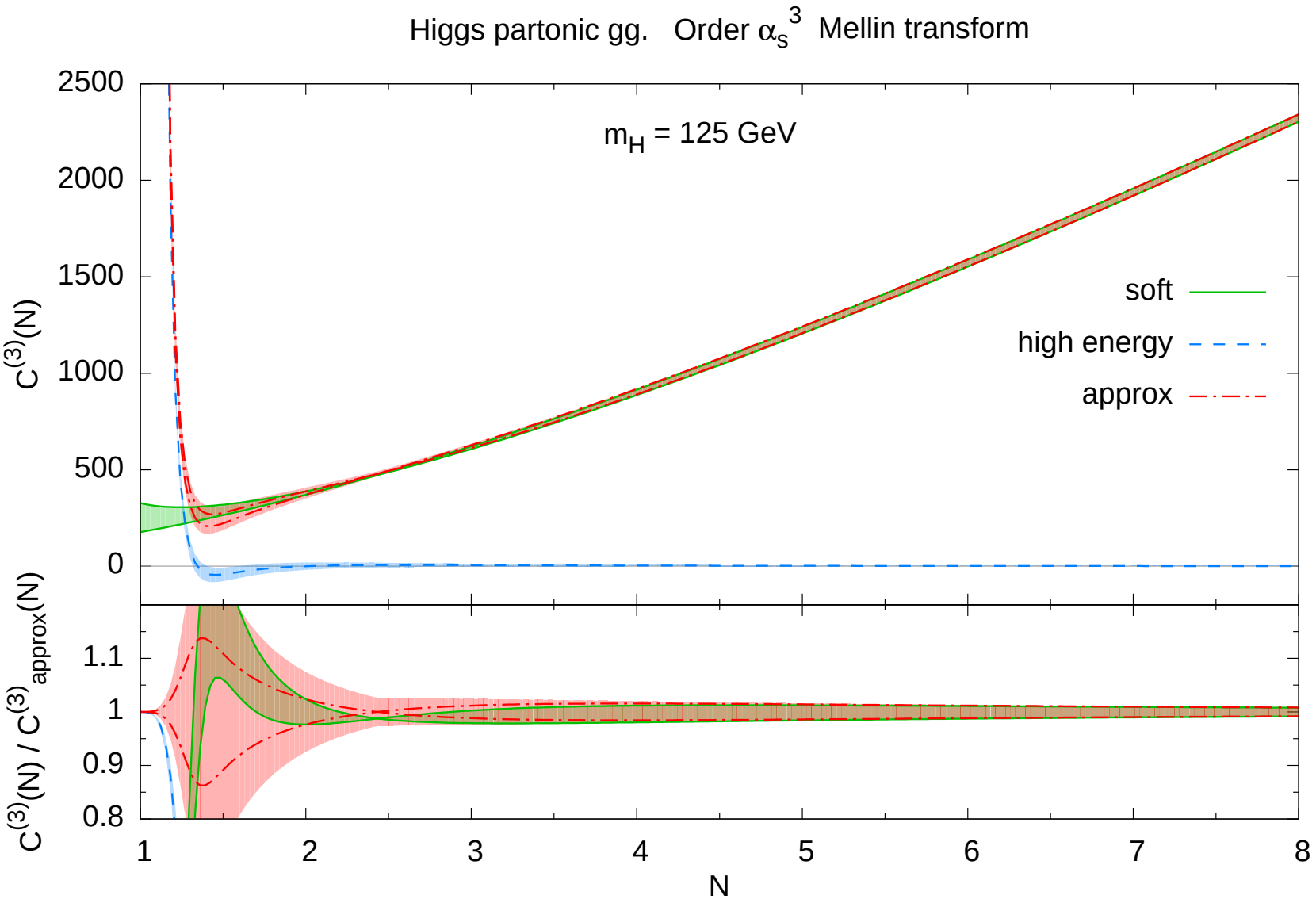

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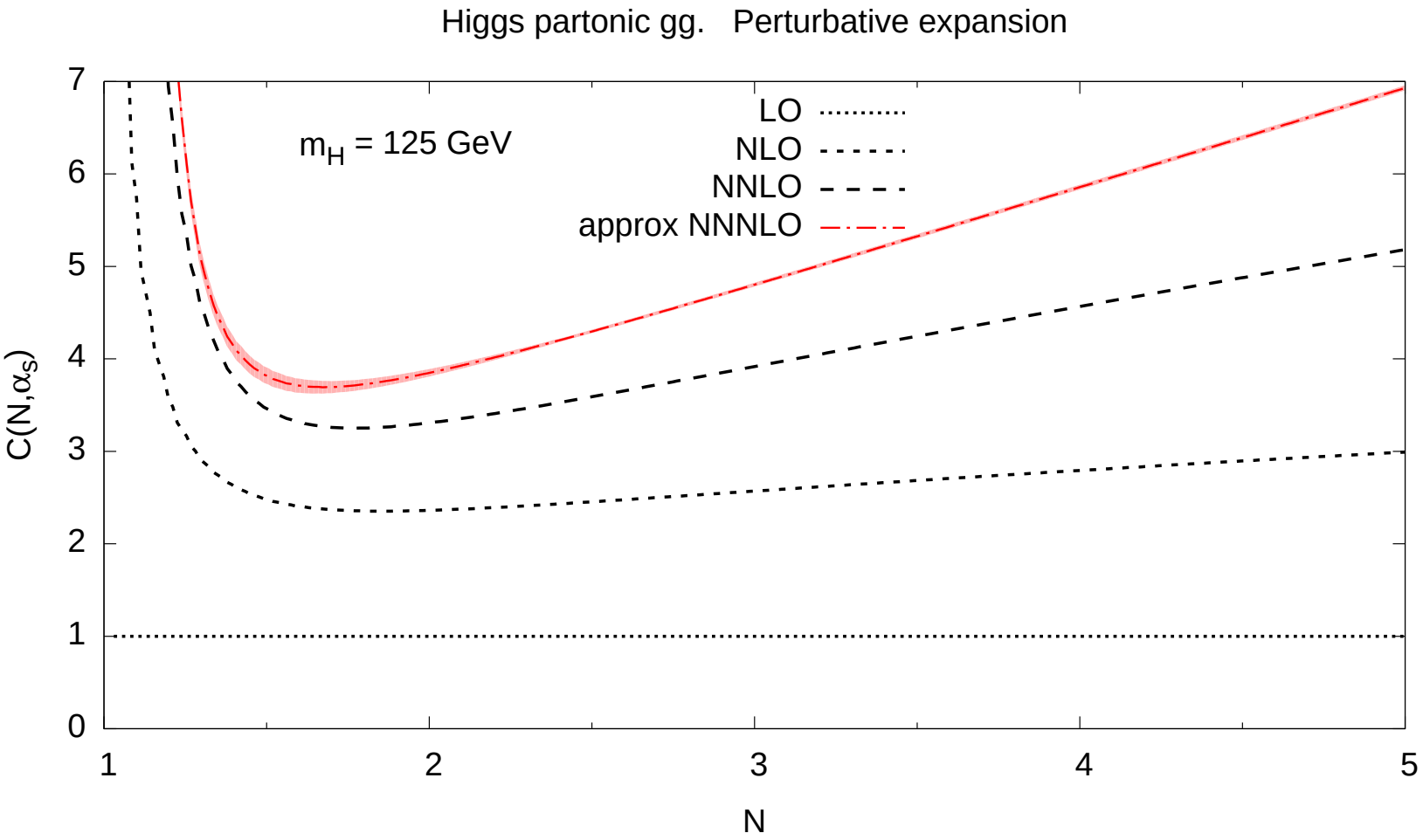
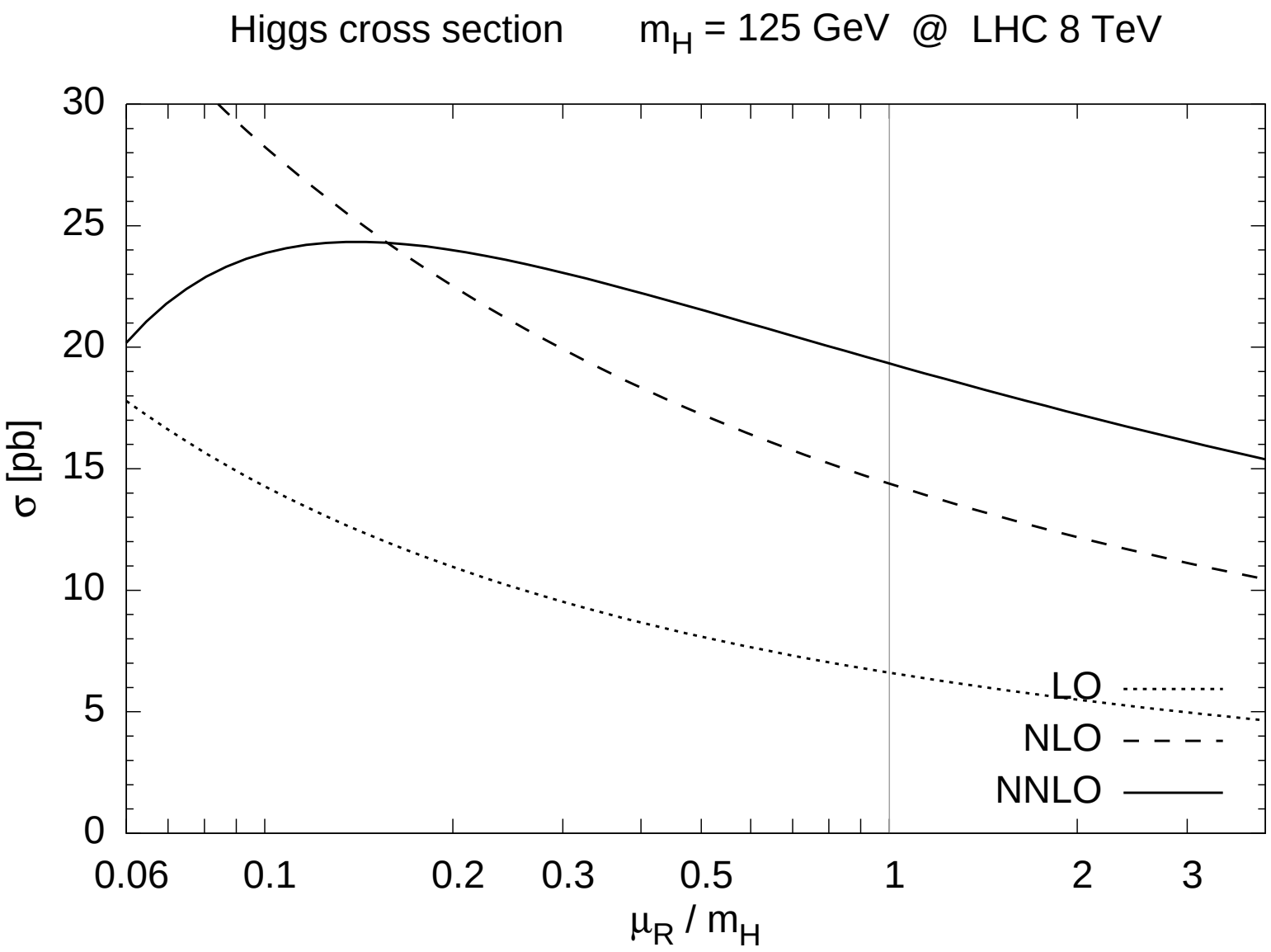


N^{3}LO prediction for Higgs production



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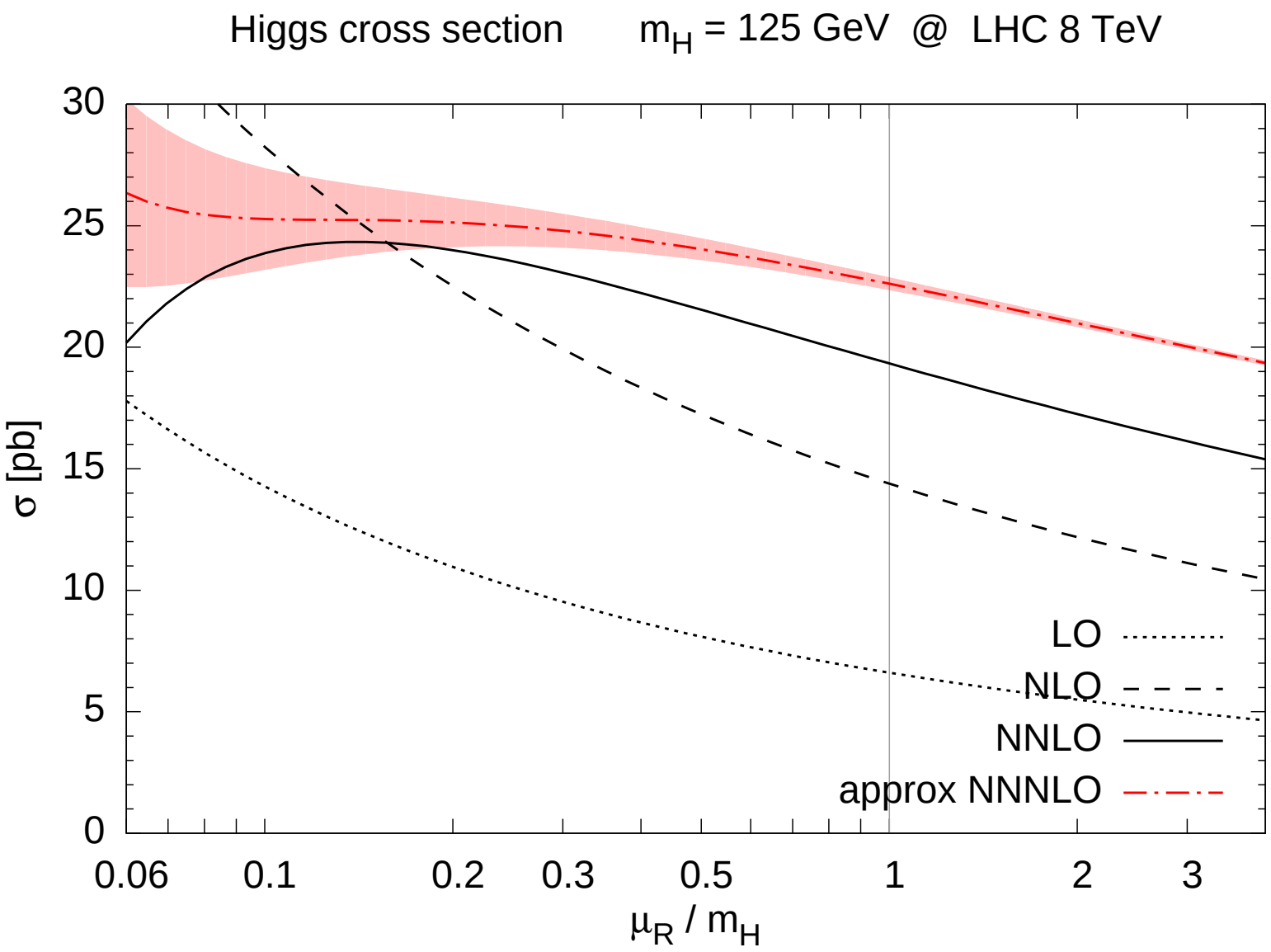

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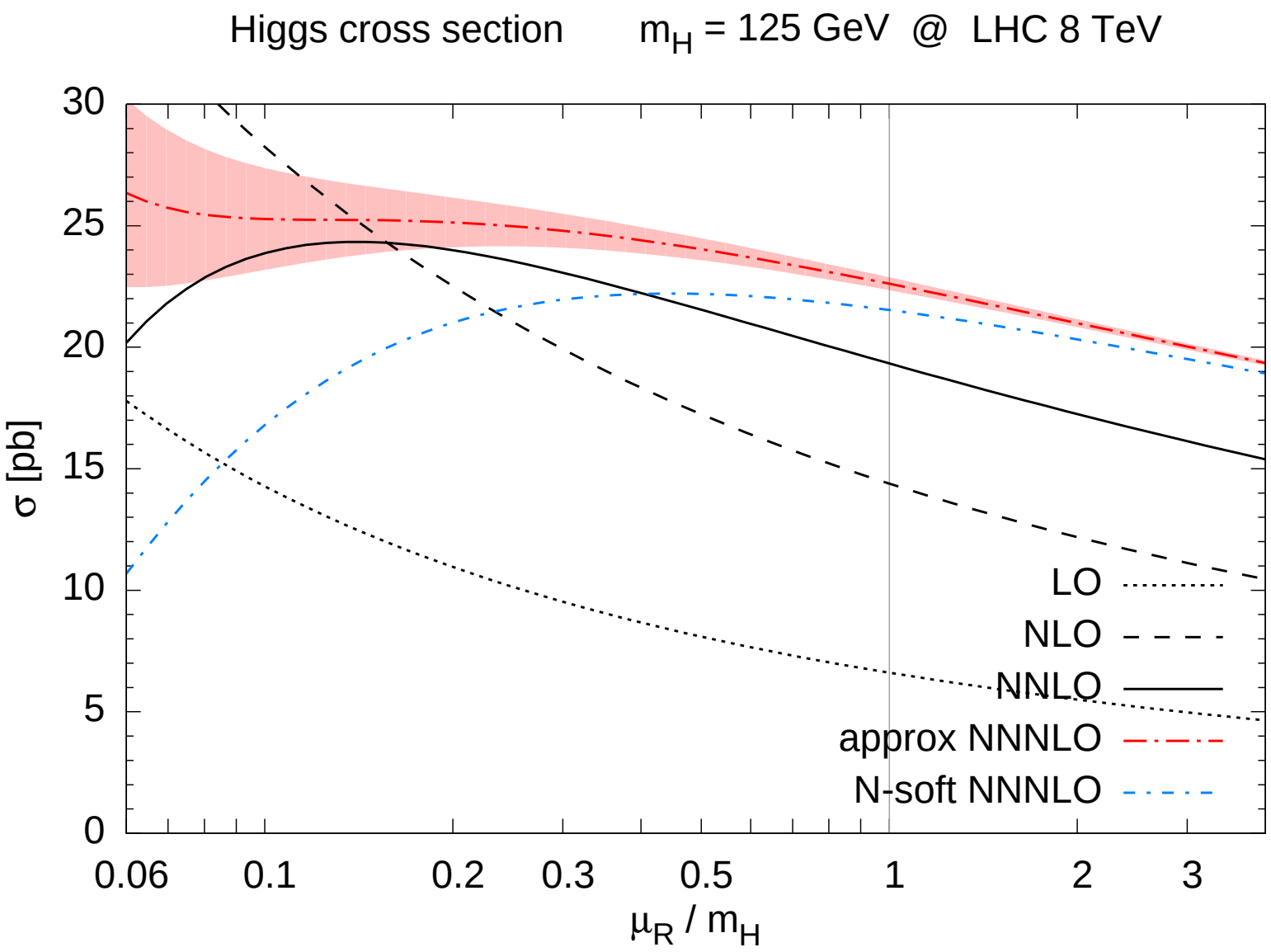

N-soft: [Moch, Vogt, 2005], expanded resummation [de Florian, Grazzini 2012]

Conclusions

- We are predicting the inclusive Higgs N^{3}LO cross section using

$$C_{gg}^{(3)}(z) \simeq C_{\text{soft}}^{(3)}(z) + C_{\text{high-energy}}^{(3)}(z)$$

with:

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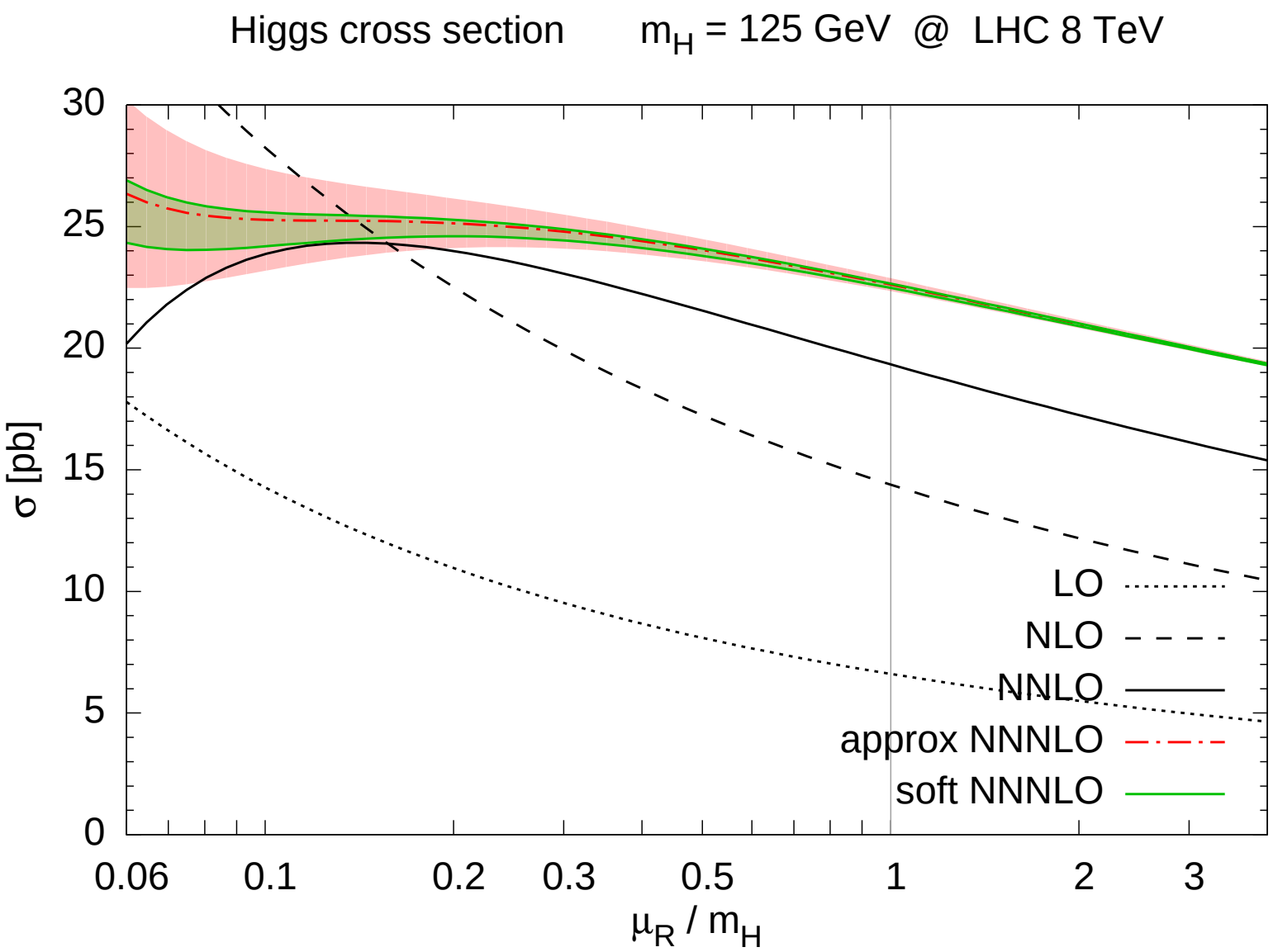
- What's next?
  - the $\delta(1-z)$ term at order α_s^3
  - subleading high-energy terms
  - other channels (at NNLO the qg channel gives a 10% contribution)
  - resummation

Backup slides

N^{3}LO prediction for Higgs production



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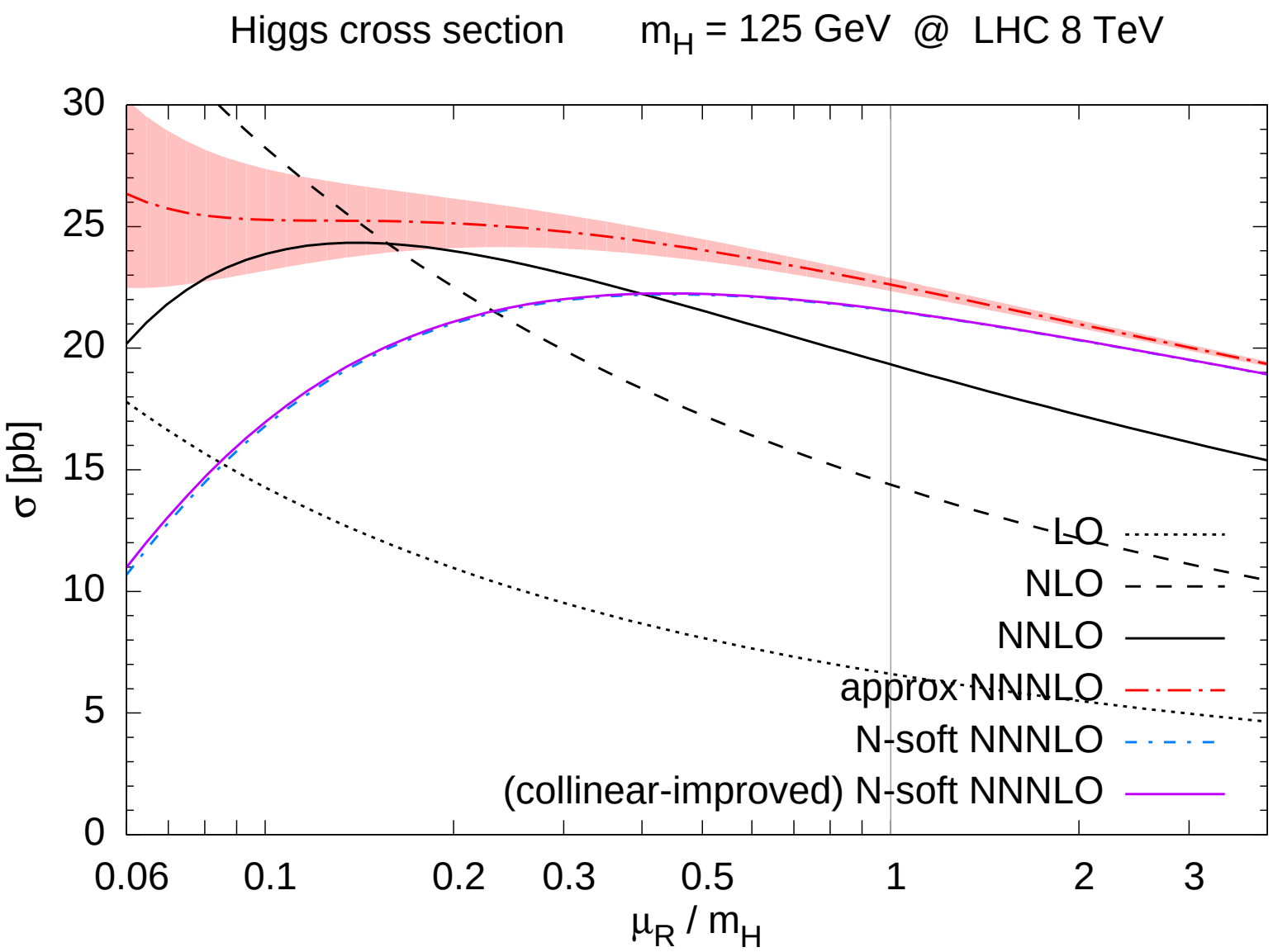
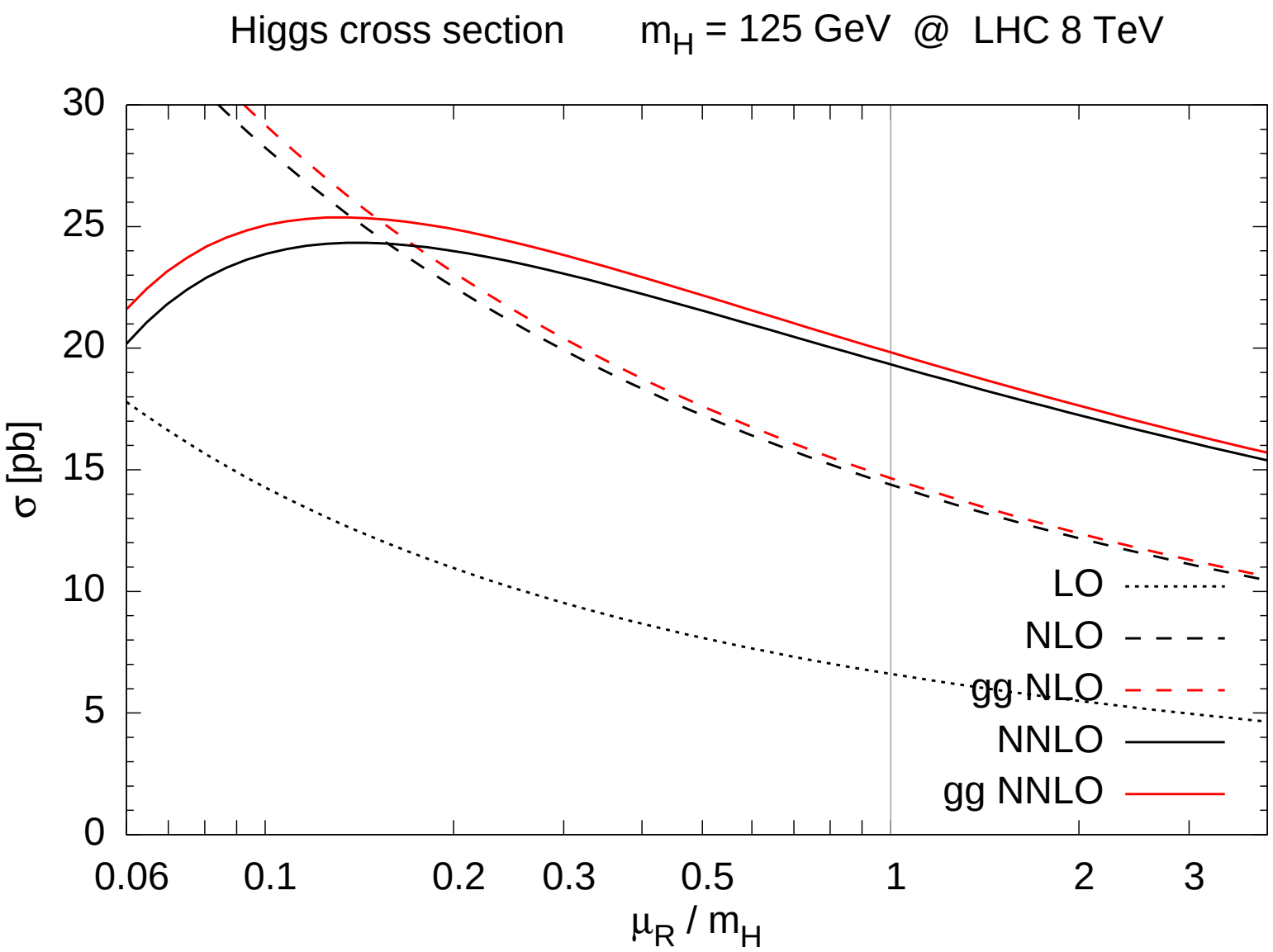


Scale dependence



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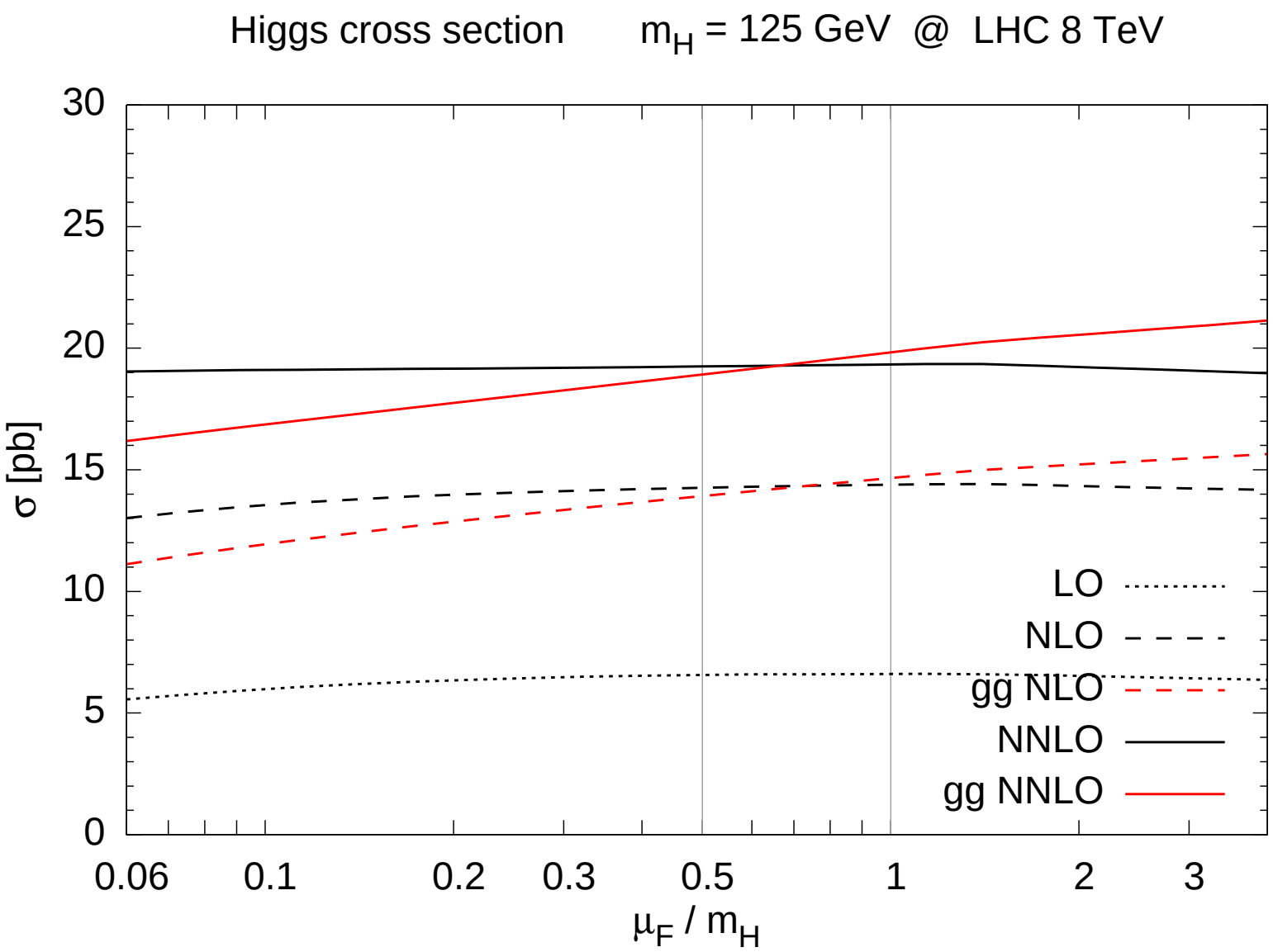
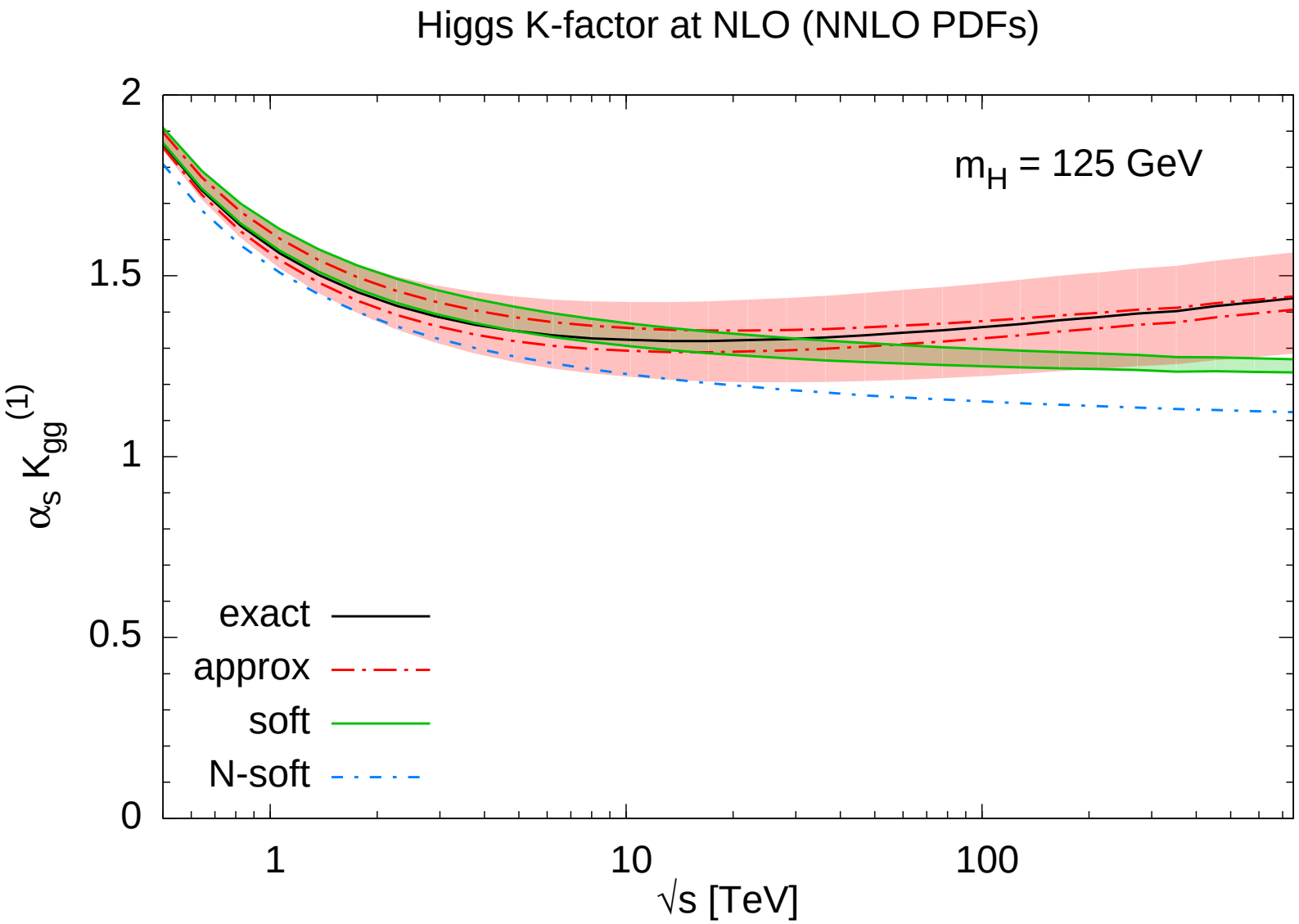


K-factors (NNLO pdfs)



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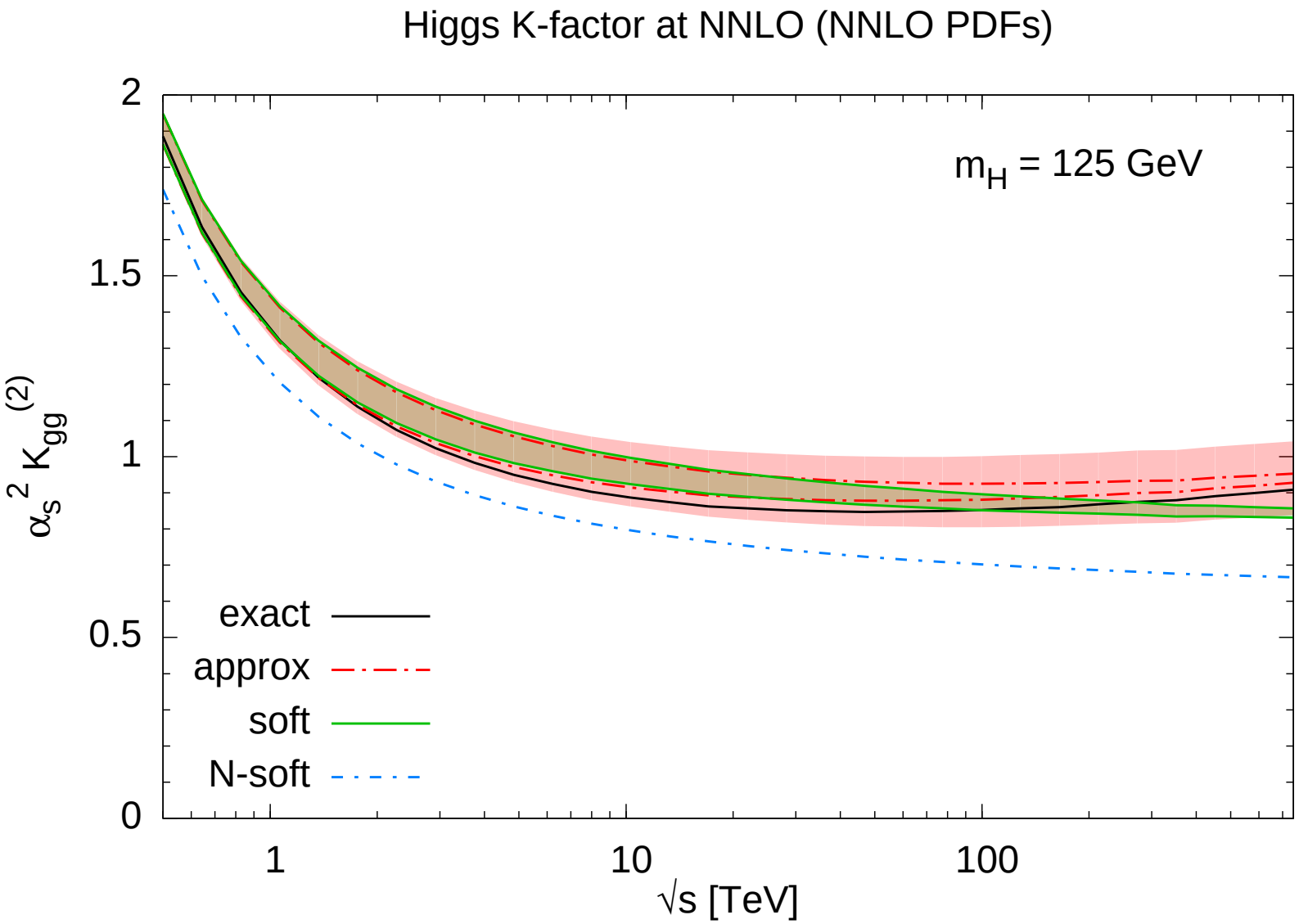

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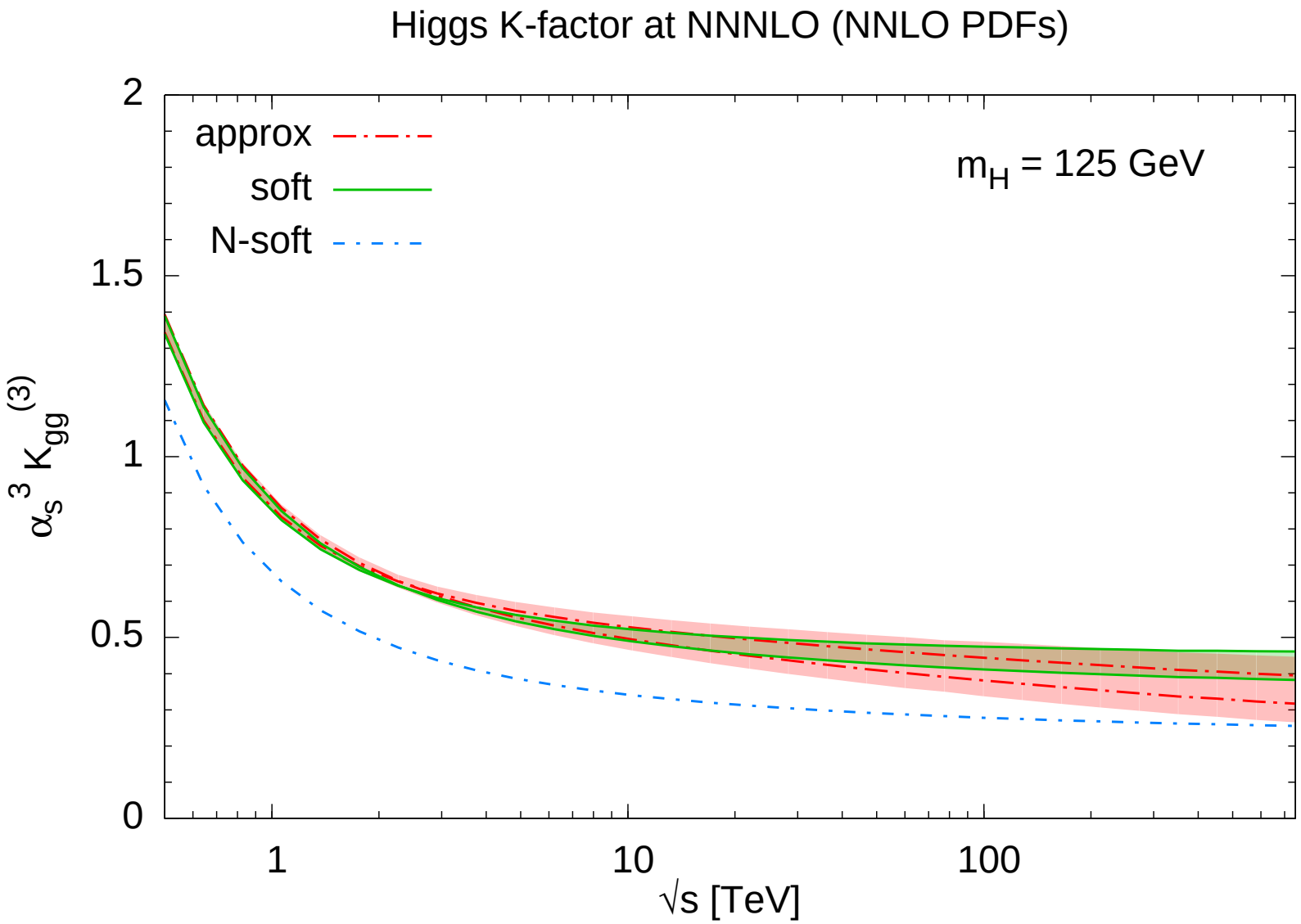
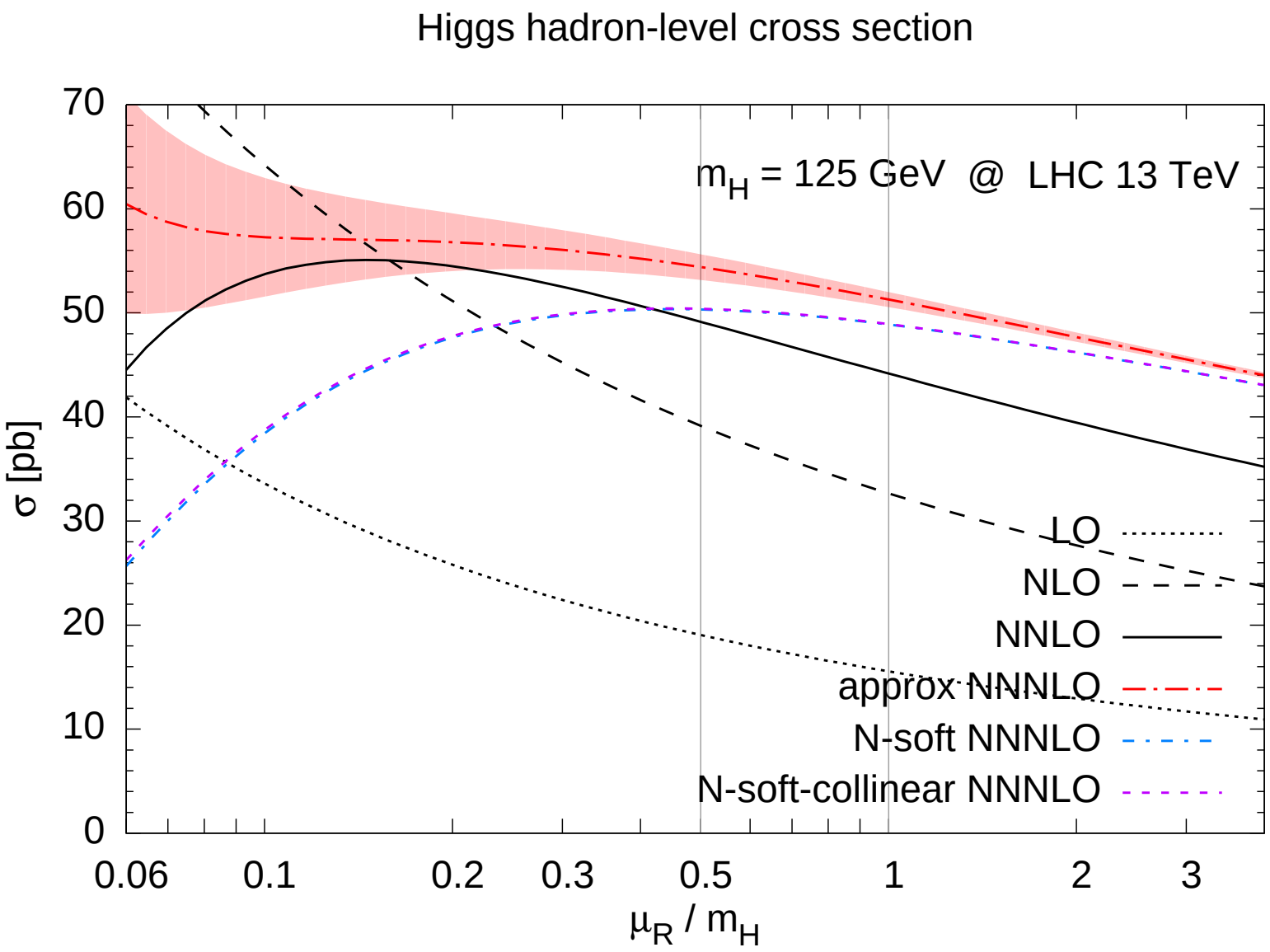
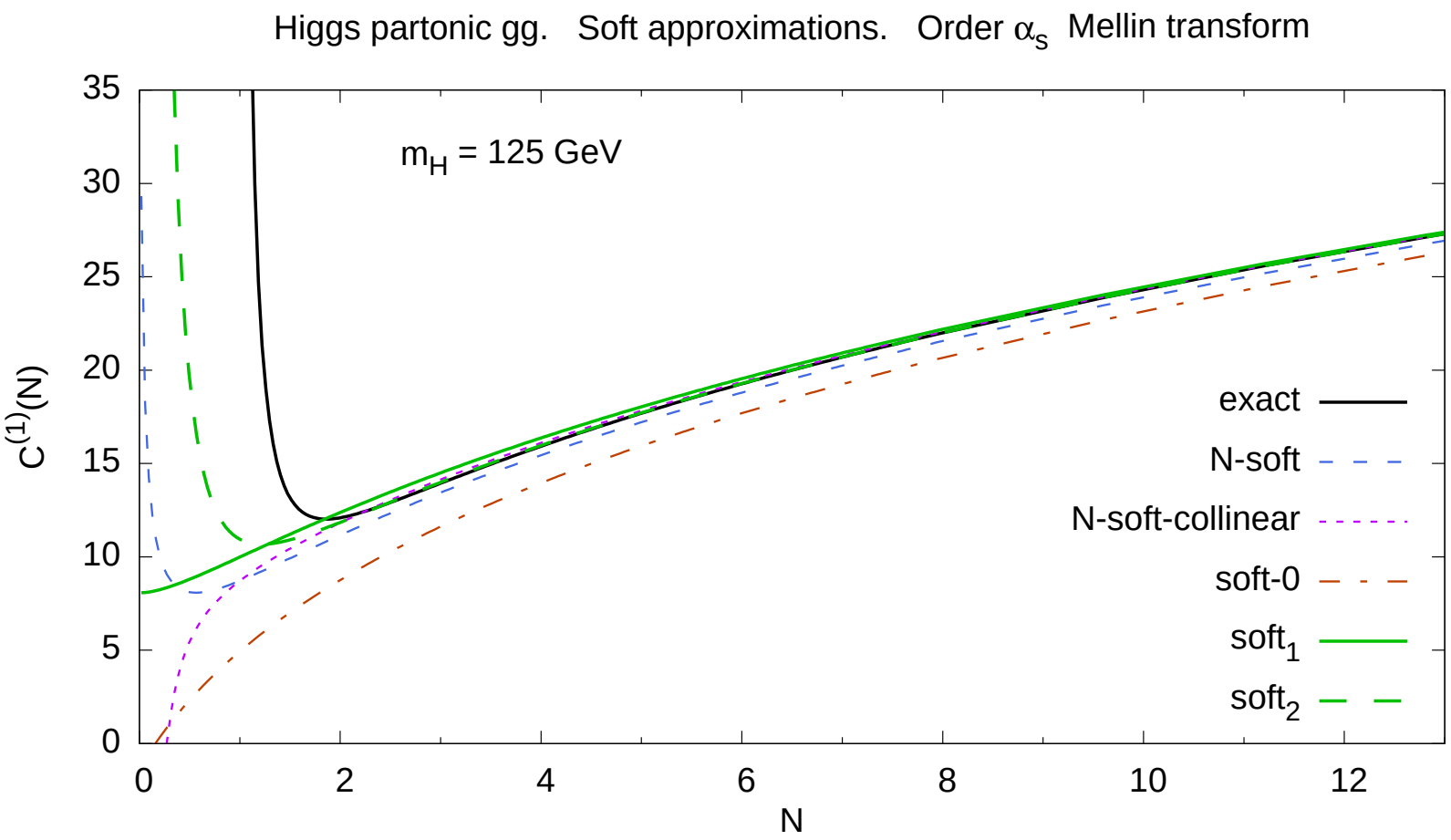


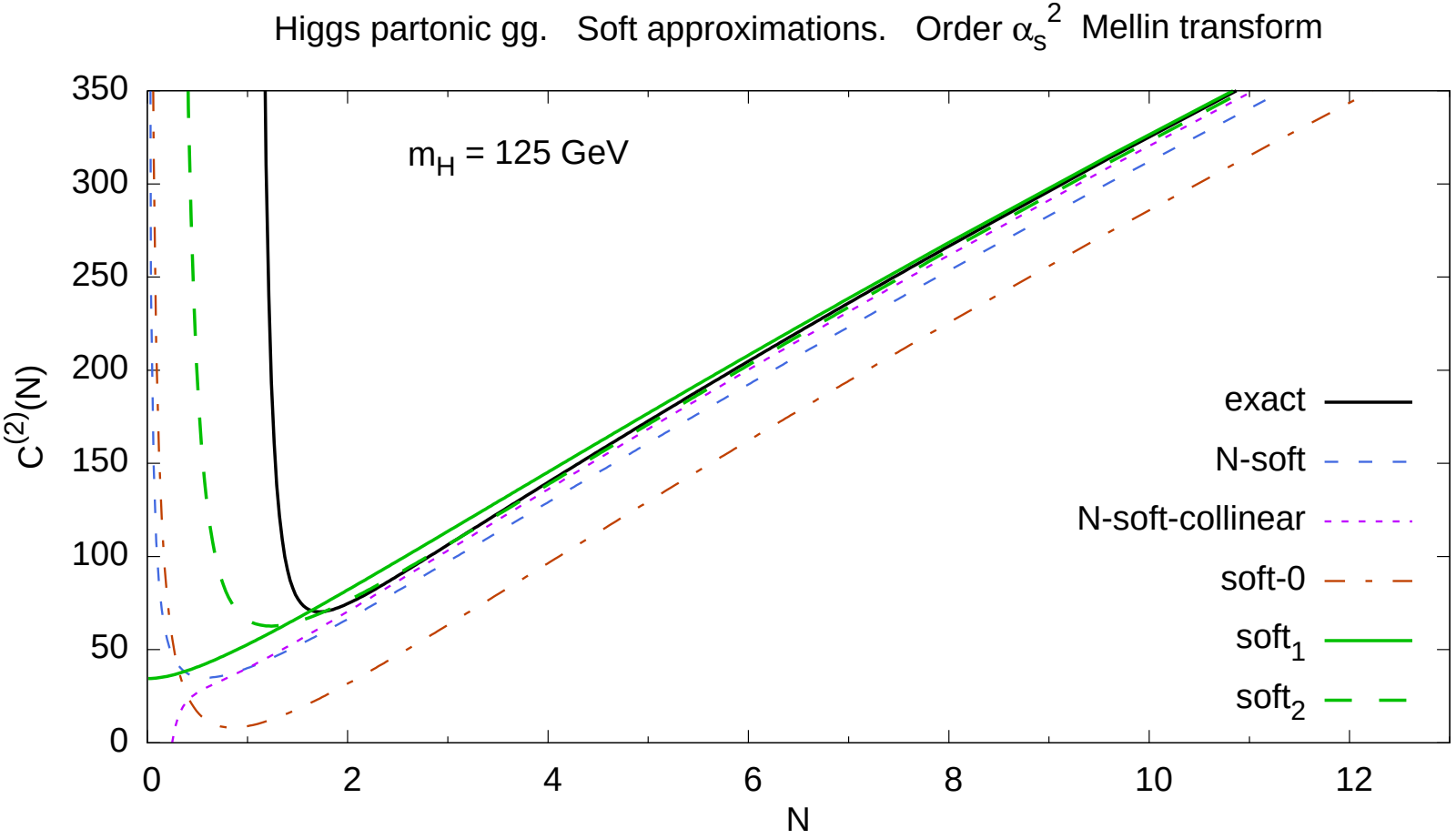
13 TeV


Higgs hadron-level cross section
m_H = 125 GeV @ LHC 13 TeV
σ [pb]
70
60
50
40
30
20
10
0
0.06
0.1
0.2
0.3
0.5
1
2
3
μ_R / m_H
LO
NLO
NNLO
approx NNNLO
N-soft NNNLO
N-soft-collinear NNNLO

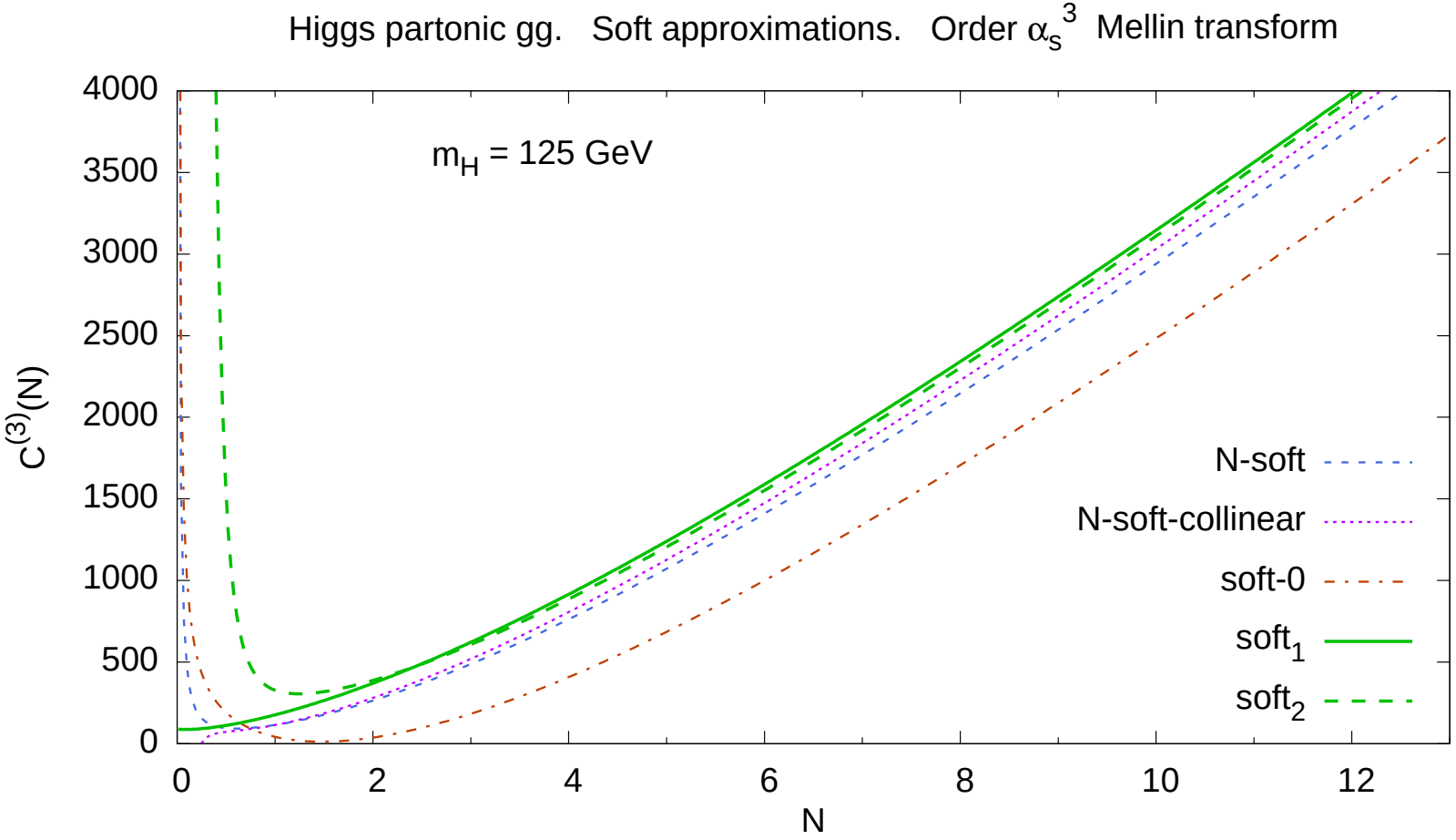
Soft approximations



Soft approximations



Soft approximations



Saddle point